Three dimensional modelling of wave set-up and longshore currents

The effect of turbulence closure models

D. Rodrigues[1]

Abstract

The purpose of this study is to analyse wave-current interactions, specially the generation of longshore currents triggered by an oblique incidence of a wave field in a long strait coast. A numerical tool was used for modelling the longshore currents. This tool consists in a fully coupled system, using the classical formulation of radiation stress (Longuet-Higgins and Stewart, 1962), between the three dimensional hydrodynamic model TELEMAC-3D (Hervouet, 2007) and the third generation spectral wave model TOMAWAC (Benoit et al., 1996). The hydrodynamic model TELEMAC-3D solves the Reynolds Averaged Navier Stokes Equations (RANS) by the finite elements method and the vertical discretization in the water column is achieved by the "sigma" coordinate. To solve the RANS equations it is necessary to use closure turbulence models to parameterize the Reynolds stress tensor and thus make the system complete. Therefore, a sensitivity analysis was done to a number of different closure turbulence models, which are embedded in TELEMAC, followed by a comparison between numerical results and experimental data obtained at the Large Scale Sediment Transport Facility (LSTF) in Vicksburg, USA. The variables compared were the longshore velocity, the mean surface level and the significant wave height.

1. INTRODUCTION

The understanding of the coastal hydrodynamic processes is essential for several applications, in particular the design of coastal protections and harbour sheltering structures. Among these processes, the combined effect of waves and currents has recently been the subject of different studies.

Longuet-Higgins and Stewart (1962, 1964) introduced the concept of radiation stress which represents the excess of momentum flux due to the waves.

In order to study the interactions between waves and currents, a fully coupled system between the three dimensional hydrodynamic model TELEMAC-3D (Hervouet, 2007) and the spectral wave model TOMAWAC (Benoit et al., 1996) was used. Turbulence modeling is a key issue in the hydrodynamic models. Accordingly, three closure turbulence models were tested. First, a constant viscosity model was applied, which is based on the Boussinesq (1877) hypothesis. This turbulence model assumes a constant turbulent viscosity and it is the simplest closure model. The second closure turbulence model is based on the gas kinetic theory and on the mixing length concept. This model was

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[1] Instituto Superior Técnico, Ulisboa, Av. Rovisco Pais, 1, 1049-001 Lisboa, Portugal, daniela.rodrigues@ist.utl.pt

developed by Prandtl (1925). Like the previous one, it is also a zero equations model. Finally, the k-ε two equations model was used. This model is based on developing evolution equations for the turbulent quantities k and ε.

The numerical simulations using the Longuet-Higgins and Stewart (1962, 1964) approach were compared with experimental data from the Large Scale Sediment Transport Facility (LSTF) at Vicksburg, USA (Hamilton and Ebersole, 2001).

2. NUMERICAL MODELS

2.1. TELEMAC-3D

2.1.1. Governing equations

The 3D hydrodynamic circulation model used in the present study was TELEMAC-3D (Hervouet, 2007), which is incorporated in the TELEMAC system, developed at EDF R&D, Chatou, France. It is an open source software. This model solves complex 3D free surface flows and can be employed from large areas (ocean basin scale) to smaller areas (estuary scale).

TELEMAC-3D solves the RANS (Reynolds Averaged Navier Stokes) equations by the finite elements method. The algorithm of this numerical model is divided into three main steps. In a first stage, the advected velocities are computed by solving the advected terms in the momentum equations. After, and taking into account the turbulent viscosity terms and the source terms in the momentum equations, new velocities are calculated. At a third stage, the water depth is computed through the vertical integration of the continuity equation and the momentum equations, but only with the pressure terms associated with the presence of the free surface.

For an incompressible flow, TELEMAC-3D solves the continuity (1) and the momentum equations (2), assuming the hydrostatic pressure hypothesis:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-g\frac{\partial Z_s}{\partial x}+\nu\Delta(u)+\ F_x$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-g\frac{\partial Z_s}{\partial y}+\nu\Delta(v)+\ F_y \qquad (2)$$

2.2. TOMAWAC

2.2.1. Wave action balance equation

The wave model used in this study was TOMAWAC (release 6.2) (Benoit et al, 1996). This model is incorporated in the TELEMAC system, developed at EDF R&D, Chatou, France. The range of applicability of this model extends from deep to shallow waters.

