

Digital Control of a DC-DC Converter

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Abstract — The goal of this project is to implement the digital control of a DC-DC converter, in this case a Buck since it's meant to decrease the input voltage of 3.3 V to 1.8 V at the output. The control of the output voltage is achieved through a non-stop monitoring of its value followed by the adjustment of the switch duty cycle. Every time the output voltage is below 1.8 V the switch duty cycle is increased so that the output voltage is increased as well. The same way, every time the output voltage is above 1.8 V the switch duty cycle is decreased so that the output voltage is decreased as well. It's also important that the DC-DC converter is independent from its load. So all the simulations of this circuit are made considering the coupling/decoupling of a load which is controlled by a 5 kHz switch. This way the robustness of the system is improved.

I - INTRODUCTION

In the last decades we have witnessed a huge growth in development of technology, changing completely our lifestyle. The devices used are meant to become more efficient, smaller and portable. The portability is achieved by using batteries which allow the devices to work while unplugged. However, those devices are designed with several different blocks that might need different input voltages. Since implementing several batteries would be infeasible, one way to do it is by using DC-DC converters.

DC-DC converters are electronic circuits that convert a source of Direct Current (DC) from one voltage level to another. There are several different topologies to implement a DC-DC converter depending on the purpose to use it. The main differences between those topologies rely on the output voltage value (V_o) being higher or lower than the input voltage value (V_i).

This work is about implementing a digital control for a Buck converter, which is meant to decrease the voltage level from the input to the output. The V_o must achieve the value of 1.8 V having an input voltage $V_i = 3.3$ V as the converter operates at a 2 MHz frequency.

The control is made by managing the switch to be ON or OFF as we need to respectively raise or lower the output voltage which is always being monitored.

In order to keep the frequency stabilized, the control system acts on the duty cycle of the switch. The higher the duty cycle is the higher the output voltage gets and vice versa.

The whole circuit is designed and simulated using the software Simulink – Matlab which incorporates a control block programmed in Verilog language. The Verilog

programming is made in Active-HDL environment. This creates a digital control block which is latter on instantiated on Simulink.

Sliding Mode Control (SMC) is used for the control of the system since it's a robust control technique suitable for nonlinear systems with high sensitivity to disturbances and plant uncertainties.

II - DC-DC CONVERTER

There is a wide range of topologies to implement a DC-DC converter. These might have different goals, differing from each other is aspects such as simplicity, specificity, robustness, efficiency and effectiveness.

Due to its generality and simplicity of concept the three main ones are as follows:

- Boost

This topology allows the output voltage V_o to become higher than the input voltage V_i .

Figure 1 illustrates the typical circuit of a boost converter.

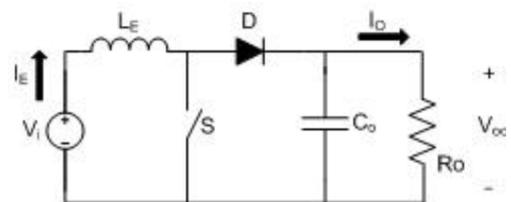


Figure 1 – Boost converter.

- Buck

Unlike the boost, the buck is a topology that is used when V_o must become lower than V_i .

Figure 2 illustrates the typical circuit of a buck converter.

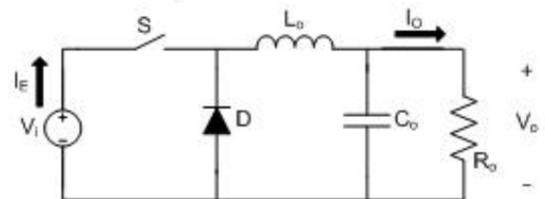


Figure 2 – Buck converter.

- Buck-Boost

As its name suggests, the buck-boost topology allows V_o to become either higher or lower than V_i .

Figure 3 illustrates the typical circuit of a buck-boost converter.

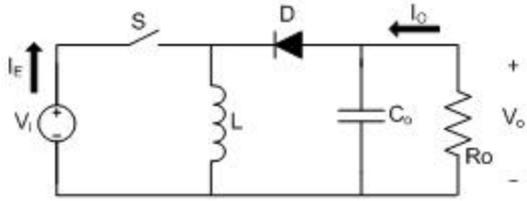


Figure 3 – Buck-Boost converter.

Since the goal of this project is to force V_o to be always lower than V_i , the obvious choice is to implement a buck converter.

