Low-energy phenomenology of neutrino mass mechanisms

André Morgado Patrício[1]
Under supervision of Filipe Rafael Joaquim[1]
[1]*Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal*
(Dated: October 2014)

In the present work, we analyse seesaw extensions of the Standard Model, which naturally accomodate tiny neutrino masses through tree-level exchange of heavy fields, which may be either fermionic singlets/triplets or scalar triplets. From an effective viewpoint, neutrino masses are generated by a dimension-five operator, common to all theories with Majorana neutrinos. However, a plethora of dimension-six operators exists. In this thesis, we derive the low-energy dimension-six operators for the basic seesaw scenarios and discuss the possibility of having observable effects of the dimension-six operators with naturally suppressed dimension-five operators.

Phenomenological consequences are explored, including a detailed analysis of charged-lepton flavour violating (CLFV) processes. Our focus relies, mainly, on predictions and constraints set on each model from $\ell_\alpha \to \ell_\beta \gamma$ and $\ell \to 3\ell'$ decays, as well as from $\mu \to e$ conversion in nuclei. The possibilities to observe these processes in present and future experiments are also considered. Besides that, a combined analysis including other electroweak decays shows departures from unitarity not larger than 2σ in the fermionic seesaw models.

Keywords: Neutrino Physics; SM extensions; Seesaw mechanisms; Phenomenology; Rare decays; Lepton flavour violation.

I. INTRODUCTION

Strong evidence that the Standard Model (SM) is not the ultimate theory of particle physics comes from neutrino oscillation experiments, which imply nonvanishing neutrino masses and mixing. The most fundamental questions regarding neutrinos still need an answer: absolute neutrino masses and their hierarchy are still unknown and we need to make clear if neutrinos are Dirac or Majorana particles. Secondly, in spite of undeniable evidence for lepton flavour violation (LFV) in neutrino oscillations, all searches for LFV in the charged sector (CLFV) have obtained negative results. A minimal extension of the SM with massive neutrinos in which total lepton number L is conserved predicts strongly suppressed CLFV rates but does not provide a natural explanation for the huge disparity between the magnitude of neutrino masses and the masses of charged fermions. This suggests that tiny neutrino masses are related to a new physics scale Λ.

A natural explanation for neutrino masses is provided by the seesaw model of neutrino mass generation. The scale Λ at which this new physics manifests itself can, in principle, be arbitrarily large, up to the GUT scale and even beyond. An interesting possibility is to have $\Lambda \sim \mathcal{O}(\text{TeV})$, in a way which naturally accomodates tiny neutrino masses. Such low-energy scenario requires a model-independent pattern [1], where predicted rates of CLFV processes can be within the reach of future experiments even when direct detection of the new particles is not allowed at the LHC. Moreover, an analysis of CLFV rates in each seesaw model could be used to identify the mechanism responsible for neutrino masses.

II. NEUTRINOS IN THE SM

The Standard Model is a gauge theory based on the symmetry group $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, where the subscripts '$c$', '$L$' and '$Y$' label degrees of freedom called colour (c), left-handed chirality and weak hypercharge (y), respectively. Our analysis will be focused on the electroweak subgroup $\mathrm{G_{SM}} = \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$.

There are $n = 3$ flavours α of leptons, whose components with left-handed chirality transform as an $\mathrm{SU}(2)_L$ doublet L_α. The right-handed components $\ell_{\alpha R}$ for charged leptons are $\mathrm{SU}(2)_L$ invariant while neutrinos ν_α do not possess right-handed components. The leptonic content of the SM is thus:

$$L_{\alpha L} = \begin{pmatrix} \nu_{\alpha L} \\ \ell_{\alpha L} \end{pmatrix}, \quad \ell_{\alpha R}\,; \qquad \alpha = e, \mu, \tau \ . \tag{1}$$

The Lagrangian of the Standard Model is

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{gauge}} + \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{f}} + \mathcal{L}_{\mathrm{Yuk}} \ , \tag{2}$$

and gauge invariance under G_{SM} requires replacing the usual partial derivative with a covariant one,

$$\partial_\mu \to D_\mu \equiv \partial_\mu - igA_\mu^k I_k - ig' B_\mu \frac{Y}{2} \ , \tag{3}$$

where g and g' are coupling constants. For each generator I_k of $\mathrm{SU}(2)_L$ and Y of $\mathrm{U}(1)_Y$, gauge invariance requires the introduction of a vector gauge boson (A_μ^k and B_μ), which will give rise to the massive $W_\mu^\pm$ and Z_μ bosons as well as to the massless photon A_μ.

The term $\mathcal{L}_{\rm Higgs}$ in Eq. (2) contains the kinetic terms for the Higgs doublet ϕ as, well as the Higgs potential

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \ , \qquad \phi = \begin{pmatrix}\varphi^+ \\ \varphi^0\end{pmatrix} . \tag{4}$$

Fermion and gauge boson kinetic terms are included in $\mathcal{L}_{\rm f}$ and $\mathcal{L}_{\rm gauge}$, respectively. Finally, the leptonic part of the Yukawa Lagrangian $\mathcal{L}_{\rm Yuk}$ describes the interactions between leptons and the Higgs doublet ϕ:

$$\mathcal{L}^\ell_{\rm Yuk} = -\left(\mathbf{Y}^\ell_{\alpha\beta}\overline{L_{\alpha L}}\,\phi\,\ell_{\beta R} + \text{h.c.}\right) \ , \tag{5}$$

which, after electroweak symmetry breaking (EWSB), generates a Dirac mass term for charged leptons:

$$\mathcal{L}^\ell_{\rm Dirac} = -\left(\mathbf{M}^\ell_{\alpha\beta}\overline{\ell_{\alpha_L}}\ell_{\beta_R} + \text{h.c.}\right) , \quad \mathbf{M}^\ell_{\alpha\beta} \equiv v\mathbf{Y}^\ell_{\alpha\beta} \ , \tag{6}$$

where $\mathbf{M}^\ell$ is the charged lepton mass matrix in this flavour basis and $v \equiv \sqrt{-\mu^2/2\lambda}$ is the vacuum expectation value (VEV) of the neutral component φ^0. A similar term can be written for quarks, with $\mathbf{M}^{u,d} = v\mathbf{Y}^{u,d}$.

Since SM neutrinos do not have right-handed components, for them no Dirac neutrino mass term can be generated upon EWSB. Hence, neutrinos are strictly massless particles in the SM. This allows one to rotate neutrino fields with the same unitary matrix $\boldsymbol{V}_L^\ell$ used to diagonalize the charged-lepton mass matrix:

$$\ell_L \to \boldsymbol{V}_L^{\ell\dagger}\ell_L, \qquad \nu_L \to \boldsymbol{V}_L^{\ell\dagger}\nu_L \ , \tag{7}$$

cancelling the effect of $\boldsymbol{V}_L^\ell$ in charged currents (CC):

$$\mathcal{L}^\ell_{\rm CC} = \frac{g}{2\sqrt{2}}\left(J_W^\mu W_\mu^- + \text{h.c.}\right) , \ \ J_W^\mu = 2\,\overline{\ell_L}\gamma^\mu\nu_L \ . \tag{8}$$

On the other hand, electromagnetic (EM) and weak neutral currents (NC) are diagonal in both basis,

$$\mathcal{L}_{\rm NC} = \frac{gZ^\mu}{2c_W}\left[\overline{\nu_L}\gamma_\mu\nu_L - \overline{\ell_L}\gamma_\mu\ell_L - 2s_W^2 J_\mu^{\rm EM}\right] , \tag{9}$$

$$\mathcal{L}_{\rm EM} = |e|J_\mu^{\rm EM}A^\mu = -|e|\overline{\ell}\gamma_\mu\ell A^\mu \ , \tag{10}$$

where $s_W \equiv \sin\theta_W$ and $c_W \equiv \cos\theta_W = g/\sqrt{g^2+g'^2}$. Therefore, in the SM there are no flavour-changing neutral currents (FCNC) and, in this basis, the SM Lagrangian exhibits the $\mathrm{U}(1)^n$ symmetry

$$\ell_{\alpha_{L,R}} \to e^{i\varphi_\alpha}\ell_{\alpha_{L,R}}, \qquad \nu_{\alpha_L} \to e^{i\varphi_\alpha}\nu_{\alpha_L} \ , \tag{11}$$

connected with conservation of flavour lepton numbers L_α and, therefore, of total lepton number $L = \sum_\alpha L_\alpha$.

Given the experimental evidence of neutrino oscillations [2], which imply tiny but nonzero neutrino masses, the absence of neutrino masses in the SM is a strong evidence that this theory must be extended. It is therefore important to understand theoretically how to describe massive neutrinos and how can their small masses be naturally explained.

