Determination of the longitudinal parameters of a three-phase cable

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Abstract — The work presented aimed to calculate the longitudinal parameters - resistance and inductance - per unit length of power cables taking into account skin effect and proximity effect.

The method used was based on the discretization of the conductor into a large number of sub-conductors with a variable rectangular section and uniform current density.

Using the MATLAB tool we developed a computer algorithm to calculate:
- the longitudinal parameters per unit length,
- the current density inside the conductors and
- the magnetic induction field inside and outside of the conductors.

The method was validated comparing simulation results with theoretical results for a cylindrical conductor and for a cylindrical conductor with a steel core.

We also applied the method to different types of cables, namely with non-cylindrical geometries, different electrical conductivities and different magnetic permeabilities.

Index Terms—Power cables, longitudinal impedance, skin effect, proximity effect and density current.

I. INTRODUCTION

This work aims to study the longitudinal parameters - resistance and inductance - per unit length, of underground cables taking into account the skin effect and the proximity effect.

The skin effect is associated with the fact that the conductor has finite conductivity and it is characterized by the tendency of electrons to circulate around the periphery of the conductor as the frequency increases, being nonexistent in DC. This effect is responsible for increasing the conductor resistance as its effective area decreases with frequency.

The proximity effect is associated with alteration of the field created by a particular conductor when there are other conductors nearby. This effect becomes stronger when the distance between conductors decreases.

II. METHODOLOGY

The work performed here follows two other thesis presented previously [2] and [3], where the same methodology based on discretization of the conductors of the system through sub-conductors was also used.

The methodology here present uses the same kind of discretization proposed in [3]. It is considered that the sub-conductors characterizing each conductor of the system have infinite length, variable rectangular cross section and uniform current density. The number of sub-conductors depends on the conductor geometry and frequency.

The main point of this methodology consists in the division of the conductors, with any geometry, in sub-conductors of smaller rectangular section. For accurate results the set of sub-conductors should characterize the shape of the real conductor and the cross section of the sub-conductors must decrease as frequency increases. In order to reduce the computational effort, in particular for high frequency, we consider that the cross-section of the sub-conductors decreases towards the surface of the conductor, ie sub conductors in the periphery of the conductor have a smaller section than in the inner zone.

Figure 1 shows a cylindrical conductor and the sub-conductors to be used for its characterization. For this example, in which we used a grid with dimension $N_d = 20$ (20x20 matrix), and we obtained a total of $N=220$ sub-conductors.

![Fig. 1. Division of a cylindrical conductor in sub-conductors with $N_d = 20$](image)
Considering one conductor discretized by \( N \) sub-conductors, being \( N \) the reference sub-conductor of the system.

The general law of induction applied to the closed path \( s \), which is in [5] is given by:

\[
\oint_s E \cdot ds = -\frac{d}{dt} \Phi_s
\]

(2)

Where \( E \) is the electric field and \( \Phi \) is the magnetic flux.

Consider now the path shown in Figure 2, running through conductor \( N \) and \( i \), among the longitudinal coordinates \( z \) and \( z+\Delta z \).

![Representation of the path \( s \) between \( i \) and one sub-conductor \( N \) of reference for application of the general law of induction.](image)

Fig. 2. Representation of the path \( s \) between \( i \) and one sub-conductor \( N \) of reference for application of the general law of induction.

We easily conclude the following final expression:

\[
-\frac{du}{dz} = E_i - E_N + j\omega(A_i - A_N)
\]

(3)

Where \( A \) is the potential vector, \( U \) is the voltage and \( \omega \) the angular frequency.