As it can be seen in Figure 2, the buck converter is composed by a DC voltage source, a switch, a capacitor, an inductor, a diode. There is also represented the load resistance.

The purpose of those components are as follows:

- DC Voltage source: Provides a DC voltage input to the circuit;
- Switch: As it switches between ON and OFF it is coupling/decoupling the voltage source to the circuit. By doing that it allows the V_o value to become respectively higher or lower.
- Capacitor: Stores energy in form of voltage and helps to control the ripple at the load;
- Inductor: Stores energy in form of a magnetic field and prevents current spikes;
- Diode: The free-wheeling diode provides a way to drain the inductor current (i_L) thus protecting the switch.

Sometimes instead of a free-wheeling diode, another switch. This implementation provides shooting through and lower the power consumption that is caused by the diode. However this solution implies a higher complexity that is not worth it considering the purpose and specifications of this project.

Table I shows the main characteristics of the components used in this project, where the parameters V, R, C and I represent respectively the voltage, resistance, capacitive and inductive properties of each component.

Table I

Component	V	R	C	I
DC V source	3.3 V	-	-	-
Capacitor	-	10 mΩ	22 μF	-
Inductor	-	10 mΩ	-	10 μH
Diode	$V_f = 0.3$	10 mΩ	250 pF	-
Switch	-	$R_d = 10 \text{ m}\Omega$ $R_{on} = 100 \text{ m}\Omega$	-	-

A buck converter can operate in two different modes, the Continuous Conduction Mode (CCM) and the Discontinuous Conduction Mode (DCM).

II.1 - CONTINUOUS CONDUCTION MODE

If the inductor current (i_L) is always above zero during the whole switching period, the converter is said to be operating in CCM.

The energy stored in an inductor is given by

$$E = \frac{1}{2} \cdot L \cdot i_L^2 . \quad (2.1)$$

In order to obtain the voltage at the inductor terminals (v_L), comes

$$v_L(t) = L \cdot \frac{di_L(t)}{dt} \Rightarrow \quad (2.2)$$

$$i_L(t) = i_L(0) + \frac{1}{L} \int_0^t v_L(\tau) \cdot d\tau.$$

Considering both states (ON and OFF):

$$\begin{cases} v_L(t) = V_{IN} - v_o(t), & 0 < t < DT \rightarrow ON. \\ v_L(t) = -v_o(t), & DT < t < DT \rightarrow OF. \end{cases} \quad (2.3)$$

By replacing (2.3) in (2.2) comes:

$$0 = \int_0^{DT} (V_{IN} - v_o(\tau)) d\tau + \int_{DT}^T -v_o(\tau) d\tau \Leftrightarrow \quad (2.4)$$

$$\int_0^{DT} v_o(\tau) d\tau + \int_{DT}^T v_o(\tau) d\tau.$$

Dividing (2.4) by T it's possible to obtain the average values for V_o and V_{IN} .

Once in steady state regime, the average value of V_o matches the desired reference value. Assuming V_{IN} to be constant, the relation between V_{REF} and V_{IN} is given by:

$$V_{IN} \cdot D = V_{REF} \Leftrightarrow D = \frac{V_{REF}}{V_{IN}} . \quad (2.5)$$

As it can be verified by (2.5), where the duty cycle is defined, when in CCM the relation between V_{REF} and V_{IN} is linear, as shown in Figure 4.

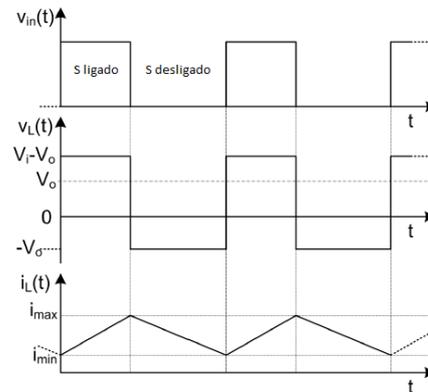


Figure 4 – Buck operating in CCM.

II.2 - DISCONTINUOUS CONDUCTION MODE

A buck converter is said to be operating in DCM when during a portion δ of the period T the inductor current i_L reaches the value of zero. When that happens the inductor becomes completely discharged

Since the converter is still operating in steady state, the energy stored at the very beginning of a period is the same as it was at the end of the previous period. In this case that corresponds to the value of zero. This means the average value of voltage across the inductor terminals (v_L) is also equal to zero.

