Development of a new wave energy harnessing system

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Abstract

This dissertation’s subject is a new system of harnessing wave energy that intends to be more economically viable and competitive than the current nowadays systems by having all the key components to work mechanically in traction. Its main aim is to deepen the knowledge about the dynamic behavior of the system when subjected to different types of waves. In order to achieve this, it was designed a computer program capable of simulating the temporal heaving motion of the floater as well as the behavior of all the components of this plant, using the Runge-Kutta method. It was also implemented a controller for electric generator of the Proportional-Integral type so as to maintain its constant speed within an acceptable range. Simulations were carried out for regular waves, which varied independently the incident wave’s Height and Period. It was also analysed a group of 14 spectra of irregular waves corresponding to a wave climate of a point off the coast of Leixes. An analysis of the impact that the variation in sizing of the three main components (ballast mass, cables diameter and generator inertia) would have on the behavior of the plant was made in a further qualitative way. It was possible to validate the operation of the program and its conformity to the theoretically predicted. It was also possible to determine the average powers and efficiencies for regular waves and the wave climate in study, having been defined in detail the influence of each component in the overall system operation.

Keywords: wave energy, hydrodynamic modeling, non-linear states model, control.

1. Introduction

This work has as its subject a new system of harnessing wave energy described in the patent [1] created by Professor Jos Maria Anda, Supervisor, and by Joo Campos Henriques, Co-supervisor of this thesis.

The aim of this work is to create a program capable of performing a computer simulation of the behavior of the mentioned wave energy power plant, for different sea states. It is intended to vary several parameters either the sea states or the plant components sizing, in order to find out what is the power that is possible to extract from this plant, what will its efficiency be and also in what range of sea states will this plant be able to operate.

Unlike the numerical method used in another dissertation [2], is intended to use the Runge-Kutta method to simulate the systems of equations that describe the behavior of all parts of this plant.

Obtaining these data is of high importance to determine the feasibility of this system, both on the technical side, regarding the energy production, and on the economical side, regarding economic sustainability.

1.1. State of the art and economic viability

Currently it has been awakened high interest in offshore applications, in waters with a few tens of meters, in floating systems for harnessing wave energy. Groupings of offshore devices deployed at depths of 40 to 80 m are currently considered as the most suitable configuration for extensive resource exploitation of wave energy.

The option for floating systems has its justification on the fact that the available power is much higher in areas of deeper waters. Allied with that fact, we have the relative simplicity of conception and construction of a floater.

Here is the point where this system [1] enters, which is the field of study of this work. It is aimed with this system to achieve improvements at two levels relative to the systems previously proposed. Firstly, the system is designed so that all components work exclusively in traction and none in compression, thus avoiding excessive scaling that is required for a component to support such compression efforts. Secondly, the commonly used hydraulic cylinders are no longer required and a ballast, attached to the floater by a cable system, which seeks to accumulate energy over several wave cycles by being lifted by cables shall be used, thereby storing energy in the form of gravitational potential energy.
and with this increased electrical output.

This all combines that, with this new system, it has been considerably reduced the complexity of the plant’s structure, as well as to the difficulties of construction. This makes the system more economically viable, this also being one of the main reasons that preclude the development of other structures previously thought. The goal is to decrease to the minimum the equipment construction price, maximizing the amount of energy it can produce, thus hoping to achieve competitiveness against other types of renewable energy.

2. Description of the wave energy

2.1. Installation’s main elements

The main elements of the installation are represented in the figure 1

![Figure 1: Scheme of the installation’s main elements [1].](image)

The station consists of a floater (1), which is subjected to hydrodynamic forces of waves and the force of a set of cables (3). The ballast (2) is connected to the floater through a set of cables and has a vertical movement that varies its direction depending on the stage it is on. This movement is guided by rods (5) which are anchored into the seabed (6) at its lower end, and is connected to a balloon (4) at the upper end, and thus will always be subjected to traction and not to compression.

The ballast is connected to its brakes (21), which allow it to be immobilized in a position without losing elevation. The cable assembly is then wrapped around a drum (101) that is connected to the electrical generator (104) through a gear ratio that allows it to have a lower rotational speed than the generator.

Connected to the drum shaft is a brake disk (hereinafter referred to as drum brake (102)) which is actuated to reduce the rotational speed of the drum or immobilize it, and is also combined with a cable winding mechanism (105). The latter is used when the cable is untensioned and needs to be wound around the drum.