III. NEUTRINO MASSES AND MIXING

A Dirac mass term for neutrinos can be defined in a minimal SM extension with three right-handed neutrinos ν_{α_R} with null hypercharge, singlets under $G_{\rm SM}$. To the SM Lagrangian (2), we must thus add the term

$$\mathcal{L}^\nu_{\rm Yuk} = -\left(\mathbf{Y}^\nu_{\alpha\beta}\overline{L_{\alpha_L}}\tilde{\phi}\nu_{\beta_R} + \text{h.c.}\right) , \quad \tilde{\phi} \equiv i\tau^2\phi^* \ , \tag{12}$$

which, after EWSB, generates a Dirac mass term,

$$\mathcal{L}^\nu_{\rm Dirac} = -\left(\mathbf{M}^\nu_{\alpha\beta}\overline{\nu_{\alpha_L}}\nu_{\beta_R} + \text{h.c.}\right) , \ \mathbf{M}^\nu_{\alpha\beta} = v\mathbf{Y}^\nu_{\alpha\beta} \ . \tag{13}$$

However, being neutrino masses much smaller than those of charged fermions, the couplings $\mathbf{Y}^\nu$ should be unnaturally smaller than $\mathbf{Y}^{\ell,u,d}$ [see Eq. (6)]. On the other hand, the Yukawa term (12) conserves total lepton number L. Thus, if we set the couplings $\mathbf{Y}^\nu$ to zero, the system does not exhibit any new symmetry and a neutrino Dirac mass term is not natural according to the 't Hooft naturalness criterium [3]. Therefore, a more natural neutrino mass term must exist.

In fact, it is possible to build a Majorana mass term for neutrinos [4]. Such construction is related with the possibility of describing a fermion ψ as a two-component spinor, identifying ψ_R with ψ_L^C (the superscript C denotes charge conjugation). The description of ψ as a two-component spinor is thus possible only if the Majorana condition $\psi^C = \psi = \psi_L + \psi_L^C$ is satisfied, which implies that only neutrinos can be Majorana particles with Lagrangian:

$$\mathcal{L}^M = \overline{\nu_L}i\partial\!\!\!/\nu_L - \frac{1}{2}\left(\overline{\nu_L^C}\mathbf{M}^\nu\nu_L + \text{h.c.}\right) \ , \tag{14}$$

where $\mathbf{M}^\nu$ is a symmetric neutrino mass matrix. The Lagrangian (14) generates lepton-number transitions $\Delta L = \pm 2$. As such, by taking the limit $\mathbf{M}^\nu \to \mathbf{0}$, one recovers total lepton number conservation and small Majorana masses are natural according to 't Hooft.

Let us now analyse the general case with $n = 3$ left-handed neutrinos $\nu_{\alpha L}$ and n' right-handed neutrinos ν_{sR} $(s = 1, ..., n')$, for a total of $n_t = n + n'$ neutrino fields. Defining the column vector N_L by

$$N_L \equiv \begin{pmatrix}\nu_L \\ \nu_R^C\end{pmatrix} , \ \ \nu_L = \begin{pmatrix}\nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L}\end{pmatrix} , \ \ \nu_R^C = \begin{pmatrix}\nu_{1R}^C \\ \vdots \\ \nu_{n'R}^C\end{pmatrix} , \tag{15}$$

the most general mass term for it is

$$\begin{aligned}\mathcal{L}^{\rm D+M}_{\rm mass} &= -\frac{1}{2}\overline{N_L^C}\mathbf{M}^{\rm D+M}N_L + \text{h.c.}\,, \\ \mathbf{M}^{\rm D+M} &\equiv \begin{pmatrix}\mathbf{M}^L & \mathbf{M}^{D^T} \\ \mathbf{M}^D & \mathbf{M}^R\end{pmatrix} ,\end{aligned} \tag{16}$$

where $\mathbf{M}^D$ is a $n' \times n$ mass matrix while $\mathbf{M}^L$ and $\mathbf{M}^R$ are $n \times n$ and $n' \times n'$ symmetric mass matrices, respectively. The diagonalisation of the mass term (16) is achieved by writing flavour fields N_L as an unitary rotation $\boldsymbol{V}_L^\nu$ of mass eigenstates ν_{kL} (roman indices):

$$\begin{aligned} N_L &= \boldsymbol{V}_L^\nu n_L, \qquad\qquad n_L = (\nu_{1L}, ..., \nu_{n_t L})^T , \\ \mathbf{M}^\nu &\equiv (\boldsymbol{V}_L^\nu)^T \mathbf{M}^{\mathrm{D+M}} \boldsymbol{V}_L^\nu = \mathbf{diag}(m_1, ..., m_{n_t}) . \end{aligned} \tag{17}$$

Defining the Majorana fields $\nu_k = \nu_{k_L} + \nu_{k_L}^C$, we finally verify that in the most general case neutrinos are Majorana particles [see Eq. (14)], with Lagrangian

$$\mathcal{L}^{\mathrm{D+M}} = \frac{1}{2} \overline{\nu_k} \left(i \partial\!\!\!/ - \mathbf{M}_{kk}^\nu \right) \nu_k \ . \tag{18}$$

In the mass basis (17), leptonic currents read

$$J_W^\mu = 2 \, \overline{L_L} \gamma^\mu \boldsymbol{U} n_L, \qquad J_{Z,\nu}^\mu = 2 \, \overline{n_L} \gamma^\mu \boldsymbol{U}^\dagger \boldsymbol{U} n_L \ , \tag{19}$$

where $\boldsymbol{U}$ is a $3 \times n$ non-unitary mixing matrix:

$$\boldsymbol{U}_{\alpha k} = \sum_{\beta = e, \mu, \tau} (\boldsymbol{V}_L^{\ell\dagger})_{\alpha\beta} (\boldsymbol{V}_L^\nu)_{\beta k} \ . \tag{20}$$

In the special case $n' = 0$, with no sterile neutrinos, the 3×3 mixing matrix is called Pontecorvo-Maki-Nakagawa-Sakata matrix $\boldsymbol{U}_{\mathrm{PMNS}} = \boldsymbol{V}_L^{\ell\dagger} \boldsymbol{V}_L^\nu$ [5, 6]. A general $n \times n$ unitary matrix can be parameterised by $n(n-1)/2 = 3$ Euler angles θ_{12}, θ_{13}, θ_{23} and by $n(n+1)/2 = 6$ phases, However $n = 3$ phases can be removed from $\boldsymbol{U}_{\mathrm{PMNS}}$ by a convenient rephasing of charged lepton fields. The same does not apply to Majorana neutrino fields because their mass term (14) does not remain invariant. Therefore, $\boldsymbol{U}_{\mathrm{PMNS}}$ is parametrised in terms of three mixing angles and $n(n-1)/2 = 3$ physical phases, one Dirac phase δ and two Majorana phases $\alpha_{1,2}$ [7]:

$$\boldsymbol{U}_{\mathrm{PMNS}} = \underbrace{\begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -s_{23} c_{12} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}}_{\boldsymbol{U}^D} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}}_{\boldsymbol{U}^M} , \tag{21}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

As a consequence of lepton mixing, neutrinos produced with definite flavour are allowed to oscillate between flavours along their trajectory. Neutrino oscillation experiments are only sensitive to mass-squared differences $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$, providing no information on the absolute neutrino mass scale. Also, transition probabilities do not depend on Majorana phases $\alpha_{1,2}$. On the other hand, since the sign of Δm_{31}^2 is unknown, whereas Δm_{21}^2 is positive, there are two possible orderings allowed by experiments: a normal ordering with $m_1 < m_2 \ll m_3$ and an inverted ordering for which $m_3 \ll m_1 < m_2$. The results from the most recent global fit to oscillation data [8] are shown in Table I.

Param.	**Global fit results at 1σ $\{3\sigma\}$**	
	Normal spectrum	Inverted spectrum
Δm_{21}^2	$7.54\,^{+\,0.26}_{-\,0.22}\,\left\{^{+0.64}_{-0.55}\right\}$	
$s_{12}^2/10^{-1}$	$3.08 \pm 0.17\,\left\{^{+0.51}_{-0.49}\right\}$	
Δm_{31}^2	$2.44^{+0.08}_{-0.06}\,\{\pm 0.22\}$	$2.40 \pm 0.07\,\left\{^{+0.21}_{-0.23}\right\}$
$s_{13}^2/10^{-2}$	$2.34^{+0.22}_{-0.18}\,\left\{^{+0.63}_{-0.57}\right\}$	$2.39 \pm 0.21\,\{\pm 0.61\}$
$s_{23}^2/10^{-1}$	$4.25^{+0.29}_{-0.27}\,\left\{^{+2.16}_{-0.68}\right\}$	$4.37\,^{+\,1.73}_{-\,0.29}\,\left\{^{+2.22}_{-0.74}\right\}$
δ/π	$1.39^{+0.33}_{-0.27}\,\{\ \ /\ \ \}$	$1.35\,^{+\,0.24}_{-\,0.39}\,\{\ \ /\ \ \}$

Table I. Results of a global 3ν oscillation analysis [8]. Δm_{21}^2 and Δm_{31}^2 are in units of 10^{-5} and 10^{-3} eV^2, respectively.

On the other hand, the most direct way to determine the absolute neutrino mass scale is by investigating the β-decay endpoint of tritium ($^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \overline{\nu}_e$). Using this technique, the Mainz [9] and Troitsk [10] experiments obtained the bounds $m_\beta < 2.3$ eV and $m_\beta < 2.05$ eV at 95 % C.L., respectively.

