Analysis and Simulation of Dispersion Compensation in Fiber Optic Systems using DCF and CFBG

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Abstract—The propagation of pulses in a single-mode optical fiber with dispersion, and the compensation through DCF and CFBG is analysed. Is also analysed the CFBG apodized and non-apodized using the program OptiGrating. With the program OptiSystem, is simulated the dispersion compensating of a link of optical fiber communication system using DCF and CFBG with different apodizations.

Keywords: Dispersion Compensating Fiber (DCF), Chirped Fiber Bragg Grating (CFBG), OptiGrating, OptiSystem, Dispersion Compensating, Apodization.

I. INTRODUCTION

In a communications system the transmitter sends a message to the receiver through a communication medium. In fiber optic communication is transmitted pulses of light through an optical fiber, where the light forms an electromagnetic carrier wave that is modulated to transport information. This way the fiber optic is the medium, and the light pulses the message. The three principal components of a fiber optic communication system are: the optical transmitter, the fiber optic and the optical receiver.

In systems of fiber optic communications, normally is made an evaluation of quality through the BER (Bit Error Rate). This parameter is the ratio between the number of bits with errors and the number of the total bits transmitted during a space of time studied.

Before the appearing of fiber optic, the communication systems transmitted information through electrical wires or used radio frequencies and microwaves through open space. However, it would be a more natural choice the use of light for communications, because, unlike electricity and radio waves, this one didn’t need to be discovered. However there was two main reasons to not choose the light in the beginning, which were the difficulty of producing a light source that could be on and off very quickly, in a way that could code information in high bit rate, and the fact that the light is obstructed by opaque objects, clouds, fog and smoke. Unlike radio waves and microwaves, light is not ideal for communications in open space.

This way, the beginning of the modern era of fiber optic communication begun in 16 of May of 1960, when Theodore Maiman made the first demonstration of a laser operation. However only in 1970, that Maurer, Donald Keck and Peter Schultz, working for Corning GlassWorks, produce a single mode fiber optic with an attenuation of 16 dB/km in the wavelength of 633 nm, making the fiber optic communication possible [1]. Since then the fiber optic system have evolved passing through different generations.

The first generation of fiber optic system appeared in the end of the 70s. The optical source were multimode lasers and LED. The fiber used in this communications was multimode fiber and operate on the first window (0.8 μm) with a bit rate of 45 Mb/s. However was necessary an Optical-Electrical-Optical (OEO) repeater in every 10 km [1].

The second generation appeared in the middle of the 80s. This system operated on the second window (1310 nm) and used lasers MLM (Multi-Longitudinal Mode) with a wide spectral width. Also was used single mode fiber, which allowed a bit rate of 1.7 Gb/s and OEO repeater in every 50 km [2].

The third generation was commercially available a functioning on a bit rate of 2.5 Gb/s in 1990 [2].

The fourth generation was commercially available in 1996 [3]. In this generation were used OEO repeater and Erbium Doped Fiber Amplifiers (EDFA). This generation also used Wavelength-Division Multiplexing (WDM) to increase the bit rate. In this generation the bit rate was 2.5 to 40 Gb/s for channel and the number of channel was 4 to 160 [4].

The fifth generation was commercially available in 2007 [3]. In this generation we have bit rate of 40 Gb/s for channel and a number a 250 channels (DWDM: Dense Wavelength-Division Multiplexing).

The future generations will have an increase of spectral efficiency (bit/s/Hz) to have a better use of the bandwidth of the optical fiber. These generations will use DQPSK (Differential Quadrature Phase Shift Keying), QPSK (Quadrature Phase Shift Keying) coherent with multiplexing of polarization and OFDM (Orthogonal Frequency Division Multiplexing) to increase the spectral efficiency. In these generations the bit rate for channel will be 100 Gb/s.