This way

$$(V_{IN} - V_0) \cdot DT - V_0 \cdot T \cdot \delta = 0. \quad (2.6)$$

So the portion of the period δ is given by

$$\delta = \frac{V_i - V_0}{V_0} D. \quad (2.7)$$

The average value of the inductor current is

$$I_L = \frac{1}{T} \left(\frac{1}{2} I_{LMAX} DT + \frac{1}{2} I_{LMAX} \delta T \right) \Leftrightarrow \quad (2.8)$$

$$I_L = \frac{I_{LMAX}(D+\delta)}{2} = I_0.$$

Where

$$I_{LMAX} = \frac{V_i - V_0}{L} DT. \quad (2.9)$$

Replacing (2.7) and (2.8) in (2.9) yields

$$I_{L0} = \frac{(V_i - V_0)DT \cdot \left(D + \frac{V_i - V_0}{V_0} D \right)}{2L} \Leftrightarrow \quad (2.10)$$

$$V_0 = V_i \frac{1}{\frac{2LI_0}{D^2 V_1 T} + 1}.$$

Equation (2.10) makes it possible to obtain the value of the output voltage for any given load while operation in DCM.

II.3 - TRANSITION BETWEEN CCM AND DCM

In order to choose the right equation to define the duty cycle, one has got to know the limit for which any of those equations are valid. This limit is given by the instant t where the inductor current reaches the value of zero.

In this situation becomes

$$I_0 = \frac{D - D^2 \cdot T \cdot V_i}{2L}. \quad (2.11)$$

II.4 - BUCK CIRCUIT EQUATIONS

Since the output voltage is the parameter to control, it's crucial to obtain an expression defining V_0 as function of the circuit components.

Figure 5 recalls the circuit shown in Figure 2 with emphasis on the currents i_L , i_C and i_R .

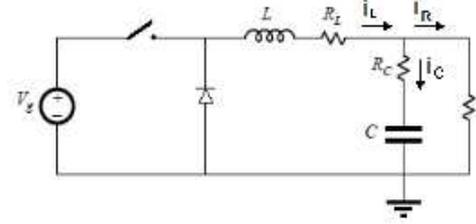


Figure 5 – KCL in buck circuit.

From Figure 5 it becomes clear that a simple KCL analysis does the trick.

$$i_L = i_C + i_R \Leftrightarrow \quad (2.12)$$

$$I_0 + \int_T \frac{v_i - v_0}{L} dt \Leftrightarrow i_L = C \frac{dv_0}{dt} + \frac{v_0}{R}.$$

From (2.12) one can obtain a second degree differential equation from which it becomes possible to calculate v_0 just by knowing the circuit components.

$$v_{in} = v_0 + \frac{R}{L} \frac{dv_0}{dt} + LC \frac{d^2v}{dt^2}. \quad (2.13)$$

IV – SLIDING MODE CONTROL

There are several control algorithms suitable for regulation of the output voltage of a DC-DC converter. Since these are nonlinear devices it's not a good approach to apply a linear control technique. However it's possible to obtain quasi-linear models on a small signal equilibrium point [1]. Even though the controllers based on these techniques are usually simple and easy to implement, they are also really dependent of the system parameters around the equilibrium point. Whenever there is a variation of those parameters the system struggles to take them into account often leading to instability issues [2].

Another possibility is the use of multiloop control techniques. The results obtained are usually good but due to its complexity they are hard to implement, specially to higher order topologies [3],

To make a control technique suitable for DC-DC converters it must be able to deal with its nonlinearity as well as the load changes, making sure the system is stable and provides a fast transient response.

Since a buck converter is a variable structure (due to the switch changes) the sliding mode control presents itself as an appropriate choice [4].

SMC is nonlinear control technique whose origin relies in the ex-URSS by the decade of 1950's. It is a Variable Structure Control (VSC) due to its discontinuities. Those discontinuities derive from the transition between the two states on which the SMC acts in order to achieve the desired balance. The frontier where the control law switches between both states is called a sliding surface.

SMC overcomes the difficulties presented by the quasi-linear model techniques. When compared with those, the SMC reveals to be particularly useful under circumstances where it's necessary to deal with disturbances and uncertainties [5].

