2.41 GHz ISM Receiver

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April 2014
To Sofia Jacinto Mendes
Acknowledgements

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Abstract

The objective of this work is to redesign a 2.41 GHz ISM receiver in 130 nm CMOS technology, focusing low area and low power. Three blocks are totally redesigned: Quadrature Cross-Coupled RC Relaxation VCO, Quadrature VCO-Mixer and Single Stage Feedback LNA.

RC oscillators have the advantage of being inductorless (i.e. low area), while typically having a high power consumption. The proposed solution is based on the single low-power cross-coupled RC relaxation oscillator developed in [1], with only one commutated current source, instead of the traditional two. As for quadrature, the coupling of two single oscillators is done with a shunt technique that uses PMOS as active loads, instead of resistors. This topology allows for a 20 % power saving, compared to commonly used soft-limiter coupling; without adding components or degrading phase noise. Additionally, MOSCap are used for voltage frequency control without extra current consumption.

The mixer is an add-on to this oscillator, taking advantage of its similarities with a single-balanced mixer. The principle is equivalent to the one in [1], although taking into account the necessity to isolate the tuning voltage. The use of the active loads increases the conversion gain.

Finally, the LNA is based on the topology of [2]. This shunt-shunt topology takes advantage of feedback use: input impedance matching without using LC networks or severely affecting the gain; gain desensitizing, increased bandwidth and nonlinear distortion reduction. The amplification stage is a common-source, while the feedback stage is a common-drain, incorporating a feedback resistor. The use of a common-drain isolates the common-source’s load from input. The innovation in this solution is the use of its load as the oscillator-mixer’s current mirror, which represents about 1/3 of the oscillator’s total power consumption. The typical load resistor is substituted by a modified diode-connected transistor. This direct feed of both DC and RF signal excludes the necessity of buffers, thus saving power and area.

Keywords

Low Area, Low Power, Relaxation Oscillator, Quadrature Oscillator, Quadrature Oscillator-Mixer, Feedback LNA
Resumo

O objectivo deste trabalho é alterar o projecto existente de um receptor a 2.41 GHz na banda ISM numa tecnologia CMOS de 130 nm, focando a obtenção de uma solução de área reduzida e baixo consumo. Três blocos foram projectados: Oscilador RC de Relaxação com Acoplamento Cruzado em Quadratura e Controlado por Tensão, Oscilador-Misturador em Quadratura e Controlado por Tensão, Amplificador de Baixo Ruído com Realimentação.

Os osciladores RC têm a vantagem de não usarem bobines (i.e. menor área), no entanto tendo um elevado consumo de potência. A solução proposta baseia-se no oscilador RC de relaxação com acoplamento cruzado de baixo consumo, desenvolvido em [1], onde apenas uma fonte de corrente comutada é usada, em detrimento das tradicionais duas. Quanto à quadratura, o acoplamento de dois osciladores é realizado através de uma técnica de shunt que recorre a PMOS como carga activa, substituindo as resistências. Esta topologia permite uma poupança de 20 % da potência, comparando com a típica utilização de soft-limiters; não adicionado componentes ou degradando o ruído de fase. Adicionalmente, são utilizados MOSCap para controlar a frequência em tensão, sem consumo extra de corrente.

O misturador é implementado sobre o oscilador, aproveitando as semelhanças com o misturador Single-Balanced. O princípio é o mesmo de [1], sendo no entanto necessário ter em conta o isolamento da tensão de controlo. A utilização de cargas activas permite ainda o aumento do ganho de conversão.

Finalmente, o amplificador de baixo ruído baseia-se na topologia de [2]. Esta topologia paralelo-paralelo retira as vantagens da realimentação: adaptação de entrada sem recurso a malhas LC ou dependência do ganho; desnobilização do ganho, aumento de largura de banda e redução da distorção não-linear. O andar de amplificação consiste num fonte-comum, enquanto a realimentação é feita através de um andar de dreno-comum que incorpora uma resistência de realimentação. O uso do dreno-comum permite que a carga do fonte-comum permaneça isolada da entrada. A inovação proposta é o uso da carga do amplificador de baixo ruído como espelho de corrente do oscilador-misturador, o que deve representar cerca de 1/3 da corrente total do oscilador. A típica resistência de carga é substituída por um transistor com ligação em diodo modificada. Este acoplamento directo de DC e RF exclui a necessidade de blocos de adaptação, reduzindo potência e área.

Palavras-chave

Área Reduzida, Baixo Consumo, Oscilador de Relaxação, Oscilador em Quadratura, Oscilador-Misturador em Quadratura, Amplificador de Baixo Ruído com Realimentação
# Table of Contents

List of Tables ............................................................................................................. xiii
List of Figures .............................................................................................................. xv
List of Acronyms .......................................................................................................... xix
List of Symbols ............................................................................................................ xxi
1.  Introduction ........................................................................................................... 1
   1.1. Background and Motivation ............................................................................ 2
   1.2. Organization of the Thesis ............................................................................ 3
2.  Receivers: Overview, Key Concepts and Topologies ............................................ 5
   2.1. Overview ........................................................................................................ 6
   2.2. Key Concepts ................................................................................................. 7
       2.2.1. Sensitivity .............................................................................................. 7
       2.2.2. Linearity ............................................................................................... 9
       2.2.3. Spurious-Free Dynamic Range .............................................................. 13
       2.2.4. Selectivity ............................................................................................. 14
   2.3. Receiver Topologies ....................................................................................... 14
       2.3.1. Heterodyne ............................................................................................ 14
       2.3.2. Homodyne ............................................................................................ 16
       2.3.3. Low-IF ................................................................................................. 19
   2.4. 2.41 GHz ISM Low-IF Receiver Frontend .................................................... 22
3.  Quadrature Cross-Coupled RC Relaxation VCO .................................................. 25
   3.1. Overview ....................................................................................................... 26
   3.1.1. Oscillator Topologies .............................................................................. 30
   3.1.1.1. MOS Oscillators .............................................................................. 30
   3.1.2. Quadrature MOS Oscillators .................................................................. 39
       3.1.3. Discussion ........................................................................................... 43
   3.2. Implemented Circuit ..................................................................................... 44
       3.2.1. Cross-Coupled RC Relaxation VCO .................................................... 46
       3.2.2. Quadrature Cross-Coupled RC Relaxation VCO ................................ 50
       3.2.3. Monte Carlo and Corners .................................................................... 55
   3.3. Conclusions ................................................................................................... 57
4.  Quadrature Single-Balanced VCO-Mixer ............................................................. 59
   4.1. Overview ..................................................................................................... 60
   4.2. Mixer Topologies ......................................................................................... 61
       4.2.1. MOS Active Mixers ........................................................................... 61
       4.2.2. Quadrature MOS Active Mixers ......................................................... 63
List of Tables

Table 1.1 - Frequency bands for Non-Specific Short Range Devices and their requirements [4] ............ 2
Table 1.2 - LNA’s existing specifications [8] ......................................................................................... 2
Table 1.3 - Quadrature cross-coupled RC relaxation oscillator’s existing specifications [1] ............... 3
Table 1.4 - Oscillator-mixer’s existing specifications [1] ................................................................... 3
Table 1.5 - Complete low-IF Rx’s existing specifications [8] ............................................................... 3
Table 2.1 - Typical specifications for a receiver .................................................................................... 22
Table 3.1 - Cross-coupled RC relaxation oscillator’s starting parameters ........................................... 46
Table 3.2 - Cross-coupled RC relaxation VCO’s parameters ............................................................... 48
Table 3.3 - Quadrature cross-coupled RC relaxation VCO’s parameters .......................................... 51
Table 3.4 - Bonding wire parameters [8] ............................................................................................... 52
Table 3.5 - ESD parameters [8] ............................................................................................................ 53
Table 3.6 - Final quadrature cross-coupled RC relaxation VCO’s parameters .................................. 53
Table 3.7 - Corners 1 to 5 ....................................................................................................................... 56
Table 3.8 - Corners 6 to 12 ..................................................................................................................... 56
Table 3.9 - Corners 1 and 2 corrected results ....................................................................................... 57
Table 3.10 - Implemented VCO’s results summary .............................................................................. 58
Table 4.1 - Quadrature single-balanced VCO-mixer’s parameters ....................................................... 67
Table 4.2 - Final quadrature single-balanced VCO-mixer’s parameters ............................................. 70
Table 4.3 - Corners 1 to 6 ....................................................................................................................... 73
Table 4.4 - Corners 7 to 12 ..................................................................................................................... 74
Table 4.5 - Corners 1 and 2 corrected results ....................................................................................... 74
Table 4.6 - Implemented VCO-mixer’s results summary ....................................................................... 75
Table 5.1 - Types of feedback amplifiers .............................................................................................. 82
Table 5.2 - Single-stage feedback LNA’s parameters .......................................................................... 92
Table 5.3 - Steering current transistors, M_{12} and M_{13}, parameters ................................................. 94
Table 5.4 - Final dimensioning of the single-stage feedback LNA parameters .................................. 96
Table 5.5 - Corners 1 to 6 ....................................................................................................................... 99
Table 5.6 - Corners 7 to 12 ..................................................................................................................... 100
Table 5.7 - Corners 3, 4, 9 and 10 corrected results ............................................................................ 100
Table 5.8 - Implemented single-stage feedback LNA results summary ............................................ 101
Table 6.1 - Implemented receiver results summary .............................................................................. 106
Table 7.1 - Implemented receiver results summary .............................................................................. 109
List of Figures

| Figure 2.1 | Common receiver block diagram (adapted from [1]) | .................................................. 6 |
| Figure 2.2 | Input resistive matching (adapted from [13]) | .................................................. 7 |
| Figure 2.3 | Input 1 dB compression point [11] | .................................................. 10 |
| Figure 2.4 | IIP3 in logarithmic scale [11] | .................................................. 11 |
| Figure 2.5 | Illustration of a system | .................................................. 12 |
| Figure 2.6 | Heterodyne receiver (adapted from [13]) | .................................................. 14 |
| Figure 2.7 | Illustration of the image frequency problem [9] | .................................................. 15 |
| Figure 2.8 | Tradeoff between the relaxation of the image-reject and channel-select filters’ requirements [17] | .................................................. 15 |
| Figure 2.9 | Homodyne receiver (adapted from [11]) | .................................................. 17 |
| Figure 2.10 | Hartley architecture [11] | .................................................. 19 |
| Figure 2.11 | Weaver Architecture [11] | .................................................. 21 |
| Figure 2.12 | Block diagram of the receiver [8] | .................................................. 22 |
| Figure 3.1 | Oscillator’s division in active (A) and passive (B) blocks [19] | .................................................. 26 |
| Figure 3.2 | Oscillator’s block diagram [19] | .................................................. 27 |
| Figure 3.3 | Equivalent electrical models for an oscillator with sinusoidal current, a), and sinusoidal voltage, b) [19] | .................................................. 27 |
| Figure 3.4 | Graphical interpretation of stability [19] | .................................................. 28 |
| Figure 3.5 | Graphical interpretation of phase noise in time, a) [8], and in the frequency spectrum, b), [19] | .................................................. 29 |
| Figure 3.6 | Cross-coupled LC oscillator [8] | .................................................. 30 |
| Figure 3.7 | Block A circuit, a) [19], and its small-signal circuit, b) | .................................................. 31 |
| Figure 3.8 | Leeson-Cutler model for phase noise [19] | .................................................. 32 |
| Figure 3.9 | Working principle of a relaxation oscillator: i) integration of a constant; ii) comparison at a certain threshold; iii) integration constant sign change [20] | .................................................. 33 |
| Figure 3.10 | High-level model of a Relaxation Oscillator [8] | .................................................. 33 |
| Figure 3.11 | Cross-coupled RC relaxation oscillator [8] | .................................................. 33 |
| Figure 3.12 | Illustration of M1 in conduction mode [20] | .................................................. 34 |
| Figure 3.13 | Schmitt-Trigger’s output waveform [8] | .................................................. 34 |
| Figure 3.14 | Oscillator’s output and integrator’s voltage signals [8] | .................................................. 35 |
| Figure 3.15 | Relaxation oscillator’s high-level noise model [23] | .................................................. 36 |
| Figure 3.16 | Sinusoidal output waveform, a), and Schmitt-Trigger’s incremental circuit, b) [8] | .................................................. 38 |
| Figure 3.17 | Equivalent impedance circuit [8] | .................................................. 39 |
| Figure 3.18 | Modified single relaxation oscillator’s high-level model (adapted from [20] and [8]) | .................................................. 40 |
| Figure 3.19 | Quadrature Relaxation Oscillator’s high-level model (adapted from [8] and [20]) | .................................................. 40 |
| Figure 3.20 | Quadrature cross-coupled RC relaxation oscillator [8] | .................................................. 41 |
| Figure 3.21 | Output waveforms | .................................................. 42 |
| Figure 3.22 | Equivalent single-ended linear model [19] | .................................................. 42 |
| Figure 3.23 | Low-power relaxation oscillator, a), and quadrature relaxation oscillator in [8], b) | .................................................. 44 |
| Figure 3.24 | Quadrature cross-coupled RC relaxation VCO | .................................................. 45 |
| Figure 3.25 | Cross-coupled RC relaxation VCO | .................................................. 46 |
| Figure 3.26 | Output voltage waveform at 2.4 GHz | .................................................. 48 |
| Figure 3.27 | Output voltage spectrum at 2.4 GHz | .................................................. 49 |
| Figure 3.28 | Phase noise at 2.4 GHz | .................................................. 49 |
| Figure 3.29 | Parametric analysis of frequency range, phase noise at 1 MHz from oscillating frequency and output flatness as a function of tuning voltage | .................................................. 49 |
| Figure 3.30 | Quadrature cross-coupled RC relaxation VCO | .................................................. 50 |
| Figure 3.31 | Final quadrature cross-coupled RC relaxation VCO | .................................................. 51 |
Figure 5.22 – Diagram of the final single-stage feedback LNA ......................................................... 94
Figure 5.23 – Single-stage feedback LNA’s layout ........................................................................... 95
Figure 5.24 – Noise figure, voltage gain and S11 ........................................................................... 96
Figure 5.25 – Input 1dB compression point ..................................................................................... 97
Figure 5.26 – IIP3 ............................................................................................................................. 97
Figure 5.27 – S11, DC output voltage, noise figure and voltage gain as a function of VDC− ......... 97
Figure 5.28 – S11, DC output voltage, noise figure and voltage gain as function of VDC+ .......... 98
Figure 5.29 – Monte Carlo results for the DC output voltage, voltage gain, noise figure and S11 ..... 99
Figure 6.1 – Complete receiver’s layout .......................................................................................... 104
Figure 6.2 – Voltage gain, noise figure and S11 ............................................................................ 105
Figure 6.3 – Oscillator’s phase noise, output voltage and output spectrum .................................... 105
Figure 6.4 – Output voltage and spectrum for a mixing example, using a 2.41 GHz RF signal mixed
with a 2.4 GHz LO’s signal ................................................................................................................ 105
Figure 6.5 – Voltage gain, noise figure, S11, phase noise and oscillating frequency ..................... 106
Figure A1.1 – Circuit that performs a 90º shift [11] ...................................................................... 113
Figure A2.1 – NMOS symbol and operating regions [34] ................................................................. 115
Figure A2.2 – Small signal model [34] ........................................................................................... 115
Figure A2.4 – Output voltage waveform at 2.4 GHz ..................................................................... 116
Figure A2.5 – Phase Noise .............................................................................................................. 116
Figure A2.6 – Frequency range, phase noise and output voltage flatness as function of the tuning
voltage ............................................................................................................................................... 116
Figure A3.1 – Differential Pair ....................................................................................................... 117
Figure A3.2 – Sign function ............................................................................................................ 118
List of Acronyms

AC – Alternate Current
ADC – Analog-To-Digital Converter
AM – Amplitude Modulation
ASIC – Application-Specific Integrated Circuit
CMOS – Complementary MOS
DC – Direct Current
DSP – Digital Signal Processor
ERP - Effective Radiated Power
ESD – Electrostatic Discharge
FoM – Figure of Merit
FSK – Frequency-Shift Keying
IC – Integrated Circuit
IF – Intermediate Frequency
IIP3 – Input Third-Order Intercept Point
IP – Intellectual Property
IQ – In-Phase and Quadrature
IRR – Image-Rejection Ratio
ISM - Industrial, Scientific and Medical Band
LNA – Low Noise Amplifier
LO – Local Oscillator
LTI – Linear Time-Invariant
MIS – Metal-Insulator-Semiconductor
MOM – Metal-Oxide-Metal
MOS(FET) – Metal-Oxide-Semiconductor Field Effect Transistor

NF – Noise Figure

PVT – Process, Voltage and Temperature

QPSK – Quadrature Phase-Shift Keying

RF – Radio Frequency

SAW – Surface Acoustic Wave

SFDR – Spurious-Free Dynamic Range

SNR – Signal-to-Noise Ratio

THD – Total Harmonic Distortion

UMC – United Microelectronics Corporation

VCO – Voltage-Controlled Oscillator
List of Symbols

F – Noise Factor

\(K_B\) – Boltzmann Constant, \(1.38 \times 10^{-23} \, [\text{J/K}]\)

P – Active Power [W]

\(Q_{\text{reactive}}\) – Reactive Power [VAR]

\(N_{\text{stage}}\) – Internal Available Noise Power [W]

S – Power Spectral Density [W/Hz] or Complex Power [VA] depending on the context

\(p(t)\) – Instantaneous Power [VA]

G – Power Gain

\(G_A\) – Available Power Gain

V – Voltage [V]

\(V_t\) – Threshold Voltage

I – Current [A]

\(A_v\) – Voltage Gain

Z – Impedance [Ω]

R – Resistance/Resistor [Ω]

\(X_{\text{reactive}}\) – Reactance [Ω]

C – Capacitance/Capacitor [F]

L – Inductance/Inductor [H]

\(r\) - Reflection Coefficient

A - Amplitude

\(A_{\text{IIP3}}\) – Amplitude for IIP3

\(A_{\text{1dB}}\) – Amplitude for Input -1dB Compression Point

Q – Quality Factor
T – Temperature [K] or Period [s] depending on the context

t – Time [s]

f – Frequency [Hz]

B – Bandwidth [Hz]

ω – Angular Frequency [rad s⁻¹]

θ – Phase Angle [rad]

ℒ – Phase Noise [dBc/Hz]

c – Speed of Light [m/s]

λ – Wave Length [m]
Chapter 1

Introduction
1.1. Background and Motivation

Since the first wireless systems, as radios or mobile phones, society has shown a growing interest in wireless solutions. Those solutions became synonymous of portability, aesthetics and optimal space management, therefore being an increasing subject of study and standardization. We will focus short-distance applications, such as routers or wireless speakers.

Regarding the specific used band, ISM band is the Industrial, Scientific and Medical band. This project can be classified as a Non-Specific Short Range Device, according to the Electronic Communications Committee [4]. This assumption obliges for a high interference tolerance, but it has the advantage of allowing multiple uses for this IP (Intellectual Property). As for 2.41 GHz option, it is justified by its versatility, having fewer restrictions than any other available range of the spectrum (Table 1.1) and leading to smaller antennas.

<table>
<thead>
<tr>
<th>Frequency Band</th>
<th>ERP</th>
<th>Duty Cycle</th>
<th>Channel Bandwidth</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>433.05 – 434.79 MHz</td>
<td>+10 dBm</td>
<td>&lt;10%</td>
<td>No limits</td>
<td>No audio and voice</td>
</tr>
<tr>
<td>433.05 – 434.79 MHz</td>
<td>0 dBm</td>
<td>No limits</td>
<td>No limits</td>
<td>≤−13 dBm/10 kHz, no audio and voice</td>
</tr>
<tr>
<td>866 – 868.5 MHz</td>
<td>+10 dBm</td>
<td>No limits</td>
<td>&lt;25 kHz</td>
<td>No audio and voice</td>
</tr>
<tr>
<td>868.7 – 869.2 MHz</td>
<td>+14 dBm</td>
<td>&lt;1%</td>
<td>No limits</td>
<td></td>
</tr>
<tr>
<td>869.3 – 869.6 MHz</td>
<td>+10 dBm</td>
<td>&lt;0.1%</td>
<td>No limits</td>
<td></td>
</tr>
<tr>
<td>869.6 – 869.9 MHz</td>
<td>+27 dBm</td>
<td>&lt;10%</td>
<td>&lt;25 kHz</td>
<td>Channels may be combined to one high speed channel</td>
</tr>
<tr>
<td>2400 – 2483.5 MHz</td>
<td>+7 dBm</td>
<td>No limits</td>
<td>No limits</td>
<td>Transmit power limit is 10-dBm ERP</td>
</tr>
</tbody>
</table>

Table 1.1 - Frequency bands for Non-Specific Short Range Devices and their requirements [4]

The objective of this thesis is to correct and redesign an already existing implementation of an ISM 2.4 GHz Low-IF Receiver Frontend in a 0.13 µm CMOS technology [1] [7]. The designed frontend is inductorless, focusing low power and low area. Three blocks were totally redesigned: oscillator, oscillator-mixer and Low Noise Amplifier (LNA). The inter-block connection between the oscillator-mixer and the Low Noise Amplifier (LNA) is improved and therefore so are their specifications. Existing figures for LNA, oscillator, oscillator-mixer and complete receiver [8], to be improved, are illustrated from Table 1.2 to Table 1.5.

<table>
<thead>
<tr>
<th>LNA</th>
<th>w/ Buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>2.68 mA</td>
</tr>
<tr>
<td>$A_\text{L} @ 2.4GHz$</td>
<td>16.5 dB</td>
</tr>
<tr>
<td>$S_11 @ 2.4GHz$</td>
<td>-11.8 dB</td>
</tr>
<tr>
<td>$NF @ 2.4GHz$</td>
<td>2.66 dB</td>
</tr>
</tbody>
</table>

Table 1.2 – LNA’s existing specifications [8]
Table 1.3 - Quadrature cross-coupled RC relaxation oscillator’s existing specifications [1]

<table>
<thead>
<tr>
<th>Current Consumption</th>
<th>990.67 μA + 990.67 μA =1.98 mA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillation Frequency [GHz]</td>
<td>2.42</td>
</tr>
<tr>
<td>Phase Noise [dBc/Hz]</td>
<td></td>
</tr>
<tr>
<td>At 1 MHz</td>
<td>-90.48</td>
</tr>
<tr>
<td>At 3 MHz</td>
<td>-102.40</td>
</tr>
<tr>
<td>At 10 MHz</td>
<td>-114.20</td>
</tr>
</tbody>
</table>

Table 1.4 – Oscillator-mixer’s existing specifications [1]

<table>
<thead>
<tr>
<th>Oscillation Frequency [GHz]</th>
<th>2.42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage Gain [dB]</td>
<td>1.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LNA</th>
<th>Buffer</th>
<th>Os/Mix</th>
<th>IQ Filter</th>
<th>Rx</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.68</td>
<td>1.44</td>
<td>1.98</td>
<td>3.95</td>
<td>13.55</td>
</tr>
<tr>
<td>Gain [dB]</td>
<td>-2.92</td>
<td>1.15²</td>
<td>13.97</td>
<td>27.5</td>
</tr>
<tr>
<td>Area [mm²]</td>
<td>0.012</td>
<td>0.013</td>
<td>0.026</td>
<td>0.39³</td>
</tr>
</tbody>
</table>

1 including references for the current mirrors
2 Voltage conversion gain: VIFrms/VRFrms (IF@10 MHz, RF@2.41 GHz)
3 Including pads

Table 1.5 - Complete low-IF Rx’s existing specifications [8]

1.2. Organization of the Thesis

This document is organized in 7 chapters, starting with this introduction (chapter 1). Chapter 2 consists on an explanation of the basic concepts that are commonly associated with receivers, namely sensitivity, linearity and selectivity. The most common receiver topologies are presented and their features discussed. The conclusion of this chapter is an overview on the receiver to be implemented.

Chapters 3 to 5, study the designed blocks: oscillator, oscillator-mixer and LNA. For each chapter there is a theoretical overview of the block, followed by an analysis of the previous implementation and a discussion on proposed improvements. The design algorithm is then described and results are obtained. The circuit robustness is ensured by Monte Carlo and corners simulations. Finally, the main results and innovations are discussed in the conclusion.

Finally, chapter 6, will be dedicated to the discussion of the complete receiver’s results, while chapter 7, discusses the thesis main contributions. Future work is proposed based on the channel-filter’s requirements.
Chapter 2

Receivers: Overview, Key Concepts and Topologies
2.1. Overview

A receiver is typically divided in two parts (Figure 2.1): an analog frontend, corresponding to the part under development in this thesis; and a Digital Signal Processing unit (DSP), corresponding to a baseband processor after Analog-to-Digital Conversion (ADC) [9].

![Common receiver block diagram](image)

**Figure 2.1 - Common receiver block diagram (adapted from [1])**

The antenna and the band-pass filter select the band of communication – band selectivity. The signal is then amplified by a Low Noise Amplifier (LNA), after which the signal is down-converted by the mixer. Amplifying before converting is a technique used to increase the sensitivity of the receiver. This down-conversion can be either to an Intermediate Frequency (IF) or directly to baseband, depending on the Local Oscillator’s reference frequency. Usually, after down-conversion, the signal is filtered again by a channel-select filter – channel selectivity - to retrieve the message signal present at the IF. The demodulation is done by a processor in the digital domain.

There are three main topologies for receivers: heterodyne, homodyne and low-IF. All these three topologies can be divided in two major categories: direct-conversion and multi-step. The topology defines the mixer and the oscillator to be used. The mixer will apply the frequency offset, given by the LO’s reference frequency, to the Radio Frequency (RF) signal.

Briefly, the objective of this chapter is to give an overview on the key aspects of the study of receivers, such as sensitivity, linearity and selectivity. These concepts allow for a better understanding of the metrics to be used and tradeoffs to be made during the project, not only as a set of individual blocks, but also as a system.
2.2. Key Concepts

2.2.1. Sensitivity

The receiver’s sensitivity is defined as the minimum signal level that can be detected under noisy conditions. The sensitivity depends mainly on the noise factor, $F$ [10]. This figure is the ratio between Signal-To-Noise Ratio (SNR) at input and output.

$$\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} \quad (2.1)$$

$$F = \frac{P_{\text{noise\_out}}}{P_{\text{noise\_in}}} = \frac{G P_{\text{signal\_out}}}{G P_{\text{signal\_in}}} = \frac{P_{\text{noise\_out}}}{P_{\text{noise\_in}}} = \frac{\text{SNR}_\text{in}}{\text{SNR}_\text{out}} \quad (2.2)$$

$P_{\text{signal\_in}}$ is the power of the signal received and $P_{\text{noise\_in}}$ is the power of the noise due to source resistance at the input. In (2.2), both the initial noise power from source and the input signal are amplified by a stage gain, $G$. Hence, the resulting noise factor value is always equal or higher than 1, where if higher than 1, the circuit generates noise.

Usually, only thermal noise is considered. Furthermore, it is assumed that this noise is white - uniformly distributed across the spectrum. In fact, noise is considered a stationary and ergodic stochastic process, commonly described by its mean-square value such as [A1.1]:

$$v_{\text{thermal\_noise}}^2 = 4K_B T R \left[ \frac{V^2}{Hz} \right] \quad (2.3)$$

Equation (2.3) is interpreted as a power spectral density and the total source’s noise power is given by its integration (2.4).

$$P_{\text{thermal\_noise}} = \int_{-\infty}^{+\infty} S(f) df = 4K_B T R B \ [W] \quad (2.4)$$

A receiver usually has input resistive matching (Figure 2.2), eliminating reflection ($Z_{in}=Z_{source}$) and maximizing power transfer ($Z_{in}=Z^*_{source}$) [11] [12].