IV. BASIC SEESAW SCENARIOS

Despite its theoretical appeal, the Majorana neutrino mass term (14) is forbidden in the SM since it has weak isospin $I_3 = 1$ and hypercharge $y = -2$. In order to build such term, a scalar triplet with $y = 2$ would be needed, which the SM does not contain. This fact suggests that the SM is a low-energy effective theory, resulting from a more complete theory at a high-energy scale Λ.

At low energies, an analysis independent of the high-energy theory can be performed in terms of an effective description, by integrating out heavy fields. The impact of the high-energy theory is then parameterised by means of an effective Lagrangian. This amounts to adding a set of non-renormalisable higher-dimension operators to the gauge-invariant SM Lagrangian:

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\mathrm{SM}} + \delta\mathcal{L}^{d=5} + \delta\mathcal{L}^{d=6} + \ldots \ , \tag{22}$$

where each operator is suppressed as $\sim \mathcal{O}\left(1/\Lambda^{d-4}\right)$. For $d = 5$, there is only one possible operator, the well-known Weinberg operator [11]:

$$\delta\mathcal{L}^{d=5} = \frac{1}{2} \boldsymbol{c}_{\alpha\beta}^{d=5} \left(\overline{L_{\alpha L}^C} \, \widetilde{\phi}^* \right) \left(\widetilde{\phi}^\dagger L_{\beta L} \right) + \text{h.c.} \ , \tag{23}$$

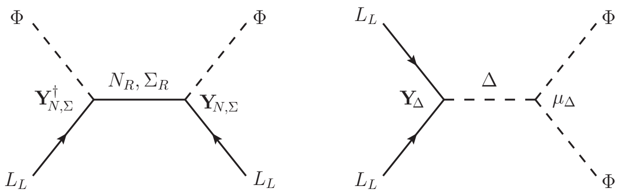

Figure 1. Basic realisations of the seesaw mechanism, with exchange of: fermionic singlets (triplets) in the type I (III) seesaw (left) or a scalar triplet in the type II seesaw (right).

where $\boldsymbol{c}^{d=5}_{\alpha\beta} \sim \mathcal{O}(1/M)$ are complex coefficients. An interesting point is that this operator induces Majorana neutrino masses after EWSB,

$$-\frac{1}{2}\mathbf{M}^{d=5}_{\alpha\beta}\overline{\nu^C_{\alpha_L}}\nu_{\beta_L} + \text{h.c.}\,, \quad \mathbf{M}^{d=5}_{\alpha\beta} = -v^2 c^{d=5}_{\alpha\beta}\,, \tag{24}$$

naturally small if Λ is high enough [12]. However, several $d=6$ operators may arise from different high-energy models [13]. Their identification is thus crucial to get some hints on the origin of neutrino masses.

In this work, we analyse seesaw extensions of the SM, where small neutrino masses result naturally from tree-level exchange of heavy fields with masses $M \sim \Lambda$. We deduce the effective Lagrangian and its phenomenological consequences for the three basic realisations of the seesaw mechanism: exchange of fermionic singlets (type I) or triplets (type III), or exchange of a scalar triplet (type II), as represented in Fig. 1.

A. Type I Seesaw Model

In the type I seesaw scenario [12, 14, 15], n' neutrino singlets $N_{sR} \sim (\mathbf{1},\mathbf{1},0)$ are added to the SM. The Lagrangian for the type I seesaw model reads

$$\begin{aligned}\mathcal{L}_\text{I} &= \mathcal{L}_\text{SM} + i\overline{N_R}\,\partial\!\!\!/\,N_R \\ &- \left(\overline{L_L}\,\widetilde{\phi}\,\mathbf{Y}^\dagger_N\,N_R + \frac{1}{2}\overline{N^C_R}\,\mathbf{M}_N\,N_R + \text{h.c.}\right),\end{aligned} \tag{25}$$

where $s = 1,...,n'$, $\mathbf{Y}_N$ is a $n' \times n$ matrix of Yukawa couplings and $\mathbf{M}_N \sim \mathcal{O}(m_N)$ is a $n' \times n'$ symmetric mass matrix. We work in a basis where both $\mathbf{M}_N$ and $\mathbf{M}_\ell$ are real and diagonal [remember Eqs. (7) and (17)].

Integrating out heavy fields, the lowest dimension effective operator arising is the dimension $d = 5$ Weinberg operator (23), which induces the Majorana mass matrix for light neutrinos:

$$\mathbf{M}_\nu \equiv -v^2\boldsymbol{c}^{d=5} = -v^2\left(\mathbf{Y}^T_N\,\frac{1}{\mathbf{M}_N}\,\mathbf{Y}_N\right)\,. \tag{26}$$

On the other hand, the second effective operator that we obtain is the $d = 6$ operator [1]

$$\delta\mathcal{L}^{d=6} = \boldsymbol{c}^{d=6}_{\alpha\beta}\left(\overline{L_{L\alpha}}\,\widetilde{\phi}\right)i\partial\!\!\!/\left(\widetilde{\phi}^\dagger\,L_{L\beta}\right)\,, \tag{27}$$

with coefficients given in terms of the full theory by

$$v^2\boldsymbol{c}^{d=6} = v^2\left(\mathbf{Y}^\dagger_N\,\frac{1}{\mathbf{M}^\dagger_N\mathbf{M}_N}\,\mathbf{Y}_N\right) \equiv \boldsymbol{\epsilon}^N\,. \tag{28}$$

After EWSB, the $d = 6$ operator rescales the kinetic terms of light neutrinos. In a mass eigenstate basis with kinetic terms canonically normalised, the lepton effective Lagrangian at order $\mathcal{O}(1/M^2)$ is then:

$$\begin{aligned}\mathcal{L}^{d\leq 6}_\text{leptons} = &\frac{1}{2}\overline{\nu}\left(i\partial\!\!\!/ - \mathbf{M}^\text{diag}_\nu\right)\nu + \overline{\ell}(i\partial\!\!\!/ - \mathbf{M}^\text{diag}_\ell)\ell \\ &+ \mathcal{L}_\text{CC} + \mathcal{L}_\text{NC} + \mathcal{L}_\text{EM}\,.\end{aligned} \tag{29}$$

In Eq. (29), $\nu = \nu_L + \nu^C_L$ are $n = 3$ light Majorana mass eigenfields and the weak currents are:

$$\mathcal{L}_\text{CC} = \frac{g}{\sqrt{2}}\overline{\ell_L}W\!\!\!\!/^-\boldsymbol{N}\nu_L + \text{h.c.}\,, \tag{30}$$

$$\mathcal{L}_\text{NC} = \frac{gZ^\mu}{2c\theta_W}\left[\overline{\nu_L}\gamma_\mu\left(\boldsymbol{N}^\dagger\boldsymbol{N}\right)\nu_L - \overline{\ell_L}\gamma_\mu\ell_L - 2s^2_W J^\text{EM}_\mu\right]\,. \tag{31}$$

Therefore, the usual mixing matrix $\boldsymbol{U}_\text{PMNS}$ in charged currents [see Eq. (21)] is replaced by a non-unitary one:

$$\boldsymbol{N} \equiv \left(\mathbf{1} - \frac{\boldsymbol{\epsilon}^N}{2}\right)\boldsymbol{U}_\text{PMNS}\,, \tag{32}$$

and there are FCNCs involving neutrinos, with mixing matrix $\boldsymbol{N}^\dagger\boldsymbol{N}$, at order $\mathcal{O}(1/M^2)$.

A consequence of the above is that the Fermi constant extracted from $\mu \to \nu_\mu e\overline{\nu_e}$, is corrected to

$$G_F = G^\text{SM}_F\sqrt{(\boldsymbol{N}\boldsymbol{N}^\dagger)_{ee}(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\mu\mu}}\,, \tag{33}$$

where $G^\text{SM}_F = \sqrt{2}g^2/(8M^2_W)$ is the SM tree-level value. Also interesting is that deviations from unitarity can be directly related to $\boldsymbol{c}^{d=6}$:

$$|\boldsymbol{N}\boldsymbol{N}^\dagger - \mathbf{1}| = v^2|\boldsymbol{c}^{d=6}| = v^2\left|\mathbf{Y}^\dagger_N\,\frac{1}{\mathbf{M}^\dagger_N\mathbf{M}_N}\,\mathbf{Y}_N\right|\,. \tag{34}$$

Finally, the counting of the number of parameters in the leptonic sector of both the full theory and its effective description shows that for $n' = n$ (or $n' < n$) the number of physical parameters is equal in both theories, thus proving that all parameters in the full theory appear in the effective description through the $d \leq 6$ operators. On the other hand, if $n' > n$, the number of parameters in the full theory is larger than in the effective theory and we would need to consider effective operators of dimension $d > 6$, in order to account for the remaining parameters.