In order to solve the system of equations involving all sub-conductors, it is necessary to add another equation which relates the total current flowing in the conductor with the current density in all sub-conductors. The total current is obtained by adding the product of the current density by the section of sub-conductor:

\[
I = \sum_{i=1}^{N} J_i 2a_i 2b_i
\]

(4)

The system of equations is complete with \( N-1 \) equations of type (3) and with equation (4).

\[
\begin{bmatrix}
-\frac{dU_1}{dz} \\
\vdots \\
-\frac{dU_{N-1}}{dz} \\
\end{bmatrix}
= \begin{bmatrix}
K_{11} & \cdots & K_{1N} \\
\vdots & \ddots & \vdots \\
K_{(N-1)1} & \cdots & K_{(N-1)N}
\end{bmatrix}
\begin{bmatrix}
J_1 \\
\vdots \\
J_{N-1}
\end{bmatrix}
= DJ
\]

(5)

\[
D = KJ
\]

The resolution of the equation system provides the current density in each sub-conductor constituting the conductor.

In the case of insulated conductor, the longitudinal impedance can be determined by dividing the value of the electric field on the conductor surface (i.e., the value of the field in the peripheral sub-conductors) by the value of the total current \( I \):

\[
Z = \frac{I_k}{\alpha l}
\]

(6)

For more complex systems, with \( N_c \) conductors, the system of equations to solve can be formulated using (5), but with some changes regarding the case of the insulated conductor.

The number of equations to be solved remains \( N+1 \), but now \( N \) is the total number of the sub-conductors featuring the \( N_c \) conductors of the system. The total current of the system (last element of the vector \( D \)) is now zero. The other elements of vector \( D \) refer to the fall of the longitudinal voltage of each conductor relative to the reference conductor. The values of \( du/dz \) are equal for the sub-conductors related to the same conductor and zero for all sub-conductors of the conductors of reference.

The matrix \( Z \) can be determined by solving (5) for different operating modes, each mode requiring only one conductor voltage and all the other connected to the reference conductor. From (5) we can deduce, to each operation mode, the currents that run through each conductor of the system. The elements of the \( Z \) matrix can then be obtained as:

\[
-\frac{du}{dz} = Z I
\]

(7)

For example, considering only three conductors in the system, where \( J_k \) is the current in the conductor \( i \) to operating in mode \( k \) (i.e., only the conductor \( k \) is in tension while the others are connected to the reference conductor) then we can write the equation (8):

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
= \begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_{11} & I_{12} & I_{13} \\
I_{21} & I_{22} & I_{23} \\
I_{31} & I_{32} & I_{33}
\end{bmatrix}
\]

(8)

Through (8) is possible to obtain all the impedances of system.
Consider now a cylindrical conductor with a core of a different magnetic material. The core has a magnetic permeability, $\mu_1$, and the outer layer has a magnetic permeability $\mu_2$. Using the above methodology, the conductor is discretized into sub-conductors of rectangular section, as shown in Figure 3.

Obtaining the final expression:

$$\frac{1}{\mu_0} \left(1 - \frac{\mu_2}{\mu_1}\right) \left(\sum_{i=1}^N B_{1t,\text{rect},i} + \sum_{j=1}^P B_{1t,\text{sup},j}\right) - J_s = 0 \quad (6)$$

The system of equations now to be solved is of the form:

$$
\begin{bmatrix}
-\frac{du_1}{dz} \\
\vdots \\
-\frac{du_{q-1}}{dz} \\
\frac{l}{I} \quad 0 \\
\vdots \\
0
\end{bmatrix}
= 
\begin{bmatrix}
K_A & K_B \\
K_C & K_D
\end{bmatrix}
\begin{bmatrix}
J_1 \\
\vdots \\
J_{N-1} \\
J_{S1} \\
\vdots \\
J_{SP}
\end{bmatrix} \quad (9)
$$

where it was necessary to introduce the current density on the boundary surface and their influence on the system.

III. RESULTS

A. Validation

To validate the proposed developed method, two examples are presented with conductors whose parameter simulation results are compared with theoretical ones, obtained using expressions published in the literature [6] and [7].

The first example considers a cylindrical isolated conductor. The impedance can be calculated using [6]:

$$Z = R_0 k r_0 J_0(k r_0)/(2 J_1(k r_0)) \quad (10)$$

Where $R_0$ is the resistance per unit length in DC, $r_0$ is the radius of the conductor, $J_0$ and $J_1$ are the first kind Bessel functions of order 0 and 1, respectively, and $k = (-j \omega \mu \sigma)^{1/2}$.