The model solves the wave action (N) conservation equation in Cartesian or spherical spatial coordinates (3).

$$\frac{\partial N}{\partial t} + \dot{x}\frac{\partial N}{\partial x} + \dot{y}\frac{\partial N}{\partial y} + \dot{k}_x\frac{\partial N}{\partial x} + \dot{k}_y\frac{\partial N}{\partial y} = Q\left(x\,, y\,, k_x\,, k_y\,, t\right) \tag{3}$$

k_x and k_y are the wave number vector components and the *dot* over the variables symbolizes the time derivative. The term Q represents the source and sink terms that can be taken into account in this model: source term from wind-driven wave generation, sink terms from whitecapping, bottom friction, depth induced wave breaking and the non-linear quadruplet and triad interactions. In the simulations showed in this work only depth induced wave breaking and bottom friction were considered.

3. LABORATORY CASE AND MODEL SETUP

The laboratory facility has been constructed at the U.S. Army Engineer Research and Development Center´s Coastal and Hydraulics Laboratory, in Vicksburg, USA. This facility intends to reproduce certain surf zone processes found on a long straight natural beach (Hamilton and Ebersole, 2001) like sand transport and beach changes processes. For the sediment transport experiments, it is necessary a method for establishing the proper longshore current. With the purpose to recreate the wave induced longshore currents, this facility is equipped with four wave drivers and an external recirculation system that allows the establishment of longshore uniform currents.

The laboratory basin has approximately 30 m in the cross-shore direction by 50 m in the alongshore direction. A brushed concrete beach, with 21 m in the cross-shore direction by 31 m in the alongshore direction, was built with a 1:30 slope developed over 18 m and 1:18 slope at the toe of the beach. The wave drivers were at 0.667 m depth and the concrete beach was characterized by straight and parallel contours. A series of tests were carried out with the generation of regular and irregular waves. The sea state, for the irregular waves case, was characterized by H_{m0} = 0.225 m and T_p = 2.5 s. Measurements of surface elevation and velocity were obtained by ten capacitance-type wave gauges and ten Acoustic-Doppler Velocitymiters (ADVs), respectively. The ADV's were set at elevations approximately one third of the water depth above the bed.

According to the LSTF dimensions, an 18 x 51 m rectangular grid was considered, with Δx = 0.3 m and Δy = 0.8 m equally spaced. It was also considered the time step equal to 0.4 s. TELEMAC-3D was configured with 10 horizontal levels along the depth.

Regarding to the wave model, it was considered the same wave parameters forced in LSTF experiments (H_{m0} = 0.225 m and T_p = 2.5 s).

The coupling system between TELEMAC-3D and TOMAWAC is done using the radiation stress concept (Longuet-Higgins and Stewart, 1962), which represents the wave forcing terms in the hydrodynamic model. By using a direct coupling between this models it is possible to represent wave current interactions in both directions: TELEMAC-3D transfers to TOMAWAC the updated values of current velocities, water depths, the vertical z levels and the Nikuradse roughness defined by the user. TOMAWAC on its turn solves the wave action density conservation equation with reference to those current and water depth values and returns to TELEMAC-3D the updated values of the wave forcing terms. Furthermore, this coupling is possible if both models run with the same horizontal mesh discretization, the same simulation time length and if the time steps set for the two simulations is equal or multiple of each other.

4. ANALYSIS OF THE RESULTS

With the purpose of analyzing the impact of the closure turbulence models in the results of the coupling between TELEMAC-3D (Hervouet, 2007) and TOMAWAC (Benoit et al., 1996), three closure turbulence models were studied. In a first step, it was tested the simplest model with the vertical turbulent viscosity set to a constant value, and the results were improved through the sequence of the following parameters: horizontal turbulent viscosity, boundary directional spread and bottom roughness. After, and take into account the best parameterization achieved for this model with respect to the referred parameters, and only in the vertical direction, were tested two types of turbulence closure models: a vertical mixing length model (with Prandtl parameterization) and the k-ε model. Inside of each closure turbulence model two approaches to reproduce the effects of depth induced wave breaking were tested: Thornton and Guza´s model (1983) and Battjes and Janssen´s model (1978).