![Figure 2.2 – Input resistive matching (adapted from [13])](image-url)
\[ R_{\text{source}} = R_{\text{in}} \, [\Omega] \rightarrow r_{\text{in}} = \frac{Z_{\text{in}} - Z_{\text{source}}}{Z_{\text{in}} + Z_{\text{source}}} = \frac{Z_{\text{source}}^* - Z_{\text{source}}}{Z_{\text{source}}^* + Z_{\text{source}}} = 0 \quad (2.5) \]

Considering those matching conditions and using (2.4), it is possible to calculate the total noise power at the input, (2.8).

\[ V_{\text{thermal noise}} \, \text{rms} = \sqrt{V_{\text{thermal noise}}^2 \cdot B} = V_{\text{source}} \, \text{rms} \quad [V] \quad (2.6) \]

\[ P_{\text{in}} = \frac{V_{\text{in}} \, \text{rms}^2}{R_{\text{in}}} = \left( \frac{V_{\text{source}} \, \text{rms} \cdot R_{\text{in}}}{R_{\text{in}}} \right)^2 = \frac{(V_{\text{source}} \, \text{rms})^2}{R_{\text{in}}} \quad [W] \quad (2.7) \]

\[ P_{\text{noise in}} = 4K_B T \cdot \frac{R_{\text{source}}}{4R_{\text{in}}} \cdot B = K_B T \cdot B \quad [W] \quad (2.8) \]

If a temperature of 300 K and a bandwidth of 1 Hz are considered, (2.8) can be rewritten as,

\[ P = 10 \log \left( \frac{P \, [W]}{1 \, \text{mW}} \right) \quad [\text{dBm}] \quad (2.9) \]

\[ \text{NF} = 10 \log (F) \quad [\text{dB}] \quad (2.10) \]

\[ P_{\text{noise in}} \big|_{B=1 \, \text{Hz}, T=300 \, \text{K}} = 10 \log (K_B \cdot T \cdot B \cdot 1000) = -174 \, \text{dBm} \quad (2.11) \]

Finally, the sensitivity expression is derived,

\[ P_{\text{signal in}} = F \cdot \text{SNR}_{\text{out}} \cdot P_{\text{noise in}} \quad [W] \quad (2.12) \]

\[ \therefore P_{\text{signal in min}} \quad [\text{dBm}] = 10 \log \left( F \cdot \text{SNR}_{\text{out min}} \cdot P_{\text{noise in}} \cdot 1000 \right) = -174 + \text{NF} + 10 \log B + \text{SNR}_{\text{out min}} \quad [\text{dBm}] \quad (2.13) \]

where \( P_{\text{signal in min}} \) is the minimum power of the signal at the input that allows detection and correct interpretation. For the use of this formula, both the channel's bandwidth, \( B \), and the minimum SNR at the output must be chosen. The noise figure value is the sum of all block contributions in the receiver chain, under input matching conditions, given by the Friis formula,

\[ F_{\text{total}} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_{A_1}} + \cdots + \frac{F_N - 1}{G_{A_1} - G_{A_N} - 1} \quad (2.14) \]

where \( F_{\text{total}} \) is the total noise factor and \( F_N \) is the noise factor of each block of the chain. \( G_{A_N} \) is the \( N \)-th's block available power gain. From this we can conclude that as the signal is amplified the sensitivity to noise, and so the noise factor's influence, decreases. Reason to why the LNA is one of the first blocks.
The Friis formula is described in terms of power, although in the case of this project, voltage signals will be considered [11] [14] [15]. Note that the SNR is a ratio between signal and noise power, meaning the SNR at the input is equal to the input available SNR, (2.15). The same applies for the SNR at the output and its output available counterpart.

\[
\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = \frac{P_{\text{signal}_A}}{P_{\text{noise}_A}} \quad (2.15)
\]

\(P_{\text{signal}_A}\) and \(P_{\text{noise}_A}\) are available signal and noise power. This is the power delivered by a source to a conjugate-matched circuit and the power delivered by the circuit to a conjugate-matched load, respectively. Hence, noise factor can be rewritten as,

\[
\therefore F = \frac{\text{SNR}_{\text{in}}}{\text{SNR}_{\text{out}}} = \frac{\frac{P_{\text{signal}_A}}{P_{\text{noise}_A}}}{\frac{P_{\text{signal}_A}}{P_{\text{noise}_A}}} = \frac{\frac{P_{\text{noise}_A} e_{\text{out}}^2}{P_{\text{noise}_A} e_{\text{out}}^2}}{\frac{P_{\text{noise}_A} e_{\text{out}}^2}{P_{\text{noise}_A} e_{\text{out}}^2}} = \frac{\frac{V_{\text{nois}_2} e_{\text{out}}^2}{V_{\text{noise}_A} e_{\text{out}}^2}}{\frac{V_{\text{nois}_2} e_{\text{out}}^2}{V_{\text{noise}_A} e_{\text{out}}^2}} \quad (2.16)
\]

where \(G_A\) is the available power gain and \(A_{\text{total}}\) is the total voltage gain from source to output.

2.2.2. Linearity

A linear function is described by a first order polynomial, \(\alpha_0 + \alpha_1 x(t)\), however nonlinear behavior usually exists,

\[
y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + ... \quad (2.17)
\]

where \(y(t)\) is the output, \(x(t)\) is the input and \(\alpha\) are the different coefficients of the system. Note that the offset \(\alpha_0\) and higher than third order terms will be ignored.

Total Harmonic Distortion

Total Harmonic Distortion (THD), is the ratio between the total power of the nonlinearities and the fundamental’s power [16],

\[
\text{THD} = 10 \log \left( \sum_{n=2}^{+\infty} V_{\text{rms}}^2 \frac{V_{\text{rms}}^2}{V_{\text{rms}}^2} \right) = 20 \log \left( \sum_{n=2}^{+\infty} V_{\text{rms}}^2 \frac{V_{\text{rms}}^2}{V_{\text{rms}}^2} \right) \text{[dB]} \quad (2.18)
\]

where \(V_1\) to \(V_n\) are the different voltage amplitudes of the harmonics and \(R\) is the circuit’s resistance value, considered equal for all harmonics.
Input 1 dB Compression Point

Input 1 dB compression point is the point where there is a 1 dB gain reduction from the expected linear gain, Figure 2.3 [15].

Figure 2.3 – Input 1 dB compression point [11]

Consider a sinusoidal signal, \( x(t) \), with a generic amplitude \( A \).

\[
x(t) = A \cos(\omega t)
\]  

(2.19)

Applying this signal to a nonlinear system, (2.17), and ignoring higher than third-order terms yields,

\[
y(t) = \alpha_1 A \cos(\omega t) + \alpha_2 A^2 \cos^2(\omega t) + \alpha_3 A^3 \cos^3(\omega t) = \frac{\alpha_2 A^2}{2} + \cos(\omega t) \left( \alpha_1 A + \alpha_3 A^3 \frac{3}{4} \right) + \frac{\alpha_2 A^2}{2} \cos(2\omega t) + \alpha_3 \frac{A^3 \cos(3\omega t)}{4}
\]  

(2.20)

where a significant distortion exists for the fundamental frequency term (signaled in red). Depending on the sign of \( \alpha_3 \), the fundamental term might be increased or decreased. Considering \( \alpha_3 < 0 \), the input 1 dB compression point is:

\[
\alpha_1 A + \alpha_3 A^3 \frac{3}{4} = \alpha_1 \frac{A}{10^\frac{1}{20}} \Leftrightarrow 20 \log|\alpha_1| + \alpha_3 A^3 \frac{3}{4} = 20 \log|\alpha_1| - 1 \text{dB} \rightarrow A_{-1 \text{dB}} = \sqrt{\left( \frac{\alpha_1}{10^\frac{1}{20}} - \alpha_1 \right) \frac{4}{3|\alpha_3|}} = \sqrt{0.145 \frac{|\alpha_1|}{|\alpha_3|}} \text{[dB]}
\]  

(2.21)

Input Third-Order Intercept Point

The Input Third-Order Intercept Point (IIP3) is the point where third-order intermodulation products increase to an amplitude level equal to the amplitude of the frequency of interest at the output, Figure 2.4. To infer the intermodulation influence on the system’s performance, a two-tone test, \( x(t) \), is applied. This test represents the interaction between an interferer and the signal.

\[
x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)
\]  

(2.22)
Applying $x(t)$ to a nonlinear system, (2.17), and through mathematical manipulation, it is possible to obtain the following terms:

- terms in $\omega_1 \pm \omega_2$:
  \[ \alpha_2 A_1 A_2 \cos(\omega_1 t + \omega_2 t) + \alpha_2 A_1 A_2 \cos(\omega_1 t - \omega_2 t) \]  
  (2.23)

- terms in $2\omega_1 \pm \omega_2$:
  \[ \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 t + \omega_2 t) + \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 t - \omega_2 t) \]  
  (2.24)

- terms in $2\omega_2 \pm \omega_1$:
  \[ \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 t + \omega_1 t) + \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 t - \omega_1 t) \]  
  (2.25)

- terms in $\omega_1$ and $\omega_2$:
  \[ \cos(\omega_1 t) \left( \alpha_1 A_1 + \frac{\alpha_3 A_1^3}{4} + \frac{A_1 A_2^3}{2} \alpha_3 \right) + \cos(\omega_2 t) \left( \alpha_1 A_2 + \frac{\alpha_3 A_2^3}{4} + \frac{A_2 A_1^3}{2} \alpha_3 \right) \]  
  (2.26)

For simplicity, the amplitudes of both signals are considered equal and $\omega_1$ and $\omega_2$ similar (e.g. two adjacent channels). If this is the case, the intermodulation frequencies that fall near $\omega_1$, $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ terms, may be down-converted to the IF.

\[ y(t) = \cos(\omega_1 t) \left( \alpha_1 A_1 + \frac{\alpha_3 A_1^3}{4} + \frac{A_1 A_2^3}{2} \alpha_3 \right) + \cos(\omega_2 t) \left( \alpha_1 A_2 + \frac{\alpha_3 A_2^3}{4} + \frac{A_2 A_1^3}{2} \alpha_3 \right) + \frac{3}{4} \alpha_3 A^3 \cos(2\omega_1 t - \omega_2 t) + (\ldots) \]  
  (2.27)

Usually, the amplitude $A$ is sufficiently small to consider: $\alpha_1 A \gg \frac{\alpha_3 A^3}{4}$. Hence, the IIP3 is obtained equating the terms in red. Notice that the highlighted terms correspond to the worst case, comparing the signal of interest, $\omega_1$, to the nearest intermodulation product, $2\omega_1 - \omega_2$.

\[ |\alpha_1|_{\text{IIP3}} = \frac{3}{4} |\alpha_3|_{\text{IIP3}} \rightarrow A_{\text{IIP3}} = \frac{4}{3} \frac{\alpha_1}{\alpha_3} \]  
  (2.28)

Equation (2.28) allows for an approximation of the IIP3 without actually reaching the necessary input amplitude, which would oblige to take into consideration higher order terms. Due to its importance, the IIP3 is commonly evaluated, not only for individual circuit blocks, but also for the complete system (Figure 2.5).
Equation (2.29) considers the individual circuit blocks in Figure 2.5 uncorrelated in terms of distortion [10]. Through inspection, it is possible to conclude that, as the signal is amplified across the system, the individual IIP3’s influence increases. For a better understanding of (2.29), let us consider two cascaded circuits, with outputs $y_1(t)$ and $y_2(t)$, and calculate their total IIP3.

$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$  

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

Substituting (2.30) in (2.31) yields:

$$y_2(t) = (x(t)\alpha_1 + x^2(t)\alpha_2 + x^3(t)\alpha_3)\beta_1 + (x(t)\alpha_1 + x(t)^2\alpha_2 + x(t)^3\alpha_3)\beta_2 + (x(t)\alpha_1 + x(t)^2\alpha_2 + x(t)^3\alpha_3)\beta_3$$

The total IIP3 is calculated using the first and third-order terms, resulting from the simplification of (2.32),

$$A_{\text{IIP3}} = \sqrt{\frac{\alpha_1}{\beta_1}} \rightarrow A_{\text{IIP3}} = \sqrt{\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_3\beta_3}}$$

or

$$\frac{1}{A_{\text{IIP3}}} = \left| \frac{3\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_3^2\beta_2}{\alpha_1\beta_1} \right| = \left| \frac{4\alpha_1\beta_1}{A_{\text{IIP3}}^3} + \frac{3\alpha_3\beta_2}{2\beta_1} + \frac{\alpha_3^2}{A_{\text{IIP3}}^3} \right|$$

where, for narrowband circuits, the term in red of (2.34) is disregarded, due to attenuation at the second block’s input. In fact, further simplification of $y_2(t)$ shows that the first block’s output term $x(t)^2\alpha_2$, (2.35) also in red, generates intermodulation frequencies that are not equal or similar to $\omega_1$. These frequencies that are applied at the second block’s input are outside its passing-band, justifying that (2.33) can be rewritten as (2.29).

$$y_2(t) = (x(t)\alpha_1 + x(t)^2\alpha_2 + x(t)^3\alpha_3)\beta_1 + (x(t)\alpha_1 + x(t)^2\alpha_2 + x(t)^3\alpha_3)\beta_2 + (x(t)\alpha_1 + x(t)^2\alpha_2 + x(t)^3\alpha_3)\beta_3$$

$$= 2x^3(t)\alpha_1\alpha_2 + (...) = 2\alpha_1\beta_2[A_3\cos(\omega_1t) + A_2\cos(\omega_2t)]$$

$$+ \left[ A_1^2 \left( \frac{\cos(2\omega_1t + 1)}{2} + \frac{\cos(2\omega_2t + 1)}{2} + \frac{2A_1A_2}{2} \left( \cos(\omega_1 + \omega_2) + \cos(\omega_1 - \omega_2) \right) \right) \right]$$

$$+ (...)$$
2.2.3. Spurious-Free Dynamic Range

The dynamic range is defined as the signal level range at the input that translates into a useful output [11][10]. In RF, the Spurious-Free Dynamic Range (SFDR) is usually used since it is necessary to take into account existent nonlinearities, even when a quasi-linear behavior is assumed. The lower bound will be determined by the sensitivity, as the minimum input tolerated. The upper end is given by a maximum two-tone input, where third-order intermodulation products do not exceed the noise floor.

The input-referred noise floor is defined as the sensitivity, \( (2.13) \), although the SNR at the output is considered unitary.

\[
\text{noise floor} = F_n \cdot P_{\text{noise in}} \ [W] = -174 + NF + 10 \log B \ [\text{dBm}] \quad (2.36)
\]

As for third-order intermodulation products, consider the following relation,

\[
\left(2.28\right) \rightarrow \frac{A_{\omega 1}}{A_{\text{IM}3}} \approx \frac{4|a_1|A}{3|a_3|A^2} = \frac{A_{\text{IIP3}}^2}{A^2} \quad (2.37)
\]

where \( A_{\omega 1} \) is the amplitude of the desired signal at the output and \( A_{\text{IM}3} \) is the amplitude of the intermodulation products, also at the output. Since the IIP3 is commonly expressed in power units, equation (2.37) is rewritten as (2.38).

\[
20 \log A_{\text{IIP3}}^2 = 20 \log A_{\omega 1} - 20 \log A_{\text{IM}3} + 20 \log A^2 \leftrightarrow 10 \log A_{\text{IIP3}}^2 = \frac{1}{2} \left( 10 \log A_{\omega 1}^2 - 10 \log A_{\text{IM}3}^2 \right) + 10 \log A^2 \leftrightarrow P_{\text{IIP3}} = P_{\text{signal in}} + \frac{P_{\text{signal out}} - P_{\text{IM3 out}}}{2} \ [\text{dB}] \quad (2.38)
\]

Furthermore, the SFDR is usually input-referred and so equation (2.38) is rearranged, considering a power gain, \( G \).

\[
P_{\text{out}} = P_{\text{in}} + G \ [\text{dB}] \quad (2.39)
\]

\[
P_{\text{IIP3}} = P_{\text{signal in}} + \frac{1}{2} \left( P_{\text{signal in}} - P_{\text{IM3 in}} \right) \leftrightarrow P_{\text{signal in}} = \frac{2P_{\text{IIP3}} + P_{\text{IM3 in}}}{3} \ [\text{dB}] \quad (2.40)
\]

Finally, the initial statement is applied, where the intermodulation products equal the noise floor. The input-referred SFDR is obtained:

\[
P_{\text{signal in}} = \frac{2P_{\text{IIP3}} + \text{noise floor}}{3} \ [\text{dB}] \quad (2.41)
\]

\[
\therefore \text{SFDR} = \frac{2P_{\text{IIP3}} + \text{noise floor}}{3} - P_{\text{signal in min}} = \frac{2}{3} \left( P_{\text{IIP3}} - \text{noise floor} \right) - \text{SNR}_{\text{out min}} \ [\text{dB}] \quad (2.42)
\]

Notice that the minimum output SNR must be defined, according to the system’s requirements.
2.2.4. Selectivity

Selectivity is described as the ability to choose a specific frequency, or frequency range, from the spectrum. It is commonly associated with filters. For instance, in the complete receiver, it is normally defined by the band-select and channel-select filters. The existence of narrowband stages also limits the system's bandwidth. Inside the passing-band, low phase and low amplitude distortion are required, being selectivity a function of the quality factor,

\[ Q = \frac{f_0}{B} \] (2.43)

where \( f_0 \) is the resonant frequency and \( B \) is the bandwidth of the filter.

2.3. Receiver Topologies

2.3.1. Heterodyne

The name heterodyne receiver, Figure 2.6, is related to the fact that the frequency of operation will not be at baseband frequency. The necessity of shifting the signal in the spectrum is a consequence of the necessity of high quality factors at very high frequencies. This is a very sensitive issue for frontend design. Taking this into account, the implementation of a heterodyne system starts with the frequency planning. Choosing the appropriate reference frequency is very important, as it will determine the IF.

![Figure 2.6 - Heterodyne receiver (adapted from [13])](image)

The signal at IF, \( y(t) \), results from mixing the RF signal, \( x_{RF}(t) \), and the local oscillator's signal, \( x_{LO}(t) \).

\[
y(t) = x_{RF}(t). x_{LO}(t) = A_{RF} \cos(\omega_{RF} t) A_{LO} \cos(\omega_{LO} t) = \frac{A_{RF} A_{LO}}{2} \cos(\omega_{RF} t - \omega_{LO} t) + \frac{A_{RF} A_{LO}}{2} \cos(\omega_{RF} t + \omega_{LO} t) \] (2.44)

\[
\omega_{IF} = \omega_{RF} - \omega_{LO} \text{ [rad.s}^{-1}] \] (2.45)
The down-conversion, (2.45), is used in receivers since it allows for better integration of the circuit, where smaller offsets require better and more complex/expensive channel-select filters – high IF. The up-conversion is filtered.

**Image Frequency**

The major drawback of this topology is the image frequency problem. In order to solve this problem, an image-reject filter must be inserted after the LNA. This filter, due to its requirements, is the topology’s main disadvantage. For instance, let us consider an image frequency at \( \omega_{\text{image}} = 2\omega_{\text{LO}} - \omega_{\text{RF}} \), Figure 2.7 a). When the RF signal is multiplied by the reference frequency, the same frequency shift as the image is obtained, Figure 2.7 b), resulting in data hazard situations, Figure 2.7 c).

![Image Frequency Problem Illustration](image1)

**Figure 2.7 - Illustration of the image frequency problem [9]**

A high IF is preferred when designing image-reject filters; however this imposes high quality factor requirements for channel-select filters, Figure 2.8 a). The inverse situation occurs if low IF is used, Figure 2.8 b), since RF and image frequency are distanced by \( 2\omega_{\text{IF}} \). There is a tradeoff between high sensitivity, for high IF, and high selectivity, for low IF.

![Image and Channel Select Filters Tradeoff](image2)

**Figure 2.8 - Tradeoff between the relaxation of the image-reject and channel-select filters’ requirements [17]**
Another inconvenient is that image-reject filters are usually external (e.g. SAW (Surface Acoustic Wave)), forcing the LNA to drive a 50 Ω impedance. To mitigate this problem, multi-step receivers can be implemented so that the first stages have high IF and the final stages have low IF.

**Half-IF**

The half-IF problem is a specific problem of this topology, resulting from second-order distortion. For instance, consider the following interferer,

\[ \omega_{\text{interferer}} = \frac{\omega_{\text{RF}} + \omega_{\text{LO}}}{2} \text{[rad.s}^{-1}\text{]} \quad (2.46) \]

which may be directly translated to the IF. This may occur due to the interferer’s second-order distortion, which is then mixed with the oscillator’s second harmonic, i); or it may be the case that second-order distortion shifts the interferer to the IF, after down-conversion, ii).

\[ i) \quad \frac{\omega_{\text{RF}} + \omega_{\text{LO}}}{2} - \frac{2\omega_{\text{LO}}}{2} = \omega_{\text{IF}} \text{[rad.s}^{-1}\text{]} \quad (2.47) \]
\[ ii) \quad \frac{\omega_{\text{RF}} + \omega_{\text{LO}}}{2} - \frac{2\omega_{\text{LO}}}{2} = \frac{\omega_{\text{RF}} - \omega_{\text{LO}}}{2} = \frac{\omega_{\text{RF}}}{2} \text{ distortion} \quad \omega_{\text{IF}} \text{[rad.s}^{-1}\text{]} \quad (2.48) \]

To mitigate this problem, differential topologies are used. In a simplified overview, consider a differential input \( x(t) \) and a differential output \( y(t) \).

\[ x(t) = x_1(t) - x_2(t) = \frac{1}{2} x(t) - \left[ -\frac{1}{2} x(t) \right] \quad (2.49) \]
\[ y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) \quad (2.50) \]
\[ \therefore y(t) = y_1(t) - y_2(t) = a_1 x(t) + \frac{a_3 x(t)^3}{4} \quad (2.51) \]

Inspecting (2.51), it is possible to observe that even-order distortion is eliminated.

### 2.3.2. Homodyne

The homodyne receivers convert the RF signal directly to its baseband frequency (i.e. the IF is zero), being the down-converted signal usually filtered with an integrated low-pass filter. This means that the oscillator must have the same frequency as the incoming signal.
Modern signals are commonly modulated in phase and frequency, meaning that a simple baseband down-conversion would result in information loss, Figure 2.9. The problem is solved using quadrature signals. The original circuit is divided in two chains; one multiplies the input signal by a cosine and the other by a sine, resulting in In-Phase (I) and Quadrature (Q) outputs, respectively.

**Quadrature Errors**

The quadrature relation between output signals must be precise, a feature difficult to obtain, especially at high frequencies due to parasitic effects. Notice that in the homodyne case, the LO’s frequency is higher than in the heterodyne case. To illustrate this problem, let us consider, a Quadrature Phase Shift-Keying (QPSK) modulation example,

\[ x_{RF}(t) = a \cos(\omega_{RF}t) + b \sin(\omega_{RF}t) \]  
\[ x_{LOI}(t) = 2A_1 \cos(\omega_{LO}t + \theta_1) \]  
\[ x_{LOQ}(t) = 2A_2 \sin(\omega_{LO}t + \theta_2) \]

where \( a \) and \( b \) have a 0 or 1 value, and \( A_1, A_2, \theta_1, \theta_2 \), represent amplitude and phase errors. For instance, the in-phase output is,

\[ y(t) = x_{RF}(t) \cdot x_{LOI}(t) = 2aA_1 \left[ \cos((\omega_{RF}+\omega_{LO})t+\theta_1) + \cos((\omega_{RF}-\omega_{LO})t-\theta_1) \right] + 2bA_1 \left[ \sin((\omega_{RF}+\omega_{LO})t+\theta_1) + \sin((\omega_{RF}-\omega_{LO})t-\theta_1) \right] \]

or after the low-pass filter,

\[ y(t) = aA_1 \left[ \cos(\theta_1) \right] - bA_1 \left[ \sin(\theta_1) \right] \]

while the expected output was \( a \).
**DC Offsets and Self-Mixing**

Homodyne receivers convert the RF signal directly to its baseband, meaning that DC offsets can be problematic. In fact, DC fluctuations may saturate components or change their expected working regions. These offsets may be caused by leakages of the LO’s signal to the mixer’s RF input port, in a process called self-mixing, mainly caused by substrate leakages or poor mixer’s LO-RF port isolation. It may also result from leakage of interferers across the receiver’s chain to the mixer’s LO port. One possible solution is using a high-pass filter, although requiring additional DC storage techniques (e.g. noise averaging out).

**Spurious Emissions**

In homodyne receivers, both LO and RF have the same frequency, meaning that leakages from the LO to the antenna, due to poor mixer’s LO-RF port isolation, will result in spurious emissions – Duality Principle.

**Flicker Noise**

The flicker noise is a colored noise, dominant at low frequencies. For instance, for MOS transistors case, the flicker noise’s power spectral density is proportional to the period of the signal [12],

\[
\frac{V_n^2}{f} = \frac{k}{C_{ox} W L} \frac{1}{\text{Hz}}
\]

(2.57)

being k a constant, dependent on device characteristics. Two possible solutions are increasing the devices or using passive components.

**Even-Order Distortion**

In the case of sinusoidal signals, the second-order distortion is the dominant even-order distortion. One possible source of distortion is second-order intermodulation. Applying a two-tone test to a nonlinear system and simplifying the resulting output, y(t),

\[
x(t) = A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)
\]

(2.58)

\[
y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + (\ldots) = \alpha_1 [A_1 \cos(\omega_1 t) + A_2 \cos(\omega_2 t)] + \alpha_2 \left[ \frac{A_1^2 \cos(2\omega_1 t) + 1}{2} + \right.
\]

\[
A_1 A_2 \left[ \cos((\omega_1 + \omega_2) t) + \cos((\omega_1 - \omega_2) t) \right] + \frac{A_2^2 \cos(2\omega_2 t) + 1}{2} + (\ldots)
\]

(2.59)

yields a low-frequency beat and two DC terms, \( \alpha_1 \frac{A_1^2}{2} \) and \( \alpha_2 \frac{A_2^2}{2} \), marked in red.
This may be the case of nonlinearities at the mixer’s output, or the result of feedthrough of the LNA’s output nonlinearities between the mixer’s RF and IF ports.

Another possible source of distortion is AM cross-modulation, for example, when there is a disturbance or fading of the signal’s amplitude during propagation.

\[ x(t) = A(t) \cos(\omega t) \]  \hspace{1cm} (2.60)

\[ y(t) = \alpha_1 A(t) \cos(\omega t) + \frac{\alpha_2 [1 + \cos(2\omega t)] A^2(t)}{2} + (\ldots) \] \hspace{1cm} (2.61)

Inspecting (2.61), it is noticeable that the second-order distortion is problematic, even when the amplitude is constant in time. A possible solution is using differential circuits.

### 2.3.3. Low-IF

The low-IF receiver down-converts the RF signal to a low IF, while using image-rejection techniques that avoid the use of external filters. The IF can be as low as \( B_{\text{channel}}/2 \), where \( B_{\text{channel}} \) is the channel’s bandwidth [17]. In order to apply the image-rejection techniques, a quadrature mixer is required. In fact, the major issue of this topology is IQ mismatch sensitivity.

Several architectures can be implemented, depending on the image-rejection method, being the most common in analog domain: Hartley, Weaver and Polyphase filters. In this section both Hartley and Weaver topologies will be studied.

**Hartley**

The Hartley architecture’s (Figure 2.10) working principle is divided in two steps: first down-conversion, where resulting outputs are in quadrature, then filtering and image-rejection. The image-rejection uses the RC filters at the end of each path to shift the in-phase and the quadrature signals by 45 and -45 degrees, respectively [A1.2].