B. Type II seesaw mechanism

In the minimal type II seesaw [15–18], we extend the SM with a scalar $SU(2)_L$ triplet $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)^T$

with hypercharge $y=2$. This flavour triplet pertains to the adjoint representation of $\mathrm{SU}(2)_L$, with generators $(I^i)^{jk}=-i\varepsilon^{ijk}$ and electric charge eigenfields given by

$$\Delta^0 \equiv \frac{\Delta_1+i\Delta_2}{\sqrt{2}}, \quad \Delta^+ \equiv \Delta_3, \quad \Delta^{++} \equiv \frac{(\Delta_1-i\Delta_2)}{\sqrt{2}} . \quad (35)$$

In a flavour basis, the type II seesaw Lagrangian is then

$$\begin{aligned}\mathcal{L}_{\rm II} = \mathcal{L}_{\rm SM} &+ \left(D_\mu \vec{\Delta}\right)^\dagger \left(D^\mu \vec{\Delta}\right) - V(\phi,\vec{\Delta}) \\ &+ \left[\widetilde{\overline{L_{\rm L}}} \mathbf{Y}_\Delta (\vec{\tau}\cdot\vec{\Delta}) L_{\rm L} + \text{h.c.}\right],\end{aligned} \quad (36)$$

with $\mathbf{Y}_\Delta$ a $n\times n$ symmetric matrix and $\widetilde{L_{\rm L}} \equiv i\tau_2 (L_L^C)$. The term $V(\phi,\vec{\Delta})$ is the most general scalar potential

$$\begin{aligned}V(\phi,\vec{\Delta}) = M_\Delta^2 &\left(\vec{\Delta}^\dagger\vec{\Delta}\right) + \mu_\Delta\left[\phi^\dagger(\vec{\tau}\cdot\vec{\Delta})\widetilde{\phi} + \text{h.c.}\right] \\ &+ \frac{\lambda_1}{2}\left(\vec{\Delta}^\dagger\vec{\Delta}\right)^2 + \lambda_2\left(\phi^\dagger\phi\right)\left(\vec{\Delta}^\dagger\vec{\Delta}\right) \\ &+ \frac{\lambda_3}{2}\left(\vec{\Delta}^\dagger I^i\vec{\Delta}\right)^2 + \lambda_4\left(\vec{\Delta}^\dagger I^i\vec{\Delta}\right)\phi^\dagger\tau^i\phi ,\end{aligned} \quad (37)$$

where M_Δ is the triplet mass, λ_i real coefficients and D_μ the covariant derivative (3). In what follows, we consider the limit $\mathcal{O}(1/M_\Delta) \ll v\mu_\Delta$.

In this case, the lowest dimensional operator which arises from the effective Lagrangian is a $d=4$ operator,

$$\delta\mathcal{L}^{d=4} = 2\frac{|\mu_\Delta|^2}{M_\Delta^2}\left(\phi^\dagger\phi\right)^2 , \quad (38)$$

whose effect is a correction to the quartic coupling λ in the Higgs potential (4):

$$\delta\lambda = -2\frac{|\mu_\Delta|^2}{M_\Delta^2} . \quad (39)$$

We also obtain the Weinberg operator (23) for $d=5$, which generates a neutrino mass matrix:

$$\mathbf{M}_\nu = -v^2 \boldsymbol{c}^{d=5} = -4\mathbf{Y}_\Delta v^2\,\frac{\mu_\Delta^*}{M_\Delta^2} \approx 4\mathbf{Y}_\Delta v_\Delta . \quad (40)$$

where $v_\Delta = \sqrt{2}\left\langle\Delta^0\right\rangle$ is the triplet VEV in the language of the full theory. Interestingly, this result shows that we can reconstruct the couplings $\mathbf{Y}_\Delta$ if we can determine the coefficients of the $d=5$ operator.

From the effective Lagrangian, we also obtain a set of three operators with dimension $d=6$:

$$\delta\mathcal{L}_{4F} = -\frac{(\mathbf{Y}_\Delta)_{\rho\sigma}(\mathbf{Y}_\Delta)^\dagger_{\alpha\beta}}{M_\Delta^2}\left(\overline{L_{L\beta}}\gamma_\mu L_{L\rho}\right)\left(\overline{L_{L\alpha}}\gamma^\mu L_{L\sigma}\right) , \quad (41)$$

$$\delta\mathcal{L}_{6\Phi} = -2\left(\lambda_2+\lambda_4\right)\frac{|\mu_\Delta|^2}{M_\Delta^4}\left(\Phi^\dagger\Phi\right)^3 , \quad (42)$$

$$\begin{aligned}\delta\mathcal{L}_{\Phi D} = 4&\frac{|\mu_\Delta|^2}{M_\Delta^4}\left(\Phi^\dagger\Phi\right)\left[\left(D^\mu\Phi\right)^\dagger\left(D_\mu\Phi\right)\right] \\ &+ 4\frac{|\mu_\Delta|^2}{M_\Delta^4}\left[\Phi^\dagger D^\mu\Phi\right]^\dagger\left[\Phi^\dagger D_\mu\Phi\right] .\end{aligned} \quad (43)$$

The first one, $\delta\mathcal{L}_{4F}$, induces a correction to the Fermi constant extracted from muon decay:

$$\delta G_F \equiv G_F - G_F^{\rm SM} = \frac{1}{\sqrt{2}M_\Delta^2}\left|\left(\mathbf{Y}_\Delta\right)_{e\mu}\right|^2 . \quad (44)$$

On the other hand, $\delta\mathcal{L}_{\phi D}$ corrects the Z boson mass as:

$$\frac{\delta M_Z^2}{M_Z^2} = 4v^2\frac{|\mu_\Delta|^2}{M_\Delta^4} . \quad (45)$$

Finally, $\delta\mathcal{L}_{6\phi}$ modifies the Higgs potential (4), which now reads

$$V = -\mu^2|\phi|^2 + \tilde{\lambda}|\phi|^4 + 2\left(\lambda_2+\lambda_4\right)\frac{|\mu_\Delta|^2}{M_\Delta^4}|\phi|^6 , \quad (46)$$

where $\tilde{\lambda} = \lambda + \delta\lambda$ is the corrected value of the quartic coupling λ [see Eq. (39)]. The combined effect of $\delta\mathcal{L}^{d=4}$ and $\delta\mathcal{L}_{6\phi}$ is thus to induce a shift in the Higgs VEV:

$$\frac{\delta v^2}{v^2} = -6v^2\frac{|\mu_\Delta|^2}{M_\Delta^4}\frac{(\lambda_2+\lambda_4)}{\lambda+\delta\lambda} . \quad (47)$$

As for the number of parameters of the type II seesaw, we can verify, by comparison of Eq. (36) with Eqs. (38) to (43), that the number of parameters in the effective theory for $d\le 6$ is smaller than the number of parameters in the full theory. In order to obtain an equal number of parameters in both theories, we would need to consider all $d\le 8$ operators [1].

C. Type III seesaw model

Let us now assume that we add n' right-handed fermionic triplets $\vec{\Sigma}_{kR} = (\Sigma^1_{kR},\Sigma^2_{kR},\Sigma^3_{kR})$ with zero hypercharge ($k=1,...,n'$) to the SM. We work in the $\mathrm{SU}(2)_L$ adjoint representation, where charge eigenfields are:

$$\Sigma^\pm = \frac{\Sigma^1 \mp \Sigma^2}{\sqrt{2}}, \qquad \Sigma^0 = \Sigma^3 . \quad (48)$$

In a flavour basis, the Lagrangian of this model reads

$$\begin{aligned}\mathcal{L}_{\rm III} = \mathcal{L}_{\rm SM} &+ i\,\overline{\vec{\Sigma}_R}\not{D}\vec{\Sigma}_R \\ &- \left[\frac{1}{2}\overline{\vec{\Sigma}_R}\mathbf{M}_\Sigma\vec{\Sigma}_R^C + \overline{\vec{\Sigma}_R}\mathbf{Y}_\Sigma\cdot(\widetilde{\phi}^\dagger\vec{\tau}L_L) + \text{h.c.}\right],\end{aligned} \quad (49)$$

where $\mathbf{Y}_\Sigma$ is a $n'\times n$ matrix and $\mathbf{M}_\Sigma$ is a $n'\times n'$ symmetric mass matrix.

