DESIGN AND ANALYSIS OF A CANTILIVERED OFFICE BUILDING

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SUMMARY

This research considers the design and analysis of a cantilevered office building compatible with the architecture design.

The structural solutions of flat slab with heads, the waffle slab and the beam slab are studied in order to choose the most suitable. For each type of structural slab, some solutions with different dimensions are chosen, in which the relative displacements are estimated at critical points. One solution of each slab type is chosen and its deflection and ultimate state are verified. Then the three solutions are compared based on the relative displacements and volume of concrete needed. The waffle slab is considered the best solution for the studied building.

The preliminary design of the vertical structural elements is done, the building is modelled with 3D finite element computation code SAP2000 and a static and dynamic structural analysis is done. The significant structural elements are designed according to Eurocodes. The Eurocode presents a set of information, including principals and application rules. The principals are obligatory while the rules of application are generally recognised. Some rules are not fulfilled. The designer can take alternative measures since well justified.

The structural elements are detailed. The drawing and the design done simultaneously is important because it allows understanding if the design is executable. The quantities work map and a budget estimate is done. It is concluded that structural solution is balanced ant the price is competitive.

The structural modifications needed to fulfil the Eurocode rules are presented.

Key-words: Design, Flat, Beam, Eurocode, SAP2000
1 Introduction

The main objectives of this research are the design of a balanced structure building compatible with the project architecture given, design and detail of the type structural elements and guarantee the security for the regulatory actions.

All phases of the project design were covered, since the structure design solution and preliminary design until the last phase of designing, including the dynamic behaviour and verification of ultimate limit state (ULS) and serviceability limit state (SLS). During this process, all challenges faced and its solutions will be referred.

The structural design must ensure some goals such as, the use, the durability, the safety and the comfort for the user. The minimization of the global volume of material must also be a principle in order to save money.

To accomplish the objectives referred, three main structural slabs are studied in order to find the most suitable for the building studied.

The structure is modelled in the 3D finite element SAP2000 code. Its results are carefully checked in order to minimize errors of modelling or design.

The estimate budget of the structure is presented to understand if the design is competitive.

In terms of regulations, the design follows the Eurocodes (EN 1990 to EN1999).

2 Constraints and Adopted Solution

The cantilevered office building is located in Cascais with seven storeys and no underground storeys, as the longitudinal section in Figure 1 shows.

![Figure 1 - Longitudinal section of the design architecture](image)

The designation balanced comes from the advance that occurs between storey 0 and 1 along all the perimeter of the building, which extends until its top.

Between other, the advance could be solved in two ways. One was to solve locally the cantilever which means to design it based on the structural functioning at one storey. The other, was to introduce columns on the edge of the slab but without link to the soil. These columns would transfer the load into beams at storey 1 which transfer the loads to the internal columns. It would be convenient the use of prestressing in these.

In figure 2 is depicted the structural type storey, used from storey 1 to technical storey, with the definition of the local axes. It is adopted the waffle slab so as it can be seen by the intercalation of the moulds and some flat zones.

The soil of foundations has an admissible stress of 400KPa. For that reason is adopted direct foundation, more precisely footings without foundation beams.
3 Materials and Actions

The strength class of concrete used according to the elements of the building is presented in table 1. The exposure class influences the cover used in the reinforcement.

Table 1 - Strength class of concrete and exposure class according to the elements

<table>
<thead>
<tr>
<th>Element</th>
<th>Strength Class</th>
<th>Exposure Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Structure</td>
<td>C30/37</td>
<td>XC1</td>
</tr>
<tr>
<td>Foundations</td>
<td>C25/30</td>
<td>XC2</td>
</tr>
<tr>
<td>Dispersion</td>
<td>C12/15</td>
<td></td>
</tr>
</tbody>
</table>

The reinforcement is considered with special ductility as it is shown by the abbreviation SD in table 2.

Table 2 - Class of the reinforcement

| Class of the reinforcement | A500 NR SD |

The main actions considered on the design are shown in table 3.

Table 3 - Dead and imposed load

<table>
<thead>
<tr>
<th>Load [KN/m²]</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead</td>
<td>2.5</td>
</tr>
<tr>
<td>Imposed</td>
<td>3</td>
</tr>
</tbody>
</table>

4 Structural Analysis of Different Kind of Slabs and Its Selection

The span lengths of the slab are illustrated in figure 3. Taking that into account, it is studied the flat slab with head, the waffle slab with and without head, and the beam slab. Each one is modelled in SAP2000.

In terms of terminology, bending moment X means that the element bends in direction X of the local axes. The same logic applies to bending moment in direction Y.