![Figure 2.10 – Hartley architecture [11]](image-url)
To illustrate the principle of operation, let us consider an input, \( x_{\text{in}}(t) \), applied to the circuit in Figure 2.10.

\[
x_{\text{in}}(t) = A_{\text{RF}} \cos(\omega_{\text{RF}} t) + A_{\text{image}} \cos(\omega_{\text{image}} t)
\]

(2.62)

where

\[
\omega_{\text{LO}} - \omega_{\text{image}} = \omega_{\text{LO}} - (2\omega_{\text{LO}} - \omega_{\text{RF}}) = \omega_{\text{RF}} - \omega_{\text{LO}} = \omega_{\text{IF}} \text{ [rad s}^{-1}] \]

(2.63)

\[
\omega_{\text{LO}} \leq \omega_{\text{RF}} \text{ [rad s}^{-1}]
\]

(2.64)

\[
x_A(t) = \frac{A_{\text{RF}}}{2} \sin[(\omega_{\text{LO}} - \omega_{\text{RF}}) t] + \frac{A_{\text{image}}}{2} \sin[(\omega_{\text{LO}} - \omega_{\text{image}}) t] = \frac{A_{\text{RF}}}{2} \sin[\omega_{\text{IF}} t] + \frac{A_{\text{image}}}{2} \sin[\omega_{\text{IF}} t]
\]

(2.65)

\[
x_B(t) = \frac{A_{\text{RF}}}{2} \cos[(\omega_{\text{LO}} - \omega_{\text{RF}}) t] + \frac{A_{\text{image}}}{2} \cos[(\omega_{\text{LO}} - \omega_{\text{image}}) t] = \frac{A_{\text{RF}}}{2} \cos[\omega_{\text{IF}} t] + \frac{A_{\text{image}}}{2} \cos[\omega_{\text{IF}} t]
\]

(2.66)

For simplicity, it is considered that a 90º shift is applied to one of the signals, instead of ± 45º to each separately. Let \( x_C(t) \) be signal \( x_A(t) \) shifted by 90º. Finally, adding \( x_C(t) \) and \( x_B(t) \) eliminates the image frequency, (2.68).

\[
x_C(t) = \frac{A_{\text{RF}}}{2} \cos[\omega_{\text{IF}} t] - \frac{A_{\text{image}}}{2} \cos[\omega_{\text{IF}} t]
\]

(2.67)

\[
\therefore y(t) = x_C(t) + x_B(t) = A_{\text{RF}} \cos[\omega_{\text{IF}} t]
\]

(2.68)

The 90º shifter is the critical component in this topology. If mismatches occur, the image frequency will only be partially cancelled. These mismatches can be either in phase or amplitude. The shifting block is tuned to \( \omega_{\text{IF}} = \frac{1}{RC} \). This resonant frequency is sensible to temperature variations and physical defects, altering the components’ parameter values. Frequency deviations prior to this block will also affect the amplitude of the output signals [A1.2]. Hence, the Weaver architecture is commonly used as an alternative.

To evaluate the obtained image-rejection, the Image Rejection Ratio (IRR) is often used. The IRR is the power ratio between the image frequency and the signal of interest both at the input and output.

\[
IRR = 10 \log \left( \frac{P_{\text{image out}}}{P_{\text{signal out}}} \right) \text{ [dB]}
\]

(2.69)

Regarding very low-IF filtering, the low-pass filters can be replaced by band-pass for a better suppression of low frequency interferences.
Weaver

The Weaver architecture’s (Figure 2.11) principle is divided in the same two steps as the Hartley architecture: first down-conversion, where resulting outputs are in quadrature; then filtering and image-rejection. In the first step, choosing the same input as in (2.62), \( x_n(t) \), yields the same expressions for \( x_A(t) \) and \( x_B(t) \), (2.65) and (2.66) respectively.

![Weaver Architecture](image)

Figure 2.11 – Weaver Architecture [11]

The second step, image-rejection, uses two additional mixers to obtain the desired 90° shift between \( x_A(t) \) and \( x_B(t) \).

\[
x_C(t) = -\frac{\Delta \text{RF}}{4} \{ \cos[(\omega_{IF1} - \omega_{LO2})t] - \cos[(\omega_{IF1} + \omega_{LO2})t] \} + \frac{\Delta \text{image}}{4} \{ \cos[(\omega_{IF1} - \omega_{LO2})t] - \cos[(\omega_{IF1} + \omega_{LO2})t] \} \tag{2.70}
\]

\[
x_D(t) = \frac{\Delta \text{RF}}{4} \{ \cos[(\omega_{IF1} - \omega_{LO2})t] + \cos[(\omega_{IF1} + \omega_{LO2})t] \} + \frac{\Delta \text{image}}{4} \{ \cos[(\omega_{IF1} - \omega_{LO2})t] + \cos[(\omega_{IF1} + \omega_{LO2})t] \} \tag{2.71}
\]

Finally, as the image is cancelled, the output terms at the second IF given by:

\[
x_D(t) - x_C(t) = \frac{\Delta \text{RF}}{2} \cos[(\omega_{IF1} - \omega_{LO2})t] \tag{2.72}
\]

However, this topology entails a secondary image problem, due to the second down-conversion, (2.74).

\[
\omega_{LO2} \leq \omega_{IF1} \tag{2.73}
\]

\[
\omega_{\text{image}_2} = 2\omega_{LO2} - \omega_{IF1} \ [\text{rad} \cdot \text{s}^{-1}] \tag{2.74}
\]

In order to avoid this problem, the low-pass filters are substituted by band-pass. Still, this topology is more versatile in terms of frequency planning. For instance, even for \( \omega_{LO1} + \omega_{LO2} = \omega_{RF} \), the spurious emissions are less significant, when compared to the homodyne topology, since each individual LO’s frequency is never equal to the RF. In this last case, the application of a band-pass filter excludes both the DC and the low-beat frequency parasitic terms that may result from self-mixing or even-order distortion. Also, the existence of a two-step conversion relaxes selectivity/sensitivity tradeoffs.
2.4. 2.41 GHz ISM Low-IF Receiver Frontend

The receiver assigned for this thesis has a low-IF topology, Figure 2.12, where image rejection techniques are implemented in the digital domain. The main objective of the frontend is to retrieve and amplify a voltage signal (under low-noise conditions), while shifting its frequency to an IF that can be read by ADC units. The output signals are in quadrature and must have low IQ error. The blocks to be designed are the LNA and the oscillator-mixer, integrated in a single ASIC (Application-Specific Integrated Circuit) in UMC 130 nm CMOS technology.

![Diagram of the receiver](image)

Figure 2.12 – Block diagram of the receiver [8]

The frequency plan sets the IF at 10 MHz and the RF at 2.41 GHz, implying that the oscillator’s reference frequency will be 2.4 GHz. This range of frequencies allows the use of lumped parameters for system design, as the operating wave length, \( \lambda \), is much larger than the circuit’s characteristic length, \( L \). The smaller operating wave length is obtained for RF, (2.75).

\[
f = \frac{c}{\lambda} \iff \lambda = \frac{c}{f} \approx \frac{3 \times 10^8}{2.41 \times 10^9} \approx 0.123 \text{ m} \quad (2.75)
\]

\[
L \sim 500 \mu\text{m} \quad (2.76)
\]

The defined frequency plan uses similar RF and LO frequencies, obliging a high LO-RF port isolation when designing the mixer. Poor isolation results in spurious emissions or self-mixing effects, due to the LO’s signal leakage. Additionally, the choice of a 10 MHz IF makes the DC decoupling at the filter’s input difficult. Hence, either large decoupling capacitors (i.e. > 10 pF) are used or the filter must support input DC fluctuations.

Regarding the complete system’s specifications, they depend on the considered standard. Typical specifications are presented in Table 2.1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>NF</td>
<td>11 dB</td>
</tr>
<tr>
<td>IIP3</td>
<td>-20 dBm</td>
</tr>
<tr>
<td>Av*</td>
<td>23 dB</td>
</tr>
</tbody>
</table>

*considering LNA and mixer only

Table 2.1 – Typical specifications for a receiver
The sensitivity of the receiver, (2.13), is a function of its total NF. According to the Friis formula, (2.14), the total NF is strongly dependant on the first block, the LNA. This block must have high gain and low individual NF. At high frequency, the noise will be approximated by the circuit's thermal noise, although at the low-IF, the flicker noise is also important.

To study the complete receiver's linearity, the most commonly used figure of merit is the IIP3. According, to (2.29), as the signal is amplified across the system, the individual circuits’ IIP3 influence increases, being highly linear mixers and filters required.

The SFDR, (2.42), is a function of both the receiver's IIP3 and sensitivity. An increase in the IIP3 increases the SFDR value, while an increase in the minimum input signal level required decreases it. In the particular case of this project, no SNR requirements were considered; meaning an exact value for this figure was not obtained.
Chapter 3
Quadrature Cross-Coupled RC Relaxation VCO
3.1. Overview

The oscillator is the first block to be designed, considering that it will incorporate the final oscillator-mixer, where reference oscillating frequency precision is required to generate down-converted quadrature output signals. In fact, the oscillator’s topology is used to implement the mixer. Furthermore, the knowledge of the oscillator-mixer’s RF input port impedance allows for better block interconnection, since the LNA can be designed for that load, avoiding the use of buffers.

The oscillator to be designed is a Quadrature Cross-Coupled RC Relaxation Voltage-Controlled Oscillator (VCO) based on the topology of [1]. The main contributions of this design are an approximate 20% power saving and the voltage frequency tuning, without extra current consumption.

The relaxation oscillators are predominantly nonlinear circuits, although for high frequencies, such as 2.4 GHz, parasitic effects produce a quasi-linear behavior. Hence, this chapter’s overview will include a characterization of quasi-linear oscillators, followed by the most commonly used figures of merit. Under this quasi-linear behavior assumption, it is useful to study and compare both the RC relaxation oscillators and the LC oscillators. The implemented circuit is then derived, based on this study. Finally, all technical options are described and results discussed in detail.

Characterization

An oscillator is a circuit that converts DC to AC energy, usually just possessing a power supply and an output signal port – autonomous circuit. It can usually be divided in an active part, related with start-up and oscillation; and a passive part related with frequency tuning. Let block A represent the active part, which depends mainly on the signal and B correspond to a passive block, whose behavior depends mainly on the frequency, Figure 3.1.

![Figure 3.1 – Oscillator’s division in active (A) and passive (B) blocks [19]](image)

The oscillator’s behavior can also be divided in two regimes: start-up and steady-state. The start-up is associated with an increase in the signal’s magnitude, until steady-state is reached and it becomes periodic. A quasi-linear oscillator might be studied as a Linear Time-Invariant (LTI) circuit during steady-state, Figure 3.2, where its behavior is described using the Laplace transform, (3.1).
Inspecting (3.1), it is possible to describe the poles by the characteristic equation,

\[ 1 - A(s_0)B(s_0) = 0 \Rightarrow |A(s_0)B(s_0)| = 1 \wedge \arg(A(s_0)B(s_0)) = 2k\pi, k \in \mathbb{Z} \quad (3.2) \]

denotes this is called the Barkhausen Criterion. Notice that the input, \( X_i \), is null, since the circuit is autonomous.

In start-up regime, positive feedback exists,

\[ |A(s_0)B(s_0)| > 1 \quad (3.3) \]

meaning the system has poles in the right-half of the s-plane, increasing the amplitude of the oscillating signal. During this regime, the circuit’s noise acts as the input, \( X_i \). The amplitude is then controlled by means of a nonlinear circuit, for instance, with variable resistance or with saturation of amplifying blocks, thus stabilizing the circuit.

Start-up and steady-state regime can also be expressed by equivalent electric models, from which circuit equations can be derived, giving a better physical interpretation. The oscillation condition for a steady-state regime can be given by a negative impedance, Figure 3.3 a), or admittance model, Figure 3.3 b), depending on whether the signal is described as a current or a voltage, respectively.

Consider that block B is a resonator with losses and block A is an amplifying block with negative resistance/conductance that compensates those losses.

At resonant frequency, \( \omega_0 \), the reactive part is null and so the oscillating condition is verified only if the real parts of blocks A and B are symmetrical. This means that the losses are compensated and the resonator may then be studied as lossless.
\[ \mathbf{V} = -Z_A(I, \omega_o) \mathbf{I} [\mathbf{V}] \cap \mathbf{V} = Z_B(\omega_o) \mathbf{I} [\mathbf{V}] \rightarrow (Z_A(I, \omega_o) + Z_B(\omega_o)) \mathbf{I} = 0 [\mathbf{V}] \land \mathbf{I} \neq 0 [\mathbf{A}] \quad (3.4) \]

\[ \mathbf{I} = -Y_A(V, \omega_o)\mathbf{V} [\mathbf{A}] \cap \mathbf{I} = Y_B(\omega_o)\mathbf{V} [\mathbf{A}] \rightarrow (Y_A(V, \omega_o) + Y_B(\omega_o))\mathbf{V} = 0 [\mathbf{A}] \land \mathbf{V} \neq 0 [\mathbf{V}] \quad (3.5) \]

However, during start-up regime and using the same equivalent circuit models, the negative resistive part of block A is dominant, increasing the amplitude of oscillation.

\[ \text{Im}(Z_B(\omega_o) + Z_A(I, \omega_o)) [\Omega] = 0 \land \text{Re}(Z_B(\omega_o)) + \text{Re}(Z_A(I, \omega_o))[\Omega] < 0 \quad (3.6) \]

\[ \text{Im}(Y_B(\omega_o) + Y_A(I, \omega_o))[S] = 0 \land \text{Re}(Y_B(\omega_o)) + \text{Re}(Y_A(V, \omega_o))[S] < 0 \quad (3.7) \]

Briefly, the circuit's energy increases, as the equivalent real part becomes negative.

\[ P = I_{\text{rms}}^2 (R_A + R_B) < 0 [W] \quad (3.8) \]

\[ P = V_{\text{rms}}^2 (G_A + G_B) < 0 [W] \quad (3.9) \]

Nonlinearities will then ensure the circuit’s stabilization, which can be graphically expressed as in Figure 3.4.

![Graphical interpretation of stability](image)

Figure 3.4 – Graphical interpretation of stability [19].

Notice that the angle, \( \psi \), should assume values between 0 and \( \pi \) radians, to verify the system’s previous conditions. If the signal increases, then the real part of block A decreases (and vice-versa); similarly, if the oscillating frequency changes the imaginary part will contradict this change as in a band-pass.

**Figures of Merit**

As for the figures of merit, an oscillator might be described by:

- Resonant Frequency, \( f_0 \); frequency of the output signal;
- Phase Noise;
- Amplitude Noise;
- Quadrature Error;
- THD: ratio between power dissipated by multiples of the fundamental \( f_0 \) and the fundamental itself, \((2.18)\);
- Pushing Effect: oscillator should not alter its working frequency due to supply variations;
- Pulling Effect: oscillator should not alter its working frequency due to load variations;
- Stability: ability to maintain its properties in time – aging – and in changing environment;
- Output Power/Voltage: output power/voltage delivered to the load at \( f_0 \).

One of the most important figures of merit is the phase noise. For its characterization, the noise is considered white and normalized for 1 Hz bandwidth, being represented by a sinusoid for each frequency of the spectrum. Hence, the phase noise is the result of the signal’s, \( x(t) \), phase distortion, \( \theta(t) \), which causes instantaneous frequency, \( f_{\text{total}} \), variations.

\[
x(t) = A(t) \cos(\omega_0 t + \theta(t)) = A(t) \cos(\psi(t)) \quad (3.10)
\]

\[
f_{\text{total}} = \frac{1}{2\pi} \frac{d\psi(t)}{dt} = \frac{1}{2\pi} \left( \omega_0 + \frac{d\theta(t)}{dt} \right) \quad [\text{Hz}] \quad (3.11)
\]

The resulting signal is a sinusoid with sidebands, Figure 3.5 b).

![Figure 3.5](image)

Figure 3.5 – Graphical interpretation of phase noise in time, a) [8], and in the frequency spectrum, b).

The phase noise figure of merit is then described as the ratio between \( P_N \), the power of the noise component at a distance \( \Delta f \) from the signal, and the power of the signal itself, \( P_S \), \((3.12)\). Considering the noise is Gaussian results in symmetric sidebands. The upper sideband is usually chosen for this metric.

\[
\mathcal{L}(\Delta f) = 10 \log \left( \frac{P_N}{P_S} \right) \quad [\text{dBc/Hz}] \quad (3.12)
\]

The case of amplitude noise is less severe, as there are usually amplitude stabilizing mechanisms, being this metric considered out of the project’s scope.
3.1. Oscillator Topologies

3.1.1. MOS Oscillators

There are several possible topologies for oscillators; in this section, transistor based oscillators are analyzed for Integrated Circuit (IC) optimization. These oscillators might be single or multi-transistor; this analysis will focus multi-transistor oscillators, allowing for a differential output, which avoids even-order distortion problems. Two topologies are chosen: cross-coupled LC oscillator and cross-coupled RC relaxation oscillator. The LC oscillators are characterized by a quasi-linear behavior, while the most common implementation of the RC relaxation oscillators is strongly nonlinear. However, for a high frequency of operation, such as 2.4 GHz, RC relaxation oscillators exhibit a quasi-linear behavior, due to parasitic effects. The oscillator to be designed takes advantage of this feature, being the comparison with the LC oscillator case useful.

Cross-Coupled LC Oscillator

The cross-coupled LC oscillator is a sinusoidal oscillator that may be divided in two blocks, recalling the analysis made for quasi-linear oscillators in the overview section. The cross-coupled differential pair corresponds to the active block A, and the LC tank has frequency-selective properties, corresponding to block B, Figure 3.6.

![Cross-coupled LC oscillator](image)

Figure 3.6 – Cross-coupled LC oscillator [8]

For simplicity, the transistors are considered ideal transconductors [A2.1] in small-signal analysis, and the LC tank lossless. Hence, the frequency of oscillation is approximately,

\[
Y_B(j\omega) = \frac{1}{j\omega L} + j\omega C = 0 \rightarrow f_0 = \frac{1}{2\pi \sqrt{LC}} [\text{Hz}] \quad (3.13)
\]

while the active admittance of block A, Figure 3.7, is given by,
Finally, regarding the phase noise, in steady-state regime the conductance of the tank, $G_B$, and the conductance of the cross-coupled differential pair, $G_A$, are symmetrical, being the losses compensated. Hence, the oscillator’s impedance can be approximated by the impedance of an ideal LC tank.

$$Z(j\omega) = \frac{1}{j\omega C + \frac{1}{G_B}} = \frac{1}{G_B[1 - \omega_0/Q_B \left(1 - \frac{\omega^2}{\omega_0^2}\right)]} \text{[\Omega]}$$  \hspace{1cm} (3.17)$$

Considering the noise at a certain frequency distance, $\Delta\omega$, from the fundamental’s signal frequency, $\omega_0$, (i.e. the upper sideband) it is possible to rewrite (3.17) as (3.20). Furthermore, (3.20) is also written as a function of the resonator’s quality factor.

$$\omega = \omega_0 + \Delta\omega \land \Delta\omega \ll \omega_0 \rightarrow \omega^2 \cong \omega_0^2 + 2\omega_0\Delta\omega \text{[rad.s}^{-1}]$$  \hspace{1cm} (3.18)$$

$$Q_{\text{tank}} = 2\pi \frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} = 2\pi f_0 \frac{\text{Energy stored}}{\text{Power dissipated}} = \omega_0 \frac{V^2_{\text{rms}}}{I^2_{\text{rms}}} = \frac{V^2_{\text{rms}}}{I^2_{\text{rms}}} = \frac{1}{\omega_0 I G_B}$$  \hspace{1cm} (3.19)$$

$$Z(\omega_0 + \Delta\omega) \cong \frac{1}{G_B \left[\omega_0 + \frac{\omega_0 \Delta\omega}{\omega_0 + \Delta\omega}\right]} \cong \frac{1}{G_B \left\{\frac{1}{\omega_0} + \frac{1}{\omega_0 + \Delta\omega}\right\}} = \frac{1}{G_B \left[\frac{\omega_0}{\omega_0 + \Delta\omega}\right]} = \frac{1}{G_B \left[\frac{2\omega_0\Delta\omega}{\omega_0 + \Delta\omega}\right]} = -j \frac{\omega_0}{G_B 2\omega_0} \text{[\Omega]}$$  \hspace{1cm} (3.20)$$

Then, considering the main noise contribution is given by the resonator’s losses, a thermal noise current source is placed in parallel with the tank.
\[
\vec{v}_n^2 = 4K_BT_GB \left[ \frac{\Delta^2}{\text{Hz}} \right] \rightarrow \vec{v}_n^2 = \vec{i}_n^2 |Z(\omega_0 + \Delta \omega)|^2 = 4K_BT_GB \left( \frac{\omega_0}{\text{g}_B 2Q \Delta \omega} \right)^2 \left[ \frac{V^2}{\text{Hz}} \right] \quad (3.21)
\]

Equation (3.21) contemplates the contributions of both the amplitude and phase noise. Hence, recalling that \(\vec{v}_n^2\) is a power spectral density [A1.1], for a 1 Hz bandwidth, its value is also equal to the total noise power. Additionally, in thermodynamic equilibrium the amplitude and phase noise have the same power [19], meaning the phase noise power is half of \(\vec{v}_n^2\) for a 1 Hz bandwidth. Applying the phase noise definition yields,

\[
\therefore L(\Delta \omega) = 10 \log \left( \frac{V_{rms}^2}{V_{rms}^2} \right) = 10 \log \left( \frac{2K_BT \left( \frac{\omega_0}{\text{g}_B} \right)^2}{V_{rms}^2 / g_B} \right) = 10 \log \left( \frac{2K_BT \left( \frac{\omega_0}{\text{g}_B} \right)^2}{P_{\text{sig}}} \right) \left[ \text{dBc} / \text{Hz} \right] \quad (3.22)
\]

where \(P_{\text{sig}}\) is the power dissipated by the tank. In order to obtain an approximated model for the whole spectrum, the Leeson-Cutler semi-empirical model is used (Figure 3.8),

\[
L(\Delta \omega) = 10 \log \left( \frac{2K_BT F}{P_{\text{sig}}} \left( 1 + \frac{\omega_0}{2Q \Delta \omega} \right)^2 \left( 1 + \frac{\Delta \omega}{\omega_0} \frac{1}{\Delta \omega} \right) \right) \left[ \text{dBc} / \text{Hz} \right] \quad (3.23)
\]

where, \(F\) is an empirical parameter and \(Q\) is the LC tank’s quality factor. In region A, the flicker noise is dominant; in region B the frequency modulation of the oscillator’s internal noise sources dominates and, in region C, only the white noise floor is relevant.

**Cross-Coupled RC Relaxation Oscillator**

The most common implementation of the cross-coupled RC relaxation oscillator exhibits a strongly nonlinear behavior. Its working principle is based on the charge and discharge of a capacitor between two reference values. The system is said to have memory since, between two thresholds, the constant of integration remains unaltered. When one of the thresholds is reached the integration sign changes, Figure 3.9.
Figure 3.9 – Working principle of a relaxation oscillator: i) integration of a constant; ii) comparison at a certain threshold; iii) integration constant sign change [20].

A high-level model of a system with these characteristics is given by a closed loop with an integrator and a Schmitt-Trigger, Figure 3.10.

Figure 3.10 – High-level model of a Relaxation Oscillator [8].

As for the practical implementation, one possibility is the cross-coupled RC relaxation oscillator in Figure 3.11.

Figure 3.11 – Cross-coupled RC relaxation oscillator [8].

The capacitor represents the integrator, being its input a current, $I_C$, and the output a voltage $V_C$. Notice that since the current is defined by the DC current sources and the capacitance is fixed, it is possible to derive a constant of integration. This results in a triangular voltage waveform.

$$I_C(t) = \frac{C dV_C(t)}{dt} \land I_C(t) = I \rightarrow \frac{dV_C(t)}{dt} = \frac{I}{C} = K_{\text{integration}} \ [V \cdot s^{-1}] \ (3.24)$$

The cross-coupled transistors and the resistors represent the Schmitt-Trigger, converting $V_C$ to $I_C$. The sign of the integration constant depends on which transistor is conducting, changing the direction of
current through the capacitor. The total current flowing through the conducting transistor is \( I \), meaning that the actual current through the capacitor is \( I \), Figure 3.12.

\[
V_C(t) = \int_{t_{\text{initial}}}^{t} K_{\text{integration}} \, dt + V_C(t_{\text{initial}}) = \frac{1}{C} \Delta t + V(t_{\text{initial}}) \quad [V] \quad (3.25)
\]

The circuit is self-biased and the conduction mode of a transistor lasts as long as \([A2.1]\),

\[V_{GS}(t) > V_i \quad [V] \quad (3.26)\]

is verified. Transistors are treated as switches, where the transition between modes of conduction is considered abrupt (i.e. an ideal Schmitt-Trigger), Figure 3.13. Now let us consider that mode 1 is given by \( M_1 \) conducting and \( M_2 \) cut-off, as in Figure 3.12.

\[
V_{GS_2}(t) = (V_{DD} - 2RI) - V_{C_+}(t) \quad [V] \quad (3.27)
\]

\[
V_{GS_1}(t) = V_{DD} - V_{C_-}(t) \quad [V] \quad (3.28)
\]
\[ V_{GS1}(t) > V_t \leftrightarrow V_{DD} - V_{C+}(t) > V_t \leftrightarrow V_{C+}(t) < V_{DD} - V_t \quad [V] \quad (3.29) \]

\[ V_{GS2}(t) < V_t \leftrightarrow (V_{DD} - 2RI) - V_{C-}(t) < V_t \leftrightarrow V_{C-}(t) > V_{DD} - 2RI - V_t \quad [V] \quad (3.30) \]

\[ \therefore V_{C_{\text{transition}}} = V_{C_{\text{max}}} - V_{C_{\text{min}}} = (V_{DD} - V_i) - (V_{DD} - 2RI - V_i) = 2RI \quad [V] \quad (3.31) \]

\[ \therefore V_{GS1} - V_{GS2} = 0 \leftrightarrow (V_{DD} - V_{C+}) - (V_{DD} - 2RI - V_{C-}) = 2RI = V_{C_{\text{transition}}} \quad [V] \quad (3.32) \]

The interpretation of (3.29) and (3.30) is that, for a constant gate voltage, the source voltage of M1, \( V_{S1} \), will rise until the condition is no longer verified and M1 enters cut-off, the opposite happening with \( V_{S2} \) and M2 that starts conducting. This transition is expressed by (3.31), which represents the maximum capacitor’s voltage. Since modes are considered symmetric (i.e. half period duration), it is possible to conclude that the minimum value should be -2RI. Therefore, the oscillation frequency is,

\[ t_{\text{initial}} = 0 \text{[s]} \land V_C(0) = -2RI \quad [V] \quad (3.33) \]

\[ V_C \left( \frac{T}{2} \right) = \frac{1}{C} \int_0^t V_C(0) \leftrightarrow 2RI = \frac{1}{C} \int_0^T 2RI \quad [V] \rightarrow f_0 = \frac{1}{8\pi RC} \quad [Hz] \quad (3.34) \]

and the output voltage signals, \( V_1 \) and \( V_2 \) of the circuit in Figure 3.11, are square waves, Figure 3.14.

\[ t_{\text{mode} \ 1} \rightarrow K_{\text{integration}} = \frac{V}{C} \land t_{\text{mode} \ 2} \rightarrow -K_{\text{integration}} = -\frac{V}{C} \quad [s] \quad (3.35) \]

\[ V_{\text{out}}(t_{\text{mode} \ 1}) = V_1(t) - V_2(t) = V_{DD} - 2RI - V_{DD} = -2RI \land V_{\text{out}}(t_{\text{mode} \ 2}) = 2RI \quad [V] \quad (3.36) \]

![Diagram](image.png)

Figure 3.14 – Oscillator’s output and integrator’s voltage signals [8].

Finally, in terms of phase noise, let \( V_{\text{out}}(t) \) be the oscillator’s output signal, which has a certain phase error, \( \theta(t) \). That error is a consequence of internal low-frequency noise modulation [21].

\[ V_{\text{out}}(t) = V \cos(\omega_0 t + \theta(t)) = V \cos(\phi(t)) \quad [V] \quad (3.37) \]
If $k_f$ represents a conversion factor in [Hz/V], a peak deviation from oscillating frequency, $\Delta f_o$, can be defined, (3.39).

$$\Delta f_o = k_f V_{\text{noise}} \, [\text{Hz}] \quad (3.39)$$

Notice that both this peak deviation, $\Delta f_o$, and the noise’s frequency, $f_{\text{noise}}$, must be small in comparison to the oscillating frequency. Additionally, for narrowband circuits: $\frac{\Delta f_o}{f_{\text{noise}}} \ll 1$ [22]. Hence, using equations (3.38) and (3.37), it is possible to rewrite the output signal.

$$V_{\text{out}}(t) = V \cos \left( \omega_o t + \frac{\Delta f_o}{f_{\text{noise}}} \sin(\omega_{\text{noise}} t) \right) = V \left\{ \cos(\omega_o t) \cos \left( \frac{\Delta f_o}{f_{\text{noise}}} \sin(\omega_{\text{noise}} t) \right) - \sin(\omega_o t) \sin \left( \frac{\Delta f_o}{f_{\text{noise}}} \sin(\omega_{\text{noise}} t) \right) \right\} \quad (3.40)$$

$$\approx V \left[ \cos(\omega_o t) - \frac{\Delta f_o}{f_{\text{noise}}} \sin(\omega_o t) \sin(\omega_{\text{noise}} t) \right] \, [V] \quad (3.41)$$

Inspecting equation (3.41), it is possible to conclude that the output signal is composed by its fundamental frequency and two sidebands. The upper band is chosen for phase noise calculations, where $f_{\text{noise}}$ is interpreted as the distance to the oscillating frequency. The high-level model in Figure 3.15 is then used for noise study [23].

