Abstract

The purpose of this work is to study the effects in the magnetic field mitigation of using horizontally buried plates between the earth/air interface and underground power cables. The proposed model implemented allows the calculation of the induction magnetic field induced by the current flowing within underground power cables taking in consideration the magnetic permeability and electrical conductivity of the soil and used plates. The frequency and plate thickness are also considered. The studied model is formulated beginning with the general solution for the magnetic vector potential inside each region (air, soil and conducting plate). The final solution is achieved by using the boundary conditions on the interfaces between each two different regions. The numerical results are obtained by implementing a computational program in order to solve the problem numerically. With this study it was possible to conclude that the depth in which the plate is buried does not influence the magnetic field value at the surface, due to the small value of the soil electrical conductivity. The frequency and plate thickness influence differently the magnetic field value, depending on the plate material characteristics. It is possible to achieve a certain magnetic field value by manipulating the plate geometry and material characteristics and/or changing the frequency.

Keywords: magnetic field mitigation, underground power cables, magnetic induction field, magnetic vector potential

1. Introduction

Magnetic fields are a common reality since men started to make use of electrical energy. Nowadays all developed countries make extreme use of electricity. In order to supply every need it becomes necessary to built up power lines to transport the energy from the source to the consumer. The final consumers make use of the energy using several kinds of appliances and domestic devices. From the power generation until the consumer magnetic fields are induced by the electrical currents.

Magnetic fields have not the same value in every place. Lines transporting more current induce higher fields. The same happens to the domestic appliances. In the last decades, the concern about the effects of magnetic fields in the human health has been growing. Several studies have been made and today many countries impose limits to the value of the magnetic induction field being this value in permanent discussion to be reduced in the future.

Overhead power lines are more common then underground ones, although underground cables are used often in places where the landscape must be preserved or in urban places. The necessity to diminish the magnetic field of underground cables becomes a matter of concern, therefore in this thesis it is studied an effective way to mitigate the magnetic fields induced by underground cables. References [1], [2] are examples of studies where the magnetic field in the air induced by underground cables is calculated, yet in this studies there is no plate between the cables and the surface. In reference [3] the magnetic field is calculated when using a non-infinite plate buried above the cables. In the present work the plate is considered to be infinite in order to solve the problem analytically.

2. Magnetic field evaluation

In order to make the desired studies it is considered a scenario where a three-phased power cable is horizontally buried at a certain depth \( h \) with distance \( s \) between two cables. A plate is buried between the cables and the surface constituted by the earth/air interface. It is possible to consider four distinct regions as displayed in figure 1.
Regions 1 and 3 represent soil and have a magnetic permeability $\mu_1$ and $\mu_3$, respectively. The buried plate is represented by region 2 with permeability $\mu_2$. Region 4 is the air and has the vacuum permeability $\mu_0$. The electrical conductivity associated to each region is $\sigma_k$ with $k = 1, 2, 3$.

2.1. Field formulation without mitigation plate

In order to reach the final solution for the magnetic field at the surface it will be used the magnetic vector potential solution $A$ inside each region.

The final solution will be achieved considering that:
- each region is a linear and homogeneous space separated from each other by a plane surface;
- the cables are buried at a constant depth;
- the magnetic field is originated by a three-phased axial current flowing through a system of three underground cables;
- the magnetic field is only dependent on the transversal coordinates;
- the quasi-static field approximation is considered;

The magnetic vector potential associated to each region can be deducted from the solution for the soil considering only one cable. From [1] the soil vector potential solution without plate is given by:

$$\mathbf{A} = \int_{-\infty}^{+\infty} \left[ F(a)e^{j\sqrt{\omega^2 - q^2}a} + \frac{1}{\pi} e^{-|y|\sqrt{\omega^2 - q^2}} G_0(a)T_k \right] e^{j\alpha x} da$$

where $F(a)$ is a function to be determined and $a$ represents the integration variable with the meaning of a space frequency, with $q_s$ and $W_0(a)$ being given by:

$$q_s = \frac{\sqrt{2}}{\delta_s} e^{-j\frac{\pi}{4}}, \quad \delta_s = \sqrt{\frac{2}{\omega \mu_\sigma}}$$

Figure 1: Illustrative figure of a system consisting in three underground cables with a plate buried between them and the surface.