2.2. General principle of operation

The process of energy production in this plant is divided into what are called energy cycles. These cycles consist of two different phases: "1 - Energy Accumulation" and "2 - Energy Production".

In the first phase the ballast is risen from a \( x_{\text{min}} \) to a \( x_{\text{max}} \) to take advantage of the hydrodynamic force exerted on the floater and in times when the cable is without tension, it is wound so as to approximate the ballast to the floater.

The second phase begins at the moment in which the ballast has reached its maximum height and thus maximum power energy. At that moment it is let down, and thus gain speed to accelerate the drum and the generator, thereby producing electrical energy.

Hereinafter will be referred to as a "energy cycle" to the set of accumulation phase and a production phase in a row. While in the phase of accumulation exist a sequence of stages that are repeated cyclically until the \( x_{\text{max}} \) is reached, in the Production there is only one sequence of stages that is executed only once.

2.2.1. Energy accumulation phase: rising the ballast

The primary objective of this phase is to raise the ballast from a minimum to a maximum elevation in the shortest possible time so as to accumulate the waves energy in the form of gravitational potential energy of the ballast.

![Figure 2: Scheme of the energy accumulation phase [1].](image)

We will consider its starting point with the floater in the cave of the wave and the ballast in the its minimum level (\( x_{\text{min}} \)), where the ballast is locked and thus remains fixed to rods. At the instant at which the floater starts to rise due to hydrodynamic force of the wave, the brake drum is actuated, thereby preventing the cable from unrolling and it starts to gain tension as the floater moves away from the bal-
last. When the force exerted on the cables exceeds in magnitude, the ballast apparent weight (weight out of the water subtracted by the impulsion on the surface of the ballast) we have that the resultant force has a positive direction and the brakes of the ballast will be disabled, allowing it to rise.

When the float reaches the crest of the wave and begins to descend, the force that the cables exert on the ballast also decreases causing it to stop climbing, and at that moment its brakes are actuated, before it starts to descend and lose elevation. Then, as the floater moves down the tension on the cable will also decrease until the point at which it vanishes. Here is then triggered the winding mechanism associated to the drum to wind the excess cable that came into existence due to the ballast being moved up, which takes advantage of the absence of tension in the cable or a value below the one this can exert. Once that, by descending, the floater is found again in the cave of the wave, it will be repeat again all the steps above until the ballast reaches the desired maximum level and reach the maximum value of the gravitational potential energy of the energy cycle.

2.2.2. Energy production phase: Ballast descending

The main objective of this phase is to convert as much of the gravitational potential energy accumulated in the previous phase, into kinetic energy of rotation of the electrical generator and its rotational inertia. The rotary inertia of the generator is what will keep it running for a longer period of time and from where a constant electrical power will be removed, thereby allowing continuous production of electricity, and not just only in the period in which the ballast is descending.

3. Numeric Modelling of the central

The equations that rule the physical phenomena observed in this plant [4] are of three types: second order equations, first order equations, and algebraic equations. Although there is not one system of equations that can correspond to the behaviour of the central during all the energy cycle, so that this is only able to be modelled by stages. Therefore, it will be here shown which are the equations that model the behaviour of each main component of the plant, and the stages in which these are applicable.

3.1. Presentation of the equations that model the power plant

In order to better describe and understand the movements that are verified at the central, here is written the variables used throughout this study.

- \( x_1 \) – Floater vertical position,
- \( x_2 \) – Ballast vertical position,
- \( \theta \) – Drum angular position,
- \( \omega \) – Electric generator angular velocity,
- \( f \) – Total force in the cables.

For a question of nomenclature and to simplify, all the variables with the lower index “1” are related to the floater, and all of those with the lower index “2” are related to the ballast.

3.1.1. Floater

The floater oscillates in heave, subjected to a hydrodynamic force \( F_{\text{wave}} \) produced by the incident wave and the radiated wave, and is also subjected to \( f \) transmitted by the cables that connect it to the ballast. It’s general equation of motion is:

\[
m_{1} \ddot{x}_1 = F_{\text{wave}} - f
\]
The force $F_{\text{wave}}$ will be approximated as a vertical force only, just studying the motion of heave and not the movements in other directions. According to the linear wave theory hydrodynamic force acting on the floater is given by:

$$F_{\text{wave}} = F_{\text{imp}} + F_{\text{rad}} + F_d,$$

(2)

The impulsion force $F_{\text{imp}}$ is given by:

$$F_{\text{imp}} = -\rho g S x_1,$$

(3)

where $\rho$ represents the density of water, $g$ is the magnitude of the acceleration of gravity and $S$ is the area of the horizontal section the floater at the level of undisturbed water free surface.

