Study about Risk Analysis of a Portfolio of Network Distribution Investments
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Abstract
This thesis studies the risk analysis of a group of projects. In order to do that, it begins by explaining how to study each project individually and how to calculate their Net Present Value (NPV) and their Value at Risk (VaR). Posteriorly, the same values are determined for a simple portfolio of non-correlated projects through convolutions. It can be concluded that the risk of a portfolio decreases with the increase in the number of similar projects that constitute the portfolio and that an approximation of the probability density function of each project to a Normal distribution is valid for portfolios with more than 20 projects. After that, the effect of correlations between projects in the return of a portfolio is explained as well as how it affects the previous risk analysis. After verifying that the correlation is an important factor in determining the risk of a portfolio and that its application to the projects that EDP made available is hard, a method of dealing with these difficulties and of calculating the correlations between projects is suggested. Finally, the proposed method is applied to the projects and the results are shown, yielding no correlation between projects with respect to the uncertainty primitives analysed and thus allowing to easily calculate the NPV and VaR of any portfolio through simple convolutions. The last section presents the results of some simulations of ways of selecting and optimizing portfolios that obey to certain imposed customizable conditions.

Keywords: Risk analysis, Distribution Network Projects, Correlations, Portfolio Selection

1. Introduction
The number of companies that use risk analysis methodologies to economically evaluate projects and to decide about investing has increased significantly. Knowing about the risk factors that affect a project, how they affect it and what their potential financial impacts are (mostly the negatives) is crucial to making good investment decisions. It is based on this information, on the financial situation of each company and on its risk policies that the projects with more priority should be defined.

In cases where the companies are involved in a lot of projects at the same time, it is also useful to understand how the risk of a portfolio varies with the different characteristics and relationships between the projects.

After knowing how to calculate the expected return and risk of a portfolio, this paper suggests a methodology that can facilitate the choice of a set of investments, chosen from a bigger one, that fit the needs of any particular company.

2. Economical Evaluation of a Project
2.1. Evaluation of a network distribution project
In a network distribution project the main cost, $C$, is the investment. These are mostly paid in the beginning of the project to allow its construction while the benefits, $B$, are received annually since the facilities start working.

To calculate the NPV it is considered that the investment is paid in year 0 and is already the full discounted value of the costs, $C_{\text{discounted}}$. The discounted benefits, $B_{\text{discounted}}$, on the other hand, have to be calculated considering the different life expectancies of each project, which can be consulted in [1].

Formally, the NPV can be described as:

$$NPV = B_{\text{discounted}} - C_{\text{discounted}}$$  \hspace{1cm} (1)

After setting the time frame for the costs and benefits it is necessary to explain how they are obtained.

The costs are easily calculated by continuing to admit that they are equal to the investment which is part of the database and given in euros.
The benefits, on the other hand, are harder to quantify. The benefits of a distribution network project have several distinct origins. This paper focuses on two portions, the benefits associated with reducing the energy losses, $B_{losses}$, and with reducing non-distributed energy, $B_{NDE}$.

None of the two portions of the benefits are calculated directly, but according to [2], it is described by a distribution due to outages during a certain time. To calculate the total benefits for the year $n$, assuming an equal valuation of $E$, we can use equation 1 to calculate the NPV.

Formally, the $E_{losses}$ is given by:

$$E_{losses} = P_{iron} + P_{conductors} = P_{iron} + I_{losses,ref}(1 + \alpha)^2 \tag{2}$$

The NDE estimates the amount of energy not distributed due to outages during a certain time. According to [2], it is described by:

$$NDE = NDE_{ref}(1 + \alpha) \tag{3}$$

To calculate the total benefits for the year $n$, assuming an equal valuation of $E_{losses}$ and $NDE$, $V_e$, the following equation can be used:

$$B = V_e \times [P_{iron} + P_{conductors}(1 + \alpha)^{n+2} + NDE(1 + \alpha)^n] = B_{NDE} + B_{losses} \tag{4}$$

2.2. Costs and benefits in the proposed method

However, the described method presents some difficulties in accessing the necessary data. So, for practical reasons, the values of the benefits and costs were calculated from data provided by EDP Distribution. After analysing the database, the following information about each project was retained: Voltage level, predicted investment, real investment, $B/C$ predicted rate, program of the project and region (the regions are divided into 6 groups according to EDP’s criteria about which “Direcção de Redes e Clientes”, DRC, they belong to).