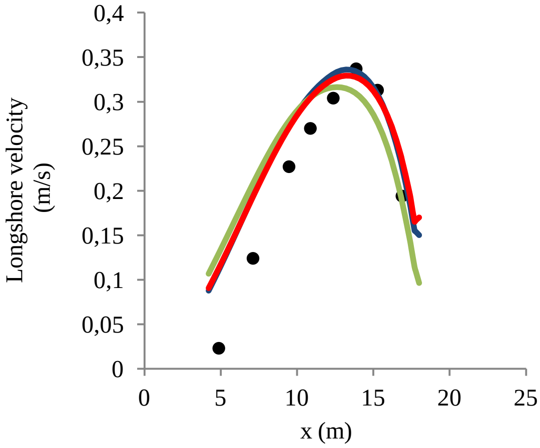
The variables that were studied were the longshore velocity, the mean surface level and the significant wave height. As stated above, the numerical results were compared with the experimental data from LSTF.

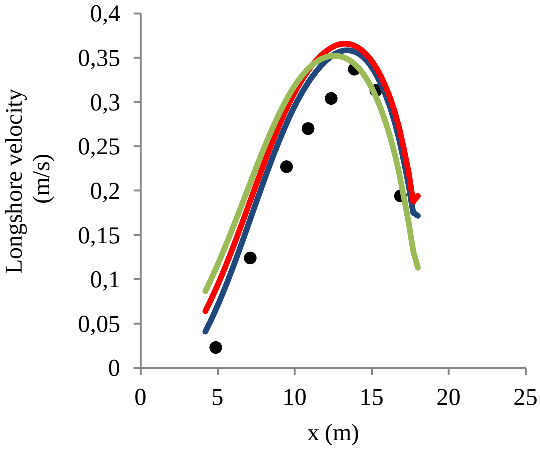
Table 1 presents the best values achieved for the different parameters. It was also concluded that the vertical turbulent viscosity can assume any value because it does not influence the results. This conclusion is a consequence of the classical formulation of Longuet-Higgins and Stewart (1962) implemented in TELEMAC-3D to reproduce the vertical variation of the radiation stress. In this approach, the radiation stress is uniformly distributed along the vertical.

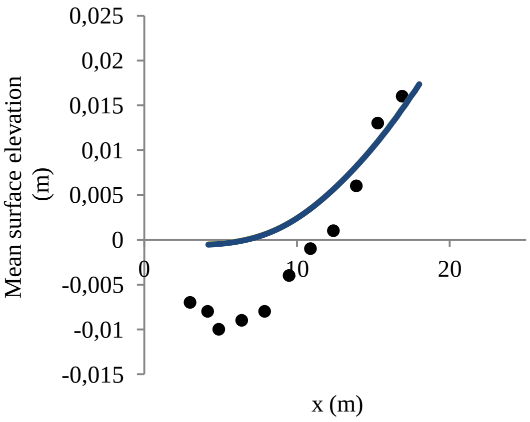
Horizontal eddy viscosity, ν_h (m^2/s)	0,05
Boundary directional spread, *s* (-)	90
Bottom roughness (m)	0,0001

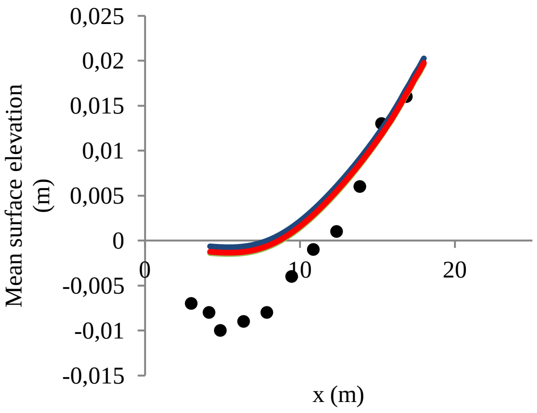
Table 1- Best values achieved for the different parameters.

Figure 1 shows the comparison between the three closure turbulence models. On the left side are presented the results for the combination of the three turbulence models and the formulation of Thornton and Guza (1983). On the right side are presented the results for the combination of the three turbulence models and the formulation of Battjes and Janssen (1978).