II. PULSE PROPAGATION IN A SINGLE MODE OPTICAL FIBER IN LINEAR REGIME

In single mode fiber there are losses (α) that reduce the transmitted pulse potency, which leads to an increase of the BER and the consequent limitation of the link length. However the dispersion is another phenomenon that affects the propagation of the pulse, which leads to an intersymbol interference (ISI) and limits the bit rate of the transmission.
A. Temporal Dispersion

In the single mode regime there is the group velocity dispersion (GVD) and the third order dispersion (TOD) [1].

The GVD on fiber optics is due to the fact of different spectral components, of the signal transmitted, propagating with different velocities, because of the variation of the refractive index with the frequency [5]. This dispersion interferes with the pulses in a fiber optic, in linear regime, with a temporal broadening, which provokes ISI and conditions the bit rate of the transmission.

Considering the fact that the frequency dependence causes a time delay, one has:

\[
D(\lambda) = \frac{1}{L} \frac{\partial \tau_g}{\partial \lambda} \tag{1}
\]

where \(D\) is the dispersion coefficient, \(L\) is the length of the connection, and \(\tau_g\) is the group delay given by:

\[
\tau_g = \frac{L}{v_g} \tag{2}
\]

where \(v_g\) represents the group velocity.

The GVD has two contributions, the material dispersion \(D_M\) and the waveguide dispersion \(D_W\) [2].

The \(D_M\) happens because the refractive index of silica, material that is used on the fabrication of fiber, changing with the optical frequency. The origin of \(D_M\) is related to the characteristic resonance frequencies for which the material absorbs the electromagnetic radiation [2]. \(D_M\) is given by:

\[
D_M = M_2 = \frac{1}{c} \frac{\partial N_2}{\partial \lambda} \tag{3}
\]

The \(D_W\) happens when the energy propagates through the cladding instead of being confined only on the core. \(D_M\) is given by:

\[
D_W = M_2 = \frac{\Delta d(vb)}{d\nu} - \frac{N_2^2 \Delta}{n_2 \lambda c} \left[ \nu \frac{d^2(vb)}{d\nu^2} \right] \tag{4}
\]

where \(c\) is the light velocity, \(N_2\) is the group index of the cladding, \(\nu\) is the normalized frequency and \(b\) is the normalized propagation constant.

In the Figure 1 is represented the material dispersion \(D_M\), the waveguide dispersion \(D_W\) and their sum \(D = D_M + D_W\) as a function of wavelength for a typical single mode optical fiber. In this figure is noticeable that \(D\) is zero for a wavelength very near to 1.31 \(\mu m\), which corresponds to the second window. This wavelength is called zero dispersion wavelength \(\lambda_{ZD}\).

However the dispersive effects don’t disappear completely for \(\lambda = \lambda_{ZD}\). The optical pulses still suffer from a broadening due to third order dispersion (TOD). This way, when the carrier is near the wavelength where \(D = 0\) or/and the signal is ultra short, is necessary to consider the TOD [2], which is governed by the dispersion slope \(S = \frac{\partial D}{\partial \lambda}\), and is equal to:

\[
S = \frac{4\pi c}{\lambda^3} \beta_2 + \left( \frac{2\pi c^2}{\lambda^2} \right)^2 \beta_3 = \frac{S_D}{4} \left[ 1 + 3 \left( \frac{\lambda_{ZD}}{\lambda} \right)^4 \right] \tag{5}
\]

where \(\beta_2 = \frac{\partial^2 \beta}{\partial \omega^2}\) is the GVD parameter responsible for the broadening of the pulse, and \(\beta_3 = \frac{\partial \beta_2}{\partial \omega}\) is the TOD parameter.

B. Pulse Propagation Equation in Linear Regime

The pulse propagation equation in linear regime is deduced in order to determine the shape of the pulse at the end of the link.

Considering the pulse \(A(0, t)\) at the beginning of the fiber optic \((z = 0)\) and that this pulse modulates a carrier of angular frequency \(\omega_0\), the field equations are:

\[
E(x, y, 0, t) = E_0 F(x, y) B(0, t) \tag{6}
\]

\[
B(0, t) = A(0, t) \exp(-i\omega_0 t) = 0 \tag{7}
\]

Because the regime is single mode the function \(F(x, y)\) represents the transversal variation of the fundamental mode LP01 and \(B(z, t)\) corresponds to the longitudinal variation of the electric field through the fiber.