From this data we can use the predicted investment to know the costs:

$$C_{discounted} = I_{predicted} \tag{5}$$

And we can calculate $B$ with the following equation:

$$B_{discounted} = I_{predicted} \times B/C \tag{6}$$

After calculating the discounted benefits and costs we can use equation 1 to calculate the NPV.

3. Risk analysis in investments

3.1. Value at Risk

In this section the concept of risk is introduced to the financial analysis. The chosen method to quantify the risk of each project was the Value at Risk, VaR. This method consists in calculating the NPV of a project in a pessimistic perspective. The VaR can be defined as the return of a project that guarantees that 95% of the time the project will have a better outcome than that value.

This value can be obtained by recalculating the values of the benefits, $B^*$, and costs, $C^*$, for the most undesirable cases.

As seen in section 2, besides the specific characteristics of each project, the benefits’ uncertainty only depends on the growth rate of consumption and the costs’ uncertainty of the investment. It can be concluded that $B^*$ and $C^*$ are the benefits and costs calculated for the pessimist growth rate of consumption, $\alpha^*$, and the pessimist investment, $I^*$, respectively. According to [3], $\alpha^*$ and $I^*$ can be calculated with the following expressions:

$$\alpha^* = \frac{1 + \alpha}{1.03} \tag{7}$$

$$I^* = \frac{I}{0.73} \tag{8}$$

Utilizing $I^*$ and the expressions 2 and 3 to obtain $B^*_{losses}$ and $B_{NDE}^*$ we can calculate the VaR using the following equation:

$$VaR = (B_{NDE}^* + B_{losses}^*) - I^* \tag{9}$$

3.2. Project Representation

Having in mind the two measures introduced (NPV and VaR) the result of the economic evaluation of each project will be represented through three Dirac deltas. The VaR will be the first Dirac delta with 5% probability, the NPV will have 90% and the third Dirac delta will have the remaining 5% and will be symmetrical to the VaR in respect to the NPV.
This representation considers that the probability of an extremely optimistic and pessimistic scenarios are the same and equal to 5%.

Based on this calculated values of each project, the definition of risk adopted in this paper, for both, individual projects and portfolios, is:

\[
Risk = \frac{NPV - VaR}{NPV}
\]

4. Evaluation of a portfolio of non-correlated investments

To evaluate the NPV and VaR of a portfolio of \( n \) projects we need to sum their density probability functions. This is done through convolution. In the case where the projects are non-correlated and represented by a discrete function (\( m_1 \) or \( m_2 \)) this can be obtained utilizing the following expression:

\[
m_3(x) = \sum_{i=1}^{k} m_1(k) \ast m_2(x - k)
\]

In the case of projects equal to the one in Figure 1 their probability density function, \( m \), can be described by:

\[
m = \begin{pmatrix}
224 & 300 & 376 \\
0.05 & 0.9 & 0.05
\end{pmatrix}
\]

Considering \( Z = m + m \) then \( Z \) is the convolution of \( m \) with itself and calculated through equation 11. Graphically, the result of this sum can be seen in Figure 2.