In a mass-eigenstate basis where $\mathbf{M}_\Sigma$ is flavour diagonal with eigenvalues of order $\mathcal{O}(M_\Sigma) \gg v$, the lowest-dimensional operator arising from the effective Lagrangian is again the Weinberg operator (23), which generates a mass matrix for light neutrinos identical to that in Eq. (26) for the type I seesaw scenario. We also obtain an effective $d=6$ operator similar to the one in Eq. (27), namely:

$$\delta\mathcal{L}^{d=6} = \mathbf{c}^{d=6}_{\alpha\beta}\left(\overline{L_{\alpha L}}\vec{\tau}\widetilde{\phi}\right) i\not{D}\left(\widetilde{\phi}^\dagger\vec{\tau}L_{\beta L}\right), \tag{50}$$

with coefficients defined as in Eq. (28). However, the usual partial derivative is now replaced with a covariant one due to the triplet (active) nature of the heavy fields. This leads to a richer interaction pattern. Going to a mass-eigenstate basis with kinetic terms canonically normalised, comes out a new feature in weak leptonic currents not present in the type I seesaw, namely FCNCs involving charged leptons:

$$\mathcal{L}_{\rm NC} = -\frac{gZ^\mu}{2\,c_W}\left[\overline{\ell_L}\gamma_\mu\left(\boldsymbol{N}\boldsymbol{N}^\dagger\right)^{-2}\ell_L\right] + \ldots\,. \tag{51}$$

An immediate consequence is a correction to the Fermi constant extracted from muon decay,

$$\begin{aligned} G_F &= G_F^{\rm SM}\sqrt{(\boldsymbol{N}\boldsymbol{N}^\dagger)_{ee}(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\mu\mu} + \frac{3}{4}(\boldsymbol{N}\boldsymbol{N}^\dagger)^{-2}_{e\mu}} \\ &\approx G_F^{\rm SM}\sqrt{(\boldsymbol{N}\boldsymbol{N}^\dagger)_{ee}(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\mu\mu}}\,, \end{aligned} \tag{52}$$

similar to that in Eq. (33) for the type I seesaw.

Finally, the number of parameters in this case is trivially obtained from that in the type I seesaw and the same conclusions apply, namely that for $n' = n$ (or $n' < n$) the number of physical parameters is equal in both the full theory and its effective description with dimension $d \leq 6$ effective operators.

V. LARGE CLFV RATES IN SEESAW MODELS

Since the effective $d=5$ operator is common to all Majorana neutrino mass models, one would need to identify the $d=6$ effective operators in order to distinguish among the three seesaw models. Charged-lepton flavour violating (CLFV) processes could grant this information. In fact, the discovery of neutrino oscillations has shown that lepton flavour is violated. This automatically leads to rare CLFV processes, such as $\ell_\alpha \to \ell_\beta\gamma$, $\ell \to 3\ell'$ or $\mu - e$ conversion in nuclei. These processes have been the subject of intense investigation for decades, and current experimental sensitivities on the various rates are expected to improve by several orders of magnitude [19], see Table II. However, in the framework of a SM extension with light Dirac neutrinos, CLFV rates are predicted to be far below expected sensitivities. In fact, we have

$${\rm BR}(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left(\frac{m_\nu}{M_W}\right)^4 \sim 10^{-52}\,, \tag{53}$$

where α is the fine-structure constant and we have assumed $m_\nu \sim 0.1$ eV for the neutrino mass scale. However, for Majorana neutrinos, the existence of additional states can potentially induce CLFV transitions not suppressed by neutrino masses and, therefore, at the reach of future experiments.

In general, large CLFV rates and the requirement of no fine-tuning demand a seesaw scale $M \sim \mathcal{O}(\text{TeV})$ [20]. From an effective viewpoint, the issue is then whether it is possible to accomodate tiny neutrino masses while keeping the $d=6$ operators (suppressed as $1/M^2$) close to observability without fine-tunings or cancellations in the heavy mass matrices or Yukawa couplings. According to the 't Hooft naturalness criterium, such decoupling of the lepton-number odd $d=5$ operator responsible for neutrino masses from the lepton-number preserving $d=6$ operators may be a natural possibility. In fact, lepton-number symmetry may be broken by a small parameter μ, while other beyond-the-SM effects of the high-energy theory are lepton-number preserving and, as such, do not need to be strongly suppressed. Such decoupling pattern for the minimal seesaw models exhibits the structure [1]:

$$c^{d=5} = f(Y)\frac{\mu}{M^2}\,, \qquad c^{d=6} = g(Y)\,\frac{1}{|M|^2}\,, \tag{54}$$

with f and g functions of the Yukawa couplings. From the $d=5$ operator we thus get a neutrino mass matrix proportional to the small parameter μ, while the effects of the $d=6$ operator may be sizable, even of $\mathcal{O}(1)$, for generic Yukawa couplings. Such feature has already been found in the type II seesaw, in Eqs. (40)-(43), thus suggesting that $\mu_\Delta \ll M_\Delta$ and $Y_\Delta \sim \mathcal{O}(1)$. An interesting and simple example is the *inverse seesaw model* [21]. Considering, for simplicity, only one left-handed neutrino ν_L and two heavy fermion singlets N_1 and N_2, this model contains one light and two heavy mass eigenvalues:

$$m_\nu \approx \frac{\mu}{M_{N_1}}\frac{m^2_{D_1}}{M_{N_1}}\frac{M^2_{N_1}}{M^2_{N_1}+m^2_{D_1}}\,, \tag{55}$$

$$m^\pm_{\rm heavy} \approx \sqrt{m^2_{D_1}+M^2_{N_1}} \pm \frac{M^3_{N_1}}{2\left(m^2_{D_1}+M^2_{N_1}\right)}\frac{\mu}{M_{N_1}}. \tag{56}$$

where $m_{D_1} \sim Y_1\,v/\sqrt{2}$ is a typical Dirac mass term. Eq. (55) shows that m_ν is suppressed by an extra factor μ/M_{N_1} with respect to the result for the minimal type I seesaw model in Eq. (26), exactly as expected from the general argument leading to Eq. (54). Therefore, in the inverse seesaw model it is possible to avoid fine-tuning ($M_{N_1} \sim 1\,\text{TeV}$) while having Yukawa couplings of order $\mathcal{O}(1)$ as long as $\mu/M_{N_1} \sim 10^{-11}$. On the other hand, the lepton-number conserving $d=6$ operators are independent of μ and low-energy effects associated to it could be discovered in the near future. Note finally, from Eq. (56), that such setup predicts a quasi-degenerate mass spectrum for heavy neutrinos, fact that will play an important role below.

Process	Present Limit		Future Sensitivity	
$\mu \to e\gamma$	5.7×10^{-13}	(MEG)	5×10^{-14}	(MEG II)
$\tau \to e\gamma$	3.3×10^{-8}	(Belle)	10^{-9}	(Super B)
$\tau \to \mu\gamma$	4.4×10^{-8}	(Belle)	$10^{-8,-9}$	(Belle II, Super B)
$\mu^- \to e^+e^-e^-$	1.0×10^{-12}	(SIN/SINDRUM)	$10^{-16,-16,-17}$	(Mu3e, MUSIC, Project X)
$\tau^- \to e^+e^-e^-$	2.7×10^{-8}	(Belle,BaBar)	10^{-10}	(Super B)
$\tau^- \to \mu^+\mu^-e^-$	2.7×10^{-8}	(Belle,BaBar)	10^{-10}	(Super B)
$\tau^- \to e^+\mu^-\mu^-$	1.7×10^{-8}	(Belle,BaBar)	10^{-10}	(Super B)
$\tau^- \to e^+e^-\mu^-$	1.8×10^{-8}	(Belle,BaBar)	10^{-10}	(Super B)
$\tau^- \to \mu^+e^-e^-$	1.5×10^{-8}	(Belle,BaBar)	10^{-10}	(Super B)
$\tau^- \to \mu^+\mu^-\mu^-$	2.1×10^{-8}	(Belle,BaBar)	10^{-10}	(Belle II, Super B)
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	4.3×10^{-12}	(SINDRUM II)	$10^{-17,-17,-18,-19}$	(COMET, Mu2e, PRISM/PRIME, Project X)
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	7.0×10^{-13}	(SINDRUM II)	$10^{-17,-17,-18,-19}$	(COMET, Mu2e, PRISM/PRIME, Project X)
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$	4.6×10^{-11}	(SINDRUM II)	$10^{-17,-17,-18,-19}$	(COMET, Mu2e, PRISM/PRIME, Project X)
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}$			10^{-16}	(COMET, Mu2e, PRISM/PRIME, Project X)

Table II. Present experimental bounds on the branching ratios of charged-lepton flavour violating processes considered in our analysis and corresponding future sensitivities. [7, 19].

VI. PHENOMENOLOGY

As discussed above, an analysis of CLFV processes, in particular the ones in Table II, could be used to identify the mechanism responsible for neutrino masses. On one hand, it allows to determine the range of Yukawa couplings and heavy masses that future experiments can reach. On the other hand, experimental bounds on CLFV decays can also be used to constrain combinations of Yukawa couplings $(\mathbf{Y}_\Delta)_{ij}$ in the type II seesaw scenario, while in the type I and type III seesaws these bounds constrain the off-diagonal elements $(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\alpha\beta}$, characterising deviations from unitarity of the leptonic mixing matrix $\boldsymbol{N}$ [see Eq. (32)]. Finally, diagonal elements $(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\alpha\alpha}$ can be constrained by a combined analysis of lepton-flavour conserving processes: W decays, Z decays and universality tests.