$$W_0(a) = \frac{j}{\sqrt{a^2 - q^2}}, \quad y > -h$$

where $\delta_s$ is the penetration depth inside the spoil, $G_0$ is achieved by applying the boundary conditions to the cable surface and as indicated in [1] has the following expression:

$$G_0 = \frac{\mu_s}{2\pi q_s r_s} H_1^{(2)}(q_s r_s) + J_1(q_s r_s) b_{0,0}$$

with

$$b_{0,0} = \frac{2j}{\pi} \int_0^{+\infty} e^{-j2q_s h \cos \theta} e^{-2q_s d} da$$

and where $H_1^{(2)}$ and $J_1$ are the Hankel function of the second kind of order one and the Bessel function of the first kind of order one respectively.

$T_k$ in (1) is the phasor of the current flowing through cable $k$.

2.2. Field formulation with mitigation plate

Region 1 is the region that contains the cable, the boundary plane with region 2 is located at $y = -(d + \frac{h}{2})$ and the cable at $y = -h$ therefore, from (1):

$$\mathbf{A}_1 = \int_{-\infty}^{+\infty} \left[ F_1(a)e^{j\sqrt{\omega^2 - q^1}a} + \frac{1}{\pi} e^{-|y+h|\sqrt{\omega^2 - q^1} |q_1| G_0(a)T_k \right] e^{j\alpha x} da$$

where $q_1$ has the same meaning of $q_s$ but now it is calculated assigning the parameters of region 1:

$$q_1 = \sqrt{\omega \mu_\sigma} e^{-j\frac{\pi}{4}}$$

Region 2 represents the mitigation plate where the boundary plane with region 3 is located at $y = -(d - \frac{h}{2})$ so, according to the presence of the upper and lower boundary planes the following result can be obtained:

$$\mathbf{A}_2 = \int_{-\infty}^{+\infty} \left[ D_1(a)e^{j\sqrt{\omega^2 - q^2}a} + D_2(a)e^{-j(\sqrt{\omega^2 - q^2})} \right] e^{j\alpha x} da$$

where

$$q_2 = \sqrt{\omega \mu_\sigma} e^{-j\frac{\pi}{4}}$$
Region 3 represents the soil again, although different electrical and magnetic characteristics may be assigned to it. The boundary plane with region 4 is located at \( y = 0 \), and an analogous result as (8) can be found:

\[
\mathcal{A}_3 = \int_{-\infty}^{+\infty} (R_1(a) e^{\sqrt{a^2 - q_3^2}} + R_2(a) e^{-\sqrt{a^2 - q_3^2}}) e^{iax} \, da 
\]  

(10)

\[
q_3 = \sqrt{\mu_0 \sigma_3} e^{\gamma_3} \frac{y}{x} 
\]  

(11)

The magnetic potential solution in the air is achieved by solving now a new fundamental field equation:

\[
\nabla^2 \mathcal{A}_4 = 0
\]

where the solution may be given by [1]:

\[
\mathcal{A}_4 = \int_{-\infty}^{+\infty} U(a) e^{-|a|} e^{iax} \, da 
\]  

(13)

With \( U(a) \) being a function to be determined by imposing the appropriate boundary conditions.

The magnetic vector potential solution for each region is now possible to achieve applying the boundary conditions on each interface.

From [1], [2] boundary conditions correspond to the following two conditions on each interface:

- the continuity of the magnetic vector potential;
- the continuity of the tangential component of the magnetic field strength:

\[
\begin{align*}
A_k &= A_{k+1} \\
\frac{1}{\mu_k} \frac{dA_k}{dy} &= \frac{1}{\mu_{k+1}} \frac{dA_{k+1}}{dy}
\end{align*}
\]  

(14)

with \( k = 1, 2, 3 \).

