Modelling of Variable-Speed Induction Machines

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Abstract — Recently, the pumping systems in dams have been using induction machines with variable speed to adjust the consumption to the operating conditions of the grid. Concerning this, it is important to develop computational models in order to perform transient regime analyses. To help in start-up process, double-cage induction machines are used. These allow a larger start-up torque and a reduction of start-up currents. For this purpose a double-cage induction machine model is developed. To allow the speed control of the machine, the model of the double-cage induction machine is approximated by a model of a single-cage induction machine. After the machine model is achieved, it is essential to connect it to the grid. This connection is performed using two converters, a rectifier, and an inverter, as well as their respective controllers. The rectifier and its controller are used in order to obtain an adequate voltage on the DC side, to keep the machine working without problems. Furthermore, the inverter and its control are also required in order to change the speed of the machine.

Keywords — Pumping systems, Double-Cage Induction machines, Single-Cage Induction machines, Inverter, Rectifier, Controller.

I. INTRODUCTION

Over the last years, the consumption of energy has been growing up leading to a necessary adaptation of the electric system. Until a recent past, consumer’s energy needs were satisfied using conventional energy sources such as, oil, coal or natural gas.

Due to recent concerns about global warming, it was necessary to find new strategies. One of the most important paths was the massive introduction of renewable energy sources in energy production. At the moment, the mostly used renewable energy is the wind, through wind turbine generators. Those use the wind as the energy source. However, the wind has the problem of being intermittent. Typically, wind magnitude is higher at night than during the day. On the contrary, the consumption of energy is higher during the day than during the night. With the massification of wind turbines we tend to have overproduction and since energy storage is too expensive to be an answer, this overproduction has to be necessarily spent.

At this moment the only economically acceptable way to store energy is using dams. The idea behind energy storage in dams is to recover the water already turbinated and storing it back in the dam reservoir. To accomplish that we need an electric pump.

In order to adjust the pump to the operational conditions of the grid we need to adjust the quantity of water that it pumps back in the dam reservoir. For this purpose variable-speed induction machines are used.

It becomes necessary to create models for these systems to use in programs of transient regime analysis of the grid in order to study the impact of these systems in the operation conditions of the grid.

In this paper, a model of a double-cage induction machine is developed and then the machine is connected to grid through a power electronic system.

II. MODEL OF A DOUBLE-CAGE INDUCTION MACHINE

A double-cage induction machine was used because it has a higher start-up torque and the currents during that period are lower than in a single-cage induction machine. This prevents possible damages to the machine.

According [1] the electrical equations in the complex form for the stator, inner and outer cage windings are respectively:

\[ \begin{align*}
    V_s &= i_s R_s + j \omega_s \varphi_s + \frac{d \varphi_s}{dt} \\
    0 &= i_{r1} R_{s1} + j (\omega_s - \omega_r) \varphi_{r1} + \frac{d \varphi_{r1}}{dt} \\
    0 &= i_{r2} R_{s2} + j (\omega_s - \omega_r) \varphi_{r2} + \frac{d \varphi_{r2}}{dt}
\end{align*} \tag{1} \]

The equivalent circuit of the double-cage induction machine can be given by:

![Fig. 1- Equivalent circuit of the double-cage induction machine](image-url)
For analysis of the circuit of Fig. 1, knowing that $i = L^{-1} \varphi$ and taking into account (1) it can be said that the differential equations that modulate the flux linkages are:

\[
\begin{align*}
\frac{d\varphi_s}{dt} &= -\frac{R_1 l_{11}}{l} \varphi_s + \frac{R_1 l_{1m} l_{22}}{l} \varphi_{r1} + \frac{R_1 l_{m1} l_{1}}{l} \varphi_{r2} \\
\frac{d\varphi_{r1}}{dt} &= \frac{R_1 l_{1m} l_{22}}{l} \varphi_s - \frac{R_1 l_{12}}{l} \varphi_{r1} + \frac{R_1 l_{m2} L_s}{l} \varphi_{r2} \\
\frac{d\varphi_{r2}}{dt} &= \frac{R_2 l_{1m} l_{1}}{l} \varphi_s + \frac{R_2 l_{m2} L_s}{l} \varphi_{r1} - \frac{R_3 l_{33}}{l} \varphi_{r2} \\
\end{align*}
\]  

(2)