For each kind of slab, it is considered several solutions with different dimensions of its elements. For each solution, it is studied the
relative displacements, for quasi-permanent action, at the points of slab identified in Figure 4. The second letter of the point's designation means the direction measurement of the relative displacement. The amount of concrete needed on each panel, identified in Figure 4, is also studied.

The main aim of this study is to find the kind of slab most suitable for the building.

The Figure 4 also illustrates the SAP2000 model of the flat and waffle slab. The red area represents the heads, the green area pictures the slab for the flat slab and the waffle area for the waffle slab. The blue region represents the hollow area.

The SAP2000 model of the beam slab is illustrated in Figure 5. Notice the presence of the beams on the cantilever area.

![Figure 4 - Model of the flat slab on SAP2000, plan X-Y, with the identification of the studied points and the definition of the panel used to calculate the amount of concrete](image)

![Figure 5 - Model 3D of the beam slab on SAP2000](image)

Based on the results, it is considered that the best solution for each kind of slab are: $(0,4 \times 0,22)m^2$ for flat slab, $(0,325 \times 0,325)m^2$ for waffle slab and $(0,7 \times 0,3 \times 0,2)m^3$ for beam slab. Notice that for beam slab, the height of the beam 0,7m includes the height of the slab 0,2m.

Each of the previous solutions are verified for ELS deformation and ELU. Relatively to ELS deformation, the displacements given by the code were the instantaneous elastic one. To take into account the creep and cracking, it is used the global coefficient method. The values obtained are compared to the L/250 limit of deformation defined in EC2. The variable L is the length of the span. All the limit states are verified.

Table 4 summarizes the results obtained. Notice that the column L.L.P means the incremental factor of the elastic displacements until the L/250 limit.
The solution that involves less concrete on each panel is the waffle slab. The beam slab is the one that controls better the relative displacements.

It is interesting to verify that the waffle slab presents similar displacements at points A, B and C, to flat slab. The heads of flat slabs controls better curvatures than rotations. Consider the next figure, which illustrates the approximate functioning at those points.

![Figure 6 - Beam deflection fully restrained-supported for a uniform linear loading](image)

The flat solution has an higher head at support 1 and for that reason controls better the curvature. The waffle solution have the same inertia at middle span but less weight, compared to the flat slab, what allows fewer rotations at support 2. That's the reason for the similarities of the displacements. But notice the fewer concrete needed for the waffle solution.

On the other hand, the span at point I functions approximately as, fully restrained-fully restrained, and the displacement is higher on waffle slab than the flat slab. That's because the displacement depends essentially of the curvatures at Support 1 and 2, and flat solution has higher head, so the results are justified.

Thereafter, it is made a study to find the most competitive solution through the equation,

\[
Rate = \text{displacement}_{\text{point}} \times \text{volume}_{\text{concrete,panel}}
\]

(1)

The results at point A, end span, and at point I, interior span, are presented in Figure 7 and Figure 8 respectively.

![Figure 7 - Rate at point A](image)

At point A, the beam slab presents excellent results.

It is well patent that the waffle slab is more efficient than the flat slab. It is recommended to compare the flat solution (0,4×0,22)m² with the waffle solution (0,425×0,325)m², both with
similar inertia. The rate difference is about two values.

![Figure 8 - Rate at point I](image)

At point I, the rate difference of both solutions is smaller, about 0.8 values. The higher difference at point A comes from the better functioning of the waffle solution at end spans, as already referred.

It is also perceived the equilibrium results of the three kinds of slabs at point I.

Table 5 presents the total volume of concrete needed for the slabs of the building according to each solution previously chosen.

<table>
<thead>
<tr>
<th>Solution Slab</th>
<th>Volume of concrete [m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam (0.7×0.3×0.2)m³</td>
<td>1179.9</td>
</tr>
<tr>
<td>Waffle (0.325×0.325)m²</td>
<td>1173.5</td>
</tr>
<tr>
<td>Flat (0.4×0.22)m²</td>
<td>1310.3</td>
</tr>
</tbody>
</table>

In terms of slabs price, the beam solution is the best one because involves the same volume of concrete compared to the waffle solution, but it has the lowest rate of reinforcement. However, it needs higher height for each storey, about 2.60m in total, what means least one storey for the building. That implies a loss of money.

The waffle solution involves less volume of concrete than the flat solution. On the other hand, the building is mainly composed by end spans. For that reasons, it is chosen the waffle slab (0.325×0.325)m².

5 Preliminary Design and Modelling

The column section is quadrangular because it is as well defined at architecture project and wall stiffness is not so different at both global directions. The preliminary design of the column is based on obtaining a value around 0.7 for the axial reduced resultant forces for fundamental combination of actions. That value is defined to guarantee a good ductility, which is necessary because the columns are part of the primary seismic elements.