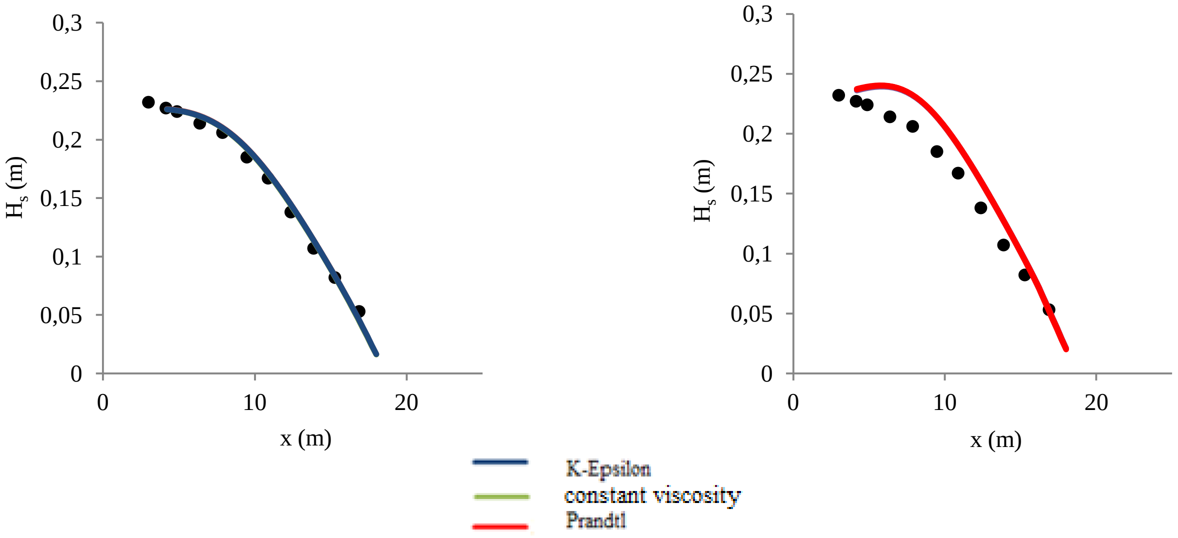


Figure 1- Cross-shore evolution of the longshore velocity at one third of the depth, mean surface level and cross-shore evolution of the significant wave height. Comparison between numerical results and LSTF measurements for the different closure turbulence models. In the left panel the formulation of Thornton and Guza (1983) was chosen and in the right panel the formulation of Battjes and Janssen (1978) was implemented. The line represents the numerical results and the dots represent the experimental data.

As shown in Figure 1, the mean surface level and the significant wave height are practically independent of the choice of the closure turbulence model, for the two wave breaking formulations.

The differences arise when the formulation of Battjes and Janssen (1978) for the representation of depth induced wave breaking is imposed. In this case, it is observed that the wave has a larger shoaling before breaking, triggering a larger wave set-up which is slightly closer to the experimental data than in the case of Thornton and Guza (1983) formulation. Regarding to the cross-shore evolution of the longshore velocity at one third of the depth, it is observed that the k-ε model is, among the different closure turbulence models, the model that best fits the experimental data. As the k-ε model is a more advanced closure turbulence model it reproduces a little better this variable, especially the value of the maximum magnitude (approximately 0.35 m/s). However, a number of discrepancies are present in the vicinity of the offshore region. These differences could be caused by a spurious longshore flow that internally recirculated in this region.

The beach profile is depicted in Figure 2 as well as the different study sections. In order to make a more detail analysis on the vertical structure of the flow, the cross-shore evolution of the longshore velocity profiles are presented in Figure 3 for different positions along the beach. It is also shown the comparison between the numerical results and the experimental data from the LSTF, applying the parametrization presented in Table 1.

On the left panel are presented the results for the combination of the three turbulence models and the formulation of Thornton and Guza (1983). On the right panel are presented the results for the combination of the three turbulence models and the formulation of Battjes and Janssen (1978).