The radiation force can be expressed by two components that are computed in the frequency domain [5]:

$$F_{\text{rad}}(\omega) = -A(\omega)\ddot{x}_1(\omega) - B(\omega)\dot{x}_1(\omega),$$

(4)

where $A(\omega)\ddot{x}_1$ represents an added mass term, $A(\omega)$ is the added mass, $B(\omega)$ the damping radiation coefficient and the term $B(\omega)\dot{x}_1(\omega)$ represents energy being transferred from the body to the water. Both are dependent on the floater’s geometry and on its frequency response.

This force can be written in the time domain by:

$$F_{\text{rad}}(t) = -\int_{-\infty}^{t} K(t-\tau)\dot{x}_1(\tau) d\tau - A_1\dot{x}_1(t),$$

(5)

where $A_1$ is the added mass coefficient of the floater and $K(t)$ represents the hydrodynamic coefficients of the floater. These hydrodynamic coefficients are not in the study area of this work and are here interpreted as an input from other previous study, and which numeric values are set to a 6 meter radius hemispherical-shaped floater.

**Diffraction force**

Since we are assuming linear water wave theory, the resulting diffraction force is obtained as a superposition of the frequency components

$$F_d(t) = \sum_{j=1}^{n} \Gamma(\omega_j) A_{\omega_j} \cos(\omega_j t + 2\pi \text{rand}()),$$

(6)

with

$$A_{\omega_j} = H_s \sqrt{2 \Delta \omega_j S^*_j(\omega_j)},$$

(7)

In the current work, real irregular waves are represented as a superposition of regular waves assuming the Pierson-Moskowitz spectrum.

$$S^*_\xi(\omega) = 262.9 \frac{H_s^2}{\omega_{\xi}^3 T_e^3} \exp\left(-\frac{1054}{\omega_{\xi}^2 T_e^4}\right),$$

(8)

For axisymmetric bodies oscillating in heave, the excitation force coefficient is related to the radiation damping coefficient by:

$$\Gamma(\omega) = \sqrt{\frac{2\rho^3 g B(\omega)}{\omega^3}}$$

(9)

A further detailed explanation about the calculation of this force is described in the dissertation [6]

### 3.1.2. Ballast

The ballast is the body that will hold the captured energy of the waves on the form of gravitational potential energy. When the ballast is suspended by the floater, its dynamic equation of motion is

$$m_2 \frac{d^2 x_2}{dt^2} = -m_2 g' + f - F_R,$$

(10)

where $g'$ denotes the equivalent gravitational acceleration, which takes into consideration the buoyancy force that the water exerts on the body of the ballast and $F_R$ represents the sum of hydrodynamic drag and friction forces on the rods.

### 3.1.3. The cables

The cables connect the ballast to the floater and have two different functions depending on the phase in which they are in the energy cycle, this means, they have the function of raising the ballast from $x_{\text{min}}$ to $x_{\text{max}}$ during the accumulation phase or accelerate the generator with the fall of the ballast during the production phase.

These are subject to elastic deformation when they are under tension, and this deformation is given by [4]:

$$\delta_C = \frac{f}{E_c A_c} (x_1 - x_2)$$

(11)

where $f$ is the force transmitted by the cables, $E_c$ the Young modulus of the cable, $A_c$ its full cross-sectional area (this means, the area of each cable multiplied by the number of cables) and $(x_1 - x_2)$ at each instant the distance between the floater and the ballast.

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The variation of the cable force with time is:

$$\dot{f} = \frac{(E_c A_c - f) (x_1 - x_2) + \dot{\theta} E_c A_c R_D}{(x_1 - x_2)}$$

(12)

### 3.1.4. Drum

The drum performs the function of winding and unwinding the cable around itself throughout the energy cycle, as well as transmitting the cable force.
to the generator through the gearbox, thus speeding it.

