Computation of the added mass matrix using a panel method

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Abstract

A 3-D potential-based boundary element method is used to compute the added mass matrix of a totally submerged body subject to accelerations. The added mass matrix is obtained from the influence coefficient matrices, which are only function of the geometric grid that models the body surface. The results obtained are compared against known analytical and numerical values and a grid size convergence study is conducted.

Keywords: Potential flow, Added mass, Panel method.

Introduction

The fluid-structure interaction is a subject of interest in the areas of propellers design and wind and tidal technologies [1,2,3]. Problems involving vibrating solids in a fluid medium are especially common due to the finite rigidity of the submerged bodies (blades) and a wide array of situation in which they operate in nonuniform wakes.

In order to do a dynamic blade stress analysis, i.e. taking into account the pressure fluctuations due to blade vibrations, it is important to obtain the added mass and damping matrices associated with the elastic blade motion. These matrices contain information that if superimposed over the structural matrices will allow stress analysis to grasp the fluid-structure interaction effects.

Previous work

Vortex-lattice methods (VLMs) and boundary element methods (BEMs) are the choices available to determine the hydrodynamic response of propellers. The VLMs were first used by Kerwin and Lee in 1978 [4] to analyse the flow around marine propulsors and the BEMs in 1985 by Hess and Valarezo [5]. VLMs and BEMs are potential flow methods but still proved to be
very efficient and reliable even when compared to viscous solvers, with BEMs being more accurate but also requiring more memory and CPU time than VLMs.

In either of these families of methods, the blades are assumed to be rigid and so the pressure fluctuations due to the small elastic deformations suffered by the propeller blades cannot be taken into account. This leads to the development of iterative procedures, e.g. Atkinson and Glover (1988) [6], and of coupled approaches as it was introduced by Kuo and Vorus in 1985 [7].

Recently, more work has been done, namely for hydroelastic analysis of surface-piercing propellers, by Dyson in 2000 [8] and cavitating propellers by Young in 2005 [3].

**Objectives of the present work**

The objective of this work is to develop a tool for computing added mass matrices with a panel method and assess the quality of the results.

**Formulation**

The starting point of this work is the continuity equation for an inviscid, incompressible and irrotational flow: the Laplace equation.

\[ \nabla^2 \Phi = 0. \] (1)

Where \( \Phi \) represents the potential of the velocities perturbations around the body.

This equation can be solved using the Green’s divergence theorem [9,10] and with the appropriate boundary conditions it yields:

\[ \Phi(P) = -\frac{1}{4\pi} \int_S \left[ \sigma \left( \frac{1}{r} \right) - \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right] dS, \] (2)

where \( \Phi(P) \) is the perturbation potential over the point \( P \) of the domain, \( \mu \) and \( \sigma \) represent the dipole and source distributions over the body.

Assuming the internal potential of the body to be zero and using the boundary condition on the body surface:

\[ \nabla \Phi \cdot n = -\mathbf{v} \cdot n \] (3)

It is possible to write equation (2) as the following Fredholm integral equation:

\[ 2\pi \mu(P) - \int_S \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) dS = \int_S \sigma \left( \frac{1}{r} \right) dS \] (4)
The previous equation can be transformed and computed as a set of \( N \times N \) algebraic equations:

\[
\sum_{k=1}^{N} \frac{1}{4\pi} \int_{\text{panel } k} \left\{ \sigma \nabla \left( \frac{1}{r} \right) - \mu \frac{\partial}{\partial n} \left( \frac{1}{r} \right) \right\} dS = 0 ,
\]

with \( N \) equal to the number of panels on the body surface.

The previous equation is evaluated at every collocation point, and captures the interaction between every pair of panels.

Equation (5) can be manipulated and written in matrix form:

\[
[A][\Phi] = [B] \left( \frac{\partial \Phi}{\partial n} \right) ,
\]

where \([A]\) and \([B]\) are, respectively, the dipole and source influence matrices.

By inverting the matrix \([A]\) it is possible to obtain the unique coefficient matrix \([C]\).

\[
[C] = [A]^{-1}[B]
\]

With the definition of \([C]\), the usage of Bernoulli’s equation and some manipulation it is possible to compute the elemental added mass matrix:

\[
[M_{ij}^A] = \rho S_i C_{ij} [T_i]^T [T_j] .
\]

**Verification**

From the developed tool it was possible to obtain the global added mass matrix of a sphere and several ellipsoids. These matrices allowed for the determination of the elemental surface forces over the submerged body due to accelerations and its integration over the body permitted the comparison with analytical and numerical global added mass values.
Defining the non-dimensional added mass coefficient (9), which can be interpreted as a force per unit of acceleration:

\[
\frac{M'_{ij}}{\rho a^3},
\]  

(9)

it is possible to compare the obtained values with the analytical values \(\overline{M}^\prime\) found in [11] and [12].

Introducing the relative error variable \(\|e_{ij}\|\) (10), we can access the error behaviour with grid size variation:

\[
\|e_{ij}\| = \frac{|\frac{M'_{ij}}{\rho a^3} - \overline{M}^\prime|}{\overline{M}^\prime},
\]  

(10)

where \(i\) and \(j\) run through the directions \(x, y\) and \(z\).

![Figure 2 – Sphere - Convergence study for the error obtain from equation (10) with grid size. \(\|e_{ij}\|\) vs (\(\sqrt{NP}\)), \(NP\) is the number of panels.](image)

![Figure 3 – Example of obtained x-force distribution over a sphere due to an unitary acceleration in the x direction.](image)
These forces distributions and relative errors were determined for all nine combinations of directions x, y and z for either a sphere and an ellipsoid of semi-axis a=1, b=0.5 and c=0.05. Examples of these results are shown in Figure 2 through 4.

![Figure 4](image)

Figure 4 – Ellipsoid (1,0.5,0.05)- Convergence study for the error obtain from equation (10) with grid size. \( ||e_j|| \) vs \( \sqrt{NP} \), \( NP \) is the number of panels.

The solid line in Figures 2 and 4 is exactly \( \left( \sqrt{NP} \right)^{-2} \), which makes easy to observe that the scheme is second order in regard to grid size.

![Illustration 5](image)

Illustration 5 – Example of obtained x-force distribution over an ellipsoid (1,0.5,0.05) due to an unitary acceleration in the z direction.
Conclusions

This work demonstrated that the applied method provides good results for the computation of added mass matrixes, and, as such, it can be a useful tool in fluid-structures interaction problems, namely in coupled approaches for dynamic blade stress analysis.

References