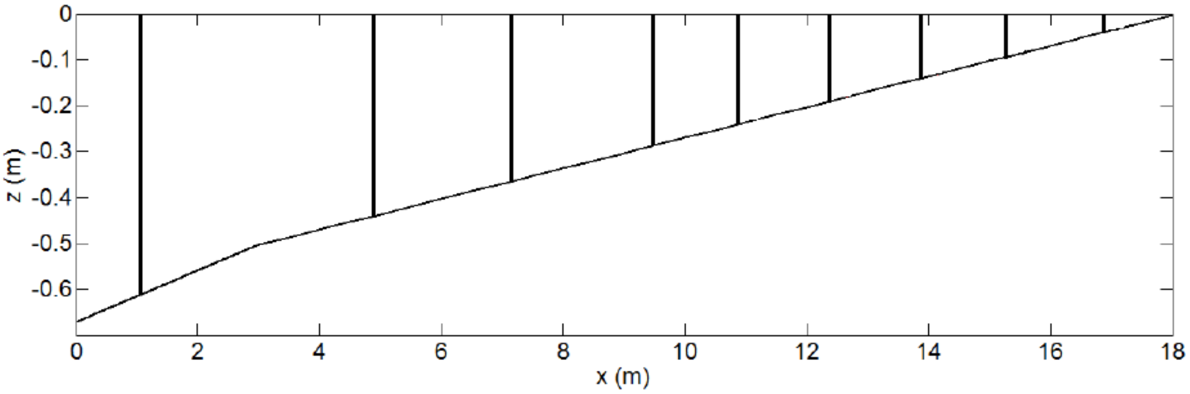
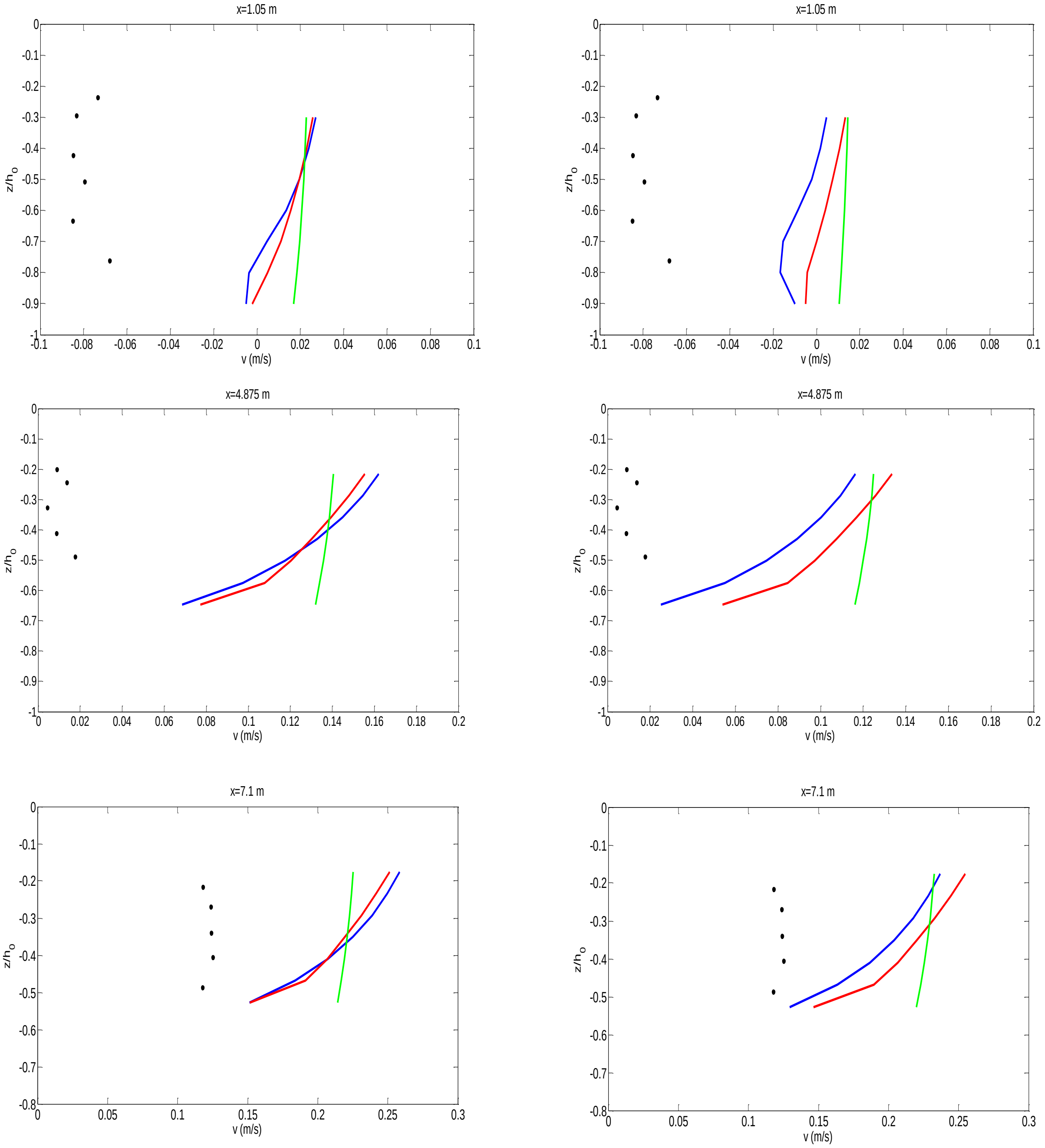
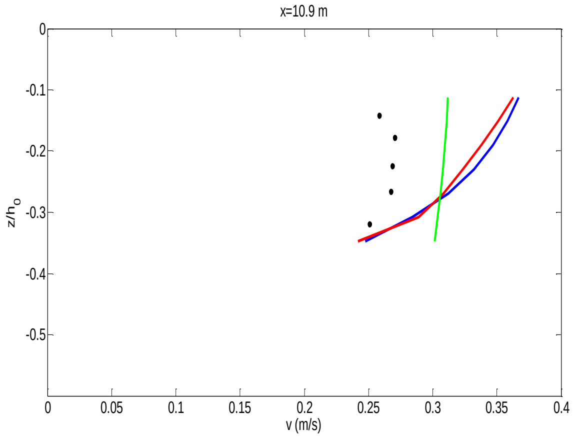