In this example a single cylindrical conductor with radius $r_0 =$5mm, electrical conductivity $\sigma=5,7 \times 10^7$ S/m and magnetic permeability $\mu=\mu_0$ was used. A grid of 135x135 ($N_d=135$) was used to perform the division of the conductor in sub-conductors of smaller size, thereby obtaining $N=9913$ sub-conductors.

Table I shows the theoretical values obtained through (10) and it also shows values obtained by simulation for several values of frequency.

![Fig. 3. Sub-conductor geometry and distribution and separation lines of a conductor with two media with different magnetic permeability for $N_d=30$.](image)

![Fig. 4. Representation of two media with the same magnetic permeability and the existence of surface current $J_s$.](image)
The second example considers an isolated conductor with a steel core. The parameter values can be obtained by the following expression [7]:

$$Z_2 = (αZ + β)/ (γZ + δ) \quad (11)$$

Where $α$, $β$, $γ$ and $δ$ are given by:

$$\begin{bmatrix}
α \\
β \\
γ \\
δ
\end{bmatrix} = \begin{bmatrix}
-jω \\
0 \\
2πkr_0 \mu_2 \\
μ_2
\end{bmatrix} \begin{bmatrix}
J_0(k_2r_0) \\
N_0(k_2r_0) \\
J_1(k_2r_0) \\
N_1(k_2r_0)
\end{bmatrix}
$$

And $N_0$ and $N_1$ are the second kind Bessel function of order 0 and 1, respectively.

A cable was used with an outer radius $r=5$ mm and an inner radius, $r_o=3.33$ mm. The core of the cable has an electrical conductivity $σ_i=2.0×10^7$ S/m and magnetic permeability $μ_i=50μ_0$, and the outer layer has an electrical conductivity $σ_2=3.5×10^7$ S/m and magnetic permeability $μ_2=μ_0$.

This conductor was simulated with $N_d=120$ which gave a division into a total of 2324 sub-conductors in the core of the cable, and 4996 sub-conductors in the outer layer.

Table II shows the theoretical values obtained through (11) and the values obtained by simulation for several values of frequency.

The results obtained show the good accuracy of the proposed method. In the first example, the relative error obtained is lower than 0.12% while in the second example it is less than 6.1%.

### B. Application examples

1) Power cable with four conductors and sheath.

The power cable studied is constituted by four aluminum conductors, three phases (with non-cylindrical geometries) and a neutral, Figure 5. We considered all conductors and sheath of aluminum with an electrical conductivity $σ=3.5×10^7$ S/m and magnetic permeability $μ_0$. This cable has an outer radius $r=2.5$ cm and an inner isolation between conductors of approximately 3 mm.

The results were obtained with $N_d=105$, which resulted in a total of 1305 sub-conductors on each non-cylindrical conductor, 890 sub-conductors in the cylindrical conductor and 1348 sub-conductors in the sheath, as shown in Figure 5.

![Fig. 5. Geometry and distribution of sub-conductors in a four conductor cable and sheath.](image-url)
Table III contains the self-resistance obtained for several values of frequency.

<table>
<thead>
<tr>
<th>frequency [Hz]</th>
<th>Self-resistance F1 [µΩ/m]</th>
<th>Self-resistance F2 [µΩ/m]</th>
<th>Self-resistance F3 [µΩ/m]</th>
<th>Self-resistance N [µΩ/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>190,024</td>
<td>190,024</td>
<td>190,024</td>
<td>235,049</td>
</tr>
<tr>
<td>50</td>
<td>201,875</td>
<td>201,353</td>
<td>201,353</td>
<td>246,815</td>
</tr>
<tr>
<td>100</td>
<td>224,132</td>
<td>223,269</td>
<td>223,269</td>
<td>268,315</td>
</tr>
<tr>
<td>500</td>
<td>336,848</td>
<td>337,260</td>
<td>337,260</td>
<td>386,472</td>
</tr>
<tr>
<td>1000</td>
<td>414,285</td>
<td>414,856</td>
<td>414,856</td>
<td>493,420</td>
</tr>
<tr>
<td>5000</td>
<td>781,410</td>
<td>783,702</td>
<td>783,702</td>
<td>984,074</td>
</tr>
<tr>
<td>10000</td>
<td>1111,166</td>
<td>1118,626</td>
<td>1118,626</td>
<td>1408,209</td>
</tr>
</tbody>
</table>