A. Type I seesaw

In the type I seesaw model, all CLFV processes are induced at one-loop level. In general, for masses $M_{N_j} \sim \mathcal{O}(\mathrm{TeV})$, the branching ratios of CLFV processes involving the same flavour transition have the form

$$\mathrm{BR}_{\ell_\alpha \to \ell_\beta} = \sum_{N_j} \frac{\left|(\mathbf{Y}_\Delta)_{j\beta}(\mathbf{Y}_\Delta^\dagger)_{\alpha j}\right|^2}{M_{N_j}^4} \left[a + b \ln\left(M_{N_j}/M_W\right)\right] \,, \quad (57)$$

where a and b are factors independent of M_{N_j} [22, 23]. While for $\mathrm{BR}(\ell_\alpha \to \ell_\beta \gamma)$ there is no logarithmic term, the same does not happen for μ-e conversion in nuclei and $\ell \to 3\ell'$ decays. Therefore, a simple analysis of these processes is not possible. However, models that naturally lead to observable CLFV rates imply a quasi-degenerate mass spectrum $m_{N_1} \approx m_{N_2} \approx ... \equiv m_N$ [see Eq. (56)]. In such scenario, the dependence of Eq. (57) on the Yukawa couplings factorises out and, therefore, an analysis based on ratios of two same-flavour transitions depends only on the mass scale m_N. This allows multiple possibilities of model-testing.

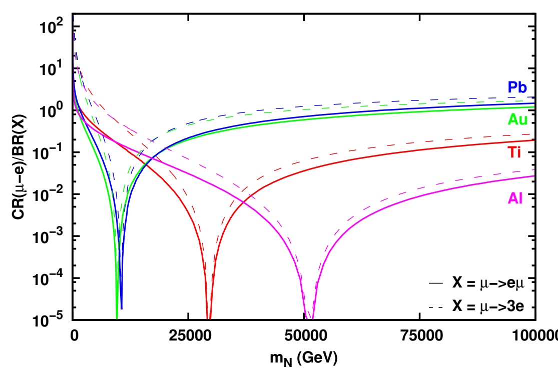

Figure 2. Ratios of the μ-e conversion rate for several nuclei and the branching ratio of $\mu \to e\gamma$ (solid lines) or $\mu \to 3e$ (dashed lines), as functions of the seesaw scale m_N.

As CLFV μ processes are presently the most constrained ones, we limit our analysis to μ-e conversion in nuclei, $\mu \to e\gamma$ and $\mu \to 3e$. In Fig. 2, we show the ratios of conversion rates in nuclei to $\mu \to e\gamma$ and $\mu \to 3e$ branching ratios. For $m_N \lesssim 10$ TeV, the observation of monotonous functions means that the measurement of this ratio would allow for a determination of m_N. In fact, the observation of a single rate that, together with the experimental upper bound on another one, leads to a ratio incompatible with Fig. 2, could be sufficient to exclude the model. Additionally, both ratios vanish at a value of $m_N \gg M_W$ dependent on the nucleus. This is explained by opposite contributions of up and down quarks to the μ-e conversion rate and shows how important the search for $\mu \to e$ conversion in several nuclei could be in testing the model [22].

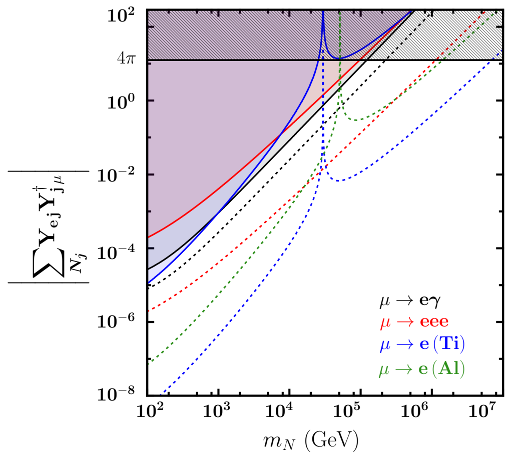

Figure 3. Ratios of μ-e conversion rates and BR($\mu \to e\gamma$) (solid lines) or BR($\mu \to 3e$) (dashed lines). Our analysis is valid in the perturbative region $\left|\sum_{N_j} \mathbf{Y}_{e\,j}\mathbf{Y}^\dagger_{j\,\mu}\right| < 4\pi$.

Present bounds and future sensitivities to Yukawa couplings are shown in Fig. 3. The results suggest that μ-e conversion experiments will become dominant in the study of flavour physics, allowing to probe $\mathbf{Y}_N \sim 10^{-3}$ if $m_N = 1\,\mathrm{TeV}$, for example. By requiring $\mathbf{Y}_N \sim \mathcal{O}(1)$, we can rephrase the bounds of Fig. 3 as upper bounds on m_N. The most stringent bound comes from $\mu - e$ conversion in ^{48}Ti,

$$m_N \lesssim 2000\,\mathrm{TeV}\cdot\left(\frac{10^{-18}}{\mathrm{CR}^{\mathrm{Ti}}_{\mu\to e}}\right)^{\frac{1}{4}} \Big|\sum_{N_j} (\mathbf{Y}_N)_{je}(\mathbf{Y}^*_N)_{j\mu}\Big|^{\frac{1}{2}} , \quad (58)$$

and shows that future experiments may probe the type I seesaw scenario beyond the ~ 1000 TeV scale.

On the other hand, in the large mass regime $m_N \gg M_W$ [$m_N \sim \mathcal{O}(\mathrm{TeV})$, for example], the $\ell_\alpha \to \ell_\beta\gamma$ branching ratio is given by the simple expression

$$\mathrm{BR}(\ell_\alpha \to \ell_\beta\gamma) \approx \frac{3\alpha}{8\pi}\frac{\left|(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\alpha\beta}\right|^2}{(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\alpha\alpha}\,(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\beta\beta}} , \quad (59)$$

which can be used to constrain the off-diagonal elements of $\boldsymbol{N}\boldsymbol{N}^\dagger$. Moreover, the diagonal elements $(\boldsymbol{N}\boldsymbol{N}^\dagger)_{\alpha\alpha}$ can be constrained by (tree-level) W and Z decays, as well as by tests on the universality of weak interactions. From all these constraints, a global fit results in the following elements of $\boldsymbol{N}\boldsymbol{N}^\dagger$ at 90% CL:

$$|\boldsymbol{N}\boldsymbol{N}^\dagger| \approx \begin{pmatrix} 0.996 \pm 0.003 & < 2.6\cdot 10^{-5} & < 1.5\cdot 10^{-2} \\ < 2.6\cdot 10^{-5} & 0.994 \pm 0.003 & < 1.7\cdot 10^{-2} \\ < 1.5\cdot 10^{-2} & < 1.7\cdot 10^{-2} & 1.001 \pm 0.004 \end{pmatrix} . \quad (60)$$

From these results, we conclude that there is a 2σ departure from unity in the diagonal elements of the matrix in Eq. (61). However, this 2σ deviation is not significant enough to be interpreted as a signal of physics beyond the SM [24, 25].

B. Type II seesaw

When compared to the type I seesaw, the type II seesaw introduces a novelty with tree-level $\ell \to 3\ell'$ decays. As a result, $\ell \to 3\ell'$ rates will be the largest. Contrarily, for radiative $\ell_\alpha \to \ell_\beta\gamma$ decays [26–28]:

$$\frac{\mathrm{BR}(\ell_\alpha \to \ell_\beta\gamma)}{\mathrm{BR}(\ell_\alpha \to e\nu_\alpha\overline{\nu_e})} \approx \frac{27\alpha}{16\pi}\frac{\left|\left(\mathbf{Y}_\Delta\mathbf{Y}^\dagger_\Delta\right)_{\alpha\beta}\right|^2}{(G_F^{\mathrm{SM}})^2 M_\Delta^4} . \quad (61)$$

From the present upper limits on BR($\ell_\alpha \to \ell_\beta\gamma$) reported in Table II, we obtain upper bounds on $|(\mathbf{Y}_\Delta\mathbf{Y}^\dagger_\Delta)_{\alpha\beta}|$ given in Table III. From the last table, we see that the most stringent bound comes from $\mu \to e\gamma$ and that, for $M_\Delta \sim \mathcal{O}(\mathrm{TeV})$, the Yukawa couplings are allowed to be of $\mathcal{O}(10^{-1})$, while they should be sizeably smaller by up to three orders of magnitude for specific flavours. Also interesting is that from the result (61) and Eq. (40) we can obtain the constraint $v_\Delta \gtrsim 1$ eV for the triplet VEV if $M_\Delta \sim \mathcal{O}(\mathrm{TeV})$. Rewriting BR($\mu \to e\gamma$) as

$$\mathrm{BR}(\mu \to e\gamma) \approx 1.4\times 10^{-13}\left(\frac{1\,\mathrm{eV}}{v_\Delta}\right)^4\cdot\left(\frac{1\,\mathrm{TeV}}{M_\Delta}\right)^4 , \quad (62)$$

we see from Table II that, for $v_\Delta \approx 1$ eV and $M_\Delta \sim 1$ TeV, the ratio BR($\mu \to e\gamma$) may reach values within the projected sensitivity of the MEG II experiment [29].