The equation that describes the mechanical motion in per-unit is:

\[
\frac{ds}{dt} = -\frac{\omega_s}{2H} (T_e - T_i)
\]

with

\[
T_e = l m (\varphi_s^* \varphi_{r1} + \frac{L_m l_1}{l} \varphi_s \varphi_{r2}^*)
\]

(3)

The equation (4) was rewritten as:

\[
T_i = C_i \omega_s^2
\]

(4)

For computational simulation, the complex variables of (2) and (6) are solved in their real $dq$ components. Defining that:

\[
f = f_d - f_q
\]

(7)

Solving (2) and (3) taking into account (7), results:

\[
\frac{d\varphi_{sd}}{dt} = -\frac{R_1 l_{11}}{l} \varphi_{sd} - \omega_s \varphi_{sq} + \frac{R_1 l_{1m} l_{22}}{l} \varphi_{r1d} + \frac{R_1 l_{m1} l_{1}}{l} \varphi_{r2d} + V_{sd}
\]

(8)

\[
\frac{d\varphi_{sq}}{dt} = -\frac{R_1 l_{11}}{l} \varphi_{sq} + \omega_s \varphi_{sd} + \frac{R_1 l_{1m} l_{22}}{l} \varphi_{r1q} + \frac{R_1 l_{m1} l_{1}}{l} \varphi_{r2q} + V_{sq}
\]

\[
\frac{d\varphi_{r1d}}{dt} = \frac{R_1 l_{1m} l_{22}}{l} \varphi_{sd} - \frac{l_{22} R_1}{l} \varphi_{r1d} - s \omega_s \varphi_{r1q}
\]

(9)

\[
\frac{d\varphi_{r1q}}{dt} = \frac{R_1 l_{1m} l_{22}}{l} \varphi_{sd} + \frac{R_2 l_{1m} l_{22}}{l} \varphi_{r1q} + \frac{R_1 l_{m1} l_{1}}{l} \varphi_{r2q}
\]

(10)

\[
\frac{d\varphi_{r2d}}{dt} = -\frac{R_2 l_{1m} l_{1}}{l} \varphi_{sq} - \frac{R_2 l_{m2} L_s}{l} \varphi_{r1q} - \frac{l_{33} R_2}{l} \varphi_{r2d} + s \omega_s \varphi_{r2q}
\]

(11)

\[
\frac{d\varphi_{r2q}}{dt} = \frac{R_2 l_{1m} l_{1}}{l} \varphi_{sq} + \frac{R_2 l_{m2} L_s}{l} \varphi_{r1q} - \frac{l_{33} R_2}{l} \varphi_{r2q} - s \omega_s \varphi_{r2d}
\]

\[
\frac{ds}{dt} = -\frac{\omega_s}{2H} \left[ \frac{L_2 l_m}{l} \varphi_{sd} \varphi_{r1q} - \frac{L_2 l_m}{l} \varphi_{sq} \varphi_{r1d} + \frac{L_1 l_m}{l} \varphi_{sd} \varphi_{r2q} - \frac{L_1 l_m}{l} \varphi_{sq} \varphi_{r2d} - T_i \right]
\]

(12)

Subsequently, a seven-order model was obtained. This means that there are seven differential equations and seven state variables, which in computational terms is very heavy to compute.

It becomes necessary to reduce the order of the model without losing the accuracy. On this type of machines the outer cage has higher resistance and lower inductance than the inner cage, in order to get a high start-up torque. This makes the transient associated to the outer cage being much faster than the inner cage transient. To neglect this transient $d \varphi_{r2}/dt$ in (2) is equal to 0.

Assuming $d \varphi_{r2}/dt = 0$ in (2) and solving the last equation in order to $\varphi_{r2}$ one obtains:

\[
\varphi_{r2} = \frac{l_{33} (1 - j s') \varphi_{s} + l_{33} (1 - j s') \varphi_{r1}}{1 + s'^2}
\]

(13)

with

\[
s' = \frac{\omega_s l_{1}}{l_{33} R_2}
\]

(14)