The footings sections are quadrangular because the correspondent columns are as well. The footings are calculated using the axial resultant forces of the characteristic combination. The areas of the footing have to guarantee a stress on soil lower than 400KPa.

The structural SAP2000 model of the building is illustrated in the next figure.

![Figure 9 - SAP2000 3D structural model of the building](image)
It is considered half value of elasticity modulus of the concrete, to take into account creeping of the concrete elements.

Slabs are modelled as shell-thick finite elements. The waffle slab is modelled as flat slab, wherein the thickness is calculated based on the inertia of the waffle slab and it is applied a weight factor reduction.

The shear walls of the core are modelled as frames finite elements and at each storey level is applied a body constraint of the six degrees of freedom.

The height columns at storey 0 are considered with an extra 1,5m compared to the dimensions of the architecture project. That's because the foundations are assumed 1,5m below the ground, to assure the mobilization of the axial tension resultant forces of the column P3B and P3C.

The footings are modelled with springs supports stiffness rotation about X and Y.

6 Seismic Analysis

The walls and the system column-slab are considered as primary seismic elements.

According to the national annex of EC8, the use of system column-slab as primary seismic element in flat slabs must be used with caution. The reason is the lack of knowledge about its behaviour facing horizontal actions and also the capacity of energy dissipation.

To consider the system as secondary seismic element, it is necessary to introduce extra walls that would change the architecture design and for that reason it is not adopted. However, it is considered some measures in the slab design to guarantee extra security.

The main three vibration modes and its frequencies are illustrated in the next table.

<table>
<thead>
<tr>
<th>Mode</th>
<th>f[Hz]</th>
<th>Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50</td>
<td>Shift Y and Torsion</td>
</tr>
<tr>
<td>2</td>
<td>0.61</td>
<td>Shift X and little Torsion</td>
</tr>
<tr>
<td>3</td>
<td>0.70</td>
<td>Shift X and little Shift Y</td>
</tr>
</tbody>
</table>

Based on art.º4.2.3.2 (6) of EC8, the building is classified as a torsionally flexible system. As a consequence, it is used a behaviour factor of q=2,0.

The seismic action is defined by Response Spectrum. It is also considered accidental torsional effects and second order effects, respectively by applying torsional moments at each storey and by a majority factor of 1,20 on the seismic efforts of first order.

The horizontal displacements for a seismic service action are positively verified.

7 Design

The design is an iterative process. Initially the wall PA4 doesn't verify the security so it is necessary to reduce its stiffness of bending and shear about 20%. The subsequent redistribution of resultant forces between walls is limited to 30% according to art.º 5.4.2.4(2) of EC8.

Columns

The columns are divided in groups with similar efforts to reduce the discrepancy of the reinforcement and to facilitate the detail.

The first storey columns are higher than the remaining storeys, therefore its resultant forces are bigger. For that reason, it is considered
different dimensions for the transversal sections as represented in the next table.

**Table 7 - Dimensions of the column transversal sections by storey**

<table>
<thead>
<tr>
<th>Storey</th>
<th>b [m]</th>
<th>h [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>1 - 6</td>
<td>0.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The seismic combination is determinant for the design compared to the fundamental combination. It is calculated the minimum and maximum axial resultant forces to design the reinforcement and to verify the condition $v_d < 0.65$, respectively. Besides axial forces, the columns are subjected to biaxial bending. The EC8 suggests as simplification to design it as bending with axial force at each direction, with a 30% reduction of the resistance. Instead, it is used a section concrete design program, named XD-CoSec.

The shear design is based on mechanism of Mørsh. To guarantee a ductility failure, the shear is calculated through the bending resistance.

Confinement is also verified according the art. 5.4.3.2.2.(8) of EC8. Notice that both interior and exterior hoops are used in the verification of shear and confinement.

**Shear Walls**

The bending diagram is obtained according the art. 5.4.2.4(5) EC8, which is based on the formation of plastic hinges on its base.

It is considered a reinforcement curtailment at storey 3 due to the rapid variation bending diagram along the height.

Each wall is designed as bending with axial force. The most efficient way to resist to bending is concentrate the reinforcement near the ends, inside boundary element, to increase the internal lever arm, $z$, as defined in the figure 10. The web reinforcement is not considered to the resistance of the bending with axial force and it is designed with minimum reinforcement. The length of the boundary elements is defined based on EC8.

![Figure 10 - Boundary element, internal level arm and web reinforcement](image)

The boundary element reinforcement is calculated based on the following equation,

$$ A_s = \frac{M}{Z - \frac{N}{f_yd}} $$

which M represents the bending moment and N the axial force.