Let consider $\theta$ as the variable which measures the angular position of the drum, and its positive sense is that in which the cable is wound. We also consider that the cable has a diameter $d_C$, the drum has an initial radius $R_0$ and that the cable is wound onto the drum around itself in one spiral. The drum’s equation of motion is:

$$I_D \ddot{\theta} = -T_D - T_{b_D} - T_W + f R_D,$$  \hspace{1cm} (13)

Where $T_D$ is the torque transmitted by the gearbox to the drum, $T_{b_D}$ is the braking torque of the drum, $T_W$ is the torque applied by the cable winding mechanism (this is one actuated when there is a slack in the cable or lack of tension, otherwise it’s value is null), and at last $f R_D$ is the torque the cable transmits to the drum, which is proportional to the cable force and the radius of the loop at that instant.

3.1.5. Gearbox or transmission box
As already said, when the clutch is actuated the generator is connected to the drum through a gearbox. So, we have the generator connected to the shaft on one side of the gearbox with speed $\omega$ and torque $T_G$; and on the drum side the shaft has rotation speed $\dot{\theta}$ and torque $T_D$. The relation between torques is given by:

$$T_G = N_{tr} T_D$$ \hspace{1cm} (14)

And the relation between speeds is:

$$\omega = -N_{tr} \dot{\theta}$$ \hspace{1cm} (15)

where $N_{tr}$ is the speed ratio between both parties said, while its value is $N_{tr} > 1$. The rotation speeds have opposite signs, given by the convention in figure 4, where it considers that the angular position of the drum $\theta$ increases when the cable is wound on the drum and decreases as it unfolds. Regarding the direction of rotation of the generator, this has a positive velocity direction in the only way it spins, which is the same sense as the drum rotates as the cable is unwound.

**Figure 4:** Senses of the variables: angular position of the drum $\theta$ and generator rotating speed $\omega$. Schematic representation of rolling the cables on the drum

3.1.6. Electric Generator
The equation of motion is given by:

$$I_G \dot{\omega} = T_G - \frac{P_G}{\omega} - T_G^R,$$ \hspace{1cm} (16)

Where $T_G$ is the torque transmitted to the generator shaft by the multiplying gearbox when the clutch is activated, $T_G^R$ representing the torque resistance by friction in the bearings of the generator and the flywheel attached to it. This binary $T_G^R$ is practically independent of the rotational speed of the generator and is therefore assumed as a constant value.

**Clutch unactuated** When the clutch is not actuated, the torque transmitted by the gearbox to the generator shaft vanishes, $T_G = 0$, and so by equation (16) the generator slows down:

$$I_G \dot{\omega} = -\frac{P_G}{\omega} - T_G^R < 0.$$ \hspace{1cm} (17)

**Clutch actuated** The production cycle begins with a small acceleration of the ballast when it is released and therefore accelerates the drum until the drum speed of rotation, affected the speed ratio imposed by the gear box, equals the rotation speed of the generator at that instant. It is at this moment that begins the main stage of the production phase, the clutch is actuated and the movement of the generator will be accelerated by the fall of the ballast. At this stage neither the brake nor the ballast of the drum are actuated, so that the only force that is exerted upon the ballast under cable is $f$ it generates binary $f R_D$ on the drum.

The equation of motion already simplified is:

$$\left(I_G + \frac{I_D}{N_{tr}}\right) \dot{\omega} = -\frac{P_G}{\omega} - T_G^R + f R_D$$ \hspace{1cm} (18)

3.2. Equations resolution method
The main objective of this work, as has been said, is to simulate the behavior of this wave energy harnessing system in order to validate its correct operation and the applicability of the equations described in its modeling. Also we want to check what is the power that it will be able to produce according to the sea state that is subjected.

For this we used a simulation performed using the program *Simulink / Matlab*. In this two state-space were used to model the corresponding systems of equations of each stage, that were solved using the Runge-Kutta method.

The system consists of equations of first and second degree strongly nonlinear, so it can not be solved by the usual methods. This leads to endeavour to reduce the degree of quadratic equations for pairs of equations of the first degree, which is deduced in the equation (1):

$$m_1 \ddot{x}_1 = F_{\text{wave}} - f$$ \hspace{1cm} (19)

from here, we get:

$$\begin{cases}
\dot{v}_1 = (F_{\text{wave}} - f)/m_1 \\
\dot{x}_1 = v_1
\end{cases}$$ \hspace{1cm} (20)
All equations are rearranged in this manner so that on the left side stays the first derivatives of the variables to be calculated and the right side remains the known variables, calculated by this method in the previous time-step. It is this procedure that will be done to all systems of equations corresponding to each stage of the energy cycle to simulate and will allow to use of the computational method described above.