For plane \( y = 0 \) the following equation system can be obtained:

\[
\begin{align*}
R_1(a) + R_2(a) &= U(a) \\
\sqrt{\frac{a^2 - q_1^2}{\mu_3}} |R_1(a) - R_2(a)| &= \frac{|a|}{\mu_0} U(a)
\end{align*}
\]

(15)

getting to:

\[
\begin{align*}
R_1(a) &= \frac{1}{2} \left( 1 - \frac{|a|}{\mu_0} \frac{\mu_3}{\sqrt{a^2 - q_1^2}} \right) U(a) \\
R_2(a) &= \frac{1}{2} \left( 1 + \frac{|a|}{\mu_0} \frac{\mu_3}{\sqrt{a^2 - q_1^2}} \right) U(a)
\end{align*}
\]  

(16)

For plane \( y = -(d - \frac{1}{2}) \):

\[
\begin{align*}
D_1(a) + D_2(a) &= R_1(a) e^{-(d - \frac{1}{2}) \sqrt{a^2 - q_2^2}} + R_2(a) e^{(d - \frac{1}{2}) \sqrt{a^2 - q_2^2}} \\
\sqrt{\frac{a^2 - q_2^2}{\mu_2}} |D_1(a) - D_2(a)| &= \left( \frac{1}{2} \right) \left( \sqrt{\frac{a^2 - q_2^2}{\mu_2}} - R_2(a) e^{(d - \frac{1}{2}) \sqrt{a^2 - q_2^2}} \\
\right)
\end{align*}
\]

(17)

replacing (16) in (17):

\[
\begin{align*}
D_1(a) + D_2(a) &= \left[ ch(\xi) + \frac{\mu_3 |a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right] U(a) \\
D_1(a) - D_2(a) &= -\frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3} \left[ sh(\xi) + \frac{\mu_3 |a|}{\mu_0 \sqrt{a^2 - q_3^2}} \right] U(a)
\end{align*}
\]

(18)

with

\[
\xi = (d - \frac{1}{2}) \sqrt{a^2 - q_2^2} 
\]  

(19)

from (21):

\[
\begin{align*}
D_1(a) &= \frac{1}{2} \left( 1 - \frac{\mu_2 |a|}{\mu_0 \sqrt{a^2 - q_2^2}} \right) ch(\xi) + \frac{\mu_3 |a|}{\mu_0 \sqrt{a^2 - q_3^2}} \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3} sh(\xi) \\
D_2(a) &= \frac{1}{2} \left( 1 + \frac{\mu_2 |a|}{\mu_0 \sqrt{a^2 - q_2^2}} \right) ch(\xi) + \frac{\mu_3 |a|}{\mu_0 \sqrt{a^2 - q_3^2}} \frac{\mu_2 \sqrt{a^2 - q_3^2}}{\mu_3} sh(\xi)
\end{align*}
\]

(20)

for plane \( y = -(d + \frac{1}{2}) \):

\[
\begin{align*}
F_1(a) + G_1(a) &= D_1(a) e^{-\xi \sqrt{a^2 - q_2^2}} + D_2(a) e^{\xi \sqrt{a^2 - q_2^2}} \\
\sqrt{\frac{a^2 - q_2^2}{\mu_1}} |F_1(a) - G_1(a)| &= \frac{1}{\mu_2} \left[ D_1(a) e^{-\xi \sqrt{a^2 - q_2^2}} - D_2(a) e^{\xi \sqrt{a^2 - q_2^2}} \right]
\end{align*}
\]

(21)

with

\[
G_1(a) = \frac{1}{\pi} e^{-(d - \frac{1}{2}) \sqrt{a^2 - q_2^2}} G_0 W_0(a) T 
\]  

(22)

replacing (20) in (21):

\[
\begin{align*}
F_1(a) + G_1(a) &= X(a) U(a) \\
\sqrt{\frac{a^2 - q_2^2}{\mu_1}} |F_1(a) - G_1(a)| &= \frac{1}{\mu_2} \left[ X(a) U(a) \right]
\end{align*}
\]

(23)
where \( X(a) \) and \( X'(a) \) are given by:

\[
X(a) = \left[ \frac{\pi}{\mu_0} \sqrt{a^2 - q_2^2} \right] ch(\nu) + \frac{\mu_2}{\mu_0} \sqrt{a^2 - q_2^2} sh(\nu) \left[ \frac{\mu_3}{\mu_0} \sqrt{a^2 - q_2^2} ch(\nu) + \frac{\mu_2}{\mu_0} \sqrt{a^2 - q_2^2} sh(\nu) \right] \nu \in \left[ 1 \right] \text{ and being given as:} \\
(24)
\]

\[
X'(a) = \left[ \frac{\pi}{\mu_0} \sqrt{a^2 - q_2^2} \right] sh(\nu) + \frac{\mu_2}{\mu_0} \sqrt{a^2 - q_2^2} ch(\nu) \left[ \frac{\mu_3}{\mu_0} \sqrt{a^2 - q_2^2} sh(\nu) + \frac{\mu_2}{\mu_0} \sqrt{a^2 - q_2^2} ch(\nu) \right] \nu \in \left[ 1 \right] \text{ and being given as:} \\
(25)
\]

where

\[
\nu = t \sqrt{a^2 - q_2^2} \quad (26)
\]