![Figure 3.15 – Relaxation oscillator’s high-level noise model [23].](image)

For the simplified model in Figure 3.15, the oscillating frequency and its peak deviation due to noise is given by (3.42) and (3.43).

$$V_C(t_{\text{initial}}) = 0 \, [V] \rightarrow V_C \left( \frac{T}{2} \right) = V = \frac{1}{T_C} \leftrightarrow f_o = \frac{1}{2CV} \, [\text{Hz}] \quad (3.42)$$

$$\Delta f_o = \frac{i_n}{2CV} \leftrightarrow \frac{i_n}{1} \, [\text{Hz}] \quad (3.43)$$
Finally, using equations (3.42) and (3.43), and calculating the output signal’s upper sideband power spectral density, $S_{\text{out}}(V_{\text{out}})_{f=fo+\Delta fo}$, it is possible to obtain both the voltage and current contributions to the total phase noise, (3.46) and (3.48), respectively [A1.1].

$$S(\Delta f_o) = \frac{|f_o|^2}{2\pi} S(i_n) \text{ [Hz]}$$

$$L(f_{\text{noise}}) = 10 \log \left( \frac{S(\Delta f_o)}{\frac{1}{2}f_{\text{noise}}} \right)^2 = 10 \log \left( 2 \frac{|f_o|^2}{2f_{\text{noise}}} S(i_n) \right) \text{ [dBc/Hz]}$$

$$\therefore L(f_{\text{noise}}) = 10 \log \left( \frac{S(i_n)}{2f_{\text{noise}}} \right)^2 \left[ \frac{\text{dBc}}{\text{Hz}} \right]$$

$$\Delta f_o = -\frac{1}{2CV^2} V_n = -\frac{f_o}{V} V_n \text{ [Hz]}$$

$$\therefore L(f_{\text{noise}}) = 10 \log \left( \frac{S(V_n)}{2V} \frac{f_o}{f_{\text{noise}}} \right)^2 \left[ \frac{\text{dBc}}{\text{Hz}} \right]$$

Cross-Coupled RC Relaxation Oscillator’s Quasi-Linear Behavior

The cross-coupled RC relaxation oscillator (Figure 3.11) is usually implemented as a nonlinear oscillator, being its output given by a square voltage waveform, although, for high frequencies, the oscillator exhibits a quasi-linear behavior due to parasitic effects. To study this behavior, where an approximately sinusoidal output is obtained, it is necessary to take into consideration the circuit’s parasitics and perform a small-signal analysis around a quiescent operating point, $Q$ [24, Figure 3.16 a). Let that operating point be the transition between the conduction of transistors $M_1$ and $M_2$, this is between modes 1 and 2.

$$I_{M_1} = I_{M_2} = I \text{ [A]}$$

$$V_{GD_1} = V_{DD} - (V_{DD} - 2RI) = 2RI \text{ [V]}$$

$$V_{GD_2} = (V_{DD} - 2RI) - V_{DD} = -2RI \text{ [V]}$$

$$|V_{GD}(t)| = |V_1(t) - V_2(t)| \text{ [V]}$$

In quasi-linear behavior, transistors operate in the saturation region, a condition that is verified for,
\[ V_{GS}(t) > V_t \wedge V_{DS}(t) > V_{GS}(t) - V_t \mapsto V_t > V_{GD}(t) \mapsto V_t > 2RI \ [V] \] (3.53)

where the maximum possible output voltage, \( V_{GD} = 2RI \), does not exceed the transistors' threshold voltage, \( V_t \). Under these conditions, the oscillator’s Schmitt-Trigger (i.e. the cross-coupled differential pair and resistors) small-signal circuit is given by Figure 3.16 b).

![Figure 3.16](image)

**Figure 3.16** – Sinusoidal output waveform, a), and Schmitt-Trigger’s incremental circuit, b) [8].

Using Kirchhoff’s current and voltage laws it is possible to find the equivalent impedance \( Z \), in parallel with the integrator [24].

\[ Z(s) = \frac{\nu}{i} = \frac{2}{i} \left( R - \frac{1}{sC_{gs} + 4C_{gd}} \right) [\Omega] \] (3.54)

Expression (3.54) can be simplified taking into consideration the circuit’s high operating frequency. This leads to an equivalent circuit with an inductor in series with a resistor.

\[ \tau_{gs} = g_m^{-1}C_{gs} \land \omega \ll \tau_{gs}^{-1} \] (3.55)

\[ \tau_{gd} = 4RC_{gd} \land \omega \ll \tau_{gd}^{-1} \] (3.56)

\[ Z(s) = 2 \left[ g_m^{-1} - R + sRg_m^{-1}\left(C_{gs} + 4C_{gd}\right) \right] = R_{\text{equivalent}} + j\omega L_{\text{equivalent}} [\Omega] \] (3.57)

\[ R_{\text{equivalent}} = 2g_m^{-1}R [\Omega] \] (3.58)

\[ L_{\text{equivalent}} = 2Rg_m^{-1}\left(C_{gs} + 4C_{gd}\right) [H] \] (3.59)

If the integrator’s capacitance \( C \) is dominant, the total oscillator’s equivalent impedance will be that of an LC resonator with losses, Figure 3.17.
Therefore, in steady-state regime and considering that the losses are compensated, the frequency expression is now given by,

\[ R_{eq} = 0 \iff g_m^{-1} - R = 0 \ [\Omega] \quad (3.60) \]

\[ s = j\omega \rightarrow \omega_o = \frac{1}{\sqrt{LC}} \ [\text{rad}\cdot\text{s}^{-1}] \quad (3.61) \]

Regarding the start-up regime, expression (3.62) must be verified.

\[ g_m^{-1} - R > 0 \rightarrow g_mR > 1 \quad (3.62) \]

Finally, the phase noise for the cross-coupled RC relaxation oscillator’s quasi-linear behavior is obtained in a similar manner to the nonlinear case, however, being the high-frequency current noise averaged out by the integrator’s capacitor. The major contribution to the oscillator’s phase noise is the voltage high-frequency component, being equation (3.48) rewritten [8],

\[ \mathcal{L}(f_{\text{noise}}) = \left(\frac{2\alpha f_o}{2\nu^2}\frac{f_o}{f_{\text{noise}}}\right)^2 \Lambda \alpha \frac{B_c}{2\nu} \quad (3.63) \]

where \( B_c \) is the effective noise conversion bandwidth.

### 3.1.2. Quadrature MOS Oscillators

The low-IF topologies require quadrature signals, existing two possible approaches regarding this requirement: either this function is incorporated in the oscillator itself or a specific block must be used. Generally, for the second solution, a frequency divider, a polyphase filter or an RC-CR network are used. The divider requires the oscillator to be designed for a higher frequency, which results in higher consumption and sensibility to parasitic effects, while usually outputting square waveforms. In the case of poly-phase filters and RC-CR networks the components’ parameter dispersion is problematic. This issue was discussed for the Hartley topology, which uses an RC phase-shifter. Additionally, using an independent circuit may require drivers to avoid pulling effect, due to load variations. In this section, a quadrature MOS oscillator, using two coupled single RC relaxation oscillators from section 3.1.1, is analyzed for both nonlinear and linear behavior.
Quadrature Cross-Coupled RC Relaxation Oscillator

In order to obtain a quadrature cross-coupled RC relaxation oscillator, it is necessary to modify the high-level model of the single RC relaxation oscillator in Figure 3.10. Consider the high-level model in Figure 3.18 a), where a soft-limiter is added to evaluate the integrator’s output, \( V_{\text{INT}} \). The soft-limiter’s output, \( V_{\text{SL}} \), is in quadrature with the Schmitt-Trigger’s output, \( V_{\text{ST}} \), Figure 3.18 b).

![Diagram](image1)

Figure 3.18 – Modified single relaxation oscillator’s high-level model (adapted from [20] and [8]).

The quadrature oscillator in Figure 3.19 a), uses two coupled single relaxation oscillators, where the soft-limiters’ output signals, \( V_{\text{SL1}} \) and \( V_{\text{SL2}} \), are added at the Schmitt-Trigger’s input of the opposite oscillator, \( V_1 \) and \( V_2 \). If the integrators’ outputs, \( V_{\text{INT1}} \) and \( V_{\text{INT2}} \), are in quadrature, this is the same as considering that the slope of the integrator’s output triangular wave is increased by a constant value, \( V_2 \) in Figure 3.19 b). This means that the signal has a higher first order derivative, resulting in faster Schmitt-Trigger’s level commutation and less sensitivity to noise at the decision level.

![Diagram](image2)

Figure 3.19 – Quadrature Relaxation Oscillator’s high-level model (adapted from [8] and [20])
A practical implementation of this high-level model is given by Figure 3.20, where two single cross-coupled RC relaxation oscillators are coupled through differential-pairs, working as soft-limiters.

Figure 3.20 – Quadrature cross-coupled RC relaxation oscillator [8].

In order to describe the circuit’s behavior, consider the left single oscillator in Figure 3.20. If the soft-limiter senses \( V_{C2^+} - V_{C2^-} > 0 \), current will be drawn at node \( V_2 \) and so \( M_1 \)’s gate voltage, \( V_{G1} \), drops. The opposite happens if the soft-limiter senses \( V_{C2^+} - V_{C2^-} < 0 \), current will be drawn at node \( V_1 \) and so \( M_2 \)’s \( V_{G2} \) drops. The soft-limiter’s decision level is given by \( V_{C2^+} - V_{C2^-} = 0 \). Hence, the soft-limiter’s current draw lowers the Schmitt-Trigger’s decision level, \( V_{C_{\text{transition}}} \), which is the same as boosting the integrator’s output voltage, \( V_C \), towards the Schmitt-Trigger’s usual input comparison level. Hence, as for the high-level model, output signals, \( V_1 \) and \( V_2 \), become aligned in quadrature.

\[
V_{GS1} - V_{GS2} = 0 \leftrightarrow (V_{DD} - RI_{\text{soft-limiter}} - V_{C+}) - (V_{DD} - 2RI - V_{C-}) = 2RI - RI_{\text{soft-limiter}} - V_{C_{\text{transition}}} \rightarrow V_{C_{\text{transition}}} = 2RI - RI_{\text{soft-limiter}} \quad [V]
\]  

(3.64)

Finally, it must be noticed, that this study assumes that both the single oscillators are symmetrical. The occurrence of mismatches results in frequency and amplitude deviations. For example, in Figure 3.20, if the capacitance \( C_2 \) is reduced, the maximum integrator’s amplitude, \( V_{C1} \), will also decrease. This increases the frequency of the left oscillator, following the frequency of the right one. Variations of charge and discharge currents result in a similar effect. The single oscillators must operate at the same frequency, being quadrature preserved. Regarding the circuit’s phase noise, it will be inversely proportional to the number of oscillators, being enhanced [25] [26].

**Quadrature Cross-Coupled RC Relaxation Oscillator’s Quasi-Linear Behavior**

It is possible to obtain a quadrature cross-coupled RC relaxation oscillator with quasi-linear behavior for high frequencies and considering the circuit’s parasitic effects. The quadrature oscillator exhibits a quasi-linear behavior, if each of the two coupled single oscillators respect the condition given by equation (3.53), ensuring that the differential cross-coupled pairs operate in the saturation working
region. The resulting differential output waveforms, \( V_{\text{out1}} \) and \( V_{\text{out2}} \), are approximately sinusoidal, Figure 3.21.

![Output waveforms.](image)

Figure 3.21 – Output waveforms.

In the case of quasi-linear oscillators, where a sinusoidal output is required, it is important to note that usually there is a tradeoff between the phase noise reduction and quadrature precision. In fact, the coupling of two oscillators alters the oscillating frequency value, even when oscillators are perfectly symmetrical. This tradeoff is analyzed using the simplified model in Figure 3.22 [19], where the outputs of each single oscillator are given by \( V_x \) and \( V_y \). Each simplified single oscillator is represented by an LC tank with losses, \( g_p \), and an active part, \( g_m \), to compensate those losses. The coupling gain is given by \( g_c \).

![Equivalent single-ended linear model](image)

Figure 3.22 – Equivalent single-ended linear model [19]

Using the Barkhausen criterion, (3.2), it is possible to prove that the signals \( V_x \) and \( V_y \) are in quadrature, (3.66).

\[
V_x = B(s)V_y \land V_y = A(s)V_x \quad [V]
\]  

(3.65)

\[
A(s) = -B(s) \rightarrow \frac{V_x}{V_y} = -\frac{V_y}{V_x} \leftrightarrow V_x^2 + V_y^2 = 0 \leftrightarrow V_x = \pm jV_y \quad [V]
\]  

(3.66)

Furthermore, in steady-state regime, it is possible to prove there is an oscillating frequency deviation due to the oscillator’s coupling, (3.70). Let \( Z_{xy} \) be the impedance of each single oscillator.

\[
g_p - g_m = 0 \quad [S] \rightarrow Z_{xy}^2(s) = \frac{1}{(s^2 + \frac{1}{\Gamma})} \quad [\Omega]
\]  

(3.67)
\[ A(s_o)B(s_o) = 1 \leftrightarrow A(j\omega_o)B(j\omega_o) = -g_c^2Z^2_{x,y}(j\omega_o) \leftrightarrow -g_c^2Z^2_{x,y}(j\omega_o) = 1 \quad (3.68) \]

\[ 4LC \gg (g_cL)^2 \rightarrow \omega_o = \frac{1+\sqrt{(g_cL)^2+4LC}}{2LC} \rho_s \frac{\sqrt{Q}}{2LC} \quad \text{[rad.s}^{-1}] \quad (3.69) \]

\[ \omega_o = \omega_{oLCTank} + \frac{g_c}{2C} \rho_s \frac{\sqrt{Q}}{2LC} \quad \text{[rad.s}^{-1}] \quad (3.70) \]

Inspecting equation (3.70), it is possible to observe that, as the coupling is reinforced, the oscillating frequency is deviated from the LC tank’s resonance, increasing the circuit’s phase noise. This effect is less significant if the tank is designed in order to have a dominant capacitance, C. In the particular case of relaxation oscillators, the equivalent LC tank depends on the circuit’s parasitics, which means that the phase-noise is less sensitive to this coupling effect than in the LC oscillators’ case, where the quality factor is severely degraded. In fact, according to [20], the phase noise will be inversely proportional to the number of coupled oscillators and enhanced by coupling strength, taking in consideration that the quasi-linear behavior is derived from an operating quiescent point of the typically nonlinear relaxation oscillator circuit.

### 3.1.3. Discussion

The quadrature cross-coupled RC relaxation oscillators are usually implemented as a nonlinear circuit, although, for high frequencies, it is possible to obtain a sinusoidal output due to parasitic effects. The RC relaxation oscillators are characterized by a reduced area, as the circuit is inductorless. The major drawbacks are the higher phase noise and power consumption.

The purity of the output sinusoid depends mainly on the quality factor, Q, of the tank. In the case of the LC oscillators, the tank is implemented with passive components, while in the relaxation oscillator’s case the tank is formed by its parasitics at high frequency. This difference is observed in the phase noise expressions, (3.22) and (3.63), for the LC and the relaxation oscillator case respectively, where an immediate difference is the Q term in the denominator.

Although, for coupled LC and RC relaxation oscillators, the phase noise becomes comparable, since the LC oscillators’ phase noise is more sensible to oscillating frequency deviations. The oscillating frequency deviations are either due to coupling gain, (3.69), or due to single oscillator mismatches. The coupled oscillators will operate at the same frequency, which is defined by the new system’s steady-state. Hence, the quality factor degradation in the LC oscillators is critical. The relaxation oscillators are less dependent on circuit mismatches, and the tradeoff between quadrature precision and phase noise is not so demanding. For instance, the overall phase noise is improved when compared to the case of a single oscillator [23].
Regarding the referred higher power consumption, theoretically, the LC oscillator’s active components are used to compensate the non-ideal losses of the tank. In relaxation oscillators however, there is current integration. This implies, as a starting point, at least a \( \frac{1}{2} CV^2 \) energy loss per charging cycle.

The power optimization issue was addressed in [8] taking into consideration that the integrated current by the capacitor is \( I \) and the total used current is \( 2I \). Hence, one of the two current sources may be removed, theoretically obtaining half of the power consumption. In order to do so, transistors \( M_3 \) and \( M_4 \), Figure 3.23 a), act as switches that change the point of current draw. The transistors’ gates are connected to the Schmitt-Trigger’s outputs, \( V_1 \) and \( V_2 \), emulating the integration constant sign change.

![Figure 3.23 – Low-power relaxation oscillator, a), and quadrature relaxation oscillator in [8], b).](image)

It is important to notice that, as one current source is removed, the conducting transistor, \( M_1 \) or \( M_2 \), sees its current reduced by half. Hence, the voltage drop at the resistor of the conducting branch is \( RI \), instead of \( 2RI \). The resulting frequency shift must then be compensated with proportional increase in resistance. The coupling used in Figure 3.23 b) uses soft-limiters, which were actually implemented with PMOS instead of NMOS due to biasing constraint.

The proposed solution is based on the low-power RC relaxation oscillators in Figure 3.23, while the coupling uses PMOS as active loads instead of soft-limiters. The working principle is based on load resistance change, instead of current draw, allowing further current saving. Additionally, it excludes the need of extra components, as the active loads substitute the resistors. Finally, the implemented oscillator will be a VCO, enhancing tuning capability, without additional current consumption.

### 3.2. Implemented Circuit

The proposed oscillator is a quadrature cross-coupled RC relaxation VCO with quasi-linear behavior, implemented in a UMC 130 nm CMOS technology for a supply voltage of 1.2 V, Figure 3.24. The VCO will be simulated at circuit level only, as it will be later implemented as part of the final oscillator-mixer. The implemented VCO is based on the coupling of two single low-power RC relaxation oscillators,
Figure 3.23 a), using PMOS as active loads, instead of soft-limiters. The working principle is based on load resistance change, varying the transistors' transconductance, instead of drawing current through soft-limiters, as in Figure 3.23 b). The studied coupling for relaxation oscillators uses the Schmitt-Trigger’s input, either by boosting the integrator’s quadrature output signal towards its comparison level or lowering that same comparison level, $V_{C_{\text{transition}}}$. In this topology, the resistance is decreased, lowering the comparison level, (3.71).

\[
V_{GS1} - V_{GS2} = 0 \leftrightarrow \left( V_{DD} - V_{C_t} \right) - \left( V_{DD} - 2(R - \Delta R)I - V_{C_t} \right) = 2(R - \Delta R)I - V_{C_{\text{transition}}} \rightarrow \]

\[
V_{C_{\text{transition}}} = 2(R - \Delta R)I \ [V]
\]

This solution saves current and avoids the need of extra components. Additionally, the implementation of a VCO, using MOSCaps, allows frequency tuning capability. Until now, frequency tuning was only possible through current source control. The VCO is designed to have, at least, a 200 MHz range, 2.4 GHz $\pm$ 100 MHz. The tuning voltage is considered limited between 0 and 1.2 V, ground and supply voltage, respectively. This wide tuning voltage range maximizes the possible frequency range, meaning the circuit can be dimensioned in order to minimize the tuning sensitivity for the chosen effective frequency range (i.e. 200 MHz). The tuning sensitivity is also a function of the variable capacitance, MOSCap, which must be as linear as possible, also determining the output flatness. Notice that the variable capacitor, MOSCap, must be the dominant capacitance of the circuit, otherwise parasitics will alter the circuit’s expected behavior. Hence, as the tuning sensitivity is reduced, the phase noise improves. In fact, since the frequency is no longer fixed and becomes increasingly dependent on a tuning variable, the purity of the output sinusoid is affected [27].

This section will describe the methodologies used to implement the proposed circuit, using the equations derived in section 3.1 for RC relaxation oscillators, which determine the parameters to adjust. The section is divided in 3 parts: Cross-Coupled RC Relaxation VCO, Quadrature Cross-Coupled RC Relaxation VCO, Monte Carlo and Corners.
3.2.1. Cross-Coupled RC Relaxation VCO

In order to design a quadrature oscillator, the single oscillators to be coupled must be primarily designed. The first step to design the proposed cross-coupled RC relaxation VCO, in Figure 3.25, is to study the oscillator's start-up, ensuring that the building oscillations will converge rapidly to a steady-state solution with an adequate signal level.

Using the starting values in Table 3.1, it is possible to study the circuit's start-up using a transient analysis and taking in consideration equation (3.72), where $g_m$ is the transconductance of the transistors' of the cross-coupled differential pair. Notice that both $g_m$ and the resistance, $R$, may be adjusted, although an increase in $g_m$ is preferred, as an increase in $R$ also increases thermal noise.

$$g_m R > 1 \quad (3.72)$$

In quasi-linear behavior it is desirable that the transistors operate in the saturation working region, meaning the conditions in equations (3.73) and (3.74) must be verified, considering an overdrive voltage of 0.1 V and a threshold voltage of 0.4 V [A2.1].

$$V_{GS} > V_t \land V_{DS} > V_{GS} - V_t [V] \quad (3.73)$$

$$V_t = 0.4 \text{ V} \land V_{OV} = V_{GS} - V_t = 0.1 \text{ V} \Rightarrow V_{GS} \geq 0.5 \text{ V} \quad (3.74)$$

The transconductance may then be expressed as a function of the current flowing through the MOS and its width-to-length ratio, (3.76) [A2.1]. Since low power consumption is an important specification of this project, it is preferable to increase the width as to obtain a higher $g_m$ value, although it is

<table>
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<th>Parameters</th>
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<tr>
<td>M_3, M_4</td>
<td>N_12_RF</td>
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</tr>
</tbody>
</table>

Figure 3.25 – Cross-coupled RC relaxation VCO.

Table 3.1 – Cross-coupled RC relaxation oscillator's starting parameters.
important to understand that the width cannot be indeterminably increased. For instance, for a constant current, as the width-to-length ratio increases, the voltages $V_{GS}$ and $V_{DS}$ are decreased, which may drive the transistor out of the saturation region, being necessary in this case to increase the current, (3.77).

$$k = \frac{\mu_n C_{ox} W}{2L} [AV^{-2}] \quad (3.75)$$

$$g_m = 2\sqrt{kI_D} [S] \quad (3.76)$$

$$I_D = k(V_{GS} - V_t)^2(1 + \lambda V_{DS}) [A] \quad (3.77)$$

Additionally, as MOS M3 and M4 are switches that operate in the triode region, they must have a high width-to-length ratio, so that the voltage drop, $V_{DS}$, does not compromise the biasing overhead for the current source to be later implemented.

$$V_{GS} > V_t [V] \land V_{DS} < V_{GS} - V_t [V] \rightarrow I_D = k(2(V_{GS} - V_t) V_{DS} - V_{DS}^2) [A] \quad (3.78)$$

The amplitude, given by equation (3.79), is then increased to 500 mV as a starting value, using the resistors, R. Notice that, due to the suppression of one current source in the typical differential cross-coupled RC relaxation oscillator, the resistor must be twice the expected value.

$$V_{out} = V_1 - V_2 = 2\frac{RI}{2} [V] \quad (3.79)$$

The circuit’s oscillating frequency is theoretically given by equation (3.80), although, the resonator’s inductance, $L$, results from parasitic effects, (3.81). Hence, the frequency will be simply considered as function of $C$ and $L$, where $L$ is controlled by the resistance value and cross-coupled pair’s transconductance value. In this case, changing the capacitance is preferable as the resistance influences the output level.

$$\omega_o = \frac{1}{\sqrt{LC}} \text{[rad.s}^{-1}] \quad (3.80)$$

$$L_{\text{equivalent}} = 2Rg_m^{-1}(C_{gs} + 4C_{gd}) \text{[H]} \quad (3.81)$$

Finally, the phase noise is adjusted taking into consideration equation (3.82) and (3.83). A common variable that decreases the resulting value of both phase noise equations is the current, I. Hence, the current is increased, noticing that the output voltage may be expressed in terms of current and resistance, as in (3.79).

$$L(f_{\text{noise}}) = 10\log\left[\frac{S_0}{2I^2} \left(\frac{f_0}{f_{\text{noise}}}\right)^2\right] \text{[dBc/Hz]} \quad (3.82)$$
\[ L(f_{\text{noise}}) = \frac{2\alpha 45(v_{\text{dc}})}{2V^2} \left( \frac{f_0}{f_{\text{noise}}} \right)^2 \left[ \frac{\text{dBc}}{\text{Hz}} \right] \land \alpha = \frac{B_c}{2f_0} \quad (3.83) \]

The obtained dimensioning values for the differential cross-coupled RC relaxation VCO in Figure 3.25 are presented in Table 3.2. Through inspection, it is possible to observe that the transistors’ width is increased by means of the number of fingers, \( n_{\text{fingers}} \), which reduces the parasitic gate resistance and lowers the total expected drain and source diffusion parasitic capacitances [28].

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>idc</td>
<td>700 μA</td>
</tr>
<tr>
<td>R</td>
<td>RNNPO_RF</td>
<td>1.2 kΩ</td>
</tr>
<tr>
<td>M₁, M₂</td>
<td>N_12_RF</td>
<td>( W = 1 , \mu m ) \ng ( L = 120 , \text{nm} ) \ng #Fingers = 12</td>
</tr>
<tr>
<td>M₃, M₄</td>
<td>N_12_RF</td>
<td>( W = 1 , \mu m ) \ng ( L = 120 , \text{nm} ) \ng #Fingers = 12</td>
</tr>
<tr>
<td>C_{\text{max}}</td>
<td>VARMIS_12_RF</td>
<td>400 fF</td>
</tr>
</tbody>
</table>

Table 3.2 – Cross-coupled RC relaxation VCO’s parameters.

This dimensioning yielded the results shown in Figure 3.26, Figure 3.27 and Figure 3.28, for a tuning voltage of 190 mV, corresponding to a 2.4 GHz oscillating frequency.

![Output voltage waveform at 2.4 GHz](image)

Figure 3.26 – Output voltage waveform at 2.4 GHz.

For the oscillating frequency, 2.4 GHz, the resulting THD is approximately -16.82 dB. Notice that even-order harmonics are practically inexistent since the topology is differential.
Regarding the circuit’s phase noise at 2.4 GHz, the results were -94.7 dBC/Hz, -104.3 dBC/Hz and -114.8 dBC/Hz, for a 1MHz, 3 MHz and 10 MHz distance from the fundamental, respectively.

Finally, Figure 3.29 shows a parametric analysis of the frequency range, phase noise at 1 MHz distance from oscillating frequency and output flatness, as a function of tuning voltage.
The proposed effective frequency range, 2.4 GHz ± 100 MHz, has an average tuning sensitivity of 2.2 MHz/mV, while the phase noise variation is less than 0.5 dBc/Hz and the total output voltage variation is approximately 30 mV. For the frequency of 2.5 GHz the tuning voltage is 226.4 mV, while for 2.3 GHz it is 136.5 mV.

Finally, the power consumption is 0.84 mW for a 700 μA current. As a measure of comparison with the previous work, the following Figure of Merit (FoM) was considered,

\[
\text{FoM} = \mathcal{L}_{\text{measured}} + 10\log\left(\frac{\Delta f^2}{P_{\text{DC}}} \right) \quad (3.84)
\]

where \( \mathcal{L}_{\text{measured}} [\text{dBc/Hz}] \) is the measured phase noise, \( \Delta f \) is the distance to the oscillating frequency, \( f \), both in [Hz]; being \( P_{\text{DC}} \) and \( P_{\text{ref}} \) the DC power consumption and reference power in [W], respectively. The reference power is usually 1 mW. This yielded a figure of merit of -163.06 dBc/Hz.

### 3.2.2. Quadrature Cross-Coupled RC Relaxation VCO

In order to obtain the quadrature cross-coupled RC relaxation VCO in Figure 3.30, two single oscillators are coupled through the load. The PMOS active load coupling is implemented as to have a DC voltage drop similar to the single case with resistive load, by controlling the transistors' width-to-length ratio. Additionally, the occurring frequency deviations, due to coupling, are compensated by an increase of the capacitors’ value.