Replacing (9) into the first two equations of (2) and (3) and taking into account (7) we obtain:

\[
\begin{align*}
\frac{d\varphi_{sd}}{dt} &= R_{11} \varphi_{sd} + l_{11} \varphi_{sq} + R_{12} \varphi_{r1d} + L_{12} \varphi_{r3q} + V_{sd} \\
\frac{d\varphi_{sq}}{dt} &= R_{11} \varphi_{sq} - l_{11} \varphi_{sd} + R_{12} \varphi_{r1q} - L_{12} \varphi_{r1d} + V_{sq}
\end{align*}
\]

\[
\frac{d\varphi_{r1d}}{dt} = R_{21} \varphi_{sd} + L_{21} \varphi_{sq} + R_{22} \varphi_{r1q} + L_{22} \varphi_{r1d} \\
\frac{d\varphi_{r1q}}{dt} = R_{21} \varphi_{sq} - L_{21} \varphi_{sd} + R_{22} \varphi_{r1q} - L_{22} \varphi_{r1d}
\]

\[
\begin{align*}
\frac{ds'}{dt} &= -\frac{\omega_s}{2H} \left[ \frac{R_{11} l}{l_{33}} \varphi_{sd} \varphi_{r1q} + \frac{R_{12} l}{l_{33}} \varphi_{sd} \varphi_{r1q} - \frac{L_{11} l}{l_{33}} \varphi_{sq} \varphi_{r1q} - \frac{L_{12} l}{l_{33}} \varphi_{sq} \varphi_{r1q} + \varphi_{sd} \varphi_{r3q} - \varphi_{sq} \varphi_{r3q} \right]
\end{align*}
\]

(15)

With

\[
R_{11} = -\frac{R_{11} l}{l_{33}} + \frac{R_{21} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(16)

\[
R_{22} = -\frac{R_{22} l}{l_{33}} + \frac{R_{22} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(17)

\[
R_{12} = \frac{R_{12} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(18)

\[
R_{21} = \frac{R_{21} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(19)

\[
L_{11} = -\omega_s \frac{R_{11} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(20)

\[
L_{22} = -\frac{R_{22} l_{11} l_{22} l_{33}}{1 + s'^2}
\]

(21)
This way, the seventh order model was reduced to a fifth order model. In [1], it can be seen that this approximation is quite reasonable.

### III. Model Transformation of a Double-Cage Induction Machine into a Model of a Single-Cage Induction Machine

With an overview on the control of the machine, the model of a double-cage induction machine was approached by a model of a single cage induction machine. That is possible by making the parallel connection between the outer and inner cages, which results in an equivalent circuit equal to the equivalent circuit of a single-cage induction machine of Fig. 2:

![Diagram of single-cage induction machine](image)

From now on, it is assumed that the equivalent circuit of the Fig. 2 represents our machine. But according to [9], the parallel connection of the cages is not a simple parallel. It must be taken into account the slip-dependence of resistance and reactance present in equivalent circuit. Thus, the equations defining the parallel are:

\[
R_r = \frac{R_s R_2 (R_1 + R_2) + s^2 (R_2 X_2^2 + R_2 X_1^2)}{(R_1 + R_2)^2 + s^2 (X_1 + X_2)} \quad (21)
\]

\[
X_r = \frac{R_1^2 X_2 + R_2^2 X_1 + s^2 X_1 X_2 (X_1 + X_2)}{(R_1 + R_2)^2 + s^2 (X_1 + X_2)} \quad (22)
\]

Note that \( L_r = X_r / \omega_b \).

With that, to describe this system, it will be necessary to find the model of a single-cage induction machine.

### IV. Model of a Single-Cage Induction Machine

According to [10] the electrical equations in complex form for the stator and the rotor are, respectively:

\[
\begin{align*}
V_s &= i_s R_s + j \omega_b \varphi_s + \frac{d \varphi_s}{dt} \\
0 &= i_r R_r + j (\omega_b - \omega_r) \varphi_r + \frac{d \varphi_r}{dt}
\end{align*} \quad (23)
\]

For analysis of the circuit in Fig. 2, knowing that \( i = L^{-1} \varphi \) and taking into account (23), it can be said that the differential equations that modulate the flux linkages are:

\[
\begin{align*}
\frac{d \varphi_s}{dt} &= -R_s (L_r + L_m) \varphi_s + R_s L_m \varphi_r - j \omega_b \varphi_s + V_s \\
\frac{d \varphi_r}{dt} &= R_r L_m \varphi_s - R_r (L_s + L_m) \varphi_r - j s \omega_b \varphi_r
\end{align*} \quad (24)
\]

The equation that describes the mechanical motion in per-unit is:

\[
\frac{d \omega_r}{dt} = \frac{\omega_b}{2H} (T_e - T_i) \quad (25)
\]

with (4) and (5).