From (23) it can be deducted:

\[
\begin{align*}
F_1(a) &= \frac{\sqrt{a^2 - q_2^2}}{\mu_1} \frac{X(a) - \sqrt{a^2 - q_2^2} X'(a)}{G_1(a)} \\
U(a) &= \frac{2 \sqrt{a^2 - q_2^2}}{\mu_1} \frac{X(a) + \sqrt{a^2 - q_2^2} X'(a)}{G_1(a)} \\
\end{align*}
(27)
\]

Replacing \( U(a) \) of (27) in (13) it is possible to obtain the final solution for the magnetic induction field in the air due to \( n_c \) cables, as it is done analogously in [1] and being given as:

\[
\begin{align*}
\pi_x &= \sum_{k=1}^{n_c} \left( -2G_0 \frac{T_k}{\pi} \int_{-\infty}^{+\infty} \frac{|a|e^{-(h_k-(a+x))^2/\sigma^2}}{\sqrt{a^2 - q_2^2} X(a) + \frac{\mu_1}{\mu_2} \sqrt{a^2 - q_2^2} X'(a)} e^{-|a|\gamma e^{(a-x_k)}} da \right) \\
\pi_y &= \sum_{k=1}^{n_c} \left( 2G_0 \frac{T_k}{\pi} \int_{-\infty}^{+\infty} \frac{ae^{-(h_k+(a+x))^2/\sigma^2}}{\sqrt{a^2 - q_2^2} X(a) + \frac{\mu_1}{\mu_2} \sqrt{a^2 - q_2^2} X'(a)} e^{-|a|\gamma e^{(a+x_k)}} da \right) \\
\end{align*}
(28)
\]

where \( (x_k, -h_k) \) are the coordinates of the cable \( k \) and \( T_k \) the current flowing in cable \( k \). \( B_x \) and \( B_y \) are the axial and transversal components of the magnetic induction field \( B \) respectively.

Finally, the root mean square of the magnetic induction field \( B_{rms} \) in the air is given by:

\[
B_{rms} = \sqrt{\frac{\pi_x^2 + \pi_y^2}{2}} = \sqrt{B_{rmsx}^2 + B_{rmsy}^2} \quad (30)
\]

where

\[ h = (c-b)/n, a_j = b + jh \text{ for } j = 0, 1, ..., n-1, n \]
\[ a_0 = b, a_n = c \]

\[ F(a) \text{ tends to 0 when } a \text{ tends to infinite which means that the value of the integral } \int^c f(a) da \text{ will} \]

\[ \text{not practically change if the upper limit } c \text{ rises. This means that it is possible to use a small value} \]

3. Implementation

In order to solve equations (28) and (29) it was used a computational program [4]. As they are Fourier integrals it was possible to solve them with a function \( IFFT \), although within these calculation is \( b_{0,0} \) that was solved numerically by using Simpson’s rule.

3.1. Simpsons rule

In order to solve the integral on equation (5) it was used the composite Simpson’s rule. This rule may be written as in [5]:

\[
\int^c f(a) da = \frac{3}{8} \left[ f(a_0) + 2 \sum_{j=1}^{n-1} f(a_{2j}) + 4 \sum_{j=1}^{n/2} f(a_{2j-1}) + f(a_n) \right] \\
(31)
\]
\( \varepsilon \) that maximizes \( F(a) \) for a certain \( a \) which will be afterwards assigned to \( c \). Using \( \varepsilon \) the upper limit \( c \) may be found as follows:

\[
f(a) = e^{−j2a}\cos a e^{-2a} = e^{-j2\frac{1}{\pi}(1-j)\varepsilon^n e^{-n}}\]

\[
|e^{-j2\frac{1}{\pi}(\varepsilon x^n - \varepsilon^n e^{-2\pi})}| \approx e^{-\frac{1}{\pi}\varepsilon^n e^{-2\pi}}
\]