However this can be a long process. In order to simplify the process of convolution, the distribution density function of the return of each project was converted into a normal distribution, like in the model suggested by Harry Markowitz in 1952 [4]. This conversion was done using the following expressions:

\[
\mu = NPV
\]

\[
\sigma_i = \sqrt{\sum_{i=1}^{n} p_i (x_i - u)^2}
\]

Which, in the case of a project defined by the 3 Dirac deltas mentioned, is the same as:

\[
\sigma_i = \sqrt{2 \ast (NPV - VaR)^2 \ast 0.05}
\]

The new VaR becomes:

\[
VaR_i = NPV_i - 1.644 \ast \sigma_i
\]

The representation of each project by a normal distribution simplified the amount of computational power needed and, in the Matlab experiment, made the convolution of 30 similar projects 81 times faster (from 20.37s to 0.25s).

However, this method only obtained accurate results in estimating the VaR for portfolios of over 20 projects as can be seen in the figures 3 and 4.
Due to the definition of risk made in equation 10 it is also expected that the value of the risk in both methods tends to the correct value (as can be seen in figure 5). In this figure it can also be observed that the "normal approximation" with similar projects is optimistic. However, this approximation becomes pessimistic when the projects are different between themselves. This can be seen in figure 6 which compares both methods doing the convolution of 30 different projects by 20 different orders.

5. Evaluation of a portfolio of correlated investments

Real portfolios are usually made of slightly correlated investments due to, for example, the economy of the country. Because of this there are a lot of studies about portfolios of correlated investments. According to [5], the total risk of a portfolio has two components: the diversified risk and the non-diversified. The first part corresponds to the effect of the individual variance of each project and the second to the correlation between projects. The total risk (the sum of both components) in a correlated portfolio is represented in figure 7.

5.1. Pearson correlation

As concluded before, the correct evaluation of a portfolio needs the correlations between all the projects involved. This is not an easy task to accomplish due to the fact that the direct application of the concept of Pearson correlation is impossible because correlations between single events (returns of individual projects) doesn’t make sense.

The projects have to be grouped in different categories according to a specified characteristic so as to calculate the correlation between categories of projects. After this grouping process, the collected data is stored in the following format:

<p>| Table 1: Data about the type 1 projects |  |  |  |</p>
<table>
<thead>
<tr>
<th>VaR (_x) (\text{predicted} )</th>
<th>(NPV _x) (\text{predicted} )</th>
<th>(NPV _x) (\text{real} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

<p>| Table 2: Data about the type 2 projects |  |  |  |</p>
<table>
<thead>
<tr>
<th>VaR (_y) (\text{predicted} )</th>
<th>(NPV _y) (\text{predicted} )</th>
<th>(NPV _y) (\text{real} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

However, this creates another problem. The general formula for the Pearson correlation (equation 16) is usually only used in paired information. The fact that the projects in each group have no defined order makes it so there are no pairs of information.

\[
\text{Cov}(X,Y) = E[(X - E[X])(Y - E[Y])] \quad (16)
\]

This problem was solved by admitting each project had a density probability distribution de-
fined by 3 Dirac deltas (similar to 1) and by manip-
ulating the expression 16 into:

\[
\text{Cov}(X, Y) = \sum_{x} \sum_{y} (x - \mu_x)(y - \mu_y)p(x, y)
\]  

Where \(\mu_x\) and \(\mu_y\) are the expected values of each 
measurement of each project of type 1 and type 2,
respectively. In other words, they are the predicted 
NPV of each project. The term \(p(x, y)\) for any \((x, y)\) 
available is a uniform probability distribution function 
that represents the fact that each pair of occurrences 
has the same impact in the covariance.

Having these assumptions in mind the covariance 
can be rewritten by the following equation:

\[
\text{Cov}(x, y) = \frac{1}{M \times N} \sum_{x} \sum_{y} (x - \mu_x)(y - \mu_y)
\]  

Applying formula 18 to tables 5.1 and 5.1 yields the 
following result:

\[
\text{Cov}(X, Y) = \frac{1}{M \times N} \left[(x_1 - \mu_{x1})(y_1 - \mu_{y1}) + (x_2 - \mu_{x2})(y_2 - \mu_{y2}) + (x_3 - \mu_{x3})(y_3 - \mu_{y3})\right]
\]  

\[
\leftrightarrow \text{Cov}(X, Y) = -4
\]

It is interesting to note that the number of terms in 
19 is 9 which is the number of all the possible combinations 
between two projects of different types. It can be concluded from this fact that this method of calculating the covariance is indifferent to the order of the projects in each group and that it can be applied to groups of different sizes.