In order to obtain the desired response, SMC changes the controller structure in response to system changes. That is reached through manipulation of the switch causing it to force the system to slide to the desired sliding surface. The systems control regime while in the sliding surface is called sliding mode. Once on this regime the system remains insensitive to parameters changes and disturbances [6].

Another advantage to the use of SMC instead of another robust control techniques is its simplicity either in analog or digital domain.

IV.1 – SMC IMPLEMENTATION ON A BUCK

The buck parameter that is to be controlled is the output voltage (v_0). That's achieved by monitoring it all the time while acting on the switch so that v_0 slides to the desired surface.

More than just monitoring v_0 it's also possible to monitor i_L as well [7]. In this case one can also add a PID controller in order to speed up the convergence. However this option requires more resources whose cost is not worth it according to the specifications of this project.

Thus SMC is the chosen option to keep monitoring v_0 , compare it to the reference value where v_0 is wanted to converge to given by (v_{REF}) and acting on the switch.

Now let us consider the expression (3.1) where x represents the system state vector, μ represents the control entry applied to the system, while A and B are the system control matrixes which are assumed to be fully controllable.

$$\dot{x} = A \cdot x(t) + B \cdot \mu \quad (3.1)$$

Once in matrix form comes

$$dx = [A]x + B\mu. \quad (3.2)$$

To keep up with the difference between the value measured in v_0 and the desired one v_{REF} comes

$$\tilde{x} = x - x_{REF} \quad (3.3)$$

In order to maintain the state variable on the desired value, that is $\tilde{x} = 0$, one must define the sliding surface on which the state variable will slide until it reaches x_{REF} .

The sliding surface is defined by

$$S(t) = \{x | s(x, t) = 0\} \quad (3.4)$$

The expression $s(x, t)$ must be chosen so that it is exponentially stable.

Equation (3.5) does the trick, leading \tilde{x} to zero.

$$S(x, t) = \left(\frac{d}{dt} + \delta \right)^{n-1} \tilde{x}(t), \delta > 0 \quad (3.5)$$

From (3.5) comes

$$\begin{aligned} S(x, t) &= \left(\frac{d}{dt} + \delta \right) \cdot \tilde{x}(t) \Leftrightarrow \\ S(x, t) &= \frac{d\tilde{x}(t)}{dt} + \delta \cdot \tilde{x}(t). \end{aligned} \quad (3.6)$$

Since a buck converter is a second order system, due to the presence of a capacitor and an inductor which are reactive components, in (3.5) will come $n=2$. Because the converter is operating at a 2 MHz frequency comes $\delta = 2 \times 10^6$.

The expression (3.6) might also be interpreted as a PD controller because it contains a derivative and a proportional constant term.

The state variables that allow to control the system are given by:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_0 \\ \dot{v}_0 \end{bmatrix} \quad (3.7)$$

Substituting (3.7) in (2.13) comes

$$\frac{d^2 x_2}{dt^2} = \frac{v_{in}}{LC} - \frac{x_1}{LC} - \frac{1}{RC} x_2. \quad (3.8)$$

And substituting (3.8) in (3.2) comes

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{RC} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} \cdot v_{in} \cdot \mu \quad (3.9)$$

This way, and recalling (3.6) and (3.7), the sliding surface is obtained as

$$S(v_0, t) = \frac{dv_0}{dt} + \delta \cdot (v_0(t) - V_{REF}). \quad (3.10)$$

To accomplish the equilibrium one must force $S(v_0, t) = 0$. Now it's possible to define the state switching conditions that lead to an equilibrium condition. These are given by:

$$\mu = \begin{cases} 1, s(v_0, t) = \frac{dv_0}{dt} + \delta \cdot (v_0(t) - V_{REF}) < 0 \\ 0, s(v_0, t) = \frac{dv_0}{dt} + \delta \cdot (v_0(t) - V_{REF}) \geq 0 \end{cases} \quad (3.12)$$

Figure 6 illustrates the convergence towards the sliding surface.

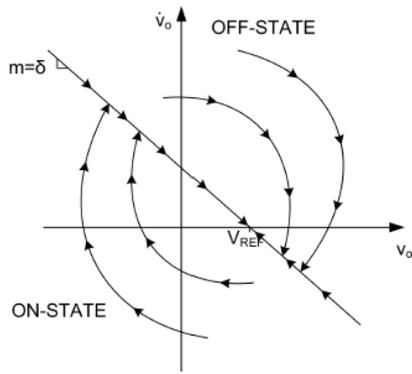


Figure 6 – Convergence towards the sliding surface.