Figure 2 - Beach profile and the different study sections.

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z/h0
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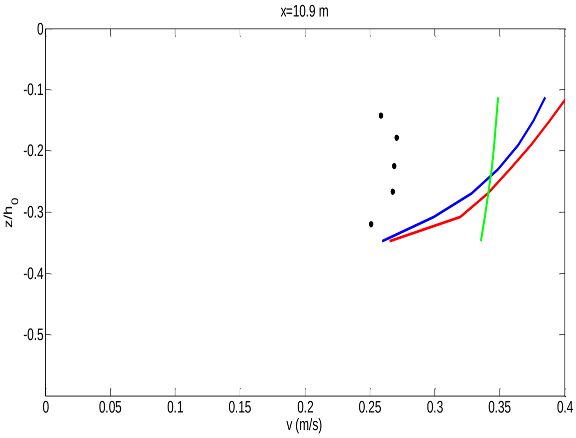
-0.1
-0.2
-0.3
-0.4
-0.5
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
v (m/s)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z/h0
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-0.1
-0.2
-0.3
-0.4
-0.5
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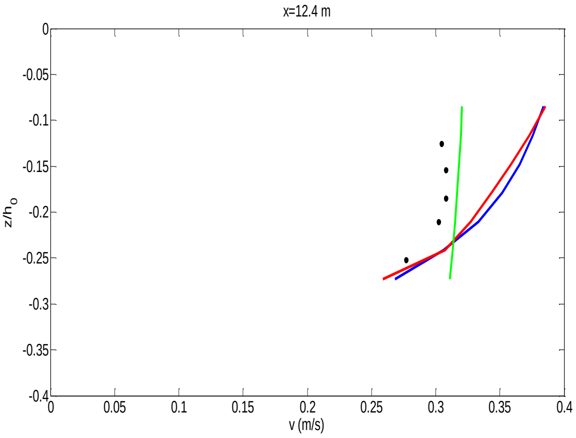
v (m/s)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z/h0
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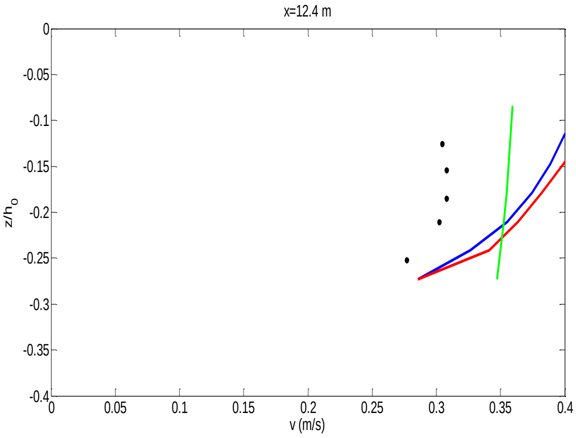
v (m/s)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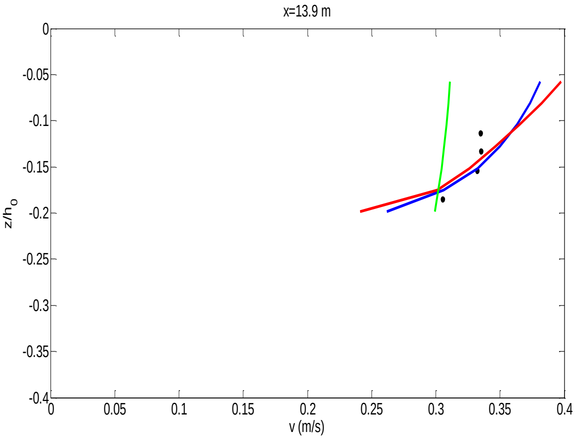
v (m/s)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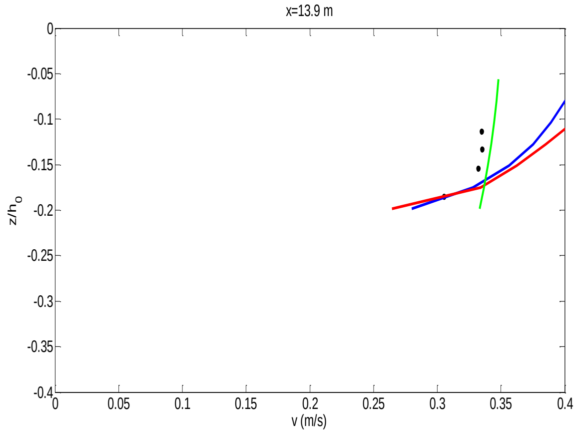
v (m/s)