Finally, after having calculated the covariance in 19 the correlation is obtained by calculating the standard deviation of both types of projects and normalizing the covariance.

\[
\sigma_1 = \sqrt{\frac{1}{3} [(5 - 4)^2 + (5 - 3)^2 + (8 - 5)^2]} = 2,1602
\]

\[
\sigma_2 = \sqrt{\frac{1}{3} [(4 - 6)^2 + (3 - 4)^2 + (2 - 5)^2]} = 2,1602
\]

\[
\text{corr} = \frac{\text{cov}(X, Y)}{\sigma_1 \sigma_2} = -0.8572
\]

5.2. Evaluation of a portfolio of correlated investments defined by normal distributions

This paper begins to study the effect of correlation in the risk of a portfolio by studying the case of a simple portfolio of two correlated projects represented by normal distribution like

Markowitz proposed [4]. According to [6], the risk of a portfolio is equal do the expected value of the square of the error between the return of the portfolio and the average return of the portfolio. This can be described by:

\[
\sigma_p^2 = E(R_p - \bar{R}_p)^2
\]  

In which \(\sigma_p^2\) is the standard deviation of the portfolio, \(R_p\) the return and \(\bar{R}_p\) the expected return of the portfolio. Defining \(R_p\) and \(\bar{R}_p\) according to the two investments portfolio described we get:

\[
R_p = R_1 + R_2
\]

\[
\bar{R}_p = \bar{R}_1 + \bar{R}_2
\]

With this definitions we can expand equation 20 to:

\[
\sigma_p^2 = E[(R_1 + R_2) - (\bar{R}_1 + \bar{R}_2)]^2
\]

\[
\sigma_p^2 = E[(R_1 - \bar{R}_1) + (R_2 - \bar{R}_2)]^2
\]

\[
\sigma_p^2 = E[(R_1 - \bar{R}_1)^2 + 2 \times (R_1 - \bar{R}_1)(R_2 - \bar{R}_2) + (R_2 - \bar{R}_2)^2]
\]

\[
\sigma_p^2 = \sigma_1^2 + \sigma_2^2 + 2 \times \rho \times \sigma_1 \sigma_2
\]

Equation 23 explains the method to determine the standard deviation of the normal distribution that results from the convolution between any two projects. As can be observed, the standard deviation of the portfolio depends on the standard deviation of each project and on the correlation between the investments, \(\rho\). It is interesting to notice that when \(\rho = 0\), the equation used to calculate uncorrelated portfolios is obtained.

5.3. The effect of correlation in a portfolio

Redefining \(R_p\) and \(\bar{R}_p\) according to equations 24 and 25:

\[
R_p = X_1R_1 + X_2R_2
\]

\[
\bar{R}_p = X_1\bar{R}_1 + X_2\bar{R}_2
\]

where \(X_1\) is the a percentage of investment in project 1 and \(X_2\) in project 2 (the sum of \(X_1\) and \(X_2\) is 1), makes it possible to obtain many different portfolios with just two investments. According to [6], with this definition of \(R_p\) and \(\bar{R}_p\) the standard deviation of the portfolio is given by:

\[
\sigma_p = \sqrt{X_1^2 \sigma_1^2 + (1 - X_1)^2 \sigma_2^2 + 2X_1(1 - X_1) \rho \sigma_1 \sigma_2}
\]  

and represented, according to different values of correlations, by figure 8.
Figure 8: Risk and return according to the % of the investment in X1 for different correlations

It can be observed in figure 8 that there are pairs of X1 and X2 that improve the Risk/return rate more then others and that the closer the correlation is to -1 the more important it is to diversify in order to decrease risk.