The equation (4) was rewritten:

\[
T_e = \text{Im} \left \{ \frac{L_m}{l} \varphi_r \varphi_s \right \} \quad (26)
\]

For digital simulation, the complex variables of (24) and (25) are solved in their real \( dq \) components, taking into account (7). We reach the following results:

\[
\begin{align*}
\frac{d \varphi_{sd}}{dt} &= -a_1 \varphi_{sd} - a_2 \varphi_{sq} + a_2 \varphi_{rd} + V_{sd} \\
\frac{d \varphi_{sq}}{dt} &= -a_3 \varphi_{sd} + a_1 \varphi_{sq} + a_2 \varphi_{rq} + V_{sq} \\
\frac{d \varphi_{rd}}{dt} &= a_3 \varphi_{sd} - a_4 \varphi_{rd} - (\omega_b - \omega_r) \varphi_{rq} \\
\frac{d \varphi_{rq}}{dt} &= a_3 \varphi_{sq} + (\omega_b - \omega_r) \varphi_{rd} - a_4 \varphi_{rq} \\
\frac{d \omega_r}{dt} &= \frac{\omega_b}{2H} \left \{ (\varphi_{sd} \varphi_{rq} - \varphi_{sq} \varphi_{rd}) - T_i \right \}
\end{align*} \quad (27)
\]

with

\[
\begin{align*}
a_1 &= \frac{R_s (L_r + L_m)}{l} \\
a_2 &= \frac{R_s L_m}{l} \\
a_3 &= \frac{R_r L_m}{l} \\
a_4 &= \frac{R_r (L_s + L_m)}{l}
\end{align*} \quad (28-31)
\]

After obtaining this model of the machine we have to link it to the grid.
V. CONNECTING THE MACHINE TO THE GRID

To connect the machine to the grid, power electronics will be used, by means of two converters. Firstly, it will be used a rectifier to convert alternating current (AC) into direct current (DC). After that, an inverter will be used to transform DC into AC, in order to connect the machine to the grid. These converters are used to control the machine.

The rectifier is controlled to supply the active and reactive power to the machine and control the voltage in DC-side to make them steady.

The inverter controls the speed of the machine through a reference power value.

Fig. 3 is a representation of the connection between the grid and the DC rectifier:

![Fig. 3 – Schematic of the connection of the grid to the DC-side of the rectifier](image)

Note that to connect the grid and the rectifier it a RL circuit is used. $r_{on}$, also present in Fig. 3, is the resistance of the switches. An abc to dq transformation is also used because the model of the machine is in dq coordinates.

The inverter is linked to the $V_{DC}$ terminals present in Fig. 3. The machine will be connected to the AC-side of the inverter.

It is important to say that the all control is realized in SI units and the machine model will be executed in SI units as well, because this way is much easier to implement the system, because the base values in the AC and in the DC sides are different in per unit.

Firstly it is necessary to describe the connection of the rectifier to the grid. From Fig. 3, we deduce:

$$L \frac{d\mathbf{i}}{dt} = -(R + r_{on})\mathbf{i} + \mathbf{V}_{ext} - \mathbf{V}_{r}$$  \hspace{1cm} (32)

Knowing that:

$$\mathbf{V}_{r} = \mathbf{V}_r e^{j(\omega_0 t + \theta_0)}$$  \hspace{1cm} (33)

$$\mathbf{V}_{ext} = \mathbf{V}_{ext} e^{j\beta}$$  \hspace{1cm} (34)

$$\mathbf{i} = \mathbf{i}_{dq} e^{j\gamma}$$  \hspace{1cm} (35)

It is possible to obtain:

$$L \frac{d\mathbf{i}_{dq}}{dt} = -J L \mathbf{i}_{dq} \frac{d\rho_r}{dt} - (R + r_{on})\mathbf{i}_{dq} + \mathbf{V}_{ext}$$  \hspace{1cm} (36)

Assuming that, the complex variables of (36) are solved in their real dq components:

$$f = f_{dq} e^{j\beta} = (f_d + jf_q) e^{j\beta}$$  \hspace{1cm} (37)

