Abstract: The main goals of the presented work are to study the influence of the proximity effect on the capacitance coefficients of a geminated overhead line and on the magnitude of the electric field at the conductors’ surfaces. The expanded multipole method in Fourier series was used to build the electric potential solution. Within this work, a computer program was developed using the tool MATLAB® from MathWorks. The results obtained were compared with the ones presented in references [2] and [3], being the difference less than 1%. Three geometries for the conductor bundle were analyzed: flat, triangular and square. The results show that the proximity effect strongly affects the maximum electric field value at the conductors’ surfaces but weakly affects the capacitance coefficients.

Key-words: Multipole method, Solution of the potential, Proximity effect, Capacitance matrix, Electric field.

1. INTRODUCTION

Bundled conductor arrangements are often utilized in high voltage (HV) transmission lines. This strategy, on one hand, allows to mitigate the phenomenon called corona effect. On the other hand, it could originate a phenomenon called proximity effect which means that the electric field originated by one conductor is deformed by the presence of the other conductors located in the neighborhood. This fact affects the transmission line parameters, reason why it is important to study the phenomenon.

The main goals of the present study are the evaluation of the capacitance coefficients and the magnitude of the electric field at conductors’ surfaces, for bundled conductor arrangements, taking into account the proximity effect. Three different conductor bundle configurations are analyzed – flat, triangular and square – and a sensitivity study related to the distance between the bundle conductors is performed.

The analytical method used is based on the following assumptions:
– The earth is considered a flat and perfect conductor, it is located far away of the conductor bundles and does not have influence on the electric field created by them.
– The sub-conductors of the bundle are cylindrical, arranged in parallel between each other, they are in electrostatic equilibrium and they have at the same voltage applied.
– The dielectric surrounding the sub-conductors of the bundle is linear, isotropic and homogeneous and it is assumed to be air.

Based on these assumptions, the expanded multipole method in Fourier series is used to build the electric potential solution. From this solution the capacitance coefficients and the electric field will be evaluated.

The method is validated, comparing the results for flat and triangular configurations, namely the equivalent capacitance and the magnitude of the conductor surface electric field, with the results presented in references [2] and [3].

This document is organized in five sections. Section 1 makes the introduction. Section 2 concerns an approach neglecting the proximity effect, to use as a benchmark. In Section 3 the electric potential solution form is presented as well as the electric field solution in a conductor surface. In Section 4 the results for the flat, triangular and square configurations are presented and compared with the ones in references [2] and [3]. Conclusions are presented in Section 5.
2. ZEROTH ORDER APPROXIMATION

Before analyzing the conductor bundles, it is necessary to establish the reference case, named by zeroth order approach. The reference case consists of a single cylindrical conductor of radius \( r_c \) above ground at height \( h \gg r_c \). The conductor to ground voltage is \( V \). For this case the results are obtained by the following expressions, Ref. [1],

Per unit length capacitance:

\[
C_0 = \frac{2\pi \varepsilon_0}{\ln(2h/r_c)}
\]

(1)

Surface electric field strength:

\[
E_a = \frac{V}{r_c \ln(2h/r_c)}
\]

(2)

Consider the three configurations shown in Fig. 1, where all the conductors are interconnected, sharing a common potential.

Assuming that \( r_c \ll d \ll h \), where \( d \) is the distance between two adjacent sub-conductors (Fig. 1), the proximity effect can be neglected. Under this condition the potential coefficients matrix p.u.l. have the following expressions:

\[
[S_0] = \left[ \begin{array}{cccc}
S_{11} & S_{12} & \cdots & S_{1n} \\
S_{21} & S_{22} & \cdots & S_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
S_{k1} & S_{k2} & \cdots & S_{kn} \\
\end{array} \right],
\]

(3)

where,

\[
S_{0kk} = \ln \left( \frac{2h}{r_c} \right)
\]

(4)

\[
S_{0kj} = \ln \left( \frac{r_{ki}'}{|\bar{w}_{ki}|} \right), \quad i \neq k
\]

(5)

where \( r_{ki}' \) is the distance between the conductor \( k \) and the location of the image of conductor \( l \) and \( \bar{w}_{ki} \) is the position complex vector of conductor \( k \) axis with respect to the conductor \( i \) axis.

The capacitance matrix is given by inversion of the potential coefficients matrix.

\[
[C_0] = [S_0]^{-1}
\]

(6)

Since a linear behavior has been assumed for the medium, the charge in each sub-conductor can be written as a linear combination of all conductor voltages [1]:

\[
[Q] = [C][V]
\]

(7)

Fig.1. Cross section of conductor bundle configurations: (a) flat, (b) triangular, (c) square bundle.