The function absolute value must be smaller than \( \varepsilon \):

\[
e^{-\frac{1}{\pi}\varepsilon^n e^{-2\pi}} < \varepsilon
\]

\[
e^{-\frac{1}{\pi}\varepsilon^n e^{-2\pi}} < \varepsilon
\]

neglecting \( e^{-2\pi} \):

\[
e^{\frac{1}{\pi}\varepsilon^n} > \frac{1}{\pi}
\]

\[
e^{\frac{1}{\pi}\varepsilon^n} > \ln \left( \frac{1}{\pi} \right)
\]

\[
e^n > \frac{1}{\pi}\ln \left( \frac{1}{\pi} \right)
\]

\[
a > \ln \left( \frac{1}{\pi}\ln \left( \frac{1}{\pi} \right) \right)
\]

Finally the upper limit of the integral is given by:

\[
c = \ln \left( \frac{1}{\pi}\ln \left( \frac{1}{\pi} \right) \right)
\]

The calculations in this work were made with \( \varepsilon = 10^{-10} \).

### 3.2. Fourier integrals

The integrals on equations (28) and (29) are Fourier integrals and can be written as:

\[
\begin{align*}
\int_{-a_{\text{max}}}^{a_{\text{max}}} \frac{|a|e^{-(b_k-(d+\frac{1}{2}))\sqrt{a^2-q_k^2N(a)}}}{\sqrt{a^2-q_k^2N(a)}} e^{-ja_x}\,d\alpha = & |a| e^{-j\frac{\pi x}{N}} e^{j\frac{\pi x}{N}} \sum_{k=1}^{N} I_k \\
\int_{-a_{\text{max}}}^{a_{\text{max}}} \frac{a e^{-(b_k-(d+\frac{1}{2}))\sqrt{a^2-q_k^2N(a)}}}{\sqrt{a^2-q_k^2N(a)}} \sum_{k=1}^{N} I_k
\end{align*}
\]

Using a function \( \text{IFFT} \) from a computational program [4] it was possible to calculate the inverse discrete Fourier transform while paying attention to the following relations:

\[
x_{\text{max}} = \frac{2\pi N}{4a_{\text{max}}} (i) \quad \delta a = \frac{2a_{\text{max}}}{N} (ii) \quad \delta x = \frac{x_{\text{max}}}{N/2} (iii)
\]

The \( \text{IFFT} \) output function will be defined between \(-x_{\text{max}}\) and \( x_{\text{max}} \) and the space between these points will be determined by \( N \) with the relation (iii). Having \( x_{\text{max}} \) and \( N \) it is possible to determine \( a_{\text{max}} \) from (i). Then, from (ii) \( \delta a \) is easily calculated. Being \( \delta a \) the discretization of the Fourier integral between \(-a_{\text{max}}\) and \( a_{\text{max}} \). For these particular problem the maximum distance \( x_{\text{max}} \) of interest is relatively small, therefore from (i) and (iii) \( a_{\text{max}} \) or \( N \) can be changed in order to obtain a better discretization. As long as \( x_{\text{max}} \) does not get to small the better option is to reduce \( a_{\text{max}} \) and fix \( N \) so that the calculation time does not rise.

### 4. Results

In this section it is made the validation of the proposed model. Several aspects such as plate positioning, plate thickness effect, frequency and plate electrical and magnetic characteristics were studied in order to understand the behaviour of the magnetic field in the surface, when a plate is buried between the power cables and the soil/air interface.

To perform the studies it was considered a 400kV cable which configuration is displayed in figure 1. The assigned parameters were:

- \( s = 21.8 cm \);
- \( r_e = 7.4 cm \);
- \( h = 1.5 m \);
- \( l = 3 m m \);
- \( d = 1.352 m \);
- \( f = 50 Hz \);
- \( \mu_1 = \mu_0 \), \( \mu_2 = \mu_0 \) (Aluminium) , \( \mu_2 = 100\mu_0 \) (Steel100) , \( \mu_2 = 500\mu_0 \) (Steel500) , \( \mu_2 = \mu_0 \) (Copper) , \( \mu_2 = \mu_0 \);
- \( \sigma_1 = \sigma_3 = 0.1 Sm^{-1} \), \( \sigma_2 = 3.5 \times 10^7 Sm^{-1} \) (Aluminium) , \( \sigma_2 = 1 \times 10^7 Sm^{-1} \) (Steel100) , \( \sigma_2 = 1 \times 10^7 Sm^{-1} \) (Steel500) , \( \sigma_2 = 504 \times 10^7 Sm^{-1} \) (Copper);
- the currents flowing through cable are given as:

\[
\begin{align*}
I_k &= \sqrt{2}I_{ef}e^{-j(k-1)\frac{\pi}{2}} \\
I_{ef} &= 1995A
\end{align*}
\]

with \( k = 1, 2, 3 \).

Some parameters such as the frequency, plate geometry and material characteristics and plate depth may change depending on the different performed studies.

### 4.1. Validation

The validation was made using an application called FEMM (Finite Element Method Magnetics) which allows solving several electromagnetic problems.

On FEMM the problem was created using a non infinite border where the region representing the air is 98m long and 60m high. Analogously the region representing the soil is 98m long and 60m high. The
boundary between the soil and air was placed exactly between the soil and air regions. All the remaining parameters were assigned and represented exactly as in the proposed model.

The magnetic induction field $B_{rms}$ calculated with the proposed model and "FEMM" is displayed in figure 2.

Figure 2: $B_{rms}$ with an Aluminium plate obtained for $y = 0$ using the proposed model and FEMM; $f = 50Hz$

The deviation between $B_{rms}$ at $x = 0m$ and $y = 0m$ in the two applications is equal to 0.0874% which means that the proposed model was successfully validated.

4.2. Influence of the plate depth
The results on figure 3 were obtained for an aluminium plate with $t = 3mm$ placed at three different depths with a soil conductivity equal to $10^{-2} Sm^{-1}$.

Figure 3: $B_{rms}$ profile for different aluminium plate depths calculated for $f = 50Hz$

The plots in figure 3 are perfectly overlapped showing that the plate positioning has no influence on $B_{rms}$ for small values of soil electrical conductivity.

4.3. Influence of the plate thickness
Figure 4 illustrate the variance of $B_{rms}$ at $x = 0m$ and $y = 0m$ for different plate materials when the plate thickness $t$ changes. This variance is implicit in $Fr$ which is equal to the ratio between $B_{rms}$ without plate and $B_{rms}$ with plate. As $d$ does not affect the results the variation of $t$ is made by fixing $d$ and changing the upper and lower side of the plate at the same time.

Figure 4: Magnetic field factor of reduction $Fr$ calculated for plate thickness between 0 and 5mm for all three materials and $f = 50Hz$.

Aluminium plate has the greater $Fr$ of the three materials until a thickness near 2.4mm but the steel500 plate has the highest $Fr$ after that thickness starting to grow faster. Until 5mm the steel100 has the worst results.

The magnetic field behaviour for the same frequency but different thicknesses is directly related to the "Penetration Depth" introduced in equation (2). Only the material properties and the working frequency affect the value of $\delta$, although if $\delta$ does not change but thickness $t$ grows, the field capability to penetrate the plate decreases. This capability decreases faster or slower depending on the ratio between $t$ and $\delta$.

4.4. Influence of frequency
In order to analyse the frequency effect, the plate thickness was fixed at 3mm. Then, a frequency variation was applied being the obtained results displayed in figure 5.

Figure 5: Magnetic field factor of reduction $Fr$ calculated for frequencies between 0 and 100Hz for all materials with $t = 3mm$. 

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For $t = 3\, \text{mm}$ Steel500 has the higher $Fr$ value even at a low frequency such as $50\, \text{Hz}$. Aluminium has a better $Fr$ then Steel100 until $100\, \text{Hz}$, even though with the enlargement of the frequency interval Steel100 starts to achieve better results.

Penetration depth $\delta$ is directly influenced by the frequency and rapidly decreases when frequency increases. Even testing a $3\, \text{mm}$ thick plate which is in general a small thickness value shows that the magnetic field is highly reduced. As seen in Thickness the ratio between $t$ and $\delta$ is determinant on the mitigation of the magnetic field, yet this time the parameter changing is $\delta$ and not $t$.