### 3.3. Central controls

In practice, monitoring of installations will be done through a small number of mechanisms to decide how this will behave, and these controls are the ON / OFF type.

- **Ballast Brake (BB)** - This is what will catch the ballast rods to prevent it from losing vertical elevation.
- **Drum brake (DB)** - Will block the drum when it wants the cable exert strain on the ballast.
- **Mechanism Winding Cape (CWM)** - Is coupled to the drum to allow rotate it and roll the cable.
- **Clutch (CLU)** - This is the mechanism that allows to couple the movements of rotation of the drum and gear box to the generator Electrical.
- **Power Generator (PG)** - This is the electrical power that will be requested the generator to provide, controlled by the power controller of PI type.

### 3.4. Stages of plant operation

A logic that controls the processes of the plant operation was defined. We know that the energy cycle is divided into two phases: Accumulation and Power Production, and each of these is divided into stages, which are characterized by a different system of equations. In Figure 5 described the flow diagram which symbolizes the logical sequence between stages and phases, as well as the necessary conditions for there to be an exchange of stage.

Figure 5: Scheme of the stages of system control

### 4. Power controller of the plant

The objective of power control is to keep the same rotational speed of the generator, in order to obtain a power output with constant characteristics, namely, always with the same electrical frequency. However, the generator speed is not constant, it is immediately set to the average rotating speed of the generator a reference value (omega-ref) 1500 [rpm] which intends to keep. As it is not possible to maintain a zero error in instantaneous power value, a PI (Proportional-Integral) controller that controls the electrical power to be drawn from the generator based on the error of the generator speed is used, and so it is intended to stabilize the speed value in a range of values considered acceptable, for example:

\[
1400 \text{rpm} \leq \omega \leq 1600 \text{rpm}, \tag{21}
\]

in order to control the variation of all the other parameters of the plant. For example, in the simulation corresponding to the referenced figure it was achieved by keeping the relative error of the angular velocity of less than 7% over the period of 12,000 seconds, which is the same period used in all other simulations.

**Calculating the Plant efficiency**

The maximum power that can be extracted by a floater in heave for a Pierson-Moskowitz spectrum for deep water is given by:

\[
P_{\text{max}} = 149.5 H_s^2 T_e^3. \tag{22}
\]

And for regular waves of height \( H \) and period \( T \) is given by:

\[
P_{\text{max}} = E_r c_g L_m. \tag{23}
\]

where \( E_r \) is the energy per unit of horizontal area, \( c_g \) the rate at which energy spreads, or the group velocity and \( L_m \) the maximum width of capture for absorbing spot.

\[
E_r = \frac{\rho g H^2}{8}, \tag{24}
\]

\[
c_g = \frac{g T}{4 \pi}, \tag{25}
\]

\[
L_m = g \left( \frac{T}{2 \pi} \right)^2. \tag{26}
\]

Computing the ratio between the power generator and the maximum power that could be extracted from this spectrum

\[
\eta = \frac{P_G}{P_{\text{max}}}, \tag{27}
\]

we define \( \eta \) as representing the performance or efficiency of the plant, or in other words, the fraction of the maximum power that the plant is producing.
5. Results Discussion

5.1. Regular waves case

For the case with regular waves, two types of simulations were made: one varying the wave period \(T\), and the other one varying de wave height \(H\). The values for the average power of the generator, maximum power of the wave, and efficiency of the plant for each simulation are described in table 1.

<table>
<thead>
<tr>
<th>(n)</th>
<th>(H[m])</th>
<th>(T[s])</th>
<th>(P_G[kW])</th>
<th>(P_{\max}[kW])</th>
<th>(\eta[%])</th>
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<tbody>
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<td></td>
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<tr>
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<td>210.7</td>
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<td>8</td>
<td>762.2</td>
<td>1529.2</td>
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5.1.1. Wave period variation \(T\)

After the analysis of the results obtained, it is possible to see that the main effect of the increasing of the power of the wave on the system is decreasing the period of each energy cycle, or the time that it takes for the ballast to go from its lowest height to the highest, coming back to the lowest. This is seen when analyzing a 1000 seconds interval: the number of cycles completed is 11, 21 and 32 for the wave periods of 6, 8 and 10 seconds, respectively. Therefore, there is less time between each energy production cycle, which results on the generator being accelerated more frequently, producing more energy.