3. PROXIMITY EFFECT

Assuming a static phenomenon, where the current is absent, the Maxwell equations can be written as:

\[
\begin{align*}
\text{curl } E &= 0; \quad E = -\nabla \phi + V \\
\text{div } D &= \rho
\end{align*}
\]

(8)

where

- \( E \) is the electric field (V/m)
- \( V \) is the electric potential (V)
- \( D \) is the electric displacement (C/m²)
- \( \rho \) is the charge density (C/m³)

As referred before, linearity and isotropy are assumed for the medium. Under these assumptions the medium equation is the following.

\[
D = \varepsilon E
\]

(9)

where, \( \varepsilon \) is the permittivity of the medium (F/m)
3.1 Potential Solution

The expanded multipole method in Fourier series was used to build the electric potential solution, Ref.[2].

\[
V = 0
\]  

(10)

Solving Eq. (10) through the variable separation method on polar coordinates \((r, \varphi)\), the potential solution for the conductor \(i\) centered on axis \(O_i\) is the following:

\[
V_i = C_{0i}^{(i)} \ln \left( \frac{1}{y} \right) + \sum_{m=0}^{+\infty} A_m \rho^{i-m} y^{im\varphi}
\]

(11)

The potential of (11) can be obtained by extraction of the real part of the complex potential, \(\bar{W}_i\), given by:

\[
\bar{W}_i(y) = C_{0i}^{(i)} \ln \left( \frac{1}{y} \right) + \frac{1}{2} \sum_{m=1}^{+\infty} A_m^{(i)} y^{-m}
\]

(12)

where

\[
y = r e^{-j\varphi}
\]

(13)

If the singularities axis is not coincident with the conductor axis, then we have to expand (12) in terms of a Taylor’s development in order to obtain a formulation centered at the conductor axis [4]. For the case when the singularities axis is the axis of conductor \(i\), \(O_i\), the following expansion, centered at the axis of conductor \(k\), \(O_k\), (Fig. 2), is obtained

\[
E_0^{(k)}(r) = C_0^{(k)} \ln \left( \frac{r_k}{r} \right) + \sum_{i=1}^{N} \sum_{m=0}^{+\infty} C_m^{(i)} \ln \left( \frac{r_k \xi_i}{r_k} \right) + P_k
\]

(14)

where \(r_k\) is the distance between \(P\) and the location of the image of conductor \(k\) relative to the soil surface. As a consequence of the Gauss theorem

\[
C_0^{(k)} = \frac{q_k}{2\pi e_0}
\]

(15)

Where \(q_k\) is the electric charge (p.u.l) in the conductor \(k\). The term \(P_k\) in Eq. (14) represents the contribution of the proximity effect and is given by

\[
P_k = \sum_{l=1}^{N} \sum_{m=1}^{+\infty} C_m^{(l)} \left( \frac{\bar{W}_k^{(l)}}{\bar{W}_k^{(l)}} \right)^{-m}
\]

(16)

\[
E_m^{(k)}(r) = \frac{(r/\bar{r}_k)^{m|l|}}{2|m|} \left( \frac{\beta_m^{(k)} + |m| C_m^{(k)} (\bar{W}_k^{(k)})^{-2|m|} + \gamma_m^{(k)} \right), m \neq 0
\]

(17)

\(E_m^{(k)}(r)\) is the nth order coefficient of the Fourier series development. In Eq.(17) \(\bar{r}_k\) is the radius of the conductor \(k\) used for normalization purposes. The coefficients \(C_m^{(k)}\), \(k = 1,...,N, m = 0, \pm 1, \pm 2,\) are the development coefficients to be determined by imposition of the boundary condition of each conductor surface. The terms, \(\beta_m^{(k)}\) and \(\gamma_m^{(k)}\) in Eq.(17) are calculated by the following expressions

\[
\beta_m^{(k)} = \sum_{i=1}^{N} \left( (-1)^m C_m^{(i)} \left( \frac{\bar{W}_k^{(k)}}{\bar{W}_k^{(k)}} \right)^{-m} \right), m > 0
\]

(18)

\[
\gamma_m^{(k)} = \sum_{i=1}^{N} \left( (-1)^m C_m^{(i)} C(p,|m|) \left( \frac{\bar{W}_k^{(k)}}{\bar{W}_k^{(k)}} \right)^{-p} \right), m > 0
\]

(19)

where

\[
C(p,|m|) = \frac{(p + |m| - 1)!}{(p - 1)!(|m| - 1)!}
\]

(20)

The complete form of the solution of the electric potential, \(V^{(k)}(r, \varphi)\), centered at the axis of conductor \(k\) due to all singularities located at the axis of all \(N\) conductors in the vicinity of each other is

\[
V^{(k)}(r, \varphi) = \sum_{m=-\infty}^{+\infty} E_m^{(k)}(r) e^{im\varphi}, \quad r = \bar{r}_k
\]

(21)