As demonstrated in [1], the equation is:

\[
\frac{\partial A}{\partial z} + \frac{\partial A}{\partial t} + i \frac{\beta_1}{2} \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_2 \frac{\partial^3 A}{\partial t^3} = 0 \tag{8}
\]
C. Numerical Simulation of the Pulse Propagation in linear regime

Now is presented the numerical resolution of Eq. 8 through FFT (fast Fourier transform) and IFFT (inverse fast Fourier transform).

Defining the dispersion length \( L_D \) by:

\[
L_D = \frac{\tau_0^2}{|\beta_2|}
\]  
(9)

and the dimensionless distance variable:

\[
\zeta = \frac{z}{L_D}
\]  
(10)

The Eq. 8 can be written by [1]:

\[
\frac{\partial A}{\partial \zeta} + i \frac{1}{2} \text{sgn}(\beta_2) \frac{\partial^2 A}{\partial \tau^2} - k \frac{\partial^3 A}{\partial \tau^3} = 0
\]  
(11)

where \( k \) is the coefficient of the dispersion of third order and is given by [1]:

\[
k = \frac{\beta_3}{6|\beta_2| \tau_0}
\]  
(12)

where \( \tau_0 \) is the pulse width.

Thereby the numerical resolution of Eq. 9 has the following steps [1]:

- To the impulse \( A(0, \tau) \) apply FFT, getting \( \tilde{A}(0, \xi) \).
- To \( \tilde{A}(0, \xi) \) multiply\( \exp \left\{ i \frac{1}{2} \text{sgn}(\beta_2) \xi^2 + k \xi^3 \right\} \)

getting \( \tilde{A}(\zeta, \xi) \).
- Applying IFFT to \( \tilde{A}(\zeta, \xi) \) we get \( A(\zeta, \tau) \).

D. Gaussian Pulse Propagation in Linear Regime

Considering the Gaussian pulse and \( \beta_3 = 0 \) [2]:

\[
A(0, t) = A_0 \exp \left[ - \frac{1 + i C}{2} \left( \frac{t}{\tau_0} \right)^2 \right]
\]  
(13)

where \( C \) represents the chirp parameter.

In the Figure 2 to 4 is visible the reduction of the amplitude of the pulse through the fiber. This phenomenon happens due to the broadening of the pulse caused by GVD. It is also noticeable that when the chirp parameter is less than zero, the amplitude of the Gaussian pulse decreases more rapidly and for a lower value than \( C=0 \). In the case of \( C > 0 \), the chirp compensates the GVD in the beginning of the fiber, however, through the fiber, this compensation disappears reaching this pulse, on the end of the fiber, a lower value in amplitude than \( C=0 \).

E. Pulse Broadening

The pulse broadening happens due to GVD and TOD [1]. The effective width of the pulse is:

\[
\sigma(z) = \sqrt{\langle t^2 \rangle - \langle t \rangle^2}
\]  
(14)
Being $V$ the normalized spectral width, considering $V \ll 1$, and ignoring the TOD for a Gaussian pulse the following equation represents the broadening factor:

$$\frac{\sigma^2}{\sigma_0^2} = \left(1 + C \frac{\beta_2 L}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_3 L}{2\sigma_0^2}\right)^2$$ \tag{15}$$

In Figure 5 is noticeable that the lower the $|C|$, the signal will have less dispersion at the end of the fiber. On the other hand, for values of $C<0$ cause more dispersion than values of $C>0$ with the same module.

**F. Figure of Merit**

Given two links, the Figure of Merit can say which one is better, this way the higher the Figure of Merit the better the link.