On the other hand, the leading contribution to $\ell \to 3\ell'$ decays is due to tree-level exchange of a doubly-charged scalar Δ^{++}. Alternatively, one can say that these processes are generated by the $d = 6$ effective operator $\delta\mathcal{L}_{4F}$ in Eq. (41). For them, we obtain:

$$\frac{\mathrm{BR}(\ell^-_\alpha \to l^+_\beta l^-_\sigma l^-_\rho)}{\mathrm{BR}(\ell^-_\alpha \to e^-\nu_\alpha\overline{\nu}_e)} = \frac{4(2-\delta_{\sigma\rho})|(\mathbf{Y}_\Delta)_{\alpha\beta}|^2|(\mathbf{Y}_\Delta)_{\rho\sigma}|^2}{M_\Delta^4\,(G_F^{\mathrm{SM}})^2} , \quad (63)$$

which leads to the upper bounds on Yukawa couplings given in Table III. The most stringent upper bound, namely $|(\mathbf{Y}_\Delta)_{\mu e}||(\mathbf{Y}_\Delta)_{ee}| \lesssim 5.8\times 10^{-6}\cdot(M_\Delta/1\,\mathrm{TeV})^2$, comes from the $\mu \to 3e$ decay and its non-observation implies a lower bound on the seesaw scale:

$$M_\Delta \geq 414\,\mathrm{TeV} , \qquad \text{for} \quad \mathbf{Y}_\Delta \sim \mathcal{O}(1) . \quad (64)$$

Furthermore, future sensitivities given in Table III indicate that planned experiments will be able to probe values of Yukawa coupling combinations at least one order of magnitude below the present bounds.

A third CLFV process which can constrain the Yukawa parameters $\mathbf{Y}_\Delta$ is μ-e conversion in nuclei. In this case, the conversion rate in light nuclei reads:

$$\mathrm{CR}\,(\mu\,\mathcal{N} \to e\,\mathcal{N}) \approx \frac{\alpha^5 m_\mu^5 Z_{\mathrm{eff.}}^4 Z}{9\pi^4\Gamma_{\mathrm{capt.}}}F^2(-m_\mu^2)\left|C^{(\mathrm{II})}_{\mu e}\right|^2 , \quad (65)$$

where ($Z_{\mathrm{eff.}}$) Z is the (effective) atomic number, F is a nuclear form factor and $\Gamma_{\mathrm{capt.}}$ is the muon

Process	Constraint On	Upp. Bound	Fut. Sens.
$\mu \to e\gamma$	$\lvert(\mathbf{Y}_\Delta \mathbf{Y}_\Delta^\dagger)_{\mu e}\rvert$	1.4×10^{-4}	4.5×10^{-5}
$\tau \to e\gamma$	$\lvert(\mathbf{Y}_\Delta \mathbf{Y}_\Delta^\dagger)_{\tau e}\rvert$	8.0×10^{-2}	1.4×10^{-2}
$\tau \to \mu\gamma$	$\lvert(\mathbf{Y}_\Delta \mathbf{Y}_\Delta^\dagger)_{\tau \mu}\rvert$	9.4×10^{-2}	1.4×10^{-2}
$\mu^- \to e^+e^-e^-$	$\lvert(\mathbf{Y}_\Delta)_{\mu e}\rvert\lvert(\mathbf{Y}_\Delta)_{ee}\rvert$	5.8×10^{-6}	5.8×10^{-8}
$\tau^- \to e^+e^-e^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau e}\rvert\lvert(\mathbf{Y}_\Delta)_{ee}\rvert$	2.3×10^{-3}	1.4×10^{-4}
$\tau^- \to \mu^+\mu^-\mu^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau \mu}\rvert\lvert(\mathbf{Y}_\Delta)_{\mu\mu}\rvert$	1.4×10^{-3}	9.8×10^{-5}
$\tau^- \to \mu^+e^-e^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau \mu}\rvert\lvert(\mathbf{Y}_\Delta)_{ee}\rvert$	1.2×10^{-3}	9.8×10^{-5}
$\tau^- \to e^+\mu^-\mu^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau e}\rvert\lvert(\mathbf{Y}_\Delta)_{\mu\mu}\rvert$	1.3×10^{-3}	9.8×10^{-5}
$\tau^- \to \mu^+\mu^-e^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau \mu}\rvert\lvert(\mathbf{Y}_\Delta)_{\mu e}\rvert$	1.6×10^{-3}	9.8×10^{-5}
$\tau^- \to e^+e^-\mu^-$	$\lvert(\mathbf{Y}_\Delta)_{\tau e}\rvert\lvert(\mathbf{Y}_\Delta)_{\mu e}\rvert$	1.3×10^{-3}	9.8×10^{-5}

Table III. Bounds on $(\mathbf{Y}_\Delta)_{ij}\left[\times\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2\right]$ from $\ell_\alpha \to \ell_\beta\gamma$ and $\ell \to 3\ell$ decays in the type II seesaw scenario.

capture rate. These nuclear parameters are reported in Ref. [30]. The quantity $\left|C_{\mu e}^{(\mathrm{II})}\right|^2$ depends on the Yukawa couplings $\mathbf{Y}_\Delta$ and, from Table II, we obtain that μ-e conversion in ^{48}Ti sets an upper bound $|C_{\mu e}^{(\mathrm{II})}| < 3.1 \times 10^{-3}$. On the other hand, using future sensitivities to μ-e conversion rates in Table II, we are also able to conclude that future experiments with ^{48}Ti will be sensitive to $|C_{\mu e}^{(\mathrm{II})}| \gtrsim 4.7 \times 10^{-6}(M_\Delta/1\,\mathrm{TeV})^2$.

We conclude by noting that future facilities open up the possibility of observing new signals of the type II seesaw. For instance, expected sensitivities to M_Δ are

$$M_\Delta \lesssim 200\,\mathrm{TeV} \cdot |(\mathbf{Y}_\Delta \mathbf{Y}_\Delta^\dagger)_{\mu e}|^{1/2} \cdot \left[\frac{10^{-14}}{\mathrm{BR}(\mu \to e\gamma)}\right]^{\frac{1}{4}}, \quad (66)$$

$$M_\Delta \lesssim 400\,\mathrm{TeV} \cdot |C_{\mu e}^{(\mathrm{II})}|^{1/2} \cdot \left[\frac{10^{-18}}{\mathrm{CR}(\mu\mathrm{Ti} \to e\mathrm{Ti})}\right]^{\frac{1}{4}}, \quad (67)$$

$$M_\Delta \lesssim 2400\,\mathrm{TeV} \cdot \sqrt{|(\mathbf{Y}_\Delta)_{\mu e}||(\mathbf{Y}_\Delta)_{ee}|} \cdot \left[\frac{10^{-16}}{\mathrm{BR}(\mu \to 3e)}\right]^{\frac{1}{4}}, \quad (68)$$

which shows that, for $\mathbf{Y}_\Delta \sim \mathcal{O}(1)$, future CLFV decay experiments may in principle probe the type II seesaw model beyond the $\sim 1000\,\mathrm{TeV}$ scale.

C. Type III seesaw

A crucial difference between the type I and type III seesaw is that for the latter there is charged-lepton flavour mixing in neutral currents [see Eq. (51)]. As a result, $\ell \to 3\ell'$ and μ-e conversion in nuclei are now tree-level processes. Still, $\ell_\alpha \to \ell_\beta\gamma$ proceeds at one-loop since the QED coupling (10) remains flavour diagonal. Even for a seesaw scale $m_\Sigma \sim 100$ GeV, in the canonical type III seesaw one would obtain $\mathrm{BR}\left(\ell_\alpha \to \ell_\beta\gamma\right) \sim 10^{-27}\,\mathrm{BR}(\ell_\alpha \to \ell_\beta \overline{\nu_\beta}\nu_\alpha)$ [31], a value far below the present bounds in Table II. However, the

Process	Upper Bound on $\lvert\boldsymbol{\epsilon}_{e\mu}^{\Sigma}\rvert$	Future Sens. to $\lvert\boldsymbol{\epsilon}_{e\mu}^{\Sigma}\rvert$
$\mu \to e\gamma$	2.3×10^{-5}	6.5×10^{-6}
$\mu \to 3e$	1.1×10^{-6}	1.1×10^{-8}
$\mu\,\mathrm{Au} \to e\,\mathrm{Au}$	1.5×10^{-7}	1.8×10^{-10}

Table IV. Most stringent bounds and future sensitivities on $\boldsymbol{\epsilon}_{\beta\alpha}^{\Sigma}$ resulting from CLFV processes in the type III seesaw.

rates can be naturally much larger if neutrino masses are proportional to a small lepton number violating parameter μ. In this case, the $d = 6$ operator (50) is greatly enhanced and, in the large mass regime, we obtain

$$\frac{\mathrm{BR}(\ell_\alpha \to \ell_\beta\gamma)}{\mathrm{BR}(\ell_\alpha \to \ell_\beta \overline{\nu_\beta}\nu_\alpha)} \approx 1.08 \times 10^{-3} \cdot \left|\boldsymbol{\epsilon}_{\beta\alpha}^{\Sigma}\right|^2. \quad (69)$$