3.2 Boundary conditions

In order to determine the field solution and the coefficients \(C_m^{(k)}\), boundary conditions must be enforced. Assuming constant potential for the surface of conductor \(k\) \((r = \bar{r}_k)\), those conditions have the form
\[ V_k = \text{constant} \Rightarrow \begin{cases} E^{(k)}_0 \mid_{r=r_k} = V_k \\ E^{(k)}_m \mid_{r=r_k} = 0 \end{cases} \] (22)

These boundary conditions allow the coefficients \( C^{(k)}_m \) to be determined by solving the following system of linear equations

\[ \beta_m + \gamma_m = -|m| C^{(k)}_m \] (23)

Once the coefficients \( C^{(k)}_m \) have been obtained, the p.u.l. voltage for each conductor can be obtained from the 0th-order development, Eq. (14), as a consequence of the boundary conditions.

3.3 Capacitance and proximity

To calculate the capacitance of the bundle, first it is necessary to obtain the potential coefficients matrix. Each column of \([S]\) is defined by

\[ S_{ki} = \frac{\phi_i}{Q_j} Q_j = 0, \forall x \] (24)

To calculate them it is necessary to have into account the system of charges. Once the potential coefficients matrix obtained, the capacitance matrix is determined like in Eq. (6). Finally, because all the conductors of the bundle are connected, the capacitance p.u.l. of the bundle can be defined as

\[ C_{eq} = \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \] (25)

and the proximity effect contribution, described in Eq. (16), is given by

\[ S_{pe} = \frac{\sum_{k=1}^{N} P_{ki} Q_i}{\sum_{i=1}^{N} Q_i} \] (26)

3.4 Electric field

The electric field solution on conductor \( k \) surface is calculated through the potential solution gradient:

\[ \frac{\partial V^{(k)}}{\partial r} = -\sum_{m=-\infty}^{\infty} \frac{\partial E^{(k)}_m(r)}{\partial r} e^{im\phi} \] (27)

4. NUMERICAL RESULTS

All the results shown and discussed here are in normalized quantities, presented against \( x = d/r_c \). It is considered that the radius, \( r_c \), of the cylindrical conductors is equal to 15 mm and the center of the bundle is assumed to be at height, \( h \), above ground equal to 20 m.

As a first step the validation of the method is discussed. According to Ref. [2], the Fourier series development may be truncated at the order \( m = 21 \) for error in the potential smaller than \( 10^{-6} \). The flat and triangular configuration results are used to validate the method. The electric field on the sub-conductor surfaces is compared with the magnetic field evaluated in Ref. [2] (under the assumption of perfect conductors). It is possible to verify that the field results are very similar. Also the maximum electric field magnitude is compared with the one evaluated in Ref. [3]. Deviations are found smaller than 1% for the maximum field magnitude.

The values for the capacitance of the bundle and the proximity (in terms of potential coefficients) are compared with the references mentioned before and the deviations are also minimal, smaller than 1%.

For the flat configuration, the normalized electric field curves \( E_{1\phi}/E_0 \) and \( E_{2\phi}/E_0 \) are different because the asymmetry of the geometry. The electric field distribution on the lateral sub-conductor surface (Fig. 3) has maximum value at \( \phi = \pi \). For the central sub-conductor the maximum value occurs at \( \phi = \pm \pi/2 \). The field on the lateral sub-conductor is bigger when compared with the central sub-conductor, this fact is linked up with the fact that \( Q_1 > Q_2 \).

The variation of \( Q_2/Q_1 \) has been plotted against \( x \) in Fig. 4, note that \( Q_2/Q_1 \) is always smaller than unity.

![Flat Configuration](image)

**Fig. 3** Distribution of the normalized electric field \( E_{1\phi}/E_0 \) and \( E_{2\phi}/E_0 \) along the periphery of two sub-conductors for the flat bundle configuration (Fig. 1). (a) refers to sub-conductor 1. (b) refers to sub-conductor 2. Curves are for \( x = 20 \) (curve 1), \( x = 10 \) (curve 2), \( x = 5 \) (curve 3) and \( x = 2.5 \) (curve 4). These results are similar to the ones presented in Fig. 5, Ref.[2].
For $x$ smaller than 10, the exact approach for $Q_2/Q_1$ differs from the zeroth order approach, this shows the influence of the proximity effect on the charge distribution.

Fig. 4 Ratio between the values of the charges in the sub-conductors 2 and 1, $Q_2/Q_1$, of the flat configuration as a function of $x$ for the zeroth order approximation and exact algorithm (including the proximity effect). These results are similar to the ones presented in Fig. 6, Ref.[2].

The bundle capacitance and the proximity effect against $x$ for the flat configuration are plotted in Fig. 5. The results show that the proximity effect does not seem to affect significantly the bundle capacitance.