The equivalent to this figure, where $R_p$ and $\bar{R}_p$ are defined in equations 21 and 22, in which we can only totally invest in a project or not invest at all, is represented in figure 9.

Figure 9: Possible portfolios for X1 and X2 with different correlations

The values of the standard deviation can be calculated through equation 23 and converted to percentage of the return dividing by 1.4.

5.4. Intraclass correlation

5.4.1 Method to calculate ICC

According to [7] there are 6 types of intraclass correlation. In this paper we will use ICC(1,1) because every result of return of a project was measured in different conditions and was not an average of many measurements. In this case the different sources of variation can are described in table 5.3.

As visible in table 5.3, this method of calculating the intraclass correlation uses two values: BMS and WMS.

Still according to [7], the ICC(1,1) coefficient is given by equation:

$$\rho = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_w^2}$$  \hspace{1cm} (29)

Utilizing table 5.3 and making de adequate calculations we can simplify equation 29 to:

$$\rho = ICC(1, 1) = \frac{BMS - WMS}{BMS + (k - 1)WMS}$$  \hspace{1cm} (30)

5.4.2 Practical example of ICC calculation

Using the model explained above on the projects represented in tables 5.1 and 5.1, utilizing all 9 combinations of possible pairs of projects and assuming each project is represented by the error between the predicted NPV and the real NPV the following matrix can be created:

$$
\begin{pmatrix}
1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 & 3 \\
1 & -1 & -3 & 2 & 2 & -2 & -1 & -3 & -3
\end{pmatrix}
$$

After compiling the information in this matrix, a one way variance analysis can be done to calculate ICC(1,1).

For $j \neq i$. Which, for the case at hand can be transformed into:

$$\sigma_p^2 = \sum_{i=1}^{N} \sigma_i^2 + \sum_{i}^{N} \sum_{j}^{M} X_i X_j \sigma_{i,j}$$  \hspace{1cm} (28)

However, to use this equation one needs to know the correlation between every project in the portfolio. In section 5.1 the correlation between projects of different categories or types were calculated, the next section explains how to calculate the intraclass correlation - or correlation between projects of the same type.
Table 3: Sources of variance in the chosen ICC model [Source: Patrick E. Shrout e Joseph L. Fleiss [1979]]

<table>
<thead>
<tr>
<th>Source of Variance</th>
<th>Square Average</th>
<th>One-way variance analyses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects (between types of projects)</td>
<td>BMS</td>
<td>( k \sigma T^2 + \sigma w^2 )</td>
</tr>
<tr>
<td>Within subjects (within projects of the same type)</td>
<td>WMS</td>
<td>( \sigma w^2 )</td>
</tr>
<tr>
<td>Between judges</td>
<td>JMS</td>
<td>-</td>
</tr>
<tr>
<td>Residual</td>
<td>EMS</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
BMS = k[(\bar{X}_{line1} - \bar{X}_{total})^2
+ (\bar{X}_{line2} - \bar{X}_{total})^2] = k[(2)^2 + (-2)^2] = 9 \times 8 = 72
\]

\[
WMS = 3(1 - \bar{X}_{line1}) + 3(2 - \bar{X}_{line1})
+ 3(3 - \bar{X}_{line1}) + 3(2 - \bar{X}_{line2}) + 3(1 - \bar{X}_{line2})
+ 3(3 - \bar{X}_{line2}) = 12
\]

Table 4: Sources of variance in the example

<table>
<thead>
<tr>
<th>Variance Source</th>
<th>( D_f )</th>
<th>Square</th>
<th>Square Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between subjects</td>
<td>1</td>
<td>72</td>
<td>BMS = 72</td>
</tr>
<tr>
<td>Within subjects</td>
<td>( 2(k - 1) = 16 )</td>
<td>12</td>
<td>WMS = 12/16 = 0.75</td>
</tr>
</tbody>
</table>

ICC(1,1) can be obtained applying expression 30:

\[
ICC(1,1) = \frac{72 - 0.75}{72 + (9 - 1)0.75} = 0.9135
\]

As expected the intraclass correlation is very high. This is true because the return of every project in type 1 was above expectation and every project in type 2 was below expectation.