Even if the power increases when the wave period increases, the same can’t be said of the efficiency of the plant, defined in equation 27, which has its maximum for the wave period of 8 seconds. This might have to do with various factors, but what is important to note is that the value of 8 seconds is the closest one to the resonance value of the floater, allowing for the extraction of a greater amount of energy from the wave.

5.2. Wave height variation \(H\)

Simulations were conducted for 5 different wave heights, in which it was verified that the wave height is related to the regular wave power, depending on it by the relation \(H^2\), as can be verified by the equations 23 to 26.

It was also seen from the results of table 1, that the wave type that has a better efficiency in the usage of the wave energy is the one with \(H = 2\), 0m, which is a median value compared with the other values tested, which makes the energy production possible for a wider value of wave powers.

The evolution of some of the variables studied for regular waves can be seen with greater detail on figure 6.

5.3. Wave behaviour representative of the sea state in a location at the sea off Leixões

A wave behaviour represents a sea state through a time span in a certain location. That behaviour can be described by a spectrum of irregular waves, with a certain power \(P_{\max}\) and a probability of happening of \(\phi_i\). In this section, it will be presented the results for the simulations of the plant behaviour with a group of spectrums of irregular waves representative of a wave behaviour for a location at the sea off Leixões, which can be seen in table 2.

<table>
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<th>(P_G[kW])</th>
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Two series of simulations were made, in which the average generator power \(P_G\) and the efficiency \(\eta\) of each spectrum are described.

The annual average efficiency for the plant can be calculated with the plant efficiency for a certain spectrum and its occurrence probability:

\[
\eta_{\text{annual}} = \sum \eta_i \phi_i. \quad (28)
\]

Similarly, the average annual power is given by:

\[
P_{G\text{ annual average}} = \sum P_G \phi_i. \quad (29)
\]

The values are presented in the last line of table 2 and they are the best indicators of performance of
the plant, as they consider the probability of occurrence of each spectrum, which will say which values of power and efficiency are expected to happen for the plant through the period of a year, if the plant was placed at the sea of Leixões.

The presented analysis was made for two different configurations of the plant, that were based on the variation of some of its dimensions and identified as "Series 1" and "Series 2".

**Parameter variation of the two analyzed series**

Initially, for the first result series, it was desired that the heights of oscillation of the ballast were comprised in an interval from -30 m to -5 m, but these were found to be very close to the heights of oscillation of the floater, which resulted in collisions for the most energetic states. On the second series of results the values were changed to prevent this, increasing the distance between the two components. The reason for trying to keep the ballast movements closer to the floater is related to the fact that it is desired that the depth of the installation location is as smaller as possible, to decrease installation costs. If the system created can only work on deep waters, it is necessary to place the plant far away from the coast, as only there deep waters will be found, increasing the cost of transmission of electricity to land with electric wires in the seabed, which might compromise the project economically.

**Friction effect on the efficiency**

A friction term was considered in the equation of the generator, which corresponds to the internal bearing resistance. This resistance was characterized in the simulations by a resistive binary \((T_R)\) that opposed the direction of rotation. Therefore, there is a minimum limit of energy in the spectrum due to that resistive term, for which the plant can produce energy. This means that when the power available on the sea state is lower than a certain value, the energy dissipated by friction is higher that the energy that can be captured by the plant, leading to the stoppage of the generator.

**Mass relation between ballast and floater**

The same way, after the first simulations, it was verified that the relation value \(\Delta\) between the ballast and the floater masses has a very relevant impact on the capacity for extracting energy of the plant. It was initially assumed a mass relation \(\Delta = 0,5\), which revealed to be a very high value, as for the lower energy spectrums it was not possible to attain enough force on the cable in order to overcome the ballast mass. The ballast brakes would never be deactivated, then, as the ballast wouldn’t move upwards. The value chosen for the mass relation was \(\Delta = 0,3\), as it already allowed for the production of energy in less energetic spectrums, and also an acceptable efficiency for less energetic spectrum.