The shear design is also based on the diagram defined by art. 5.4.2.4(8) EC8. A 45° inclination of the compressive strut is considered to limit its value.

In terms of confinement, initially is calculated the critical height according the art. 5.4.3.4.2(1) EC8.

The walls identification scheme is bellow illustrated.

![Figure 11 - Identification scheme of the walls](image)

The walls PA1 and PA2 don't verify the maximum critical height defined in EC8, which
is a detail recommendation for local ductility. The solution would be increasing the length of the walls, about 0.85m, what would change substantially the architecture project and for that reason it is not adopted. However it is taken attention to the compression strut and to the confinement reinforcement.

**Slabs**

The resultant forces are obtained at the first storey.

The negative and positive bending moments over the solid areas of the slab are governed by the seismic combination, while the positive bending moments over the waffle area are governed by the fundamental combination.

The punching shear is verified and its reinforcement is located between 0.5d and 2d, wherein d is the effective depth of the slab. The reinforcement designed is superior to what the calculation revealed in order to increase the security relatively to the brittle failure and to confer greater ductility. That’s because, as already said, the flat slab capacity dissipation of energy is reduced.

The supporting reinforcement is also designed which its function is to assure a secondary mechanism of resistance in case of failure by punching shear.

The control of deformation is carefully verified at the points with maximum relative displacements.

The serviceability limit of cracking is also verified.

The lack of resistance to fire is one of the main disadvantages appointed to the waffle slab. Therefore, the structural fire design is also made according the table 5.11 of EC2 1-2. The results are presented at the Table 8.

### Table 8 - Structural fire design

<table>
<thead>
<tr>
<th>Minimum dimensions [mm] for the combination 2 e 5</th>
<th>Dimensions used [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{mn} = 120$</td>
<td>$b = 120$</td>
</tr>
<tr>
<td>$a = 35$</td>
<td>$a = 40$</td>
</tr>
<tr>
<td>$h_s = 100$</td>
<td>$h_s = 100$</td>
</tr>
<tr>
<td>$d_b = 15$</td>
<td>$d_b = 50$</td>
</tr>
</tbody>
</table>

**Foundations**

The height of the footings must have a minimum value to assure the adequate stiffness. The fundamental combination is the conditioning. It is verified the stress acting on soil and designed the reinforcement.

### 8 Budget Estimative

It is considered for the budget, the price of concrete, reinforcement, formwork, rigging and soil. The estimated price is 93€/m², what indicates that the design effectuated is competitive.

### 9 Necessary Structural Modifications to be According National Annex of EC8

According to the NA.4 art.º4.2.d) of EC8, except in cases of low seismicity, which is not the case, the flat slab must be considered as secondary seismic element, which by definition doesn’t resist to the seismic action.

It is also stated that the lateral stiffness of the total secondary seismic elements must not be superior to 15% of the total primary seismic elements.

In the next figure is illustrated the structural modifications introduced in order to fulfil the suggestions above. The length of the square section of the column is reduced to 0.5m and
is introduced four walls, developed on axe Y, each one with a cross section of (0,3×3)m.

One of the four walls introduced

![Figure 12 - New structural solution](image)

At the next table is presented the base shear and its percentage absorbed by each structural element.

**Table 9 - Percentage of base shear absorbed by each structural element**

<table>
<thead>
<tr>
<th>Direction</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_b$ [KN]</td>
<td>4862.8</td>
<td>4720.1</td>
</tr>
<tr>
<td>Columns [KN]</td>
<td>499.9</td>
<td>605.0</td>
</tr>
<tr>
<td>Walls [KN]</td>
<td>4362.9</td>
<td>4115.1</td>
</tr>
<tr>
<td>Columns/Walls [%]</td>
<td>11.5</td>
<td>14.7</td>
</tr>
</tbody>
</table>

**10 Detailing**

All the major structural elements are detailed. As example, it is presented the detailing of the column P3B, storey 0, submitted to axial tension.

![Figure 13 - Detailing column P3B, storey 0](image)

**11 Conclusions**

The developed design allowed reaching a balanced structural solution compatible with architecture project. The security of the major structural elements is achieved and the estimated price aims to a competitive solution designed.

This work prompts the following general conclusions:

i. The beam slab is the best solution in terms of total price of construction slabs but looses for the flat slab because it steals considerable height at each storey.

ii. The heads of flat slab loose efficiency at end spans, functioning as fully restrained-supported.

iii. The waffle slab has a better structural functioning at end spans, functioning as fully restrained-supported, than the fat slab

iv. The best slab solution for this building conditionings is the waffle slab

v. The use of flat slab, as primary seismic element at relevant seismic zones, must be with caution

**12 References**