![Figure 3.30 – Quadrature cross-coupled RC relaxation VCO](image)

Notice that the MIS variable capacitors (VARMIS), used in the single case, are substituted by PMOS MOSCaps. This technical option improves the linearity of the circuit’s variable capacitance. For instance, if the typical DC gate voltage is 0.25 V and the technology’s MOS threshold voltage is 0.4 V,
the MOS's carrier channel exists for a 1.2 V to 0.65 V effective tuning voltage range. The previous effective tuning voltage range was only 430 mV.

The circuit is then adjusted, as in the single case. The obtained phase noise at a 1 MHz distance from the fundamental, 2.4 GHz, is -98.49 dBc/Hz; a 3 dB improvement justified by the coupling of two oscillators. The total power is now 1.68 mW for a total current of 1.4 mA. Hence, the FoM is -163.9 dBc/Hz. Graphical results of this intermediate design step may be consulted in [A2.2], being the dimensioning parameters shown in Table 3.3.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Idc</td>
<td>700 μA</td>
</tr>
<tr>
<td>M₅, M₆</td>
<td>Pₑ₁₂_RF</td>
<td>W = 2.4 μm, L = 120 nm, #Fingers = 16</td>
</tr>
<tr>
<td>M₁, M₂</td>
<td>Nₑ₁₂_RF</td>
<td>W = 1 μm, L = 120 nm, #Fingers = 12</td>
</tr>
</tbody>
</table>

Table 3.3 - Quadrature cross-coupled RC relaxation VCO’s parameters.

It is now necessary to substitute the ideal current sources in Figure 3.31, where M₉ transistors are those sources and M₁₀ is the current mirror, with reference current Iᵣₑᶠ. Considering an overdrive voltage of 0.1 V, the gate-source voltage, Vₑₛ, is 0.5 V. The width-to-length ratio is then dimensioned for the desired current, 700 μA, considering a 1:1 current ratio in (3.86), and compensating channel modulation effects.
\[ V_{DS10} = V_{GS10} = V_{GS9} \text{ [V]} \quad (3.85) \]

\[ \frac{I_{\text{OUT}}}{I_{\text{REF}}} = \frac{k_9 (1 + \lambda V_{DS9})}{k_{10} (1 + \lambda V_{DS10})} \cong \frac{W_9}{W_{10}} \frac{L_9}{L_{10}} \quad (3.86) \]

Finally, the complete oscillator must contemplate the external connections and respective Electrostatic Discharge (ESD) protections, Figure 3.32. The external connections are the supply voltage (VDD), ground (GND), tuning voltage (VTUNE) and the current mirror (Mirror1).

![Diagram of complete VCO](image)

Figure 3.32 – Diagram of complete VCO

Each connection is associated with a pad from the technology (PAD_RF) and a bonding wire with the ideal model shown in Figure 3.33 and respective parameters, Table 3.4 [8].

![Bonding wire model](image)

Figure 3.33 – Bonding wire model [8].

Table 3.4 – Bonding wire parameters [8].

Regarding ESD protections, only the current mirror and the tuning voltage external connections require ESD protections to avoid oxide disruption. The protection circuit is shown in Figure 3.34 and its respective parameters in Table 3.5 [8].
Additionally, two DC decoupling capacitors, MOMCAPS_RF, are used between VDD and GND, minimizing the pushing effect of the power supply. Hence, undesired oscillating frequency changes and phase noise degradation, due to noise injection, are avoided. Their values are 1.137 pF and 135 fF, stabilizing the current at two different time constants. The final dimensioning is shown in Table 3.6, after fine tuning of the circuit.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁, M₂</td>
<td>N₁₂_RF</td>
<td>W = 1 μm, L = 120 nm</td>
<td>M₇, M₈</td>
<td>P₁₂_RF</td>
<td>W = 9.6 μm, L = 360 nm, #Fingers = 19, Mult. = 2</td>
</tr>
<tr>
<td>M₃, M₄</td>
<td>N₁₂_RF</td>
<td>W = 2 μm, L = 120 nm</td>
<td>M₉, M₁₀</td>
<td>N₁₂_RF</td>
<td>W = 1.5 μm, L = 210 nm, #Fingers = 16</td>
</tr>
<tr>
<td>M₅, M₆</td>
<td>P₁₂_RF</td>
<td>W = 2.4 μm, L = 120 nm, #Fingers = 16</td>
<td>RMirror₁</td>
<td>res</td>
<td>675 Ω</td>
</tr>
</tbody>
</table>

Table 3.6 – Final quadrature cross-coupled RC relaxation VCO’s parameters.

This dimensioning yielded the results shown in Figure 3.35, Figure 3.36 and Figure 3.37, for a tuning voltage of 913 mV, corresponding to a 2.4 GHz oscillating frequency.

Figure 3.35 – Output voltage waveform at 2.4 GHz
For the oscillating frequency, 2.4 GHz, the resulting THD is approximately -20.4 dB. Notice that even-order harmonics are practically inexistent since the topology is differential.

![Output Spectrum](image)

**Figure 3.36 – Output voltage spectrum**

Regarding the circuit’s phase noise at 2.4 GHz, the results were -92.4 dBc/Hz, -105.1 dBc/Hz and -117.7 dBc/Hz, for a 1 MHz, 3 MHz and 10 MHz distance from the fundamental, respectively.

![Phase Noise](image)

**Figure 3.37 – Phase Noise at 2.4 GHz**

Finally, Figure 3.38 shows a parametric analysis of the frequency range, phase noise at 1 MHz distance from oscillating frequency and output flatness, as a function of tuning voltage.

![Parametric Analysis](image)

**Figure 3.38 – Parametric analysis of frequency range, phase noise at 1 MHz from oscillating frequency and output flatness as a function of tuning voltage.**
The proposed effective frequency range, 2.4 GHz ± 100 MHz, has an average tuning sensitivity of 0.8 MHz/mV, while the phase noise variation is 1.6 dBc/Hz and the total output voltage variation is approximately 4 mV. For the frequency of 2.5 GHz the tuning voltage is 867 mV, while for 2.3 GHz it is 1.118 V.

The effective frequency range is considered to be 200 MHz, although, through inspection of Figure 3.38, it is possible to observe that the circuit is robust enough to operate beyond those values. Furthermore, the current source can still be used for frequency tuning purposes. Hence, it is plausible to say that, although pulling effect is not directly evaluated (as the filter’s input load is unknown), the circuit will be able to support frequency deviations caused by an output load.

Finally, the power consumption is 3.29 mW for a 2.745 mA current. The obtained FoM is -156.9 dBc/Hz, where for a fair comparison with [8] the current mirrors are not considered. A current of 1.71 mA is considered instead.

3.2.3. Monte Carlo and Corners

The stability of an oscillator is described as the ability to maintain its properties both in time and in a changing environment. The stability and robustness of the proposed VCO are tested using Monte Carlo and corners simulations. The Monte Carlo simulation is a statistical study on parameter mismatches and process variations, according to the technology standard deviations. The simulation results shown in Figure 3.39, evaluate the phase noise at 1 MHz from the fundamental tone, oscillating frequency and output quadrature relation, for a constant tuning voltage of 913 mV. The obtained results validate the design, with an average of -92.4 dBc/Hz for phase noise and 2.4 GHz for the oscillating frequency. Existent frequency deviations can then be controlled both by the MOSCap and, if needed, by the current source.

![Figure 3.39 – Monte Carlo results for phase noise at 1 MHz from oscillating frequency, oscillating frequency and output quadrature, for a constant tuning voltage.](image-url)
Regarding the corners simulation, it is characterized by an extreme parameter variation for different Process, Voltage and Temperature (PVT) conditions. The selected corners are the same as in [8], being the results shown in Table 3.7 and Table 3.8.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Corner 1</th>
<th>Corner 2</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.2</td>
<td>1.32</td>
<td>1.32</td>
<td>1.08</td>
<td>1.08</td>
<td>1.32</td>
</tr>
<tr>
<td>ESD</td>
<td>nom</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>nom</td>
<td>ff</td>
<td>ff</td>
<td>ss</td>
<td>ss</td>
<td>fs</td>
</tr>
<tr>
<td>PAD</td>
<td>nom</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>nom</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>Temperature</td>
<td>27</td>
<td>-30</td>
<td>-30</td>
<td>120</td>
<td>120</td>
<td>-30</td>
</tr>
</tbody>
</table>

Table 3.7 – Corners 1 to 5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 6</th>
<th>Corner 7</th>
<th>Corner 8</th>
<th>Corner 9</th>
<th>Corner 10</th>
<th>Corner 11</th>
<th>Corner 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.32</td>
<td>1.08</td>
<td>1.08</td>
<td>1.32</td>
<td>1.32</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>fs</td>
<td>fs</td>
<td>fs</td>
<td>ss</td>
<td>ss</td>
<td>fs</td>
<td>fs</td>
</tr>
<tr>
<td>PAD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Temperature</td>
<td>-30</td>
<td>120</td>
<td>120</td>
<td>-30</td>
<td>-30</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

Table 3.8 – Corners 6 to 12

Inspecting Table 3.7 and Table 3.8, it is possible to observe that the worst cases are corners 1, 2, 3 and 4. The first pair shows a frequency increase due to both PMOS and NMOS being in fast-fast mode. The second pair shows the opposite situation, MOS in slow-slow mode. Since the VCO is implemented using a MOS-only topology, it also becomes more sensible to transistor parameter variation. Corners 3 and 4 are not possible to correct, due to the imposed 10 % bias variation. As for
corners 1 and 2, their correction is possible by increasing the current mirror’s reference resistor to 1.475 kΩ and using a 1.2 V tuning voltage, Table 3.9.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal</th>
<th>Corner 1</th>
<th>Corner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.2</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>ESD</td>
<td>nom</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>nom</td>
<td>ff</td>
<td>ff</td>
</tr>
<tr>
<td>PAD</td>
<td>nom</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>nom</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Temperature</td>
<td>27</td>
<td>-30</td>
<td>-30</td>
</tr>
</tbody>
</table>

**Table 3.9 – Corners 1 and 2 corrected results**

3.3. Conclusions

The proposed oscillator is a quadrature cross-coupled RC relaxation VCO, Figure 3.31, operating in quasi-linear behavior, due to high frequency operation (i.e. 2.4 GHz). The RC relaxation oscillators are associated with a reduced area, as the circuit is inductorless, being their major drawback the higher phase noise and power consumption, when compared with the LC oscillators. The power optimization issue is addressed using the low-power single RC oscillator topology in [8] (Figure 3.23 a)), where instead of having the usual two current sources, only one is used. The main contribution of this work is the coupling using PMOS as active loads, which substitute the typical soft-limiters. The working principle is based on load resistance change, instead of current draw, allowing further current saving and excluding the need of extra components. Additionally, the implementation of a VCO using MOSCaps, allows flexible frequency tuning without current increase. Until now, frequency tuning was only possible through current source control. The maximum frequency tuning range, given by variable capacitance control, is 900 MHz, from 2.3 GHz to 3.2 GHz, Table 3.10.

Finally, the resulting FoM at 2.4 GHz is -156.7 dBc/Hz, where a 2 dB gain is obtained in comparison to the previous design in [8] (-154.7 dBc/Hz). However, the FoM calculation does not take current mirrors in consideration, where the present solution avoids the extra current consumption due to the soft-limiters’ current reference. Hence, the total power consumption is 3.29 mW, representing a 20 % power save. Table 3.10 summarizes the most relevant results.
<table>
<thead>
<tr>
<th>Block</th>
<th>Implemented VCO</th>
<th>*Oscillator from [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>2.4 GHz</td>
<td>2.63 GHz</td>
</tr>
<tr>
<td>Phase Noise @ $f_0+1$ MHz</td>
<td>-92.4 dBc/Hz</td>
<td>-93.1 dBc/Hz</td>
</tr>
<tr>
<td>THD</td>
<td>-20.4 dB</td>
<td>-20.8 dB</td>
</tr>
<tr>
<td>$I_{Total}$</td>
<td>2.75 mA</td>
<td>2.75 mA</td>
</tr>
<tr>
<td>$P_{DC}$</td>
<td>3.29 mW</td>
<td>3.29 mW</td>
</tr>
<tr>
<td>FoM</td>
<td>-156.7 dB</td>
<td>-156.3 dB</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>approx. 2.3 GHz</td>
<td>to 3.2 GHz</td>
</tr>
</tbody>
</table>

* Pre-layout

Table 3.10 – Implemented VCO’s results summary.
Chapter 4

Quadrature
Single-Balanced
VCO-Mixer


4.1. Overview

In this chapter the implementation of the VCO-mixer is discussed, where the mixing function is directly implemented on the quadrature cross-coupled RC relaxation VCO developed in chapter 3. Hence, it is important to ensure that the VCO operation is not affected. The single-balanced mixer topology is considered the best solution, taking into account the design constraints. This work’s main contribution is the increase of the conversion gain, due to the use of active loads, instead of the typical resistors. Therefore, the use of the VCO-mixer, to perform frequency generation and mixing, reduces the total power consumption and area, while maintaining the output quadrature precision.

This chapter's overview characterizes the ideal mixing operation and the most common figures of merit. The single-balanced mixer and its implementation on the quadrature oscillator-mixer developed in [1] is analyzed and discussed. The present circuit is based on that study. Finally, all technical options are described and the results discussed.

Characterization

The mixer is responsible for translating the RF signal to an IF, being associated with a time-varying or a nonlinear behavior. In this work, switched mixers are considered. The mixer block has applications both in the receiver or transmitter, depending whether down-conversion or up-conversion is required. Ideally, the mixer behaves as an analog multiplier, Figure 4.1. Hence, if an RF signal, \( x_{RF}(t) \), and an LO’s signal, \( x_{LO}(t) \), are mixed, the result is the signal \( y_{IF}(t) \) at the IF.

\[
y_{IF}(t) = x_{RF}(t).x_{LO}(t) = A_{RF} \cos(\omega_{RF}t)A_{LO} \cos(\omega_{LO}t) = \frac{A_{RF}A_{LO}}{2} \cos(\omega_{RF}t - \omega_{LO}t) + \frac{A_{RF}A_{LO}}{2} \cos(\omega_{RF}t + \omega_{LO}t)
\]

\[\omega_{IF} = \omega_{RF} - \omega_{LO} \quad \omega_{RF} \geq \omega_{LO} \text{ [rad/s]}\]

Figure 4.1 – Mixer’s functional representation, a), input spectrum, b) and output spectrum, c).

Figures of Merit

As for the figures of merit, a mixer might be described by:

- Conversion Gain: The gain from the RF input to the IF output;

\[
A_{VC} = \frac{y_{IF}}{y_{RF}} \quad (4.3)
\]
- Scattering Parameter $S_{11}$: Measure of the input port matching, (2.5);

- Scattering Parameter $S_{22}$: Measure of the output port matching, (2.5);

- Mixer’s LO-RF Port Isolation: Prevention of LO’s signal leakage to the RF port;

- Mixer’s LO-IF Port Isolation: Prevention of the LO’s signal feedthrough to the IF port;

- Input 1 dB Compression Point: Input signal level for which the linear gain is reduced by 1 dB, (2.21);

- IIP3: Input signal level for which the third-order intermodulation products reach an amplitude level equal to the amplitude of the fundamental (2.28);

- Quadrature Error;

- Noise Figure: Measure of the noise generated by the circuit, (2.10);

However, the IIP3 and the input 1 dB compression point are considered out of the project’s scope, as the oscillator-mixer’s input is not matched to the LNA’s output, since an integrated receiver is implemented.

### 4.2. Mixer Topologies

#### 4.2.1. MOS Active Mixers

There are several possible topologies for mixers; in this section, only active switched transistor based mixers are analyzed, considering IC optimization. One of the advantages of the active mixers is the existence of conversion gain, which enhances the output signal level and the system’s noise figure, (2.14). Two very common topologies are the single-balanced and the double-balanced mixers.

The single-balanced mixer, Figure 4.2, is a 2 quadrants multiplier, since the transistor $M_3$’s biasing requires that $v_{RF}(t)>V_t$. The RF input, $v_{RF}(t)$, is single-ended, while the LO’s input, $v_{LO}(t)$, is differential. Its output, $v_{IF}(t)$, is also differential. This mixer is characterized by good mixer’s LO-RF port isolation, but poor LO-IF port isolation.

![Figure 4.2 – Single-balanced mixer [8].](image_url)
In order to explain this mixer’s behavior, consider that M₃ is an ideal current source with current I, and that the differential pair is in the saturation working region. Therefore, it is possible to describe the output voltage, vᵢᶠ(t), using the differential pair currents, iₘ₁(t) and iₘ₂(t), as in (4.6) [A3.1].

\[ i_{M_1}(t) = \frac{1}{2} + v_{LO}(t) \sqrt{\frac{kl}{2} - \frac{k^2}{4} v_{LO}^2(t)} [A] \] \( \land \) \[ i_{M_2}(t) = \frac{1}{2} - v_{LO}(t) \sqrt{\frac{kl}{2} - \frac{k^2}{4} v_{LO}^2(t)} [A] \] (4.4)

\[ v_{IF}(t) = (V_{DD} - i_{M_1}(t) R) - (V_{DD} - i_{M_2}(t) R) = -2R v_{LO}(t) \sqrt{\frac{kl}{2} - \frac{k^2}{4} v_{LO}^2(t)} [V] \] (4.5)

\[ I \gg \frac{k}{2} v_{LO}^2(t) \rightarrow v_{IF}(t) = -2R v_{LO}(t) \sqrt{\frac{kl}{2}} [V] \] (4.6)

Since the current source is actually implemented by M₃ in saturation working region, it is possible to rewrite equation (4.6) as (4.8) [A2.1].

\[ I = i_{M_3}(t) = k_3 (v_{RF}(t) - v_i)^2 [A] \] (4.7)

\[ \therefore v_{IF}(t) = -R v_{LO}(t) (v_{RF}(t) - v_i) \sqrt{2k_3} [V] \] (4.8)

Equation (4.8) has cross products that yield the desired result, plus an undesired term at the LO’s frequency. This undesired term justifies the poor mixer’s LO-IF port isolation.

\[ v_{IF}(t) = -\sqrt{2k_3 R v_{LO}(t)} v_{RF}(t) + \sqrt{2k_3 R v_{LO}(t)} v_i [V] \] (4.9)

Hence, for sinusoidal signals the conversion gain is given by equation (4.10).

\[ \therefore A_V = \frac{v_{IF}}{v_{RF}} = -\frac{R v_{LO}}{\sqrt{2}} \sqrt{\frac{kl}{2}} \] (4.10)

Although, it might be the case that the LO’s signal is strong enough to force the mixer to operate in switching mode. In this case, transistors M₁ and M₂ switch between cutoff and triode. Notice that in this mode, commutation must be fast enough to avoid losses. To illustrate this operation, consider an ideal switching response to an LO’s Sign function input [A3.2]. Additionally, consider that M₃ is an ideal transconductor for the small RF signal, vᵣ(t).

\[ \text{Sign}[\cos(\omega_{LO} t)] = \frac{4}{\pi} \cos(\omega_{LO} t) - \frac{4}{3\pi} \cos(3\omega_{LO} t) + (...) \] (4.11)

\[ i_{M_3}(t) = I_{DC} + I_{rf} \cos(\omega_{RF} t) = I_{DC} + g_m v_{rf} \cos(\omega_{RF} t) [A] \] (4.12)

\[ i_{M_1}(t) = i_{M_3}(t) \frac{1}{2} [1 + \text{Sign}[\cos(\omega_{LO} t)]][A] \land i_{M_2}(t) = i_{M_3}(t) \frac{1}{2} [1 - \text{Sign}[\cos(\omega_{LO} t)]] [A] \] (4.13)
\[ v_{IF}(t) = (V_{DD} - i_{M_1}(t)R) - (V_{DD} - i_{M_2}(t)R) = -R_i M_3(t) \text{Sign} [\cos(\omega_{LO} t)] [V] \quad (4.14) \]

From equation (4.14) it is possible to obtain the mixer’s conversion gain in switching mode.

\[ v_{id}(t) = -\frac{2}{\pi} R g_m V_{RF} \cos(\omega_{IF} t) [V] \quad (4.15) \]

\[ \therefore A_{VC} = \frac{v_{id}}{v_{RF}} = \frac{2}{\pi} R g_m \] (4.16)

Finally, the double-balanced mixer is characterized as a 4 quadrant multiplier. In this case, both RF, \( v_1(t) \), and LO’s, \( v_2(t) \), inputs are differential, Figure 4.3. Therefore, the circuit’s linearity is increased, in particular, by the reduction of the even-order distortion. Additionally, it also increases the conversion gain, enhancing the system’s noise figure. Regarding the mixer’s LO port isolation, this topology has better LO-IF port isolation than the previous single-balanced mixer, while maintaining a good LO-RF port isolation. For instance, the crossed pseudo-differential pairs, controlled by the LO, generate two equal currents except for the RF differential pair’s term. Thus, implying that the LO’s output signal is cancelled. Therefore, the double-balanced mixer cannot be implemented on the oscillator-mixer, being the single-balanced mixer used instead.

![Figure 4.3 – Double-balanced mixer [30]](image)

**4.2.2. Quadrature MOS Active Mixers**

The low-IF receiver architectures require IQ signals for image-rejection. There are two commonly used solutions: designing two mixers to accommodate quadrature signals or incorporating the mixing function directly on the oscillator. The first solution, where two independent mixers are used, entails some disadvantages, for example, additional power consumption, additional area and higher quadrature error. Hence, the quadrature MOS oscillator-mixer developed in [1] is analyzed, where the single-balanced mixer is implemented directly on the quadrature cross-coupled RC relaxation oscillator, Figure 4.4.
Figure 4.4 – Quadrature oscillator-mixer implemented in [1].  

The circuit is practically identical to the original oscillator of Figure 3.23 b), except for the current mirror. In this case, the RF signal is added to the current sources’ DC gate voltage (M_{14} and M_{15}). Additionally, the capacitor C_D is used to decouple the LNA’s DC output, while the low-pass filter, given by R_F and C_F, filters the current mirror’s AC component (M_{13}). Therefore, AC noise introduction is avoided and the current mirror’s reference precision is maintained.

According to [23], the mixing function does not affect the duty-cycle, being the quadrature error a product of two relative errors, where the coupling strength is determinant for the quadrature precision.

4.2.3. Discussion

In this work, the mixing function is directly implemented on the quadrature cross-coupled RC relaxation VCO developed in chapter 3. According to [23], the mixing function does not affect the duty-cycle, being the quadrature error a product of two relative errors, where the coupling strength is determinant for the quadrature precision. This technical option takes advantage of the VCO topology already in place, being advantageous in terms of area, power saving and quadrature precision. The mixing function is based on the single-balanced mixer’s operation, where the RF signal is added to the current sources’ DC gate voltage, as in the circuit of Figure 4.4.

The equations that describe the single-balanced mixer are compatible with those derived for the quadrature cross-coupled RC relaxation VCO. For instance, both the transistors of the LO’s input differential pairs are in saturation working region, instead of being switched; increasing the conversion gain, (4.10), and reducing the distortion due to commutation. Finally, the proposed solution uses
PMOS as active loads, instead of the traditional resistors (Figure 4.4), increasing the expected conversion gain.

4.3. Implemented Circuit

The proposed quadrature single-balanced VCO-mixer, Figure 4.5, results from the direct implementation of the mixing function on the quadrature cross-coupled RC relaxation VCO designed in chapter 3. Hence, the design is compliant with the design steps used for the VCO, in quasi-linear behavior. This technical option allows for power saving, area reduction and increased quadrature precision. The circuit is designed in UMC 130 nm CMOS technology, for a supply voltage of 1.2 V.

![Figure 4.5 – Implemented quadrature single-balanced VCO-mixer.](image)

The mixing function is based on the operation of the single-balanced mixer, where both the transistors of the LO’s differential pairs, M₁ and M₂, are in the saturation working region to maximize the conversion gain and minimize the commutation distortion. However, PMOS active loads, M₅ and M₆, are used to further increase the conversion gain, substituting the traditional resistors. The active loads have higher incremental impedance for the same DC voltage drop as the resistors. The VCO-mixer’s quasi-linear behavior is explained by the same analysis made for the cross-coupled RC relaxation oscillator, in section 3.1.1, being the resistors substituted by the active loads’ source-drain incremental resistance. Additionally, the resistors R₁ are used to isolate the AC signal from the frequency tuning DC voltage source, VTUNE in Figure 4.5. Ideally, choking inductors are used, although the area constraints eliminate this option.

Finally, the RF signal is injected at the current sources’ gate, being the capacitor C₂ used to decouple the LNA’s DC output, and the low-pass filter, given by R₂ and C₁, used to decouple the current mirror’s AC noise. Notice that both the DC decoupling capacitor, C₂, and the AC decoupling capacitor, C₁,
must be large; although, the resistor, $R_2$ must be carefully chosen as to avoid a severe thermal noise increase, (2.3).

This section will describe the methodologies used to implement the mixing function on the quadrature cross-coupled RC relaxation VCO, using the equations derived in section 4.2 for the single-balanced mixer. The section is divided in 3 parts: Quadrature Single-Balanced VCO-Mixer, Layout and Monte Carlo and Corners.

### 4.3.1. Quadrature Single-Balanced VCO-Mixer

The similarities between the existent quadrature cross-coupled RC relaxation VCO and the single-balanced mixer circuits allow a direct implementation of the mixing function; where the design steps shown in section 3.2, for the VCO, are still valid for the mixer. Therefore, the only modification in the topology is the input of the RF signal at the current sources’ gates, added to the DC reference ($M_9$ in Figure 4.5). Hence, the capacitor $C_2$ is used to decouple the LNA’s DC output, while the low-pass filter, given by $R_2$ and $C_1$, is used to decouple the current mirror’s AC noise. The capacitor $C_2$ must be large enough to avoid RF signal loss; $C_1$ must be large enough to decouple the AC noise, and the resistor $R_2$ must have a high value to choke the AC signal, without severely increasing the thermal noise. The same principle is applied to the resistor $R_1$ that isolates the AC signal from the frequency tuning DC voltage source, $V_{TUNE}$. The dimensioning values are presented in Table 4.1.

Regarding the conversion gain, (4.17), its value can be improved by an increase of the oscillator’s output signal, $V_{lo}$, load resistance, $R$, or width-to-length ratio of either the cross-coupled differential pairs or current sources, $k_{1,2}$ and $k_9$, respectively.

\[
A_{VC} = \frac{v_{HF}}{v_{ref}} = -\frac{RV_{lo} \sqrt{k_{1,2}k_9}}{\sqrt{2}} \quad (4.17)
\]

\[
k = \frac{\mu_n \cdot C_{ox} \cdot W}{2L} [AV^2] \quad (4.18)
\]

The current sources’ width-to-length ratio, $k_9$, is considered the most suitable variable to increase, as the design of the VCO’s current sources, in section 3.2.2, did not focus their transconductance maximization. Therefore, for a constant DC gate voltage, a slight current increase is expected, (4.19), leading to a reduction of the current sources’ drain-source overhead voltage.

\[
I_D = k(V_{GS} - V_t)^2(1 + \lambda V_{DS}) [\text{A}] \quad (4.19)
\]

In order to compensate the circuit’s higher DC voltage drop, the width-to-length ratio of both the PMOS active loads and the cross-coupled differential pairs is also increased; which additionally reinforces the coupling and improves the quadrature precision. The maximum RF input signal is 100
mV, as in the case of the VCO’s current sources, where an overdrive voltage of 0.1 V and a 1:1 current mirror ratio is considered.

Finally, the MOSCaps capacitance is adjusted to correct the oscillating frequency value. The diagram of the complete quadrature single-balanced VCO-mixer is represented in Figure 5.6, resulting from the introduction of the RF input in the VCO’s diagram in Figure 3.32.