6. Optimizing the portfolio

After calculating all the necessary correlations and thus being able to calculate the VaR and NPV of any group of projects, this paper tries to optimize the resulting portfolio.

In order to do this, the Matlab script done allows the imposition of certain conditions to the resulting portfolio:

- Minimum NPV - Ensures that the portfolio has at least this value as NPV.
- Minimum VaR - Ensures the portfolio has a VaR above this value.
- Maximum risk - Forces the risk of the resulting portfolio to be the minimum risk of any of the evaluated portfolios.
- Maximum investment - Ensures that the total investment is lower than this value.

Besides all these possible restrictions, it is also possible to apply a utility function, in order to obtain a more customizable optimization.

6.1. Utility Functions

Most companies are not concerned with money in itself but on the utility it provides.

The expected utility criteria is the most accepted form to chose between risk investments (for example, between two lotteries). The expected utility, \( U \), is calculated very similarly to the expected value and can be calculated using the expression:

\[
U = \sum_{i=1}^{n} P_i \cdot U(X_i) = p_1 U(x_1) + p_2 U(x_2) + p_3 U(x_3) \ldots
\]

(31)

In this criteria the company chooses the portfolio with the highest utility instead of the highest NPV. It is interesting to notice that this function is invariance to linear transformations in the portfolio.

Usually, more money brings more utility, however, this does not have to be a linear relationship. Since most investors are risk averse, most utility functions are convex and become linear when the expected returns become higher.

The most common utility functions utilized in investment risk analysis are the following:

- \( U(x) = \ln(x) \) ; \( U(x) = \sqrt{x} \)
- \( U(x) = x^a \) ; \( U(x) = 1 - e^{-ax} \)

However, not all investors are risk averse. Some prioritize the expected value (risk takers) and some are indifferent to risk (risk-neutral). In these cases the utility functions are concave or linear, respectively. Figure 11 represents these types of investors.
7. Results

7.1. Database and sampling

This paper utilized the stratified sampling method. This method was chosen for three main reasons:

- It allowed the divisions of the projects into categories, making it possible to draw conclusions in respect to specific chosen characteristics.
- Considering that the proportions between the sample and the population are maintained this method is as effective a simple random sampling.
- When the information is already divided into classes, it is not needed to join data from different files.

To obtain the proportions to create the sample, 6871 projects of EDP were analysed obtaining the following results:

<table>
<thead>
<tr>
<th>Voltage level</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>High Voltage</td>
<td>1%</td>
</tr>
<tr>
<td>Medium Voltage</td>
<td>15%</td>
</tr>
<tr>
<td>Low Voltage</td>
<td>84%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DRC</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Norte</td>
<td>18.56%</td>
</tr>
<tr>
<td>Porto</td>
<td>12%</td>
</tr>
<tr>
<td>Mondego</td>
<td>12%</td>
</tr>
<tr>
<td>Tejo</td>
<td>30%</td>
</tr>
<tr>
<td>Lisboa</td>
<td>12%</td>
</tr>
<tr>
<td>Sul</td>
<td>6%</td>
</tr>
</tbody>
</table>

The chosen sample was composed of 42 projects per DRC, totalling 252 projects. From these, 180 were of low voltage (71.5%), 54 of Medium Voltage (21.5%) and only 18 of high voltage (7%). The difference between the relative quantity of projects in the high voltage class is due to the fact that at least 3 projects per category were needed to calculate correlations between classes.