Fig. 5 Flat bundle configuration. (a) Normalized capacitance $C_{eq}/C_0$ and zeroth order approximation against $x$. (b) Normalized contribution of the proximity effect for the potential coefficient $S_p/s_0$ against $x$.

For the triangular configuration, the normalized electric field curves are coincident for the three sub-conductors due to symmetry. The electric field has a maximum at $\varphi = \pi$.

Fig. 6. Comparing the field results with the flat configuration, it is possible to conclude that the magnitude of the electric field decreases with the geometric reorganization.

Fig. 6 Distribution of the normalized electric field $E_{eq}/E_0$ along the periphery of a sub-conductor for the triangular bundle configuration. Curves 1, 2, 3 and 4 are for $x = 20$ (curve 1), $x = 10$ (curve 2), $x = 5$ (curve 3) and $x = 2.5$ (curve 4). These results are similar to the ones presented in Fig. 4, Ref.[2].

Fig. 7 Triangular bundle configuration. (a) Normalized capacitance $C_{eq}/C_0$ and zeroth order approximation against $x$. (b) Normalized contribution of the proximity effect for the potential coefficient $S_p/s_0$ against $x$.

The bundle capacitance and the proximity effect for the triangular configuration have been plotted in Fig. 7. Comparing these results with the ones plotted in Fig. 5 it is verified that: the bundle capacitance decreases and the proximity effect is much more intense on the triangular bundle. These facts are linked up with the fact that in triangular configuration each sub-conductor pair is separated by a distance $d$ while in the flat configuration one
conductor is at a distance $d$ but the other is at a double distance $2d$.

Next, attention is paid for the square configuration results. Fig. 8 represents the normalized electric field along the periphery of a sub-conductor. The field on the square configuration is smaller when compared with the one present on the previous configurations. This was expected because the square bundle has more sub-conductors. As a result, each sub-conductor has less electrical charge and the field magnitude in each sub-conductor decreases.

The curves shown in Figs. 3-9 are useful for describing the distinct features of the flat, triangular, and square configurations. However, if the reader wishes to confirm the results, tabular form results with numerical entries will be more convenient. With this in mind, Tables 1 to 3 were produced, where normalized results are shown concerning the maximum electric field magnitude, the capacitance and the contribution of the proximity effect to the potential coefficient, for each bundle configuration.

### Table 1 – Flat bundle

<table>
<thead>
<tr>
<th>$x$ = 2.5</th>
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<th>$x$ = 10</th>
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<td>0.6303</td>
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<tr>
<td>$E_{2\text{mix}}/E_0$</td>
<td>0.3953</td>
<td>0.3425</td>
<td>0.3516</td>
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<td>1.1228</td>
<td>1.1914</td>
<td>1.2768</td>
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<td>$Q_2/Q_1$</td>
<td>0.4776</td>
<td>0.6160</td>
<td>0.7608</td>
</tr>
<tr>
<td>$S_{p eq}/S_0$</td>
<td>$-4.6718$</td>
<td>$-1.1435$</td>
<td>$-0.2869$</td>
</tr>
<tr>
<td>$C_{0eq}/C_0$</td>
<td>1.1155</td>
<td>1.1888</td>
<td>1.2760</td>
</tr>
</tbody>
</table>

### Table 2 – Triangular bundle

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<tr>
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<td>$-15.449$</td>
<td>$-4.7181$</td>
<td>$-1.2456$</td>
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<tr>
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<td>1.0839</td>
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### Table 3 – Square configuration

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<tbody>
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<td>0.4570</td>
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<tr>
<td>$S_{p eq}/S_0$</td>
<td>$-16.433$</td>
<td>$-5.1946$</td>
<td>$-1.3939$</td>
</tr>
<tr>
<td>$C_{0eq}/C_0$</td>
<td>1.1088</td>
<td>1.1962</td>
<td>1.2985</td>
</tr>
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### 5. Conclusions

This paper studies the influence of the proximity effect on the capacitance coefficients of cylindrical conductor
bundles of overhead transmission lines. The magnitude of the electric field at the conductors’ surface is also evaluated. The cylindrical conductors bundle analyzed are in flat, triangular and square configurations.

The expanded multipole method in Fourier series was used to build the electric potential solution. The method was validated using results already published in references [2] and [3].

The results show that the electric field magnitude is strongly affected by the proximity effect. The geometric reorganization of the bundle, from flat configuration to triangular configuration, decreases the electric field amplitude. With the proximity decrease the field has tendency to have a uniform distribution on the sub-conductors surfaces.

It was also concluded that the capacitance weakly depends on the proximity. Even with a big increase of the proximity, like the case of the flat configuration compared to the triangular or square configuration the capacitance doesn’t suffer a relevant the difference between the zeroth order approach and the exact approach.

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REFERENCES