**Numerical analysis**

Analyzing the results obtained, in table 2 it can be seen that, as opposed to what would be expected, the spectrum that most contributed for the increasing of the annual average power was not the one that showed a higher efficiency \((n=8)\) but the a more energetic spectrum with a similar occurrence probability \((n=9)\). For \(n=8\), it is possible to get more energy from the wave in a more efficient way, being higher the percentage of available energy captured. However, in absolute values, and taking into account the occurrence probability, the energy captured by spectrum \(n=9\) is the highest of all the spectrums. With this, it can be concluded that it is important not only to optimize the plant dimensions taking into account the efficiency increasing, but also to maximize the value of the product of the produced power and the occurrence probability.

**Graphical analysis of the results**

It is presented the graphic for spectrum \(n=8\), which was the one that obtained the best efficiency. Even if all the simulations were done for a total period of 12000 seconds, the results are presented only for the last 2000 seconds so that it is possible to see the analyzed variables with detail.

![Figure 7: Evolution of the angular velocity and generator power, positions and velocities of the ballast and floater, for an irregular wave spectrum with \(T_e = 8,03\) s and \(H_s = 3,06\) m, (Series 2, n=8).](image)

After the analysis of figure 7, it is important to note the irregularity of the floater and ballast movements, opposed to what happened with the regular waves, due to the irregularity and the strength of the incident wave diffraction, which best represents what happens in nature. As for figure 7 corresponding to spectrum \(n=8\), the analysis is very similar to the one for spectrum \(n = 9\), differing primarily on the range of motion of the float which is directly connected to the power available for the spectrum, and as a consequence to the power extracted. Note however that this is the spectrum...
which showed higher yield of all the analyzed spectrums, and probably the entire configuration of the plant is more optimized for spectrums similar to this one.

5.4. Influence of the dimension of some components on the dynamic behaviour of the plant

Here a study was conducted to illustrate the importance of optimization of all components of the central regarding sizing, since it strongly influences the performance and the dynamic behaviour of the plant. It will only be studied the effect of the variation of three characteristics of the plant: the mass of the ballast, the diameter of the cables and the inertia of the generator. The simulation made with a regular wave of \( H = 2 \text{m} \) and \( T = 8 \text{s} \) was chosen to serve as default state, from which the variations of the parameters will be done. This wave was chosen for being the regular wave for which we obtained better performance of the plant, and also because being a regular wave, allows us to anticipate and predict some expected behaviours, and thus compare the results obtained with these.

The results for the default state are presented. These charts show the variations in the positions and speeds of the float and ballast, the diffraction force and the rotational speeds of the generator: a reference and an instant speed.

![Figure 8: Default state for the variations of the parameters in study. Linear wave with \( T = 8 \text{s} \) and \( H = 2 \text{m} \).](image)

5.4.1. Effect of the variation of the mass of the ballast

To study the effect of varying the mass of the ballast, two comparisons were performed with the default value. The values used for the mass ratio of the float and ballast (\( \Delta \)) were \( \Delta = 0, 3 \) for the default state and \( \Delta = 0, 1 \) and \( \Delta = 0, 5 \) for the following simulations, where the mass ratio \( \Delta \) is given by:

\[
\Delta = \frac{m_2}{m_1 + A_1^\infty}.
\] (30)

After analysis of the variations of mass calculated, one can conclude that the change in mass of the ballast has two major effects: its increase causes the power accumulation cycle to take a longer time to be concluded, as seen in the simulation for \( \Delta = 0, 1 \), each energy cycle takes 30 seconds, while for \( \Delta = 0, 5 \) this cycle already takes about 90 seconds. The other effect that it is easily visible is the variation in rotational speed of the generator that occurs during the production phase. As the accumulation cycle takes longer for a larger weight of the ballast, it also leads to the accumulation of more potential energy (because remember that \( E_{\text{POTENTIAL}} = mg \Delta h \)), giving it a greater acceleration of the generator in the production phase.

5.4.2. Effect of the variation of the diameter of the cables

It was studied here the influence that the variation of the diameter of the cables connecting the ballast to the floater has on their dynamic behaviour. Although it may not seem, from all the parameters studied this is the one with the biggest impact on the movement of both bodies, because it is the cables that make the connection. The default value used in the diameter of the cables was \( D_c = 30 \text{mm} \), and therefore it is presented here the results for the diameters of \( D_c = 60 \text{mm} \) and \( D_c = 10 \text{mm} \). It was tried to vary considerably the numerical value of each of the simulations against the default value so that it was quite obvious and visible the effect that the changes may generate.