4.5. Influence of the ratio $t/\delta$

In order to study the impact of $t/\delta$ on the field mitigation, $Fr$ was calculated for all three materials varying $t/\delta$. This variation was made by fixing $t$ on $3\, \text{mm}$ and then calculating the respective $f$ to ensure the previously fixed ratio. The obtained results are displayed in figures 6 and 8.

From figure 6 it becomes clear that aluminium makes a better mitigation for $t/\delta$ higher than a few hundredths. Steel500 as a higher $Fr$ than steel100 until a value near 0.6 but the behaviours invert after that. In figure 7 the results show that for high penetration depths the materials with higher magnetic permeabilities are better for mitigation and for small $\delta$ values the mitigation is higher for small $\mu$ materials. This results show that for a high $\mu$ and $\delta$ the magnetic field lines tend to penetrate de plate but do not cross it. For small $\mu$ and $\delta$ the field lines tend to stay bellow the plate.

The plot in figure 8 was obtained for an Aluminium plate and a Copper plate. The difference between these two materials is the electrical conductivity. The curves are overlapped showing that the plate $\sigma$ does not affect the results.
4.6. Influence of the height to earth surface

When trying to ensure a certain $B_{rms}$ value for some specific height, the reduction factor $Fr$ along the height $y$ becomes an important aspect.

$Fr'$ is the ratio between $B_{rms}$ without plate at $(x = 0, y = 0)$ and $B_{rms}$ with plate at $(x = 0, y \geq 0)$.

$Fr'$ was calculated for different materials and thickness varying $y$ from $0m$ to $4m$. Frequency was maintained constant at $50Hz$. Results are displayed in figures 9, 10 and 11.

![Figure 9: $Fr'$ calculated for $y$ between 0 and 4m for all materials. $t = 1mm; f = 50Hz$.](image)

![Figure 10: $Fr'$ calculated for $y$ between 0 and 4m for all materials. $t = 3mm; f = 50Hz$.](image)

![Figure 11: $Fr'$ calculated for $y$ between 0 and 4m for all materials. $t = 5mm; f = 50Hz$.](image)

Figures 9, 10 and 11 show that the magnetic field value for a particular $y$ is influenced by the plate’s material and thickness. In this particular case $\delta$ does not change for each material as frequency stays at $50Hz$, a fact that explains the variance between results when $t$ changes as explained in sections the previous subsections.

It was already seen that keeping the plate thickness constant and changing the operating frequency was another way to mitigate the magnetic field. Figure 12 illustrate the changes on $Fr'$ for $f = 100Hz$ and $t = 3mm$.

![Figure 12: $Fr'$ calculated for $y$ between 0 and 4m for all materials. $t = 3mm$ and $f = 100Hz$.](image)

5. Conclusions

With this work it was possible to study and analyse a procedure that allows the mitigation of the magnetic induction field at the surface, created by underground cables carrying a certain amount of electrical current. It becomes clear that the effectiveness of the procedure, which consists on burying a plate between the underground power cables and the surface is directly related to several aspects such as: plate thickness; plate magnetic permeabi-
ity and electrical conductivity; frequency.

As expected and was afterwards confirmed the plate thickness has a major role on the field mitigation. The mitigation of the magnetic field becomes higher when the plate thickness grows. Although the mitigation is not the same in all the cases. It was possible to conclude that if the frequency for a certain material remains constant the penetration depth does not change. So for small thickness values the magnetic permeability of the plate is determinant and the materials with lower magnetic permeabilities are better to mitigate the field. On the other side increasing thickness and maintaining $f$ reveals that materials with higher permeabilities are better.

Frequency is directly related to the penetration depth of the magnetic field into a certain material. When the frequency increases the mitigation of the field becomes higher for all materials, yet it grows faster in the materials with higher $\mu$.

Varying the ratio between the plate thickness and penetration depth revealed that for small ratios the higher the magnetic permeability the better the material, although as the value $\frac{t}{\delta}$ starts to increase this behaviour changes and the materials with small permeabilities become better for mitigation.

Another conclusion concerns the possibility to manipulate the plate characteristics in order to achieve a certain magnetic induction field value $B_{rms}$. This aspect does not apply only to 0 meters but to any height above ground.

It was also concluded that the depth at which the plate is buried does not influence the mitigation result. It was demonstrated that this statement is valid for small values of soil electrical conductivity.

References