Admitting a broadening coefficient of:

$$\eta = \frac{\sigma}{\sigma_0}$$ \tag{16}$$

corresponding a bit rate $B$. The product $B^2 L$ corresponds to the Figure of Merit and is dimensionless. In the Figure 6 is represented the influence of the chirp in the product $B^2 L$ for $\beta_2 < 0$ and $\beta_2 > 0$. It is noticeable that the curves for $\beta_2 < 0$ and $\beta_2 > 0$ are symmetric, and when the $|C|$ is lower the Figure of Merit is bigger, which leads to a better connection.

**G. Effects of the Third Order Dispersion**

The effects of TOD are relevant if the wavelength is equal to the zero dispersion wavelength ($\lambda_D$) and $\beta_2 \approx 0$. For ultra short pulses is also necessary to include the TOD effects [6]. Considering:

$$L_D' = \frac{\tau_0^3}{|\beta_3|}$$ \tag{17}$$

In Figure 7 there are three Gaussian impulses, one for $z=0$ (initial impulse), other for $\beta_2 = 0$ ps$^2$/km and an impulse for the value of $\beta_2$ satisfies the condition $L_D' = L_D$. In this Figure were considered the following values: link distance $z = 5L_D'$, chirp equals to zero, $\beta_3 = 0.09$ ps$^3$/km and $\tau_0 = 1$ ps.

For $\beta_2 = 0$ ps$^2$/km is possible to observe the strong oscillations. This fact happens due to the fact of TOD distorting the pulse in such way that it has an asymmetrical oscillatory shape. However these oscillations could appear on the front or the back of the impulse, depending if the $\beta_3$ is positive or negative, respectively.

For $L_D = L_D'$ ($\beta_2 = \beta_3/\tau_0$), it happens the same phenomenon described on the previous paragraph for $\beta_3$ positive and negative. However in this case the oscillations are lower. For values of $\beta_2$ bigger, in such a way that $L_D \ll L_D'$, the pulse becomes almost Gaussian meanwhile the TOD has less and less influence.
III. DISPERSION COMPENSATING FIBER

The optical amplifiers resolve the problem of losses in fiber optics, however the problem of dispersion get worse, due to the fact of they don’t restore the signal amplified. This way the signal transmitted accumulates dispersion along the multiples optical amplifiers [7].

The normal DCF has a negative dispersion high in module, this way they have a negative chromatic dispersion much bigger than the positive chromatic dispersion of the transmission fiber. DCF also have a smaller length because, normally they have more attenuation than the transmission fibers. DCF also have a smaller effective area, which made them more vulnerable to non linear effects, this way they are installed at the end of the transmission fiber, where the lower potency reduce the non linear effects [8].

In order to have a perfect GVD compensation the next condition must be satisfied:

\[ \beta_{21}L_1 + \beta_{22}L_2 = 0 \]  

where \( \beta_{21} \) and \( \beta_{22} \) is the GVD parameter for the transmission fiber and DCF, respectively, and \( L_1 \) and \( L_2 \) is the length of the transmission fiber and DCF, respectively.

In order to have a perfect TOD compensation the next condition must be satisfied:

\[ \beta_{31}L_1 + \beta_{32}L_2 = 0 \]  

where \( \beta_{31} \) and \( \beta_{32} \) is the TOD parameter for the transmission fiber and DCF, respectively, and \( L_1 \) and \( L_2 \) is the length of the transmission fiber and DCF, respectively.

However the perfect compensation of both GVD and TOD is very difficult.

In Figure 8 to 10 is presented the compensation of the GVD parameter, TOD parameter and the GVD and TOD through a DCF.

Analyzing the Figure 8 and 9 is visible that the DCF totally compensate the GVD and TOD effect, respectively. However, when the signal has both (GVD and TOD), the DCF can’t compensate the broadening of the signal, so this one has the original shape.