V - SMC IMPLEMENTATION

The software programs used to implement and simulate the circuit and respective control technique are Simulink – Matlab and Active-HDL.

Simulink allows the design and simulation of the electronic circuit while Active-HDL is used to write the digital control program in Verilog language. The control block from Active-HDL is then instantiated on Simulink allowing a co-simulation between both software programs.

Figure 7 shows the circuit designed on Simulink.

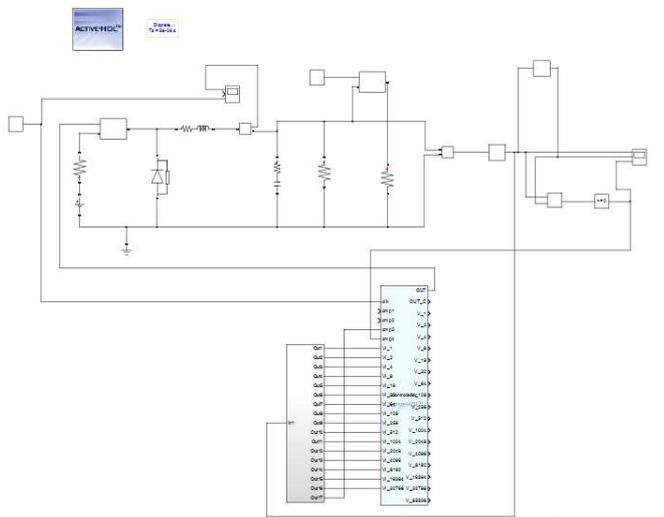


Figure 7 – Complete circuit designed on Simulink environment.

In order to simulate load changes, simulations are made considering an “extra-load” that is coupled/decoupled do the actual load with the help of another switch operating at a 1 kHz frequency. Also in attempt to simulate a more realistic environment instead of an ideal one, there is added a delay of 500 ns to purposely create a mismatch between the signals that are being read and the ones that are being acted on.

When it comes to implement the SMC, a derivative must be calculated. In attempt to reduce the necessary hardware to

obtain it, it is approximated by just multiplying the sign of the output voltage (which is positive when it’s raising and negative when it’s lowering) by a constant (k) whose value was obtain experimentally.

There is also an ADC designed to minimize the hardware necessary to accomplish a result that meets the requirements of the project. That is achieved by reducing considerably the number of bits of the ADC and adjusting the comparison levels to meet the specifications of the project.

Then there is the control block, which is the one that actually implements the digital SMC. This is the one programmed in Verilog language in Active-HDL environment.

Since the implementation of the SMC is made through Verilog coding, instead of showing all the code on the program, a better way to illustrate the implementation of the SMC is the diagram block presented by Figure 8.

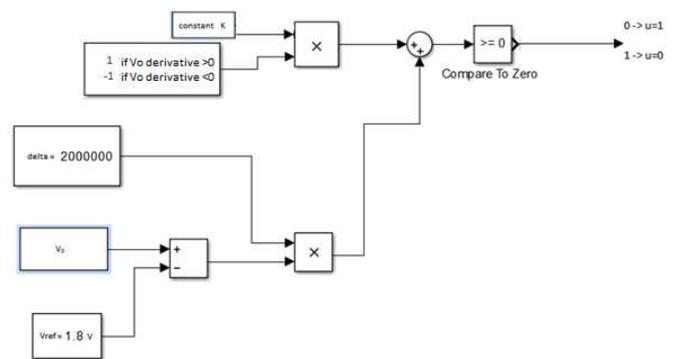


Figure 8 – Illustration of the SMC implementation.

Figure 9 shows the result of a simulation for load (R=100 Ω).

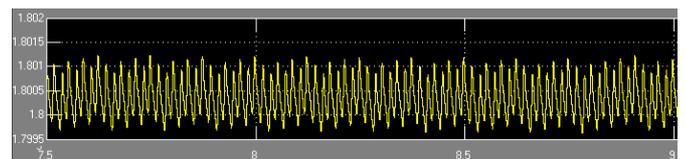


Figure 9 – v_o signal obtained by simulation.

Since the goal of this project is to obtain a v_o value equal to 1.8 V from an input voltage of 3.3 V, the simulation shown in Figure 9 accomplishes the goal. The maximum error verified is lower than 1.5 mV.

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