The conclusion after analyzing the results is that the diameter of the cable, which along with the number of cables is what defines the total number of cables, is an element that contributed greatly to the rigidity of the link which is given by:

\[
K_{eq} = E_c A_c.
\] (31)

That said, it is noticed that for \( D_c = 60 \text{mm} \) there is an extremely rigid connection and due to the cable being a type of bond that only transmits tensile forces, it happens that whenever the cable is stretched an extremely high force is generated for extremely short time interval which in the limit could be described as almost a shock. Thus, we have very high and intermittent forces in the cable, which suggest that in this case there would be a large vibration in the handle and a small capacity to absorb impacts.

On the other hand the connection cable of \( D_c = 10 \text{mm} \) would be much more elastic, yielding much lower forces in comparison with the previous case, but also leading to a greater deformation of the cable, which may preclude the generation of energy in the case of low energy sea states in which the floater has very small oscillations.

5.4.3. Effect of the inertia of the generator

The latter parameter studied was the inertia of the generator, which has an effect similar to that de-
scribed on the weight of the ballast. As the mass of the ballast stores the gravitational potential energy during lifting, the inertia of the generator stores the potential kinetic energy of rotation when the angular acceleration of the generator happens.

Knowing this, it was compared the amplitude of the variation of the angular velocity of the generator through each energetic cycle, for a total period of 12000 seconds. The values of inertia used were $I_G = 20,000 \text{[Kg m}^2\text{]}$ for the default state, of $I_G = 40,000 \text{[Kg m}^2\text{]}$ and $I_G = 5,000 \text{[Kg m}^2\text{]}$. Analyzing the results of the study with the highest value of inertia, the variation of the angular velocity through an energetic cycle was on the order of 25 rpm, while the same variation for an inertia of $I_G = 5,000 \text{[Kg m}^2\text{]}$ was on the order of 175 rpm.

6. Conclusion

In the present work it was successfully achieved the implementation and validation of a computational program capable to simulate the dynamic behaviour of the new type of wave energy harnessing system. The correct working of the program was validated with both regular waves and irregular wave spectra, including the complete simulation of a annual wave climate of a point off the coast of Leixões. Finally it was made an analysis of the impact the sizing of some critical components had over the general behaviour of the plant.

The code was implemented and reviewed initially with a simpler system, described in the article [5]. This was two body energy converter system with a power-take-out mechanism simplified as a spring-dumper connection between both bodies.

It was refined the former idea of control algorithm that existed over the plant. The main aspects changed were the criteria variables for changing between stages. Whilst the previous thoughts on this subject were all focused on absolute coordinates of the ballast and floater ($x_1$ and $x_2$), this revealed to be not robust enough, being changed to the difference in length between the two bodies ($x_1 - x_2$).

From the regular wave analysis it was concluded that despite the fact the power of a wave increases with both wave height ($H$) and period ($T$), the wave parameters to which the plant has a better efficiency are variable and depend on the plant’s geometries. For the values tested, the most efficient regular wave was $H = 2 \text{m}$ and $T = 8 \text{s}$.

Regarding the analysis for the irregular wave spectrum and wave climate of a point off the coast of Leixões there were some important conclusions to remember. Firstly, between the two series of geometries studied the variation of the range of depths the ballast was oscillating around generated a considerable difference in their efficiency, being the series which oscillated the ballast in deeper positions the one to achieve a higher efficiency. Secondly, it was important to notice that the spectrum with the greatest efficiency was not the one that contributed the most for the increase of the annual average power. With this it would be a suggestion for a future work to do a optimization of station’s parameters regarding not the increase of efficiency, but the increase of the annual average power produced, as well as getting a geometry optimized to have a broader range of wave power capture, because the current geometry didn’t allow to produce anything from the very high and low power waves.

To conclude, there were several relations perceived between the variation of some main parameters, as the ballast mass, cable diameter and generator inertia, with the functioning, behaviour and even stability of the power plant as a whole. To summarize: the increase of the ballast mass would increase the amount of energy captured by each accumulation cycle as well as increasing it’s time duration; an analogy with this can be made for the generator inertia with the production phase; the system complete stability is very dependent on the cable total area.

References