It’s possible to obtain:

$$L \frac{d\mathbf{i}_d}{dt} = L \frac{d\rho_r}{dt} i_q - (R + r_{on})i_d + V_{td}$$

$$- V_r \cos(\omega_0 t + \theta_0 - \rho_r)$$  \hspace{1cm} (38)

$$L \frac{d\mathbf{i}_q}{dt} = -L \frac{d\rho_r}{dt} i_d - (R + r_{on})i_d + V_{tq}$$

$$- V_r \sin(\omega_0 t + \theta_0 - \rho_r)$$  \hspace{1cm} (39)

Introducing a new variable:

$$\frac{d\rho_r}{dt} = \alpha_{re}$$

And noticing that:

$$V_{rd} = V_r \cos(\omega_0 t + \theta_0 - \rho_r)$$  \hspace{1cm} (40)

$$V_{rq} = V_r \sin(\omega_0 t + \theta_0 - \rho_r)$$  \hspace{1cm} (41)

The equations that define the connection of the rectifier to the grid are:

$$L \frac{d\mathbf{i}_d}{dt} = L \alpha_{re} i_q - (R + r_{on})i_d + V_{td} - V_{rd}$$  \hspace{1cm} (42)

$$L \frac{d\mathbf{i}_q}{dt} = -L \alpha_{re} i_d - (R + r_{on})i_q + V_{tq} - V_{rq}$$

It must be then necessary to implement the controllers of the converters.

VI. RECTIFIER CONTROL

According to [12], the control of the rectifier is described by:

![Fig. 4 – Rectifier control](image)
Attending to (37), the real and reactive powers are:

\[ P_s = \frac{3}{2} (V_{rd} i_d + V_{rq} i_q) \quad (43) \]
\[ Q_s = \frac{3}{2} (-V_{rd} i_q + V_{rq} i_d) \]

To control the active and reactive power it’s assumed that the system is in a steady state.

If the system is in a steady state, (43) becomes:

\[ P_s = \frac{3}{2} V_{rd} i_d \quad (44) \]
\[ Q_s = -\frac{3}{2} V_{rd} i_q \]

Based on (44), the currents that will control the system are:

\[ i_{d\text{ref}} = \frac{2}{3} V_{rd} P_{\text{ref}} \quad (45) \]
\[ i_{q\text{ref}} = \frac{2}{3} V_{rq} Q_{\text{ref}} \]

However, (45) is only true in a steady state. The steady state is characterized by a \( V_{rq} \) equal to zero. To ensure that (45) is true, \( V_{rq} \) must be zero and a PLL (Phase-Locked Loop) must be used, as described in Fig. 4.

The PLL is going to transform abc to dq coordinates and it will have a loop in \( V_{rq} \) to pull it to zero. The PLL is described in the following figure:

![PLL schematic](image)

According to [12], the transfer function that characterizes \( H(s) \) in Fig. 5 is:

\[ H(s) = \frac{h_r}{V} \frac{s^2 + (2 \omega_0)^2}{s^2 + p/\alpha} \quad (46) \]

with

\[ \alpha = \frac{1 + \sin(\delta_m)}{1 - \sin(\delta_m)} \quad (47) \]
\[ p = \omega_c \sqrt{\alpha} \quad (48) \]

For simulation, it’s used \( \omega_c = 200 \text{ rad/s} \) and \( \delta_m = 45^\circ \).

It is also introduced a saturation at \( \omega_a \) to prevent that in the start-up \( \omega_a \) becomes a sinusoid.

With the input currents we have to define the controller block present in Fig. 4. Through this block, we must obtain the modulation indices of the rectifier.

If its defined that \( \omega_{r\text{e}}(t) = \omega_0 \) and considering that the relationship between the AC-side and the DC-side of the rectifier according to [12] is:

\[ \frac{V_{AC}}{V_{DC}} = \frac{V_{DC}}{2} m_r \quad (49) \]

The modulation indices of the rectifier are given by:

\[ m_{d_r} = \frac{2}{V_{DC}} \left( \frac{L \frac{di_d}{dt} + (R + r_{on}) i_d - L \omega_0 i_q + V_{rd}}{V_{rd}} \right) \quad (50) \]
\[ m_{q_r} = \frac{2}{V_{DC}} \left( \frac{L \frac{di_q}{dt} + (R + r_{on}) i_q + L \omega_0 i_d + V_{rq}}{V_{rq}} \right) \]

If two new control variables are defined:

\[ u_{d_r} = L \frac{di_d}{dt} + (R + r_{on}) i_d \quad (51) \]
\[ u_{q_r} = L \frac{di_q}{dt} + (R + r_{on}) i_q \]