IV. DISPERSION COMPENSATION WITH FIBER BRAGG GRATING

Despite the improvements in the fabrication of fiber optics and the advances in the area, the basic components such as mirror, filters and reflectors were difficult to integrate on fiber optics. However, everything changed with the capacity of changing the refractive index of the core through the UV light absorption. This photosensibility of fiber optics allows the possibility of creating phase structures on the core of the fiber. These phase structures are obtained changing constantly the refractive index according to a periodic pattern along the core of the fiber. A periodic modulation of the refractive index of the core of the fiber makes it behaves like a selective mirror with a wavelength that satisfies the Bragg condition [9]. Thus they form the Fiber Bragg Grating, that is a periodic refractive index perturbation along the fiber which is formed by an exposition of the core to an intense optical interference [10]. The FBGs have many applications due to their proprieties, versatility and the variety of controlled parameters, such as filters and demultiplexers. However in this paper the application studied for FBG is the dispersion compensation [11].
Between the different dispersion compensators used actually, the Chirped Fiber Bragg Gratings (CFBG), are one of the most important. Since their first use in dispersion compensation, suggested by Ouellette in 1987, they have been widely used in dispersion compensation. This type of Bragg can be designed to compensate a wide range of wavelengths, or only one channel. The CFBG have a non uniform period along their length.

A. Apodization of Fiber Bragg Gratings

The FBG doesn’t have an infinite length, in other words, they have a beginning and an end. This way these Gratings begin and end abruptly, which makes that exist side lobes on the reflection spectrum. However, if the refractive index of amplitude modulation, in the beginning and end of the FBG, begins and ends gradually is possible to reduce the side lobes.

B. Simulation of Fiber Bragg Gratings using OptiGrating

The OptiGrating uses the Coupled Mode Theory to model the light and enable analysis and synthesis of gratings. This is a powerful tool of analysis of coupling and reflexion along the fiber. OptiGrating also has specialized modules to simulate physical conditions, such as temperature and deformations on the FBG.

Simulating a CFBG with no apodization, to obtain the potency spectrum of a CFBG with no apodization (Figure 11), is visible the oscillations of the potency spectrum of transmission represented by the colour red, and the potency spectrum of reflexion represented by the colour blue. However, these oscillations could be removed if the CFBG is apodized (as shown in Figure 12). The Figure 12 shows a potency spectrum of a CFBG with Gaussian apodization where the potency spectrum of transmission represented by the colour red, and the potency spectrum of reflexion represented by the colour blue.

Is also interesting the comparison between Figure 13 and 14. In both figures is visible the reflexions of the different wavelengths along the CFBG. The shorter wavelength has a reflexion on the end of the CFBG and the longer have the reflexion on the beginning. This fact makes the CFBG a dispersion compensator. However the Figure 13 and 14 differs from the fact of Figure 14 being apodized and Figure 13 not. This way in the apodized CFBG the index increases slowly at the beginning and decreases slowly at the end of the CFBG, which gives that form for Figure 13.

The influence of apodization is also noticed comparing the Figure 15 and 16. Both represent a delay spectrum, where the red is representing the different delays of the wavelengths on transmission, and blue is representing the different delays of the wavelengths on reflexion. Once again is visible the
suppression of oscillations by the apodization. However, analysing only the Figure 16 is visible a bigger delay for the shorter wavelengths, and a smaller delay for the larger wavelengths. This makes that the group delay is compensated and the CFBG compensates dispersion.

![Figure 15 – Delay spectrum of a CFBG with no apodization](image)

V. ANALYSIS THE DISPERSION COMPENSATION OF A LINK OF OPTICAL FIBER COMMUNICATION SYSTEM USING OPTISYSTEM

OptiSystem is an innovator simulator of optical communications systems, which projects, tests and optimizes virtually any optical fiber link. This program also can be used with the program OptiGrating.

In all the next simulations used the following parameters:
- Bit rate: 10 Gb/s.
- Extintion ratio of MachZehnder modulator: 30 dB.
- SMF attenuation: 0.2 dB/km.
- SMF dispersion: 18 ps/nm/km.

The parameters of the fiber SMF were obtained through the Corning SMF-28e+ datasheet [12].

The eye diagrams amplitude is in arbitrary units, because this value is only to compare simulation in the same environment.

The comparison values for the distinct simulation used was the BER and the Q factor.