The self-resistance of the conductor increases with the increase of frequency. Another conclusion that can be drawn is that the resistances of the three phase conductors are very similar, while the resistance of the neutral conductor remains somewhat higher over the whole frequency range.

Analyzing Table IV, we can see that self-inductance decreases with the increase of frequency.

<table>
<thead>
<tr>
<th>frequency [Hz]</th>
<th>Self-inductance F1 [µΩ/m]</th>
<th>Self-inductance F2 [µΩ/m]</th>
<th>Self-inductance F3 [µΩ/m]</th>
<th>Self-inductance N [µΩ/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>221,365</td>
<td>221,365</td>
<td>221,365</td>
<td>266,683</td>
</tr>
<tr>
<td>50</td>
<td>206,580</td>
<td>207,876</td>
<td>207,876</td>
<td>251,495</td>
</tr>
<tr>
<td>100</td>
<td>181,463</td>
<td>184,033</td>
<td>184,033</td>
<td>226,438</td>
</tr>
<tr>
<td>500</td>
<td>110,558</td>
<td>113,208</td>
<td>113,208</td>
<td>160,315</td>
</tr>
<tr>
<td>1000</td>
<td>92,288</td>
<td>94,772</td>
<td>94,772</td>
<td>139,319</td>
</tr>
<tr>
<td>5000</td>
<td>69,210</td>
<td>71,667</td>
<td>71,667</td>
<td>108,101</td>
</tr>
<tr>
<td>10000</td>
<td>62,442</td>
<td>64,852</td>
<td>64,852</td>
<td>99,267</td>
</tr>
</tbody>
</table>

The self-inductance of the conductor decreases with the increase of frequency. Another conclusion that can be drawn is that the inductances of the three phase conductors are very similar, while the inductance of the neutral conductor remains somewhat lower over the whole frequency range.

Analyzing the contours of current density at 50Hz, represented in Figure 8, we can verify the differences between the phase conductors and the neutral conductor, as well as the different intensity of current density in the conductors.
Finally, Figure 9 represents the magnitude of the magnetic induction field caused by the cable and Figure 10 represents the direction of this field. The magnitude has an irregular appearance in the cable due to the geometry of the conductors that constitute it, and attenuate out of the cable.

IV. CONCLUSION

This paper proposes a method for calculating the longitudinal per unit length parameters - the inductance and resistance – for a system consisting of multiple conductors, with different permeabilities, taking into account the skin effect and the proximity effect.

With this methodology it is possible to characterize systems with various geometries of conductors besides the circular cylindrical, the influence of frequency, the distance between conductors and the existence of media with different magnetic permeabilities.

The method validation was performed based on analytical expressions published in the literature for two types of cables: a cylindrical isolated conductor and a cylindrical core conductor.

For a cylindrical conductor, comparing the simulation and theoretical results we reached less than 0.12% relative error in the whole range of frequency studied. In the case of cylindrical core conductor with a different magnetic permeability the obtained results were always with less than 6.1% relative error.

The relative errors could potentially be smaller, increasing the number of sub-conductors used in discretization of the conductors of the system in both cases.

Besides the values of these parameters, the algorithm developed allows to obtain graphs of current density within the conductor and allows to obtain the intensity and direction of magnetic induction field inside and outside conductors.

The skin effect is clearly visible in the graphs which represent the current density in the conductors when comparing the same cable at 50Hz or 500Hz and 1kHz. An increase in current density at the surface of the conductor at higher frequencies is visible. The proximity effect is present in all cases with more than one conductor. The increased current in the nearest surface of the other conductor is visible when they are nearby.
REFERENCES