![Figure 4.6 – Diagram of complete quadrature single-balanced VCO-mixer.](image)

The dimensioning values obtained for the quadrature single-balanced VCO-mixer are presented in Table 4.1.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁, M₂</td>
<td>N₁₂_RF</td>
<td>W = 2 μm L = 120 nm #Fingers = 16</td>
<td>M₉, M₁₀</td>
<td>N₁₂_RF</td>
<td>W = 1.8 μm L = 120 nm #Fingers = 16</td>
</tr>
<tr>
<td>M₃, M₄</td>
<td>N₁₂_RF</td>
<td>W = 2 μm L = 120 nm #Fingers = 16</td>
<td>R₁, R₂</td>
<td>RNHR1000_MML130E</td>
<td>R = 1 kΩ</td>
</tr>
<tr>
<td>M₅, M₆</td>
<td>P₁₂_RF</td>
<td>W = 2.5 μm L = 120 nm #Fingers = 16</td>
<td>C₁</td>
<td>MOMCAPS_RF</td>
<td>C = 2.26 pF</td>
</tr>
<tr>
<td>M₇, M₈</td>
<td>P₁₂_RF</td>
<td>W = 9 μm L = 360 nm #Fingers = 24</td>
<td>C₂</td>
<td>MOMCAPS_RF</td>
<td>C = 1.5 pF</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>RMirror1</td>
<td>res</td>
<td>560 Ω</td>
</tr>
</tbody>
</table>

Table 4.1 – Quadrature single-balanced VCO-mixer’s parameters.
This dimensioning yielded the results shown in Figure 4.7 at a 10 MHz IF. A tuning voltage of 965 mV is used, corresponding to a 2.4 GHz oscillating frequency that is mixed with a 2.41 GHz RF signal.

![Figure 4.7 - Voltage conversion gain and noise figure at a 10 MHz IF; mixer’s LO-RF port isolation.](image)

A conversion gain of 6.87 dB and a noise figure of 10.61 dB are obtained. Notice that only the mixer’s LO-RF port isolation is simulated, as the single-balanced topology does not have LO-IF port isolation, yielding a value of -118.7 dB. Regarding S11 and S22, these figures are not contemplated in this study, as lumped parameters are considered for the IC.

In Figure 4.8, the parametric analysis shows the oscillator’s frequency and its respective phase noise at 1 MHz distance; the mixing conversion gain and noise figure, at a 10 MHz IF; all as a function of the tuning voltage.

![Figure 4.8 - Parametric analysis of the oscillator’s frequency and respective phase noise at 1 MHz distance; the mixing conversion gain and noise figure, at the IF; all as a function of the tuning voltage.](image)

Considering the same effective frequency range as for the VCO in chapter 3, 2.4 GHz ± 100 MHz, results in an average tuning sensitivity of 1.3 MHz/mV, while the phase noise varies from -86 to -88 dBc/Hz. The conversion gain has an approximate variation of 0.2 dB, while the noise figure varies 0.4 dB, inside the effective frequency range. It is noticeable that the introduction of the mixing function
slightly increases the phase noise, in comparison to the VCO case, where simple DC current sources are used.

Finally, an example of the mixing function is shown in Figure 4.9, where a 2.41 GHz RF signal is mixed with the 2.4 GHz LO’s signal.

![Figure 4.9 – Output voltage and spectrum for a mixing example, using a 2.41 GHz RF signal mixed with a 2.4 GHz LO’s signal.](image)

Regarding the total power consumption, 3.78 mW are used for a current of 3.15 mA (1.92 mA without current mirror references).

### 4.3.2. Layout

The layout of RF blocks is critical, especially in the quadrature single-balanced VCO-mixer’s case, since it is very sensitive to parasitic capacitances and different delay paths. (1) Hence, it is important to design a symmetric layout, having paths and components share similar delays and parasitics. For instance, the components’ connections must distribute the signals symmetrically, using multiple contacts to reduce the parasitic resistance and improve reliability. Additionally, matching components must be close, to avoid mismatches.

(2) The paths must be short, as resistance increases with length, and their width must be proportional to their total passing current, to avoid the electromigration problem. The empirical rule of 1 μm/mA is considered for the metal layers’ width. Furthermore, paths associated with AC signals use higher layers, being overlapping considered undesired, as to decrease parasitic capacitances. The opposite is wanted for DC signals, where AC noise is decoupled.

(3) Finally, a VDD and GND frame is used to shorten the biasing paths and homogeneously polarize the substrate, working as a guard-ring of the entire block. In fact, guard-rings are used throughout the design, to reduce the noise and the cross-talk. The designed layout is shown in Figure 4.10, where the
current mirror reference branch \((M_{10}, C_1, R_2 \text{ and } C_2 \text{ in Figure 4.5})\) is not designed at layout level, as the proposed LNA will incorporate this function.

![Figure 4.10 - Quadrature single-balanced VCO-mixer's layout.](image)

As expected, after layout implementation, the oscillating frequency decreases due to parasitic effects. Thus, it is necessary to reduce the MOSCaps’ capacitance and adjust the current mirror’s reference, \(R_{\text{Mirror1}}\), to 557 \(\Omega\). Additionally, the supply’s AC decoupling is improved by an extra 10 pF MOMCAP that uses the available space between the circuit and the pads. The final dimensioning for the VCO-mixer in Figure 4.5 is shown in Table 4.2.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_1, M_2)</td>
<td>\text{N_12_RF}</td>
<td>(W = 2 \ \mu m) \n (L = 120 \ \text{nm}) \n #Fingers = 16</td>
<td>(M_9, M_{10})</td>
<td>\text{N_12_RF}</td>
<td>(W = 1.8 \ \mu m) \n (L = 120 \ \text{nm}) \n #Fingers = 16</td>
</tr>
<tr>
<td>(M_3, M_4)</td>
<td>\text{N_12_RF}</td>
<td>(W = 2 \ \mu m) \n (L = 120 \ \text{nm}) \n #Fingers = 16</td>
<td>(R_1, R_2)</td>
<td>\text{RNHR1000_MML130E}</td>
<td>(R = 1 \ \text{k}\Omega)</td>
</tr>
<tr>
<td>(M_5, M_6)</td>
<td>\text{P_12_RF}</td>
<td>(W = 2.5 \ \mu m) \n (L = 120 \ \text{nm}) \n #Fingers = 16</td>
<td>(C_1)</td>
<td>\text{MOMCAPS_RF}</td>
<td>(C = 2.26 \ \text{pF})</td>
</tr>
<tr>
<td>(M_7, M_8)</td>
<td>\text{P_12_RF}</td>
<td>(W = 9 \ \mu m) \n (L = 360 \ \text{nm}) \n #Fingers = 19</td>
<td>(C_2)</td>
<td>\text{MOMCAPS_RF}</td>
<td>(C = 1.5 \ \text{pF})</td>
</tr>
<tr>
<td>(R_{\text{Mirror1}})</td>
<td>res</td>
<td>557 (\Omega)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2 – Final quadrature single-balanced VCO-mixer’s parameters.
The post-layout results obtained for the dimensioning in Table 4.2 are shown in Figure 4.11, Figure 4.12 and Figure 4.13. In Figure 4.11, the voltage conversion gain, noise figure and mixer’s LO-RF port isolation are analyzed for a tuning voltage of 935 mV, corresponding to a 2.4 GHz oscillating frequency that is mixed with a 2.41 GHz RF signal.

![Figure 4.11](image1.png)

**Figure 4.11 – Voltage conversion gain and noise figure at a 10 MHz IF; mixer’s LO-RF port isolation.**

A conversion gain of 7.83 dB, a noise figure of 10.11 dB and a mixer’s LO-RF port isolation of -81 dB are obtained.

In Figure 4.12, the parametric analysis shows the oscillator’s frequency and its respective phase noise at 1 MHz distance; the mixing conversion gain and noise figure, at a 10 MHz IF; all as a function of the tuning voltage.

![Figure 4.12](image2.png)

**Figure 4.12 - Parametric analysis of the oscillator’s frequency and respective phase noise at 1 MHz distance; the mixing conversion gain and noise figure, at the IF; all as a function of the tuning voltage.**

Considering the same effective frequency range as the VCO in chapter 3, 2.4 GHz ± 100 MHz, results in an average tuning sensitivity of 1.8 MHz/mV, while the phase noise varies from -86.2 to -87.6 dBC/Hz. The conversion gain has an approximate variation of 0.1 dB, while the noise figure varies 0.4 dB, inside the effective frequency range.
Finally, an example of the mixing function is shown in Figure 4.9, where a 2.41 GHz RF signal is mixed with the 2.4 GHz LO’s signal.

![Figure 4.13 - Output voltage and spectrum for a mixing example, using a 2.41 GHz RF signal mixed with a 2.4 GHz LO's signal.](image)

Regarding the circuit’s power consumption and area, 3.8 mW are used for a current of 3.17 mA (1.92 mA without current mirror references), while occupying an area of 0.01422647 mm² (103.24 µm x 137.8 µm), including the supply decoupling capacitors in Figure 4.6.

### 4.3.3. Monte Carlo and Corners

In this chapter, the mixing function is implemented on the quadrature cross-coupled RC relaxation VCO developed in chapter 3, while not degrading its performance. The stability and robustness of the extracted quadrature single-balanced VCO-mixer are tested using Monte Carlo and corners simulations. The Monte Carlo simulation studies the parameters’ mismatches and process variations, according to the technology standard deviations; yielding the results in Figure 4.14, for the VCO-mixer’s oscillating frequency and respective phase noise at a 1 MHz distance, output quadrature, mixing conversion gain and noise figure, both at the IF. All the results consider the use of a 935 mV constant tuning voltage. The obtained results validate the extracted design, with an average of -87 dBc/Hz for the phase noise and 2.4 GHz for the oscillating frequency, while the conversion gain and the noise figure have an average of 7.84 dB and 8.98 dB, respectively. Existent frequency deviations can be controlled both by the MOSCap and, if needed, by the current source.
Figure 4.14 – Monte Carlo results for the phase noise at 1 MHz from the oscillating frequency, output quadrature, conversion gain, noise figure, and oscillating frequency, using a 935 mV tuning voltage.

Regarding the corners simulation, extreme PVT parameter variations are studied for the same variables as in the Monte Carlo simulation, yielding the results in Table 4.3 and Table 4.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 1</th>
<th>Corner 2</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 5</th>
<th>Corner 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.32</td>
<td>1.32</td>
<td>1.08</td>
<td>1.08</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>ESD typ</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOS ff</td>
<td></td>
<td></td>
<td></td>
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</tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>Temperature</td>
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<td>120</td>
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<td>-30</td>
</tr>
</tbody>
</table>

Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 1</th>
<th>Corner 2</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 5</th>
<th>Corner 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oscillator's Phase Noise @ 1 MHz [dBc/Hz]</td>
<td>-85.44</td>
<td>-86</td>
<td>-90.3</td>
<td>-90.9</td>
<td>-86.11</td>
<td>-86.68</td>
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<tr>
<td>Quadrature [º]</td>
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<td>90.1</td>
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<tr>
<td>Gain @ IF [dB]</td>
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<td>8.675</td>
</tr>
</tbody>
</table>

Table 4.3 – Corners 1 to 6.

Inspecting Table 4.3 and Table 4.4, it is possible to observe that the worst cases are corners 1, 2, 3 and 4. The first pair shows a frequency increase due to both PMOS and NMOS being in fast-fast mode. The second pair shows the opposite situation, MOS in slow-slow mode.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 7</th>
<th>Corner 8</th>
<th>Corner 9</th>
<th>Corner 10</th>
<th>Corner 11</th>
<th>Corner 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.08</td>
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<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
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<tr>
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<td>max</td>
<td>min</td>
<td>max</td>
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<td><strong>Output</strong></td>
<td>Corner 7</td>
<td>Corner 8</td>
<td>Corner 9</td>
<td>Corner 10</td>
<td>Corner 11</td>
<td>Corner 12</td>
</tr>
<tr>
<td>Oscillator's Phase Noise @ 1 MHz [dBc/Hz]</td>
<td>-86.96</td>
<td>-87.54</td>
<td>-85.54</td>
<td>-86.18</td>
<td>-86.35</td>
<td>-86.98</td>
</tr>
<tr>
<td>Oscillator's Frequency [Hz]</td>
<td>1.987G</td>
<td>1.988G</td>
<td>3.039G</td>
<td>3.039G</td>
<td>2.064G</td>
<td>2.065G</td>
</tr>
<tr>
<td>LO_RF Isolation [dB]</td>
<td>-82.4</td>
<td>-82.77</td>
<td>-77.6</td>
<td>-76.61</td>
<td>-83.08</td>
<td>-82.92</td>
</tr>
<tr>
<td>Quadrature [°]</td>
<td>89.68</td>
<td>270.3</td>
<td>89.66</td>
<td>89.65</td>
<td>89.66</td>
<td>89.66</td>
</tr>
<tr>
<td>Gain @ IF [dB]</td>
<td>6.221</td>
<td>6.089</td>
<td>9.076</td>
<td>8.86</td>
<td>5.682</td>
<td>5.57</td>
</tr>
<tr>
<td>NF @ IF [dB]</td>
<td>10.35</td>
<td>10.67</td>
<td>8.699</td>
<td>8.831</td>
<td>10.63</td>
<td>10.88</td>
</tr>
</tbody>
</table>

Table 4.4 – Corners 7 to 12.

Since the VCO-mixer’s is based on a MOS-only topology, it becomes more sensible to transistor parameter variation. The corners 3 and 4 are not possible to correct, due to the imposed 10 % bias variation, while the corners 1 and 2 are corrected by increasing the current mirror’s reference resistor to 1.249 kΩ and using a 1.2 V tuning voltage, Table 4.5.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 1</th>
<th>Corner 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>ff</td>
<td>ff</td>
</tr>
<tr>
<td>Resistor</td>
<td>res_max</td>
<td>res_min</td>
</tr>
<tr>
<td>PAD</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>Temperature</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>Corner 1</td>
<td>Corner 2</td>
</tr>
<tr>
<td>Oscillator's Phase Noise @ 1 MHz [dBc/Hz]</td>
<td>-83.32</td>
<td>-83.95</td>
</tr>
<tr>
<td>Oscillator's Frequency [Hz]</td>
<td>2.4G</td>
<td>2.402G</td>
</tr>
<tr>
<td>LO_RF Isolation [dB]</td>
<td>-82.49</td>
<td>-80.61</td>
</tr>
<tr>
<td>Quadrature [°]</td>
<td>89.66</td>
<td>89.66</td>
</tr>
<tr>
<td>Gain @ IF [dB]</td>
<td>9.529</td>
<td>9.416</td>
</tr>
<tr>
<td>NF @ IF [dB]</td>
<td>7.16</td>
<td>7.463</td>
</tr>
</tbody>
</table>

Table 4.5 – Corners 1 and 2 corrected results.
4.4. Conclusions

The proposed oscillator-mixer is a quadrature single-balanced VCO-mixer, Figure 4.5, resulting from direct implementation of the mixing function in the quadrature cross-coupled RC relaxation VCO developed in chapter 3. A technical option that saves power, reduces the total area and maintains the quadrature precision. Hence, the VCO-mixer is based on the single-balanced mixer topology, with both the differential pair's transistors operating in the saturation working region, to maximize the conversion gain and reduce both the distortion due to commutation and the even-order distortion. The substitution of the typical resistors by PMOS active loads increases the voltage conversion gain, yielding a gain of 7.827 dB. Additionally, the single-balanced mixer topology allows for a high mixer’s LO-RF port isolation, -81 dB, that avoids possible spurious emissions or self-mixing effects due to the similarity of the RF and LO’s frequency. Finally, the circuit's total power consumption is 3.8 mW, while using a total area of 0.01422647 mm², already including the decoupling capacitors in Figure 4.6. Table 4.6 summarizes the most relevant results of the extracted circuit.

<table>
<thead>
<tr>
<th>Block</th>
<th>Implemented VCO-Mixer</th>
<th>Oscillator-Mixer from [1]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_0 )</td>
<td>2.4 GHz</td>
<td>2.42 GHz</td>
</tr>
<tr>
<td>Phase Noise @ ( f_0 + 1 ) MHz</td>
<td>-87.1 dBC/Hz</td>
<td>-87.23 dBC/Hz</td>
</tr>
<tr>
<td>( A_v ) @ 10 MHz</td>
<td>7.83 dB</td>
<td>7.87 dB</td>
</tr>
<tr>
<td>( NF ) @ 10 MHz</td>
<td>10.11 dB</td>
<td>10.18 dB</td>
</tr>
<tr>
<td>( I_{\text{Total}} )</td>
<td>3.17 mA</td>
<td>3.17 mA</td>
</tr>
<tr>
<td>( P_{\text{DC}} )</td>
<td>3.8 mW</td>
<td>3.8 mW</td>
</tr>
<tr>
<td>Area</td>
<td>0.01422647 mm²</td>
<td>0.01422647 mm²</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>approx. 2.25 GHz – 3 GHz</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 – Implemented VCO-mixer’s results summary.
Chapter 5

Single-Stage Feedback LNA
5.1. Overview

The last block to be implemented is the LNA, being designed to maximize the specifications for the already known VCO-mixer’s RF input port impedance, while avoiding the use of buffers. In fact, the proposed implementation is also used as the quadrature single-balanced VCO-mixer’s current mirror reference, reducing the total area and the power consumption. Therefore, approximately 1/3 of the total VCO-mixer’s power consumption is reduced. The implemented topology is based on a single-stage shunt-shunt feedback amplifier, where the input is matched to 50 Ω, without the use of a passive LC network.

Therefore, this chapter’s overview includes a brief characterization of tuned LNAs and their main figures of merit. The single-stage shunt-shunt feedback amplifier topology implemented in [2] is analyzed and discussed, being the present circuit based on this study. Finally, all technical options are described and the results discussed in detail.

Characterization

A low-noise amplifier must linearly amplify the input signal, without significant noise addition. According to the Friis formula, (2.14), the first block’s noise specifications are determinant for the total system’s noise figure and sensitivity. The LNA’s input is typically single-ended and matched to 50 Ω, which usually requires a passive LC network. The 50 Ω impedance matching avoids reflection (\(Z_{in}=Z_{source}\)) and maximizes the power transfer (\(Z_{in}=Z_{source}\)), (2.5). A second stage, consisting of a common-drain (i.e. buffer), might also be used to match the output. However, lumped parameters will be considered after the input matching, since the operating wavelength, \(\lambda\), is much larger than the circuit’s characteristic electrical length, \(L\).

\[
f = \frac{c}{\lambda} \leftrightarrow \lambda = \frac{c}{f} \approx \frac{3 \times 10^8}{2.41 \times 10^9} \approx 0.123 \text{ m} \quad (5.1)
\]

\[
L \sim 500 \mu \text{m} \quad (5.2)
\]

Therefore, the circuit’s voltage gain can be considered instead of the power gain. The signal level range is usually defined by the spurious-free dynamic range, (2.42), which takes in consideration the existent nonlinearities. However, as referred in chapter 1, the exact value for the SFDR is not evaluated, as no output SNR specifications are defined. Hence, the linearity is studied using the 1 dB compression point and the IIP3.

Finally, the LNA circuit is considered stable if its poles are in the left-half of the s-plane. Considering the amplifier is an LTI circuit, the frequency response might be generically described by equation (5.3),

\[
A(s) = A_o \frac{s}{s+\omega_l} \frac{1}{1+\frac{s}{\omega_p}} \bigg|_{s=j\omega} \quad (5.3)
\]
where $A_0$ is the passing-band gain, $\omega_L$ is the dominant lower frequency pole (e.g. decoupling capacitor) and $\omega_H$ is the dominant higher frequency pole. Figure 5.1 illustrates that frequency response.

![Figure 5.1 – Generic LNA frequency response.](image)

In the case of the feedback LNAs, Figure 5.2, the stability threshold is expressed by equation (5.5), where negative feedback must be ensured.

![Figure 5.2 – Feedback amplifier block diagram.](image)

\[
\frac{X_o(s)}{X_i(s)} = \frac{A(s)}{1+A(s)B(s)} \quad (5.4)
\]

\[
1 + A(s_o)B(s_o) = 0 \rightarrow |A(s_o)B(s_o)| = 1 \land \arg(A(s_o)B(s_o)) = k\pi, k \in \mathbb{Z} \quad (5.5)
\]

Therefore, the gain and phase margins, $A_M$ and $\phi_M$, are commonly evaluated as in Figure 5.3, where the amplifying stage $A(s)$ and the feedback stage $B(s)$ are considered stable, and a dominant pole exists. Additionally, to define these margins without ambiguity, the amplitude and phase curves must decrease monotonically with the increase of the frequency.

![Figure 5.3 – Illustration of the amplitude and phase margins [2].](image)

However, since the source impedance is 50 Ω and the load impedance is already known (i.e. the VCO-mixer’s RF port impedance), it is enough to guarantee that the real part of the input impedance is never negative [15] [11]. The frequency response may then be controlled, using the circuit’s poles and
zeros. In an asymptotic analysis, a pole reduces the signal’s amplitude by 20 dB per decade and produces a negative 90 degree phase shift. The phase shift starts one decade earlier than the pole’s frequency and ends one decade later. In the case of the zeros, the amplitude will rise 20 dB per decade, while a positive 90 degree phase shift is produced instead.

**Figures of Merit**

As for the figures of merit, an LNA might be described by:

- Voltage Gain;
- Scattering Parameter S\(_{11}\): Measure of the input port matching, (2.5);
- Scattering Parameter S\(_{22}\): Measure of the output port matching, (2.5);
- Input 1 dB Compression Point: Input signal level for which the linear gain is reduced by 1 dB, (2.21);
- IIP3: Input signal level for which the third-order intermodulation products reach an amplitude level equal to the amplitude of the fundamental, (2.28);
- Noise Figure: Measure of the noise generated by the circuit, (2.10);

5.2. **LNA Topologies**

5.2.1. **Common-Gate**

There are several topologies for the LNAs, however, only the single-stage topologies will be discussed in this section. In particular, the topologies that allow for the input impedance matching without the use of an LC passive network or inductive degeneration, to reduce the total area. One possible implementation is the common-gate topology, in Figure 5.4 a).

![Common-gate topology](image)

**Figure 5.4** – Common-gate topology, a), and small-signal circuit with noise sources, b) [15].
Input impedance:

Considering the small-signal circuit in Figure 5.4 b), the input impedance is given by equation (5.7).

\[
\mathbf{v}_{\text{in}} = -\mathbf{v}_{\text{gs}} \rightarrow \mathbf{i}_{\text{in}} = \mathbf{v}_{\text{in}} \mathbf{S}_{\text{Cgs}} + \mathbf{v}_{\text{in}} \mathbf{g}_{\text{m}} \quad [A] \quad (5.6)
\]

\[
\therefore Z_{\text{in}} = \frac{\mathbf{v}_{\text{in}}}{\mathbf{i}_{\text{in}}} = \frac{1}{\mathbf{g}_{\text{m}} + sC_{\text{gs}}} \cong \frac{1}{\mathbf{g}_{\text{m}}} \quad [\Omega] \quad (5.7)
\]

Voltage gain:

If the input impedance is matched to 50 Ω, the total gain is expressed by equation (5.9).

\[
\frac{\mathbf{v}_{\text{S}} - \mathbf{v}_{\text{in}}}{R_{\text{S}}} = \mathbf{i}_{\text{in}} \leftrightarrow \mathbf{v}_{\text{S}} = \mathbf{i}_{\text{in}} Z_{\text{in}} + \mathbf{v}_{\text{in}} \leftrightarrow \mathbf{v}_{\text{in}} = \frac{\mathbf{v}_{\text{S}}}{2} [V] \quad (5.8)
\]

\[
A_{\text{V total}} = \frac{\mathbf{v}_{\text{out}}}{\mathbf{v}_{\text{S}}} = \frac{-\mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}} Z_{\text{L}}}{-2 \mathbf{g}_{\text{gs}}} = \frac{\mathbf{g}_{\text{m}} Z_{\text{L}}}{2} \quad (5.9)
\]

Noise factor:

The noise factor is calculated for the small-signal circuit in Figure 5.4 b), where only the MOS channel noise and the source resistance thermal noise are represented, being the input impedance considered conjugate-matched. Using the superposition theorem, the total output noise current is given by,

\[
\mathbf{i}_{\text{in}} + \mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}} + \mathbf{i}_{\text{nd}} = 0 \leftrightarrow -\frac{\mathbf{v}_{\text{gs}}}{R_{\text{S}}} = \mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}} + \mathbf{i}_{\text{nd}} \leftrightarrow \mathbf{i}_{\text{nd}} = -2 \mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}} \quad [A] \quad (5.10)
\]

\[
\therefore \mathbf{i}_{\text{noise out}} = -(\mathbf{i}_{\text{nd}} + \mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}}) = -\frac{\mathbf{i}_{\text{nd}}}{2} \quad [A] \quad (5.11)
\]

where, \( \mathbf{V}_{\text{rms}} \) and \( \mathbf{I}_{\text{nd}} \) are uncorrelated noise sources. Therefore, the noise factor is expressed by equation (5.12),

\[
F = \frac{\mathbf{v}_{\text{noise out}}}{\mathbf{v}_{\text{noise source}}^2} \frac{1}{|A_{V \text{ total}}|^2} = \frac{\mathbf{v}_{\text{noise out}}^2}{\mathbf{v}_{\text{noise source}}^2 |A_{V \text{ total}}|^2} = 1 + \frac{\mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}}^2 Z_{\text{L}}^2}{4 \mathbf{R}_{\text{S}} Z_{\text{L}}^2} + \frac{\mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}}^2 Z_{\text{L}}^2}{4 \mathbf{R}_{\text{S}} Z_{\text{L}}^2} + \frac{\mathbf{g}_{\text{m}} \mathbf{v}_{\text{gs}}^2 Z_{\text{L}}^2}{4 \mathbf{R}_{\text{S}} Z_{\text{L}}^2} \quad (5.12)
\]

\[
\therefore F = 1 + \frac{2 R_{\text{S}} \gamma}{\mathbf{g}_{\text{m}}} \quad (5.13)
\]

where, \( \gamma \) is a constant that depends on the channel’s length. For long channels \( \gamma=2/3 \) is used.

Circuit biasing:

The current source \( I_{SS} \), represented in Figure 5.4 a), is usually implemented with a MOS, \( M_2 \). Hence, all the circuit’s transistors must operate in the saturation working region and verify equation (5.14). The maximum output voltage will be the supply voltage, \( V_{DD} \), and the minimum voltage corresponds to

81
the sum of the overdrive voltages of $M_1$ and $M_2$. The optimum DC output voltage is then expressed by (5.15) [A2.1].

$$V_{DS_{1,2}} > V_{GS_{1,2}} - V_t \land V_{GS_{1,2}} > V_t \rightarrow V_{OVD_{1,2}} = V_{GS_{1,2}} - V_t \ [V] \quad (5.14)$$

$$V_{OUT_{DC}} = \frac{V_{DD} - (V_{OVD_{1}} + V_{OVD_{2}})}{2} \ [V] \quad (5.15)$$

Finally, the biasing gate voltage of $M_i$ is given by equation (5.16).

$$V_{GG} = V_t + V_{DS_1} + V_{DS_2} \ [V] \quad (5.16)$$

### 5.2.2. Single-Stage Feedback Amplifiers

The use of feedback in amplifiers has several advantages: gain desensitizing, extended bandwidth, nonlinear distortion reduction and input/output impedance control. There are four main feedback topologies: shunt-series, shunt-shunt, series-series and series-shunt. Their use depends on the input and output variables, existing a suitable matrix for each topology, Table 5.1 [2].

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Input</th>
<th>Output</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>Shunt</td>
<td>Series</td>
<td>Dual Hybrid (H')</td>
</tr>
<tr>
<td>Transimpedance</td>
<td>Shunt</td>
<td>Shunt</td>
<td>Admittance (Y)</td>
</tr>
<tr>
<td>Transconductance</td>
<td>Series</td>
<td>Series</td>
<td>Impedance (Z)</td>
</tr>
<tr>
<td>Voltage</td>
<td>Series</td>
<td>Shunt</td>
<td>Hybrid (H)</td>
</tr>
</tbody>
</table>

Table 5.1 – Types of feedback amplifiers

The LNA developed in [2] is a shunt-shunt amplifier, Figure 5.5, used to obtain a low input impedance matching (50 Ω) and an amplified voltage output.

![Figure 5.5](image)

Figure 5.5 – Single-stage feedback amplifier developed in [2].