In general, all CLFV rates in the type III seesaw do not involve logarithmic terms and exhibit the form

$$\mathrm{BR}_{\ell_\alpha \to \ell_\beta} = g(M_W, m_\ell) \cdot \left|\boldsymbol{\epsilon}_{\beta\alpha}^{\Sigma}\right|^2, \quad (70)$$

with $g(M_W, m_\ell)$ being a function of M_W and m_ℓ. Therefore, the ratio of processes with the same flavour transition is predicted to a fixed value. Namely, we obtain:

$$\mathrm{BR}(\mu \to e\gamma) = 1.3 \times 10^{-3} \cdot \mathrm{BR}(\mu \to eee), \quad (71)$$

$$\begin{aligned} \mathrm{BR}(\tau \to \mu\gamma) &= 1.3 \times 10^{-3} \cdot \mathrm{BR}(\tau \to \mu\mu\mu) \\ &= 2.1 \times 10^{-3} \cdot \mathrm{BR}(\tau^- \to e^-e^+\mu^-), \end{aligned} \quad (72)$$

$$\begin{aligned} \mathrm{BR}(\tau \to e\gamma) &= 1.3 \times 10^{-3} \cdot \mathrm{BR}(\tau \to eee) \\ &= 2.1 \times 10^{-3} \cdot \mathrm{BR}(\tau^- \to \mu^-\mu^+e^-), \end{aligned} \quad (73)$$

and, for ratios involving μ-e conversion in nuclei (in $^{48}_{22}$Ti, for example),

$$\mathrm{BR}(\mu \to e\gamma) = 3.1 \times 10^{-4} \cdot \mathrm{CR}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}), \quad (74)$$

$$\mathrm{BR}(\mu \to eee) = 2.4 \times 10^{-1} \cdot \mathrm{CR}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}). \quad (75)$$

From these results, we see that $\ell \to 3\ell'$ and μ-e conversion rates should be much larger than $\mathrm{BR}(\ell_\alpha \to \ell_\beta\gamma)$. This is so because $\ell_\alpha \to \ell_\beta\gamma$ proceeds at loop level while $\ell \to 3\ell'$ and μ-e conversion are tree level processes. The most stringent bounds come from $\mu \to e$ transitions and are presented in Table IV. We see that the bounds on $|\boldsymbol{\epsilon}_{e\mu}|$ coming from $\mu \to e\gamma$, are not as constraining as the ones resulting from $\mu \to 3e$, and that the same applies to future sensitivities. As can be seen from Table II, the same conclusion holds for general $\ell_\alpha \to \ell_\beta\gamma$ and $\ell \to 3\ell'$ decays. Nonetheless, the most stringent bound comes from μ-e conversion in nuclei (in ^{197}Au) for which experimental sensitivities are expected to improve by several orders of magnitude. Therefore, the observation of $\ell_\alpha \to \ell_\beta\gamma$ close to present bounds would rule out the type III seesaw since it would be incompatible with bounds arising from $\ell \to 3\ell'$ or μ-e conversion in nuclei [see Eqs. (74)-(75)].

Finally, an unitarity analysis identical to the one performed in the type I seesaw allows to obtain the following elements of $\boldsymbol{N}\boldsymbol{N}^\dagger = \mathbb{1} - \boldsymbol{\epsilon}^\Sigma$, at 90% CL:

$$|\boldsymbol{N}\boldsymbol{N}^\dagger| \approx \begin{pmatrix} 1.004 \pm 0.011 & < 1.5 \cdot 10^{-7} & < 3.2 \cdot 10^{-4} \\ < 1.5 \cdot 10^{-7} & 0.993 \pm 0.011 & < 2.6 \cdot 10^{-4} \\ < 3.2 \cdot 10^{-4} & < 2.6 \cdot 10^{-4} & 1.014 \pm 0.012 \end{pmatrix}. \quad (76)$$

As in the type I seesaw [in Eq. (61)], small deviations from unity in the diagonal elements are not significant enough to be understood as a signal of new physics. Note also that, since CLFV processes are now allowed at tree level, the bounds on off-diagonal elements are stronger than those obtained in the type I seesaw.

VII. CONCLUSIONS

In this work, we have investigated the three canonical seesaw mechanisms of neutrino mass generation. From an effective viewpoint, neutrino masses are generated by a unique dimension-five operator. However, a plethora of dimension-six operators exists and, therefore, they are a crucial tool to distinguish between various models. In a nutshell, effective operators in fermionic seesaw models induce non-unitary lepton mixing, while in the scalar triplet model dimension-six operators induce exotic four-fermion couplings and corrections to gauge and Higgs potential parameters.

We have also discussed the possible values of the heavy mass scale M. An unsatisfactory aspect of seesaw models pertains to a worsening of the electroweak hierarchy problem, which could be soften if M is close to the TeV scale. This scenario is indeed possible if neutrino masses are proportional to a small lepton number violating parameter μ. Such pattern is already realised in the minimal type II seesaw and also in fermionic seesaw models, such as the inverse seesaw. Tiny neutrino masses can then be naturally accomodated, while keeping the effects of $d=6$ operators close to observability.

Finally, we have analysed CLFV processes, such as $\ell_\alpha \to \ell_\beta \gamma$, $\ell \to 3\ell'$ and μ-e conversion in nuclei, for the three canonical seesaw models. In the type I seesaw, our results show that the CLFV predicted rates are within the sensitivities of future experiments. Still, the rates of $\mu \to e$ conversion in nuclei could vanish at a value $m_N \sim 10$ TeV of the heavy mass scale. Constraints on the dimension-six operator coming from a combined analysis of other lepton flavour conserving processes also predict a 2σ departure from unitarity of the mixing matrix. In the type II scenario, we have set bounds and analysed future sensitivities to Yukawa coupling combinations. The results show that, for a seesaw scale of $\mathcal{O}(\text{TeV})$, $\mathcal{O}(1)$ Yukawa couplings are allowed by present bounds and that, for a triplet VEV $v_\Delta \sim 1$ eV, the rates of CLFV could be within expected sensitivities. Finally, in the type III seesaw scenario, the ratio of two same-flavour transition processes is a constant. Combined with the presence of tree-level FCNCs and, therefore, large $\mu \to 3e$ and $\mu \to e$ conversion rates this offers a clear possibility for model-testing.

[1] A. Abada *et al.*, JHEP **0712**, 061 (2007).
[2] Y. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **81**, 1562 (1998).
[3] G. 't Hooft, NATO Sci.Ser.B **59**, 135 (1980).
[4] E. Majorana, Nuovo Cim. **14**, 171 (1937).
[5] B. Pontecorvo, Sov.Phys.JETP **6**, 429 (1957).
[6] Z. Maki, M. Nakagawa, and S. Sakata, Progress of Theoretical Physics **28**, 870 (1962).
[7] K. Olive *et al.*, Chin. Phys. C **38**, 090001 (2014).
[8] F. Capozzi *et al.*, Phys.Rev. **D89**, 093018 (2014).
[9] C. Kraus *et al.*, Eur.Phys.J. **C40**, 447 (2005).
[10] V. Lobashev *et al.*, Phys.Lett. **B460**, 227 (1999).
[11] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
[12] T. Yanagida, Conf.Proc. **C7902131**, 95 (1979).
[13] W. Buchmuller and D. Wyler, Nucl.Phys. **B268**, 621 (1986).
[14] M. Gell-Mann, P. Ramond, and R. Slansky, Conf.Proc. **C790927**, 315 (1979).
[15] R. N. Mohapatra and G. Senjanovic, Phys.Rev.Lett. **44**, 912 (1980).
[16] J. Schechter and J. Valle, Phys. Rev. D **22**, 2227 (1980).
[17] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl.Phys. **B181**, 287 (1981).
[18] F. del Aguila and J. Aguilar-Saavedra, Nucl.Phys. **B813**, 22 (2009).
[19] R. H. Bernstein and P. S. Cooper, Phys.Rept. **532**, 27 (2013).
[20] J. Casas, J. Espinosa, and I. Hidalgo, JHEP **0411**, 057 (2004).
[21] M. González-Garcia and J. Valle, Phys.Lett. **B216**, 360 (1989).
[22] R. Alonso, M. Dhen, M. Gavela, and T. Hambye, JHEP **1301**, 118 (2013).
[23] D. Dinh, A. Ibarra, E. Molinaro, and S. Petcov, JHEP **1208**, 125 (2012).
[24] S. Antusch *et al.*, JHEP **0610**, 084 (2006).
[25] S. Antusch and O. Fischer, JHEP **1410**, 94 (2014).
[26] A. Akeroyd, M. Aoki, and H. Sugiyama, Phys.Rev. **D79**, 113010 (2009).
[27] M. Raidal and A. Santamaria, Phys.Lett. **B421**, 250 (1998).
[28] E. Ma, M. Raidal, and U. Sarkar, Nucl.Phys. **B615**, 313 (2001).
[29] A. Baldini *et al.*, (2013), arXiv:1301.7225.
[30] R. Kitano, M. Koike, and Y. Okada, Phys.Rev. **D66**, 096002 (2002).
[31] A. Abada *et al.*, Phys.Rev. **D78**, 033007 (2008).