x=13.9 m
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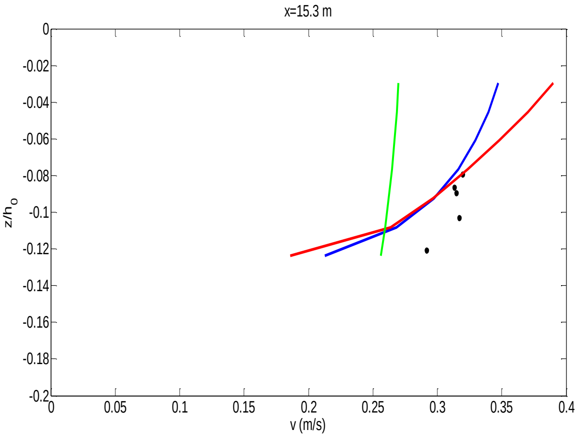
v (m/s)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-0.08
-0.1
-0.12
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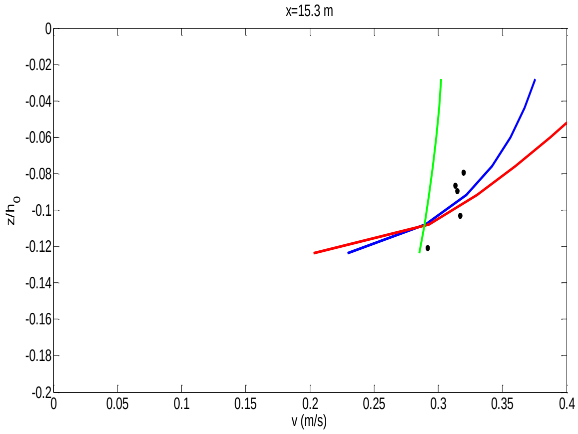
-0.2
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4
v (m/s)


x=15.3 m
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v (m/s)