The shunt-shunt topology might be generally represented by Figure 5.6, existing two blocks that represent amplification and feedback, as A and B in Figure 5.2.
Figure 5.6 – Shunt-shunt topology [2].

\[ Y_{x,y} = \frac{i_x}{v_y} \big|_{v_{xy}=0} \text{[S]} \]  

(5.17)

Particularizing for the LNA in Figure 5.5, the amplifying block A is essentially given by the cascode stage, formed by \( R_D, M_4 \), and \( M_1 \); while the feedback block B is done through \( R_{fb} \). Notice that a common-drain, \( M_2 \) and \( M_3 \), is used to isolate the cascode load from the input. The Mirror 2 is responsible for the current steering that provides more current to \( M_1 \), increasing \( g_m \) value without a higher voltage drop, which could put \( M_4 \) out of the saturation working region. Therefore, if the incremental model in Figure 5.7 is considered for the MOS transistors, it is possible to study the incremental circuits of block A and B, represented in Figure 5.8 a) and Figure 5.8 b), respectively. In this analysis, \( R_{cap} \) is considered an AC open-circuit, since it is used as a DC pull-up resistor, having a high value. Additionally, the \( C_{cap} \) capacitor is used for DC decoupling, which means it has a high value and can be considered an AC short-circuit. Finally, the resistor \( R_T \) in the circuit of Figure 5.8 a) represents the parallel between \( R_D \) and the cascode output impedance, which can be approximated by the value of \( R_D \).

Figure 5.7 – MOS incremental model [2].

Figure 5.8 – Blocks A and B incremental models, a) and b), respectively [2].

Hence, for block A it is possible to write,

\[ Y_{11A} = \frac{i_1}{v_1} \big|_{v_2=0} = \frac{v_{gs1}+C_{gd1}}{v_1} = s \left( C_{gs1} + C_{gd1} \right) \text{[S]} \]  

(5.18)
\[ Y_{21A} = \frac{I_2}{V_1} |_{V_2=0} = \frac{V_1 (g_m - sC_{gd})}{V_1} = g_m - sC_{gd} \quad [S] \quad (5.19) \]

\[ Y_{12A} = \frac{I_1}{V_2} |_{V_1=0} = -\frac{V_2 sC_{gd}}{V_2} = -sC_{gd} \quad [S] \quad (5.20) \]

\[ Y_{22A} = \frac{I_2}{V_2} |_{V_1=0} = \frac{V_2 - I_1}{V_2} = \frac{1}{R_T} + sC_{gd} \quad [S] \quad (5.21) \]

while block B is described by equations (5.22) to (5.25).

\[ Y_{11B} = \frac{I_1}{V_1} |_{V_2=0} = -\frac{V_{gS} (g_m + sC_{gs})}{I_{fb} - V_{gS}} = \frac{g_m + sC_{gs}}{1 + s(g_m + sC_{gs})R_fb} \quad [S] \quad (5.22) \]

\[ Y_{21B} = \frac{I_2}{V_1} |_{V_2=0} = \frac{V_{gS} sC_{gs}}{V_{gS} + sC_{gs}} = -sC_{gs} \quad [S] \quad (5.23) \]

\[ Y_{12B} = \frac{I_1}{V_2} |_{V_1=0} = -\frac{V_{gS} (g_m + sC_{gs})}{V_{gS} - R_m I_1} = \frac{-g_m + sC_{gs}}{1 + s(g_m + sC_{gs})R_m} \quad [S] \quad (5.24) \]

\[ Y_{22B} = \frac{I_2}{V_2} |_{V_1=0} = \frac{V_{gS} sC_{gs} + sC_{gs} R_m V_{gS}}{V_{gS} [1 + (g_m + sC_{gs})R_m]} = \frac{sC_{gs} + sC_{gs} R_m V_{gS}}{1 + (g_m + sC_{gs})R_m} \quad [S] \quad (5.25) \]

However, the shunt-shunt topology in Figure 5.6 can be simplified as in Figure 5.9.

![Diagram](image)

Figure 5.9 – Simplified shunt-shunt topology (adapted from [2]).

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
= \begin{bmatrix}
Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\
Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B}
\end{bmatrix} \quad [S] \quad (5.26)
\]

**Input impedance:**

Inspecting Figure 5.9, it is possible to express the input impedance by equation (5.27). However, the direct substitution of the calculated admittances, in (5.18) to (5.25), yields a complex expression. According to [2], a first order approach is still valid, where the MOS incremental model is given by an ideal transconductor (i.e. \(C_{gs}\) and \(C_{gd}\) in the circuit of Figure 5.7 are null). Additionally, the load
admittance, \( Y_L \), is considered capacitive, as it corresponds to the VCO-mixer’s current sources gates. If \( Y_L \) is also neglected, the approximate input impedance is given by equation (5.27).

\[
Z_{in} = \frac{Y_1}{i_1} = \left( Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L} \right)^{-1} \approx \frac{1 + g_m R_{fb}}{R_{m2}(1 + g_m R_T)} \quad [\Omega] \quad (5.27)
\]

Notice that the gains of blocks A and B are expressed by,

\[
B = \left. \frac{i_1}{v_2} \right|_{v_1=0} = Y_{12} \quad (5.28)
\]

\[
A = \left. \frac{v_2}{i_1} \right|_{v_1=0} = -\frac{Y_{21}v_1}{v_2 Y_{11}} = -\frac{Y_{21}}{(Y_{22} + Y_L)Y_{11}} \quad (5.29)
\]

meaning that equation (5.27) can be rewritten as (5.30). The original input impedance, \( Z_{Ain} \), is decreased due to the feedback, justifying the low input impedance value for the shunt-shunt topology.

\[
Z_{in} = \frac{Z_{Ain}}{1 + AB} = \frac{\frac{1}{Y_{11}}}{1 - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L} Y_{11}} \approx \left( Y_{11} - \frac{Y_{12}Y_{21}}{Y_{22} + Y_L} \right)^{-1} \quad [\Omega] \quad (5.30)
\]

Voltage gain:

Inspecting Figure 5.9 and using the same simplifications as for the input impedance, the voltage gain can be expressed by equation (5.31).

\[
A_v = \frac{v_{out}}{v_{in}} = \frac{v_2}{v_1} = -\frac{Y_{21}}{Y_{22} + Y_L} \approx -R_T g_m \quad (5.31)
\]

Noise factor:

To study and minimize the circuit’s noise factor, it is important to determine the dominant noise sources. In the LNA of Figure 5.5, the main noise sources are considered to be: \( R_{fb}, R_T, M_1 \) and \( M_3 \), as in Figure 5.10 a). These sources are thermal noise sources, considered uncorrelated. Notice that the circuit in Figure 5.10 a) considers the resistor \( R_{cap} \) as an open-circuit and the capacitor, \( Cap \), as a short-circuit. Additionally, \( R_T \) represents the parallel between \( R_D \) and the cascode output impedance.

Figure 5.10 – AC noise circuits for the LNA developed in [2]: a) initial; b) step 1; c) step 2 [2].
Step 1: The first step is to transform the noise voltage source $V_{n,RT}$ into its Norton equivalent, while a voltage-current transformation is applied to the sources $V_{n,Rg3}$ and $V_{n,Rg1}$.

$$i_{step_1} = i_{n,d1} + v_{n,Rg1}g_{m1} \ [A] \quad (5.32)$$

$$i_3 = i_{n,d3} + g_{m3}v_{n,Rg3} \ [A] \quad (5.33)$$

Step 2: The second step transforms the noise current source $i_3$ into its Thévenin equivalent, considering the source resistance $R_S$. Additionally, a voltage-current transformation is applied to the voltage source $V_{n,fb}$, as in equation (5.34), being then added to the $i_{n,RT}$ current source.

$$i_{step_2} = i_{step_1} - \frac{v_{n,RT}}{R_T} + g_{m1}v_{n,fb} \ [A] \quad (5.34)$$

$$v_{aux} = v_{n,fb} + i_3R_S \ [V] \quad (5.35)$$

Step 3: It is now possible to use the chain matrix to obtain the input referred noise sources, $V_{n,in}$ and $i_{n,in}$, as in Figure 5.11.

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix} \quad (5.36)$$

$$T_{12} = \frac{v_{1}}{i_{2}} \bigg|_{v_{2}=0} = -\frac{1}{V_{21}} \equiv -g_{m1} \quad (5.37)$$

$$T_{22} = \frac{i_{1}}{v_{2}} \bigg|_{v_{2}=0} = -\frac{y_{11}}{Y_{22}} = -\frac{g_{m2}}{(1+g_{m3}R_B)g_{m1}} \quad (5.38)$$

$$\therefore i_{n,in} = -T_{22}i_{step_2} \ [A] \quad (5.39)$$

$$\therefore v_{n,in} = -v_{aux} - T_{12}i_{step_2} \ [A] \quad (5.40)$$

Step 4: Finally, the Wiener-Khintchine theorem is applied to the total input noise voltage source, $V_{n,in\text{total}}$, obtaining the total input power spectral density, (5.44) [A1.1].

$$\overline{v^2_{n,x}} = 4K_BT_Rx \ [V^2] \quad (5.41)$$

$$\overline{i^2_{n,dx}} = 4K_BT_yg_{m_x} \ [A^2] \quad (5.42)$$
\[ v_{n,\text{total}} = i_{n,\text{in}}R + v_{n,\text{in}} = -v_{\text{aux}} - \text{step2}(T_{12} + R_{22}) = -v_{\text{aux}} - \text{step2} \alpha [V] \quad (5.43) \]

\[ \therefore \frac{v_{n,\text{total}}^2}{g_{m1}} + \frac{v_{n,\text{RT}}^2}{R_{12}} + \frac{v_{n,\text{di}}^2}{R_{12}} + 2 \text{step2} \alpha [V] \quad (5.44) \]

Using equation (5.44), it is possible to understand the different contributions of the main noise sources to the circuit’s total noise figure.

Circuit biasing:

Regarding the circuit biasing constraints, there are two critical DC paths, associated with the amplifying and the feedback stage, Figure 5.12. The circuit’s transistors must operate in the saturation working region, respecting the conditions in equation (5.45). Therefore, the minimum output DC voltage is defined by the minimum drain-source voltages that maintain \( M_4 \) and \( M_1 \) in the saturation working region. These values are given by the chosen overdrive voltages, while the maximum DC output voltage is \( V_{\text{DD}} \) [A2.1]. Therefore, the optimum DC output voltage is given by (5.46).

\[ V_{\text{DS}} > V_{\text{GS}} - V_t \land V_{\text{GS}} > V_t \rightarrow V_{\text{OV}} = V_{\text{GS}} - V_t [V] \quad (5.45) \]

\[ V_{\text{OUT,DC}} = \frac{V_{\text{DD}} - (V_{\text{OV,1}} + V_{\text{OV,4}})}{2} [V] \quad (5.46) \]

Regarding the transistor \( M_4 \) biasing gate voltage, it is expressed by equation (5.47).

\[ V_{\text{GG}} = V_t + V_{\text{DS,4}} + V_{\text{DS,4}} [V] \quad (5.47) \]

Finally, the DC input voltage that maximizes the input range is given by (5.48).

\[ V_{\text{IN,DC}} = \left[ \frac{V_{\text{DD}} - (V_{\text{OV,1}} + V_{\text{DD}}) - V_{\text{t1}}}{2} \right] [V] \quad (5.48) \]
5.2.3. Discussion

In this chapter, two topologies that allow for a 50 Ω input impedance matching, without the use of LC passive networks, were studied: the common-gate and the single-stage feedback LNA. These topologies are both inductorless, resulting in a significant area reduction. However, the use of feedback, in the second case, has the following advantages: gain desensitizing, extended bandwidth, nonlinear distortion reduction and input/output impedance control. In fact, in the common-gate topology, the input impedance and the voltage gain are related, presenting difficult tradeoffs, as observed in equations (5.9) and (5.7). In the single-stage feedback amplifier case, the voltage gain, (5.31), and the input impedance, (5.27), can be more independently controlled. Hence, the main drawback of this topology is the extra noise generation due to the use of the feedback resistor (Figure 5.5).

The LNA developed in [2] uses a shunt-shunt topology, allowing for a low input impedance matching and output voltage amplification. The amplifying block uses a cascode stage, while the feedback uses a feedback resistor. However, the cascode load is isolated from the input by a common-drain, used as a buffer.

The proposed solution is based on the topology in Figure 5.5, although, taking advantage of the knowledge of the VCO-mixer’s RF input port impedance. In fact, the LNA’s load will be used as the current mirror reference of the VCO-mixer developed in chapter 3, saving approximately 1/3 of the total VCO-mixer’s power, excluding the need of buffers and reducing the overall area.

5.3. Implemented Circuit

The implemented solution, in Figure 5.13, is based on a PMOS mirrored version of the LNA in Figure 5.5. The proposed LNA is a single-stage feedback LNA designed in a UMC 130 nm CMOS technology for a supply voltage of 1.2 V. The topology takes advantage of the already known VCO-mixer’s RF port impedance, being the LNA designed to maximize the specifications for that same load. In fact, the load is also used as the VCO-mixer’s current mirror reference, saving about 1/3 of the VCO-mixer’s power, excluding the necessity of buffers and decreasing the overall area.

![Figure 5.13 – Implemented single-stage feedback LNA.](image-url)
Therefore, the topology in Figure 5.13, substitutes the typical resistive load by a modified current source, $M_3$. Usually, a current source consists of a MOS in diode connection, where the incremental model is approximately given by a $1/g_m$ resistance, Figure 5.14 a). However, if a resistor $R$ chokes the AC signal from the drain to the gate, the DC voltage is still sampled, while the incremental resistance is higher than it would be for a load resistor with the same DC voltage, Figure 5.14 b).

![MOS in diode connection and modified current source](image)

Figure 5.14 – MOS in diode connection, a), and modified current source, b).

Additionally, to ensure the correct DC reference for the VCO-mixer’s current mirror, the AC noise is decoupled by the MOSCap $M_4$, while the desired RF signal is shunted to the output. The LNA provides both the necessary DC output and the amplified RF signal, avoiding the need for inter-block DC decoupling capacitors.

Regarding the circuit biasing conditions, the use of the modified current source load requires a higher DC voltage drop, since $M_3$ must operate in the saturation working region. Therefore, due to the biasing constraints, $M_2$ is used as a resistor instead of part of a cascode.

This section will describe the methodologies used to implement the proposed circuit, being divided in 4 parts: Single-Stage Feedback LNA, Modified Single-Stage Feedback LNA, Layout, Monte Carlo and Corners.

### 5.3.1. Single-Stage Feedback LNA

The implementation of the proposed single-stage feedback LNA, in Figure 5.13, uses a semi-empirical design, where the first step is to obtain the modified current source’s incremental resistance, $r_0$, (5.49). The DC output voltage is defined by the VCO-mixer’s current mirror requirements (defined in chapter 3), being a value of 500 mV considered. Using an initial current of 2.6 mA for the amplifying block, $I_A$, in Figure 5.15 a), it is possible to adjust the DC output voltage and estimate the incremental resistance [A2.1]. Notice that $V_A$ is a technology dependant parameter, related with the channel-length modulation effect. The output voltage is adjusted to 500 mV, using the transistor’s width-to-length ratio as in equation (5.52). Regarding the choking resistors, $R$, and the AC decoupling capacitor, $C$, their initial values are 1 kΩ and 1 pF, respectively.

$$r_0 = \frac{V_A + V_{DS}}{I_A} \quad [\Omega] \quad (5.49)$$

$$I_{DS3} = k(V_{GS3} - V_t)^2 \quad [A] \land V_{DS3} = V_{GS3}[V] \quad (5.50)$$
\[ k = \frac{\mu_n C_{ox} W}{2L} [AV^{-2}] \quad (5.51) \]

\[ V_{DS3} = \sqrt{\frac{\rho_{DS3}}{k}} + V_i [V] \quad (5.52) \]

![Figure 5.15 – Modified current mirror, a), and initial LNA circuit, b).](image)

Considering the incremental resistance to be 300 Ω and defining that the voltage gain must be 20 dB, it is possible to determine the necessary transconductance \( g_{m1} \) of the common-source transistor \( M_1 \), (5.53). The MOS transconductance is then controlled by the width-to-length ratio, as the drain-source current is considered constant in equation (5.54).

\[ A_v \equiv -r_0 g_{m1} \leftrightarrow g_{m1} = -\frac{A_v}{r_0} [S] \quad (5.53) \]

\[ g_m = 2\sqrt{R_{DS}} [S] \quad (5.54) \]

According to [2], the common-drain’s transconductance \( g_{m8} \) can be considered 5 times smaller than \( g_{m1} \), reducing the size of \( M_8 \) and the total required current, while maintaining an effective feedback.

\[ g_{m8} = \frac{8g_{m1}}{5} [S] \quad (5.55) \]

Therefore, for a \( g_{m1} \) of 33 mS and a \( g_{m2} \) of 7 mS, the required feedback resistance value, \( R_{fb} \), is 400 Ω, (5.56).

\[ Z_{in} = 50 = \frac{1+g_{m2}R_{fb}}{g_{m2}(1+g_{m1}R_T)} \leftrightarrow R_{fb} = \frac{50[8g_{m2}(1+|A_v|)]^{-1}}{8g_{m2}} [\Omega] \quad (5.56) \]

A first draft may then be obtained, if the circuit in Figure 5.15 b) is considered. The resistor \( R_A \) has an initial value of 100 Ω, considering the available biasing overhead; and the common-drain’s current source, \( I_B \), has a value of 650 μA, a current 4 times smaller than \( I_A \), to reduce the power consumption. During this step, it is important to define the overdrive voltage for \( M_1 \), which must be at least 50 mV to accommodate the RF input signal, while avoiding a significant current increase in the common-source. The pull-down resistor, \( R_8 \), and the feedback DC decoupling capacitor, \( C_{fb} \), have a starting value of 1 kΩ and 200 fF, respectively.
To obtain the final circuit, in Figure 5.16, the resistors \( R_A \) and \( R_{fb} \) are substituted by the transistors \( M_2 \) and \( M_9 \) operating in the triode working region, which are adjusted to have the same DC voltage drop as the resistors, \((5.57)\) [A2.1]. The pull-down resistor, \( R_8 \), is also substituted by the transistor \( M_7 \) in the cut-off working region (i.e. \( V_{GS} < V_t \)). Additionally, the MOSCaps \( M_4 \) and \( M_5 \) are used to decouple the AC noise; and the auxiliary transistor \( M_6 \) is used to bias the gates of \( M_9, M_2 \) and \( M_7 \), while avoiding the disruption of the oxide due to ESD.

![Circuit Diagram](image)

**Figure 5.16 – Implemented single-stage feedback LNA.**

\[
R_{\text{equivalent}} = \frac{1}{2k(V_{GS} - V_t)} \ [\Omega] \ (5.57)
\]

The ideal current source, \( I_B \), is also substituted by \( M_{10} \) and \( M_{11} \), considering a 1:1 current ratio and a \( V_{SG} \) voltage of 0.5 V (i.e. an overdrive voltage of 0.1 V), which are adjusted by their width-to-length ratio, \((5.58)\).

\[
\frac{I_B}{I_{\text{REF}}} = \frac{k_{10}(1+\lambda V_{DS10})}{k_{11}(1+\lambda V_{DS11})} \cong \frac{W_{10}}{L_{10}} \frac{V_{DS10}}{V_{DS11}} \ (5.58)
\]

Finally, the noise figure must be adjusted considering equation \((5.44)\). It is particularly important to guarantee that the choking resistors, \( R \), have a value that avoids signal loss and does not significantly increase the thermal noise. Additionally, the transistors’ width should be increased by the number of fingers, to reduce the MOS parasitic gate resistance.

The complete diagram of the single-stage feedback LNA is shown in Figure 5.17, where the external connections are the supply (VDD), ground (GND), RF input (IN) and the current mirror (Mirror1). Each connection is associated with a pad from the technology (PAD_RF) and a bonding wire with the ideal model in Figure 3.33 (as in chapters 3 and 4). Notice that ESD protections are used for the current mirror and the RF input to avoid the oxide disruption, where the circuit in Figure 3.34 and its respective parameters are used. Additionally, two AC decoupling capacitors, MOMCAPS_RF, are used between VDD and GND to avoid undesired parasitic AC loops \([11]\). Their values are 1.137 pF and 135 fF, stabilizing the current at two different time constants. For this circuit, a 1.137 fF AC decoupling capacitor will also be used for the current mirror, to reduce possible noise injection.

91
The obtained dimensioning is shown in Table 5.2.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
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</thead>
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<td>$M_1$</td>
<td>$P_{12_RF}$</td>
<td>$W = 2.3 , \mu m$ L = 120 nm #Fingers = 32 Mult. = 5</td>
<td>$M_8$</td>
<td>$P_{12_RF}$</td>
<td>$W = 2 , \mu m$ L = 120 nm #Fingers = 27</td>
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<td>$P_{12_RF}$</td>
<td>$W = 2 , \mu m$ L = 120 nm #Fingers = 20</td>
<td>$M_9$</td>
<td>$P_{12_RF}$</td>
<td>$W = 2 , \mu m$ L = 120 nm #Fingers = 5</td>
</tr>
<tr>
<td>$M_3$</td>
<td>$N_{12_RF}$</td>
<td>$W = 1.8 , \mu m$ L = 120 nm #Fingers = 16 Mult. = 3</td>
<td>$M_{10}, M_{11}$</td>
<td>$P_{12_RF}$</td>
<td>$W = 2.2 , \mu m$ L = 120 nm #Fingers = 3</td>
</tr>
<tr>
<td>$M_4, M_5$</td>
<td>$P_{12_RF}$</td>
<td>$W = 9.6 , \mu m$ L = 360 nm #Fingers = 32 Mult. = 2</td>
<td>$R$</td>
<td>$RNPPO_{RF}$</td>
<td>$R = 1.023 , k\Omega$</td>
</tr>
<tr>
<td>$M_6, M_7$</td>
<td>$N_{12_RF}$</td>
<td>$W = 0.9 , \mu m$ L = 360 nm #Fingers = 4</td>
<td>$C$</td>
<td>$MOMCAPS_{RF}$</td>
<td>$C = 142.25 , fF$</td>
</tr>
<tr>
<td></td>
<td>$R_{Mirror1}$</td>
<td></td>
<td></td>
<td>$res$</td>
<td>$1.07 , k\Omega$</td>
</tr>
</tbody>
</table>

Table 5.2 – Single-stage feedback LNA’s parameters.

This dimensioning yielded the results shown in Figure 5.18, Figure 5.19 and Figure 5.20.
At the 2.41 GHz RF, the obtained noise figure is 2.85 dB for a voltage gain of 17.7 dB, while the scattering-parameter S11 is -14 dB. Regarding the linearity of the circuit, the obtained input 1 dB compression point is -16.8 dBm.

The IIP3 is calculated using a two-tone test, where the interferer is at 2.42 GHz and the RF signal at 2.41 GHz. The upper sideband of the third-order intermodulation products, $2\omega_\text{signal} - \omega_\text{interferer}$, is considered, yielding an IIP3 of -3.43 dBm.

Finally, the total power consumption is 5.5 mW for a current of 4.59 mA (4.12 mA without the current source reference).
5.3.2. Modified Single-Stage Feedback LNA

The load of the proposed single-stage feedback LNA, in Figure 5.16, is used as the VCO-mixer's current mirror reference, which means that the DC output must be accurate. Therefore, a current steering technique is designed to make fine adjustments possible, Figure 5.21. The implemented transistors, $M_{12}$ and $M_{13}$, must be small, as to avoid undesired parasitic effects, without compromising the current steering function. Their dimensioning is given in Table 5.3. The current steering is controlled by the gate voltages $V_{DC-}$ and $V_{DC+}$. Regarding the chosen nodes, the most suitable node to inject the current is considered to be the LNA’s output, avoiding an extra DC voltage drop due to the current increase. The current will be drawn at $M_1$ drain to improve its transconductance, increasing the gain of the LNA. For instance, if the DC output voltage decreases, the current is injected by $M_{13}$, to increase that voltage. However, if the DC output voltage increases, the current is then drawn by $M_{12}$ to decrease the DC voltage. The complete diagram of the modified single-stage feedback LNA is shown in Figure 5.22.

Figure 5.21 – Final single-stage feedback LNA.

Table 5.3 – Steering current transistors, $M_{12}$ and $M_{13}$, parameters.

<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
</table>
| $M_{12}$ | $P_{12\_RF}$ | $W = 0.9 \ \mu m$  
$L = 360 \ \text{nm}$  
#Fingers = 4  
Mult. = 5 |
| $M_{13}$ | $P_{12\_RF}$ | $W = 2 \ \mu m$  
$L = 120 \ \text{nm}$  
#Fingers = 5 |

Figure 5.22 – Diagram of the final single-stage feedback LNA.
5.3.3. **Layout**

The layout of RF blocks is critical, since it is very sensitive to parasitic capacitances and different delay paths. (1) Hence, it is important to design a symmetric layout, having paths and components share similar delays and parasitics. For instance, the components’ connections must distribute the signals symmetrically, using multiple contacts to reduce the parasitic resistance and improve reliability. Additionally, matching components must be close, to avoid mismatches.

(2) The paths must be short, as resistance increases with length, and their width must be proportional to their total passing current, to avoid the electromigration problem. The empirical rule of 1 μm/mA is considered for the metal layers’ width. Furthermore, paths associated with AC signals use higher layers, being overlapping considered undesired, as to decrease parasitic capacitances. The opposite is wanted for DC signals, where AC noise is decoupled.

(3) Finally, a VDD and GND frame is used to shorten the biasing paths and homogeneously polarize the substrate, working as a guard-ring of the entire block. In fact, guard-rings are used throughout the design, to reduce the noise and the cross-talk. The designed layout is shown in Figure 5.23.

![Figure 5.23 – Single-stage feedback LNA’s layout.](image)

The obtained results for the extracted LNA are similar to those of the simulation of the schematic, being the final dimensioning shown in Table 5.4, where the current mirror’s reference resistor, $R_{\text{mirror1}}$, is adjusted to compensate the slight post-layout current deviation. The active area is 0.01858222 mm$^2$ (133.120 μm x 139.590 μm).
<table>
<thead>
<tr>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
<th>Element</th>
<th>Component</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁</td>
<td>P_12_RF</td>
<td>W = 2.3 μm L = 120 nm #Fingers = 32 Mult. = 5</td>
<td>M₈</td>
<td>P_12_RF</td>
<td>W = 2 μm L = 120 nm #Fingers = 27</td>
</tr>
<tr>
<td>M₂</td>
<td>P_12_RF</td>
<td>W = 2 μm L = 120 nm #Fingers = 20</td>
<td>M₉</td>
<td>P_12_RF</td>
<td>W = 2 μm L = 120 nm #Fingers = 5</td>
</tr>
<tr>
<td>M₃</td>
<td>N_12_RF</td>
<td>W = 1.8 μm L = 120 nm #Fingers = 16 Mult. = 3</td>
<td>M₁₀, M₁₁</td>
<td>P_12_RF</td>
<td>W = 2.2 μm L = 120 nm #Fingers = 3</td>
</tr>
<tr>
<td>M₄, M₅</td>
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<td>W = 9.6 μm L = 360 nm #Fingers = 32 Mult. = 2</td>
<td>M₁₂</td>
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<td>W = 0.9 μm L = 360 nm #Fingers = 4 Mult. = 5</td>
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<td>M₆, M₇</td>
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<td>res</td>
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</table>

Table 5.4 – Final dimensioning of the single-stage feedback LNA parameters.

The post-layout results obtained for the dimensioning in Table 5.4 are shown in Figure 5.24, Figure 5.25 and Figure 5.26. In Figure 5.24, the noise figure, voltage gain and S11 parameter are analyzed, considering the input at the 2.41 GHz RF. The simulation of the extracted circuit yielded a value of 3.16 dB for the noise figure, 16.8 dB for the voltage gain and -15 dB for the S11.

![Noise Figure, Voltage Gain, S11](image)

Figure 5.24 – Noise figure, voltage gain and S11.

Regarding the linearity of the circuit, the obtained input 1 dB compression point is -16.27 dBm.
The IIP3 is calculated using a two-tone test, where the interferer is at 2.42 GHz and the RF signal at 2.41 GHz. The upper sideband of the third-order intermodulation products, $2\omega_{\text{signal}} - \omega_{\text{interferer}}$, is considered, yielding an IIP3 of -2.67 dBm.