7.2. Correlations

7.2.1 Correlation in respect to DRC

Applying the method described in section 5.4 the intraclass correlation obtained after dividing the projects into 6 groups according to their DRC was:

$$ICC = 0.00057 \approx 0 = \rho_{11} = \rho_{22} = \rho_{33} = \rho_{44} = \rho_{55} = \rho_{66}$$

The Pearson correlations obtained are in Table 6.

As can be observed, the results show that there are no significant correlations.

It is important to notice that the intraclass correlation value is negative but should be read as 0. According to [8], when estimating the ICC, the results can be negative but there are no two events more non-correlated than two independent events.

7.2.2 Correlation in the level of voltage

Applying the same method as above in respect to the level of voltage of each project, the Intraclass and Pearson correlations were:

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Low</th>
<th>Medium</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>-0.000006</td>
<td>-0.0806</td>
<td>-0.0439</td>
</tr>
<tr>
<td>Medium</td>
<td>-0.0806</td>
<td>-0.000006</td>
<td>0.0576</td>
</tr>
<tr>
<td>High</td>
<td>-0.0439</td>
<td>0.0576</td>
<td>-0.000006</td>
</tr>
</tbody>
</table>

$$ICC \approx 0 = \rho_{11} = \rho_{22} = \rho_{33} = \rho_{44} = \rho_{55} = \rho_{66}$$
The conclusions are the same as for the DRC division. These were as expected because most of the deviations in the NPVs calculated are mostly due to the errors in investment and that type of error has little correlation to the region or voltage of the project.

7.3. Portfolio Optimization

After knowing that the correlation between the projects in respect to the variables analysed were 0, this paper focused on optimizing a portfolio of non-correlated investments.

The results of applying the methods described in section 6 to real portfolios showed that the investment is the criteria with the biggest impact in the resulting portfolio. These tests also confirmed that without that criteria, most optimal portfolios are made of the biggest number of projects available in order to minimize risk, unless one of them has a very high risk.

8. Conclusions

This paper had the objective of finding a method to quantify the risk of a group of projects allowing to chose the best available portfolio for each company.

Initially it was explained how to evaluate each project individually. From this study it was concluded that the NPV criteria was suited for this specific risk analyses. It was verified that the NPV was calculated through $B_{\text{losses}}$ and $B_{\text{END}}$ and that these were dependent on $\alpha$ and $I$. These variables are uncertain and forced the calculation of the VaR of each project. The conclusions of this study were that the VaR was the NPV calculated for a new growth rate, $\alpha^*$, and Investment, $I^*$, according to equations 7 and 8.

Posteriorly, this paper focused on analysing the evolution of risk when participating in various projects simultaneously. First, when studying portfolios of non-correlated projects it was concluded that the risk decreases with the increase in the number of similar projects of the portfolio. It was also important to notice how as the number of projects increased the resulting probability density function was more similar to a normal distribution and that, for portfolios made of more than 20 projects, the approximation of each project to a normal distribution was valid.

In section 5 the risk analyses was expanded for a portfolio of correlated investments. It was observed that the correlation between projects was a very important factor in the total risk of a portfolio. It was also concluded that to utilize the Pearson correlation concept the projects would have to be grouped according to a certain characteristic. After applying the proposed method to calculate the Pearson and Intraclass correlations it was concluded that the projects were non-correlated in respect to the two characteristics analysed.

It is very important to empathize the fact that this method of calculating the correlations can be applied to any type of project and can analyse any desired characteristic.

The results of the tests using non-correlated projects verified that the risk of a portfolio decreases with the increase in the number of projects, except for the cases where one project had a very high individual risk.

In the future, this work can be expanded by analysing different characteristics (besides DRC and voltage level) and by substituting the current algorithm with a genetic algorithm that is faster and has the capacity to process bigger portfolios.

The development of this methodology of risk analyses in projects of network distribution facilitates the work of planers and diminishes the resources needed, improving, at the same time, the quality of the decisions taken.

References

[3] M. I. Verdelho, “Metodologias de anlise de risco de projectos de investimento em redes de dis-