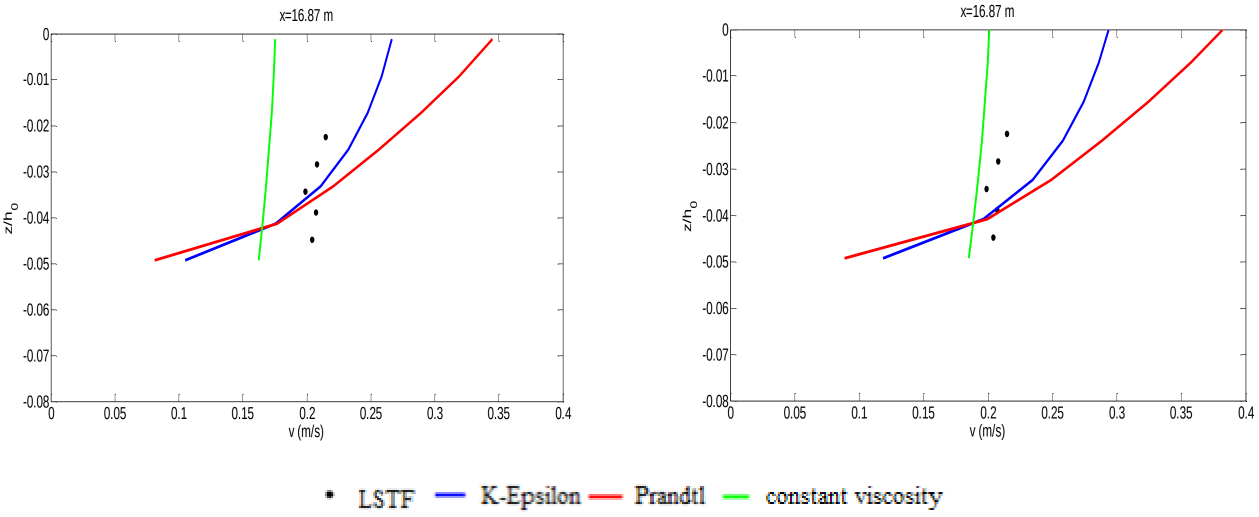

Figure 3 – Beach profile. Comparison between numerical results (line) and the experimental data from the LSTF (dots). On the left panel are presented the results for the combination of the three turbulence models and the formulation of Thornton and Guza (1983). On the right panel are presented the results for the combination of the three turbulence models and the formulation of Battjes and Janssen (1978). h_0 represents the offshore water depth.

As can be seen (Figure 3), the vertical profile of the longshore velocity is sensitive to the choice of the closure turbulence model. Depending of the chosen closure turbulence model, the turbulent viscosity is differently distributed along the vertical which influences the velocity profile.

It is verified that the k-ε model is, in general, the closure turbulence model that better fits the experimental data, except in the offshore locations, for the two wave breaking formulations.

Moreover, it is observed that the numerical results deviate much from laboratory data in the offshore ADV´s. The found differences are due to the internal offshore recirculation, previously mentioned. It is also noted that the velocity of the current is always positive and until x = 12.4 m all closure turbulence models overestimate the magnitude of the velocity.

As regards to the two breaking formulations, there is no clear conclusion to reach. Sometimes the formulation of Battjes and Janssen (1978) is marginally better and other times the formulation of Thornton and Guza (1983) is performing better.

Relatively to the vertical structure of the cross-shore velocity, there is no variation of the cross-shore component of the current velocity, u, due to the simple formulation used.

Thus, it is concluded that the coupling system between the two models satisfactorily reproduce the experimental data.

5. CONCLUSIONS

In order to get a full description of the wave-currents interactions at a regional scale, it is fundamental to couple directly a hydrodynamic model with a spectral wave model. In the present study it was coupled the hydrodynamic model TELEMAC-3D (Hervouet, 2007) with the spectral wave model TOMAWAC (Benoit et al., 1996), through the radiation stress concept (Longuet-Higgins and Stewart, 1962, 1964). The latter was uniformly distributed along the vertical, as a first and simplified option to get a 3D description over the water column.

The hydrodynamic models that are typically based on RANS equations depend on the type of the closure turbulence model adopted. Thus, three closure turbulence models were studied here and two depth-induced wave breaking formulations, Thornton and Guza (1983) and Battjes and Janssen (1978), were jointly tested. In a first stage, the constant viscosity model was used. Secondly, it was employed a vertical mixing length model (with the Prandtl parameterization) and finally it was tested the k-ε model.

The capabilities of the 3D coupled system were tested with comparisons with experimental data obtained at the Large Scale Sediment Transport Facility (LSTF) (Hamilton and Ebersole, 2001).

Regarding to the results, it was found that the mean surface level and the significant wave height are almost independent of the choice of the closure turbulence model, for both breaking formulations. Furthermore, it is also observed that the Battjes and Janssen (1978) approach induces a larger shoaling before the wave breaking, and thereby the wave set-up is larger, fitting better the experimental data, compared to the Thornton and Guza wave breaking model (1983). As regards to the evolution of the longitudinal velocity, the k-ε model is the closure turbulence model that better reproduces the experimental data. However, in all closure turbulence models, and for the two breaking formulations, is visible in the offshore region an underestimation of the laboratory data with respect to longshore velocity. Such evidence can be connected to an internal recirculation that occurs within this area.

A detailed analysis was carried out in relation to the vertical longshore velocity profiles. It was concluded that the vertical profile depends of the choice of the closure turbulence model. For both breaking formulations, the k-ε model is, and once again, the closure turbulence model that better fits the experimental data, except in the offshore region. Analogously, the above mentioned internal recirculation in the offshore zone causes the largest discrepancies.

Concerning to the two breaking formulations, it is not possible to take clear cut conclusions about the superiority of one over the other.

In general, the coupling system between the two models proved to be effective since the results obtained were satisfactory.

To briefly summarise, and in order to improve the results, the application of more sophisticated closure turbulence models (two equations models) are recommended since they provide a more complete tool for the numerical simulation of the turbulent flows.

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