In Figure 5.27 and Figure 5.28, two parametric analyses are used to study the current steering transistors that allow the control of the DC output voltage. Therefore, the S11 parameter, DC output voltage, noise figure and voltage gain are studied as a function of the control voltages $V_{\text{DC-}}$ and $V_{\text{DC+}}$. 
Regarding the NMOS transistor $M_{12}$, as the control voltage $V_{DC-}$ increases, the DC output voltage will decrease. The maximum DC voltage variation is approximately 138 mV. A higher voltage gain and a lower noise figure are expected, as the modified current source $M_3$ incremental resistance and the $g_{m1}$ transconductance increase. Notice that the current steering is only possible after $V_{GS12}$ reaches the necessary threshold voltage (i.e. $V_{GS}=0.4$ V).

![Figure 5.28 – S11, DC output voltage, noise figure and voltage gain as function of $V_{DC+}$.
](image)

In the case of the PMOS transistor $M_{13}$, as the control voltage $V_{DC+}$ decreases, the output DC voltage increases, for a maximum variation of approximately 76 mV. However, the voltage gain decreases and the noise figure increases, as the incremental resistance of both $M_{13}$ and the modified current source $M_3$ decreases. Notice that no current is drawn until $V_{DC+}$ reaches the necessary threshold voltage to allow the transistor conduction (i.e. $V_{SG}=0.4$ V). Finally, the average circuit's power consumption is 5.49 mW for a current of 4.57 mA (4.11 mA without the current source reference).

### 5.3.4. Monte Carlo and Corners

The extracted single-stage feedback LNA is used to amplify the RF signal and as the VCO-mixer's current mirror reference. Therefore, it is important to ensure that the circuit maintains its specifications when parameter mismatches and process variations occur. The Monte Carlo simulation studies the parameters' mismatches and process variations, according to the technology standard deviations; yielding the results in Figure 5.29, for the DC output voltage, voltage gain, noise figure and S11. All the results consider that no current steering is used to control the DC output voltage. The obtained results validate the design, with an average of 16 dB for the voltage gain, 3.42 dB for the noise figure and -14.15 dB for the S11 parameter. The average DC output voltage matches the required reference value, while possible voltage deviations can be controlled both by the adjusting the $V_{DC+}$ and $V_{DC-}$ control voltages and, if needed, by adjusting the current source's resistor, $R_{\text{mirror1}}$. 

98
Figure 5.29 – Monte Carlo results for the DC output voltage, voltage gain, noise figure and S11.

Regarding the corners simulation, extreme PVT parameter variations are studied for the same variables as in the Monte Carlo simulation, yielding the results in Table 5.5 and Table 5.6.

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<thead>
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<th>Corner 4</th>
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<table>
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<th>Corner 3</th>
<th>Corner 4</th>
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<td>7.02</td>
<td>2.36</td>
<td>2.23</td>
<td>17.59</td>
<td>17.26</td>
</tr>
<tr>
<td>Noise Figure [dB]</td>
<td>6.41</td>
<td>6.42</td>
<td>13.52</td>
<td>13.51</td>
<td>3</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Table 5.5 – Corners 1 to 6.

Inspecting the obtained results in Table 5.5 and Table 5.6, it is possible to observe that the worst cases are the corners 3, 4, 7, 8, 9 and 10. The corners 7 and 8 are not possible to correct, considering that the LNA is also used as the VCO-mixer’s current mirror reference, and the output DC voltage never reaches its original expected value. Regarding the remaining cases, corners 9 and 10 are adjusted using 1.2 V for the control voltage VDC- and 412 Ω for the current mirror’s resistor, $R_{\text{Mirror1}}$; the corners 3 and 4 are adjusted using 300 mV for the control voltage VDC+ and 4.5 kΩ for the $R_{\text{Mirror1}}$. 
### Table 5.6 – Corners 7 to 12.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 7</th>
<th>Corner 8</th>
<th>Corner 9</th>
<th>Corner 10</th>
<th>Corner 11</th>
<th>Corner 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.08</td>
<td>1.08</td>
<td>1.32</td>
<td>1.32</td>
<td>1.08</td>
<td>1.08</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>fs</td>
<td>fs</td>
<td>sf</td>
<td>sf</td>
<td>sf</td>
<td>sf</td>
</tr>
<tr>
<td>PAD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>Temperature</td>
<td>120</td>
<td>120</td>
<td>-30</td>
<td>-30</td>
<td>120</td>
<td>120</td>
</tr>
</tbody>
</table>

### Table 5.7 – Corners 3, 4, 9 and 10 corrected results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 9</th>
<th>Corner 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>1.08</td>
<td>1.08</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOS</td>
<td>ss</td>
<td>ss</td>
<td>sf</td>
<td>sf</td>
</tr>
<tr>
<td>PAD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>MOMCap</td>
<td>max</td>
<td>min</td>
<td>max</td>
<td>min</td>
</tr>
<tr>
<td>ESD</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
<td>typ</td>
</tr>
<tr>
<td>Temperature</td>
<td>120</td>
<td>120</td>
<td>-30</td>
<td>-30</td>
</tr>
</tbody>
</table>

### Output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 9</th>
<th>Corner 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Output [V]</td>
<td>328.1m</td>
<td>328.1m</td>
<td>653.4m</td>
<td>653.4m</td>
</tr>
<tr>
<td>Voltage Gain [dB]</td>
<td>10.45</td>
<td>10.08</td>
<td>11.55</td>
<td>11.21</td>
</tr>
<tr>
<td>Noise Figure [dB]</td>
<td>7.45</td>
<td>7.57</td>
<td>4.29</td>
<td>4.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Corner 3</th>
<th>Corner 4</th>
<th>Corner 9</th>
<th>Corner 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC Output [V]</td>
<td>507m</td>
<td>507m</td>
<td>507.6m</td>
<td>507.6m</td>
</tr>
<tr>
<td>Voltage Gain [dB]</td>
<td>12.69</td>
<td>12.45</td>
<td>17.1</td>
<td>16.8</td>
</tr>
<tr>
<td>Noise Figure [dB]</td>
<td>6.3</td>
<td>6.33</td>
<td>2.5</td>
<td>2.54</td>
</tr>
</tbody>
</table>
5.4. Conclusions

The proposed single-stage feedback LNA, in Figure 5.21, uses a shunt-shunt topology to allow the 50 Ω input impedance matching without the use of passive LC networks. Therefore, it is an inductorless circuit, which reduces the necessary layout area. Additionally, the use of feedback has the following advantages: gain desensitizing, extended bandwidth, nonlinear distortion reduction and input/output impedance control.

The designed LNA amplifies the RF signal and its load is also used as the current mirror reference of the VCO-mixer developed in chapter 3. The load is a modified current source, where the substitution of the resistor by an active load, allows for a higher incremental resistance for the same DC voltage drop. This technical option saves approximately 1/3 of the total VCO-mixer’s power, excluding the need of buffers and reducing the overall receiver’s area. For instance, the use of buffers at the output of the LNA developed in [2] reduces the gain by approximately 3 dB, while increasing the power consumption. This solution uses a total power of 5.48 mW, which represents approximately less 20% power. Additionally, it is also designed for the already known VCO-mixer’s RF input port impedance, improving the linearity and input matching. Finally, the circuit occupies an active area of 0.0185822 mm², meaning a 37.4 % area increase, in comparison to the circuit in [2]. However, instead of 6 biasing pads, the present LNA operates with only 2, reducing the overall receiver’s area, which is limited by the pad frame. The most relevant results of the extracted LNA are summarized in Table 5.8.

<table>
<thead>
<tr>
<th>Block</th>
<th>Implemented LNA</th>
<th>LNA from [2]</th>
<th>LNA with buffer from [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_v$</td>
<td>16.8 dB @ 2.41 GHz</td>
<td>16.5 dB @ 2.4 GHz</td>
<td>13.47 dB @ 2.4 GHz</td>
</tr>
<tr>
<td>NF</td>
<td>3.16 dB @ 2.41 GHz</td>
<td>2.66 dB @ 2.4 GHz</td>
<td>2.67 dB @ 2.4 GHz</td>
</tr>
<tr>
<td>$S_{11}$</td>
<td>-15.04 dB @ 2.41 GHz</td>
<td>-11.8 dB @ 2.4 GHz</td>
<td>-10.2 dB @ 2.4 GHz</td>
</tr>
<tr>
<td>IIP3</td>
<td>-2.67 dBm</td>
<td>-4.97 dBm</td>
<td>-</td>
</tr>
<tr>
<td>$P_{1dB}$</td>
<td>-16.3 dBm</td>
<td>-21.8 dBm</td>
<td>-</td>
</tr>
<tr>
<td>$P_{DC}$</td>
<td>5.48 mW</td>
<td>5.11 mW</td>
<td>7 mW</td>
</tr>
<tr>
<td>$I_{Total}$</td>
<td>4.57 mA</td>
<td>4.26 mA</td>
<td>5.84 mA</td>
</tr>
<tr>
<td>Area</td>
<td>0.0185822 mm²</td>
<td>-</td>
<td>0.011625 mm²</td>
</tr>
</tbody>
</table>

Table 5.8 – Implemented single-stage feedback LNA results summary.
Chapter 6

Complete Receiver
6.1. Overview

The 2.41 GHz ISM receiver implemented in this thesis uses a low-IF topology, where the image rejection is implemented in the digital domain. Hence, the main objective is to guarantee the linear amplification of the input signal, with low noise addition, while shifting its RF to a 10 MHz IF. The designed analog frontend incorporates a single-stage feedback LNA and a quadrature VCO-mixer, focusing a low power and low area solution. The present solution results from careful design of the individual blocks to maximize the specifications of the complete system.

6.2. Implemented Circuit

The receiver is implemented in a UMC 130 nm CMOS technology for a 1.2 V supply voltage. The proposed circuit is a low power and low area solution, where the individual LNA and VCO-mixer, developed in chapters 5 and 4, are designed to maximize the specifications of the complete system. Therefore, the final layout is shown in Figure 6.1, occupying a total area of 0.14477 mm$^2$ (383.81 μm x 377.20 μm) and using a total of 9 pads. Additionally, the layout is planned to allow the future integration of the IF filter and the available space between the pad frame and the blocks' active area is used to increase the total supply decoupling capacitance.

![Figure 6.1 – Complete receiver's layout.](image)

The following results, in Figure 6.2 to Figure 6.3, are obtained for a constant VCO-mixer's tuning voltage of 915 mV, where the total power consumption is 7.6 mW, for a current of 6.32 mA. In Figure 6.2, a voltage gain of 22.46 dB, a noise figure of 10.9 dB and a S11 parameter of -10.44 dB are obtained for the downconversion of a 2.41 GHz RF signal to a 10 MHz IF.
Therefore, the oscillating frequency has to be 2.4 GHz as in Figure 6.3, where a phase noise of -87.28 dBc/Hz is obtained at a 1 MHz distance from the oscillating frequency. The obtained THD is -17.4 dB.

An example of the complete receiver’s operation is shown in Figure 6.4, where a 2.41 GHz RF signal is mixed with a 2.4 GHz LO’s signal.

Finally, in Figure 6.5, a parametric analysis is used to study the receiver’s voltage gain, noise figure, S11, phase noise and oscillating frequency, as a function of the VCO-mixer’s tuning voltage.
The parametric analysis yielded a tuning sensitivity of 2.45 MHz/V for an effective frequency range of 2.4 GHz ± 100 MHz. However, a wider frequency range can be considered, approximately from 2.2 GHz to 2.8 GHz, where the variation of the remaining parameters is acceptable.

### 6.3. Conclusions

The 2.41 GHz ISM receiver implemented in this thesis uses the single-stage feedback LNA and the quadrature VCO-mixer developed in chapters 5 and 4, taking advantage of their careful design to maximize the system’s specifications. The design of this receiver is focused on a low area and low power solution, where the obtained area for the receiver is 0.14477 mm$^2$, being the total power consumption 7.6 mW. Therefore, the most relevant results are summarized in Table 6.1.

<table>
<thead>
<tr>
<th>Block</th>
<th>Implemented Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>f$_{RF}$</td>
<td>2.41 GHz</td>
</tr>
<tr>
<td>f$_0$</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>A$_v$ @ 10 MHz</td>
<td>22.46 dB</td>
</tr>
<tr>
<td>NF @ 10 MHz</td>
<td>10.9 dB</td>
</tr>
<tr>
<td>S$<em>{11}$ @ f$</em>{RF}$</td>
<td>-10.44 dB</td>
</tr>
<tr>
<td>Phase Noise @ f$_0$+1 MHz</td>
<td>-87.28 dBc/Hz</td>
</tr>
<tr>
<td>P$_{DC}$</td>
<td>7.6 mW</td>
</tr>
<tr>
<td>I$_{Total}$</td>
<td>6.32 mA</td>
</tr>
<tr>
<td>Area</td>
<td>0.14477 mm$^2$</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>approx. 2.2 GHz – 2.8 GHz</td>
</tr>
</tbody>
</table>

Table 6.1 – Implemented receiver results summary.
Chapter 7

Conclusions
7.1. Conclusions

The objective of this thesis is to totally redesign an already existing implementation of an ISM 2.41 GHz Low-IF Receiver Frontend, using a 130 nm CMOS technology for a 1.2 V supply voltage. The image frequency rejection will be done in the digital domain, being necessary to obtain precise quadrature outputs. Three blocks are designed: Quadrature Cross-Coupled RC Relaxation VCO, Quadrature Single-Balanced VCO-Mixer and Single-Stage Feedback LNA.

The implemented receiver is inductorless, focusing a low area and low power solution. The first block to be designed is the quadrature cross-coupled RC relaxation VCO, operating in quasi-linear behavior due to the high frequency of operation. These oscillators are usually associated with a reduced area, although, having a higher phase noise and power consumption in comparison to the LC oscillators. However, the coupling of two oscillators affects the oscillating frequency, even when symmetrical circuits are considered. Therefore, the quality factor of the LC oscillators will be severely degraded, when compared to the relaxation case. The consumption issue is addressed by coupling two single low-power cross-coupled RC relaxation VCOs, where instead of having the usual two current sources, only one is used. However, the main contribution of this work to the VCO block is the use of PMOS active loads for the coupling, substituting the typical soft-limiters. This coupling is based on load resistance change, instead of current draw, resulting in a 20 % power saving. Additionally, the implementation of a VCO, using MOSCaps, allows a flexible frequency tuning without current increase.

The mixing function is directly implemented on the quadrature cross-coupled RC relaxation VCO. This technical option saves power and reduces the total area, while maintaining the quadrature precision. Hence, the single-balanced mixer topology’s working principle is applied, where both the differential pair’s transistors operate in the saturation working region, to maximize the conversion gain and reduce the distortion due to commutation and even-order distortion. The implementation of the mixing function does not significantly affect the VCO’s behavior. Additionally, the use of the PMOS active loads instead of resistors increases the conversion gain.

Finally, the proposed single-stage feedback LNA uses a shunt-shunt topology to allow the 50 Ω input impedance matching without the use of passive LC networks. The use of an LNA with feedback has the following advantages: gain desensitizing, extended bandwidth, nonlinear distortion reduction and input/output impedance control. The designed LNA amplifies the RF signal, being its load also used as the current mirror reference of the VCO-mixer. The load is a modified current source, where the substitution of the resistor by an active load, allows for a higher incremental resistance for the same DC voltage drop. This technical option saves approximately 1/3 of the total VCO-mixer’s power, excluding the need of buffers and reducing the overall receiver’s area.

Therefore, the implemented receiver occupies a total area of 0.14477 mm² and has a consumption of 7.6 mW, being the obtained results summarized in Table 7.1.
<table>
<thead>
<tr>
<th>Block</th>
<th>Implemented Receiver</th>
<th>*Receiver from [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{RF}$</td>
<td>2.41 GHz</td>
<td>2.41 GHz</td>
</tr>
<tr>
<td>$f_0$</td>
<td>2.4 GHz</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>$A_v$ @ 10 MHz</td>
<td>22.46 dB</td>
<td>27.6 dB</td>
</tr>
<tr>
<td>NF @ 10 MHz</td>
<td>10.9 dB</td>
<td>-</td>
</tr>
<tr>
<td>$S_{11}$ @ $f_{RF}$</td>
<td>-10.44 dB @ 2.41 GHz</td>
<td>-</td>
</tr>
<tr>
<td>Phase Noise @ $f_0$+1 MHz</td>
<td>-87.28 dBc/Hz</td>
<td>-</td>
</tr>
<tr>
<td>$P_{DC}$</td>
<td>7.6 mW</td>
<td>16.26 mW</td>
</tr>
<tr>
<td>Area</td>
<td>0.14477 mm$^2$</td>
<td>0.39 mm$^2$</td>
</tr>
<tr>
<td>Frequency Range</td>
<td>approx. 2.2 GHz – 2.8 GHz</td>
<td>-</td>
</tr>
</tbody>
</table>

*Including IF filter

Table 7.1 – Implemeted receiver results summary.

### 7.2. Future Work

In order to fully test and fabricate implemented receiver, a channel-select filter must be designed. This filter will require high selectivity and linearity. The selectivity is rather important in this case, since the previous block is the VCO-mixer, which does not have LO-IF port isolation. The LO’s signal acts as a strong interferer, not only in comparison with desired signal, but also influencing the strong signal analysis. Hence, the filter must support a large input signal range and filter the signal without altering the frequency of the oscillator. Finally, the IF filter should be differential to avoid even-order distortion.
Annexes
A1.  Annex 1

A1.1 Noise Mean-Square Value and Power Spectral Density

Usually noise is described by its mean-square value, which is interpreted as a power spectral density normalized to 1 Ω resistance and 1 Hz bandwidth. Actually, noise is a stochastic process that can be considered both stationary and ergodic in the mean. This means that, although being a random signal, its stochastic properties will remain unchanged in time, \((A1.3)\), and the time average of a sample function, \(x(t)\), of the process, \(X(t)\), approaches the ensemble-averaged mean as the observation interval tends to infinity, \((A1.4)\), respectively:

\[
\overline{x(t)} = E[x(t)] = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt = \frac{1}{2T} \int_{-T}^{T} x(t) dt \quad (A1.1)
\]

\[
< x(t) > = \frac{1}{2T} \int_{-T}^{T} x(t) dt \quad (A1.2)
\]

\[
E[< x(t) >] = \frac{1}{2T} \int_{-T}^{T} E[x(t)] dt = \overline{x(t)} \quad (A1.3)
\]

\[
\lim_{T \to \infty} < x(t) > = \overline{x(t)} \quad (A1.4)
\]

Keeping these properties in mind it is now possible to prove the initial hypothesis. Recalling Parseval’s Theorem and the Fourier Transform, \(X(j\omega)\) [32]:

\[
X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (A1.5)
\]

\[
x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df \quad (A1.6)
\]

\[
z.z^* = (a + ib)(a - ib) = a^2 + b^2 = |z|^2 \quad (A1.7)
\]

\[
\text{Energy} = \int_{-\infty}^{\infty} x(t)x^*(t)dt = \int_{-\infty}^{\infty} x(t) \left[ \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} d\omega \right] d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \left[ \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right] d\omega \to \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega \quad [1] \quad (A1.8)
\]

Although, to apply the Fourier Transform, it must be guaranteed that the function is absolutely integrable:

\[
\int_{-\infty}^{\infty} |x(t)| dt < \infty \quad (A1.9)
\]
To respect this condition, let \( x_T(t) \) be a truncated version of the sample function \( x(t) \) of the process \( X(t) \):

\[
x_T(t) = x(t), |t| \leq T \land x_T(t) = 0, \text{otherwise}
\]

\[
x(t) = \lim_{T \to \infty} x_T(t)
\]

\[
\frac{1}{2T} \int_{-\infty}^{\infty} |x_T(t)|^2 dt = \frac{1}{2T} \int_{-\infty}^{\infty} |X_T(f)|^2 df \leftrightarrow \mathbb{E} \left[ \frac{1}{2T} \int_{-\infty}^{\infty} |x_T(t)|^2 dt \right] = \mathbb{E} \left[ \frac{1}{2T} \int_{-\infty}^{\infty} |X_T(f)|^2 df \right] \leftrightarrow \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} |x(t)|^2 dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} |X_T(f)|^2 df \quad [W]
\]

If we also recall that the process is stationary, meaning its expected value is constant in time, and that the signal’s energy divided by its period represents Average Power:

\[
\because |x(t)|^2 = \int_{-\infty}^{\infty} \lim_{T \to \infty} \mathbb{E} \left[ |X_T(f)|^2 \right] df \quad [W] \rightarrow S(f) = \lim_{T \to \infty} \frac{1}{2T} \int_{-\infty}^{\infty} |X_T(f)|^2 df \quad [W/Hz]
\]

### A1.2 RC Phase-Shifter

![RC Phase-Shifter Circuit](image)

Figure A1.1 – Circuit that performs a 90º shift [11].

Applying Kirchhoff’s Current Law:

\[
\frac{V_{out1}}{R} = \frac{V_{in}}{R + j\omega C} \leftrightarrow \frac{V_{out1}}{V_{in}} = \frac{R}{R + j\omega C} = \frac{j\omega RC(1-j\omega RC)}{1 + \omega^2 R^2 C^2} \quad (A1.14)
\]

\[
V_{out2}(\omega C) = \frac{V_{in}}{R + j\omega C} \leftrightarrow \frac{V_{out2}}{V_{in}} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{R^2\omega^2 C^2 + 1} \quad (A1.15)
\]

We conclude that the difference between the two signals is 90º. In terms of amplitude for resonant frequency, \( \omega = 1/RC \),

\[
\frac{|V_{out1}|}{V_{in}} = \frac{\omega RC}{R^2\omega^2 C^2 + 1} \sqrt{1 + \omega^2 R^2 C^2} \quad (A1.16)
\]

\[
\frac{|V_{out2}|}{V_{in}} = \frac{1}{R^2\omega^2 C^2 + 1} \sqrt{1 + R^2 C^2 \omega^2} \quad (A1.17)
\]
Now considering the phase:

\[
\arctg\left(\frac{1}{x}\right) = \frac{\pi}{2} - \arctg(x), \text{ if } x > 0 \tag{A.18}
\]

\[
\arg\left(\frac{V_{\text{out}1}}{V_{\text{in}}}\right) = \arctg\left(\frac{\omega RC}{1 + R_2 C_2}\right) = \arctg\left(\frac{1}{\omega RC}\right) = \frac{\pi}{2} - \arctg(\omega RC) \tag{A.19}
\]

\[
\arg\left(\frac{V_{\text{out}2}}{V_{\text{in}}}\right) = \arctg\left(\frac{-R\omega C}{1 + \omega^2 R_2 C_2}\right) = -\arctg(R\omega C) \tag{A.20}
\]

If \(\omega \neq 1/RC\) then the amplitudes are not equal and the quadrature relation is not precise.
A2. Annex 2

A2.1 MOS

For MOS transistors, considering the long channel approximation:

\[ k = \frac{\mu}{2L} \left[ AV^{-2} \right] \] (A2.1)

1) Cut-off:
\[ V_{GS} < V_t \Rightarrow I_D \equiv 0 \] (A2.2)

2) Triode:
\[ V_{GS} > V_t \land V_{DS} < V_{GS} - V_t \Rightarrow I_D = k(2(V_{GS} - V_t)V_{DS} - V_{DS}^2) \] (A2.3)

3) Saturation:
\[ V_{GS} > V_t \land V_{DS} > V_{GS} - V_t \Rightarrow I_D = k(V_{GS} - V_t)^2(1 + \lambda V_{DS}) \] (A2.4)

Notice that \( \mu \) represents the carriers’ mobility in \([m^2/V\cdot s]\) and \( C_{ox} \) the oxide capacitance in \([F/m^2]\). As for small signals analysis in saturation:

\[ i_D = I_D + i_{id} \equiv k(V_{GS} + v_{gs} - V_t)^2 = k(V_{GS} - V_t)^2 + 2k(V_{GS} - V_t)v_{gs} + kv_{gs}^2 \equiv I_D + g_m v_{gs} \] (A2.5)

Notice that when a small term is squared it becomes smaller, being the case of the last term of (A2.5).

\[ g_m = \frac{\partial I_D}{\partial V_{GS}} \bigg|_{V_{GS} = V_t} = 2k(V_{GS} - V_t) \left[S^{-1}\right] \] (A2.6)

\[ r_0^{-1} = \frac{\partial I_D}{\partial V_{DS}} \bigg|_{V_{GS} = V_t, V_{DS} = V_{DS}} = \frac{I_D}{V_A + V_{DS}} \] (A2.7)

Notice that in this analysis parasitic capacitances were ignored and so was body effect.
A2.2 Quadrature RC Relaxation VCO Results

Figure A2.3 – Output voltage waveform at 2.4 GHz.

Figure A2.4 – Phase Noise.

Figure A2.5 – Frequency range, phase noise and output voltage flatness as function of the tuning voltage.
A3. Annex 3

A3.1 Differential-Pair Currents

Assuming MOS are in saturation,

\[ I_{M_1}(t) = K(V_{GS_1}(t) - V_t)^2 \text{ [A]} \land I_{M_2}(t) = K(V_{GS_2}(t) - V_t)^2 \text{ [A]} \]  \hspace{1cm} (A3.1)

\[
\begin{align*}
V_D(t) &= V_{G_1}(t) - V_{G_2}(t) = (V_{GS_1}(t) - V_t) - (V_{GS_2}(t) - V_t) = \sqrt{\frac{I_{M_1}(t)}{K}} - \sqrt{\frac{I_{M_2}(t)}{K}} \text{ [V]} \\
\sqrt{\frac{I_{M_2}(t)}{K}} &= \sqrt{\frac{I_{M_1}(t)}{K}} - V_D(t) \leftrightarrow I_{M_2}(t) = I_{M_1}(t) + KV_D(t)^2 - 2K\sqrt{\frac{I_{M_1}(t)}{K}}V_D(t) \text{ [V]} \\
\end{align*}
\]  \hspace{1cm} (A3.2)

\[
I_{M_1}(t) + I_{M_2}(t) = I \text{ [A]} 
\]  \hspace{1cm} (A3.3)

\[
I_{M_1}(t) + I_{M_1}(t) + KV_D(t)^2 - 2K\sqrt{\frac{I_{M_1}(t)}{K}}V_D(t) = I \leftrightarrow I_{M_1}(t) - V_D(t)\sqrt{\frac{KI_{M_1}(t)}{2}} - \frac{1}{2} + \frac{K}{2}V_D^2(t) = 0 \text{ [A]} 
\]  \hspace{1cm} (A3.4)

\[
\sqrt{I_{M_1}(t)} = \frac{V_D(t)\sqrt{2(V_D(t))^2 - 2K(V^2_D(t) - V_D)^2}}{2} 
\]  \hspace{1cm} (A3.5)

Choosing the real solutions:

\[
\therefore I_{M_1}(t) = \frac{V_D^2(t)K}{4} + \frac{1}{2} \left( \frac{V_D^2(t)K}{4} + 2V_D(t)\sqrt{\frac{KI}{2}} - \frac{K^2V_D^2(t)}{2} \right) = \frac{1}{2} + V_D(t)\sqrt{\frac{KI}{2}} - \frac{K^2V_D^2(t)}{4} \text{ [A]} 
\]  \hspace{1cm} (A3.6)

\[
\therefore I_{M_2}(t) = \frac{1}{2} - V_D(t)\sqrt{\frac{KI}{2}} - \frac{K^2V_D^2(t)}{4} \text{ [A]} 
\]  \hspace{1cm} (A3.7)
A3.2 Fourier Series

A periodic signal respecting Dirichlet conditions may be described by its Fourier Series. This implies having a finite number of discontinuities, limited number of maximums and minimums and being absolutely integrable, (A1.9). [16]

\[ s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t) \quad (A3.9) \]

\[ a_n = \frac{2}{T_0} \int_{T_0/2}^{T_0} s(t) \cos(n\omega_0 t) \, dt \quad (A3.10) \]

\[ b_n = \frac{2}{T_0} \int_{T_0/2}^{T_0} s(t) \sin(n\omega_0 t) \, dt \quad (A3.11) \]

For example,

\[ \text{sign}[\cos(\omega_0 t)] = \frac{4}{\pi} \cos(\omega_0 t) - \frac{4}{3\pi} \cos(3\omega_0 t) + (\ldots) \quad (A3.12) \]

Notice that this function is even, so the terms \( b_n \) should be null.
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