![Figure 16 – Delay spectrum of a CFBG apodized](image)

A. Analysis of a 10 Gbit/s link without dispersion compensation

The analysed project is represented in Figure 17.

In this analysis was considered a BER inferior to $10^{-11}$ as acceptable, a potency of CW laser of 1 dBm and a pump power of EDFA of 20 dB. For this analysis it was proven that the link needed dispersion compensation for a distance of 35 km, considering BER < $10^{-11}$. In this simulation is important to compare the Figure 18 and 19. Despite of the great eye diagram obtained in the Figure 18, the link is not well projected, because it spend more resources than it should be.

![Figure 17 – Schematic of the project on OptiSystem of a communication system without dispersion compensation](image)

![Figure 18 – Eye diagram for a link of 10 km without dispersion compensation](image)
B. Analysis of a 10 Gbit/s link with dispersion compensation using CFBG

The analysed project is represented in Figure 20.

In this analysis was considered a BER inferior to $10^{-11}$ as acceptable, a potency of CW laser of 1 dBm and a pump power of EDFA of 20 dB. The CFBG is linear with a linearity parameter of 0.0001 µm.

In this analysis was made simulations with three types of apodization: Uniform, Tan Hiperbolic and Gaussian. In these simulations it was also varied the length of the CFBG. The results obtained in [13] showed that the Gaussian apodization had the worst results of the three. On the other hand the choice between the Uniform and Tan Hiperbolic apodization depended on the link that was being projected. Because in some links the Uniform apodization got better results in others Tan Hiperbolic had better results.

It is presented in Figure 21 a good project link, where $BER = 6.86 \times 10^{-12}$ which is no much smaller than the maximum $10^{-11}$. This way the resources spent are just enough to assure the requirements need for the correct function of the communication system.

C. Analysis of a 10 Gbit/s link with dispersion compensation using DCF

The analysed project is represented in Figure 22.

In this analysis was considered a BER inferior to $10^{-11}$ as acceptable, a potency of CW laser of 1 dBm and a pump power of EDFA of 20 dB.

Has mentioned in part III the Equation 20 must be satisfied in order to completely compensate the dispersion on an optical link.

$$D_1 L_1 + D_2 L_2 = 0 \quad (20)$$

where $D_1$ and $D_2$ are the dispersion of the SMF and DCF, respectively, and $L_1$ and $L_2$ are the length of the SMF and DCF, respectively.

However OptiSystem simulates a real system so the dispersion could not be completely compensated.

However it always tried to get the minimum length of the
DCF. Analysing the results in [13], the DCF can compensate all the connections that CFGB with different apodizations did. Although for the link of 10 km the DCF needs to increase the CW laser potency from 1 dBm to 3 dBm.

VI. CONCLUSION

In this paper it was made an analysis for the material dispersion and waveguide dispersion and was conclude that the total dispersion is zero when the wavelength is near 1.31 $\mu$m. This wavelength is called zero dispersion wavelength. It was also conclude that the smaller $|C|$ less dispersion the signal will have at the end of the fiber.

In the analysis of the dispersion compensation using DCF was conclude that when is ignored the GVD or the TOD, the original signal could be recovered. Although, when the both are considered (GVD and TOD) the pulse can not be completely compensated.

In the analysis of the FBG for dispersion compensation was conclude that CFGB is the best solution. The benefits of apodization were stated and shown. Was also shown, with the help of the program OptiGrating the properties of CFGB that make them a dispersion compensator.

At last was simulated the dispersion compensation of fiber optic communication system using DCF and CFGB with different apodizations through OptiSystem. In this analysis was conclude that the Gaussian apodization had the worst results. Although the choice between Uniform apodization and Tan Hiperbolic apodization depends on the link that is being analyzed.

This paper presented two technics for dispersion compensation, and by the results of V the DCF need more potency to achieve the same results as CFGB.

Having all this in count and also that DCF has more insertion losses, non linear effects and much bigger size than CFGB is conclude that the choice for CFGB is more attractive.

However the choice of the apodization of the CFGB depends on the link that is being projected.

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