Substituting (51) in (50), we obtain:

\[ m_{d_r} = \frac{2}{V_{DC}} \left( u_{d_r} - L \omega_0 i_q + V_{rd} \right) \quad (52) \]
\[ m_{q_r} = \frac{2}{V_{DC}} \left( u_{q_r} + L \omega_0 i_d + V_{rq} \right) \]

According to (52) the next figure shows the controller block of Fig. 4:

![Controller block](image)

![Schematic of the PLL](image)

According to [12], the transfer functions schematized in Fig. 6 are:

\[ G_{df}(s) = \frac{1}{8 \times 10^{-6} s + 1} \quad (53) \]
with \( k_p = \frac{i}{\tau_{ir}} \) and \( k_i = \frac{R + r_m}{\tau_{ir}} \).

In simulation it was used a \( \tau_{ir} = 1 \times 10^{-3} \).

According to (52), it is also possible to show, in the next figure, the means of calculation of the next values of \( i_d \) and \( i_q \):

\[
\begin{align*}
  dV_{dc}^2 \quad dt &= -2 \frac{C}{2} P_{ext} - 2 \frac{C}{2} P_{loss} - \frac{2}{C} \left[ P_s + \frac{2L}{3V_{rd}} \right] Q_s \frac{dQ_s}{dt} \\
  &- \frac{2}{C} \left[ \frac{2L}{3V_{rd}} \right] Q_s \frac{dQ_s}{dt}
\end{align*}
\]

(V7)

VII. INVERTER CONTROL

The schematic of the connection of the machine to the DC port is presented in Fig. 4, while the controller of the inverter is schematized in the next figure:

The \( i_d \) and \( i_q \) values calculated in Fig. 7 are the values which will be entering in the next step, in Fig. 6.

The transfer function of Fig. 4, according to [12] is:

\[
K_v(s) = 1868 \frac{s + 19}{s(s + 2077)}
\]

(55)

Through Fig. 4, it can be seen that \( P_{ext} \) and \( Q_{sref} \) are the active and reactive power of the machine and \( V_{Dc-ref} \) is the value that we want to achieve with \( V_{DC} \). According to [13] that value has to satisfy the next relationship to allow the machine to work:

\[
V_{DC} = \frac{2\sqrt{2}}{\sqrt{3}} V_{AC}
\]

(56)

To characterize all the control of the rectifier, we have to find the equation that models the DC voltage. Looking at the DC-side of Fig. 4, it can be said that:

\[
P_{ext} - P_{loss} - \frac{d}{dt} \left( \frac{1}{2} CV_{DC}^2 \right) = P_{DC}
\]

(58)

Developing (58) and according to [12], the differential equation that modulates the \( V_{DC} \) port is:

\[
\begin{align*}
  V_{DC} &= \frac{k_p s + k_i}{s} \frac{1}{L_s + (R + r_m)} \\
  &+ \frac{1}{L_s + (R + r_m)} \frac{1}{L_s + (R + r_m)}
\end{align*}
\]

Fig. 7 – Control of Rectifier currents

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(58)

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\[
\begin{align*}
  V_{DC} &= \frac{k_p s + k_i}{s} \frac{1}{L_s + (R + r_m)} \\
  &+ \frac{1}{L_s + (R + r_m)} \frac{1}{L_s + (R + r_m)}
\end{align*}
\]

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\]

(55)

Through Fig. 4, it can be seen that \( P_{ext} \) and \( Q_{sref} \) are the active and reactive power of the machine and \( V_{Dc-ref} \) is the value that we want to achieve with \( V_{DC} \). According to [13] that value has to satisfy the next relationship to allow the machine to work:

\[
V_{DC} = \frac{2\sqrt{2}}{\sqrt{3}} V_{AC}
\]

(56)

To characterize all the control of the rectifier, we have to find the equation that models the DC voltage. Looking at the DC-side of Fig. 4, it can be said that:

\[
P_{ext} - P_{loss} - \frac{d}{dt} \left( \frac{1}{2} CV_{DC}^2 \right) = P_{DC}
\]

(58)

Developing (58) and according to [12], the differential equation that modulates the \( V_{DC} \) port is:
Substituting (62) in (23) we obtain:
\[
\vec{V}_s = i_s R_s + j \omega_s L_m \left[ (\sigma_s + 1) \vec{i}_s + \vec{i}_{sr} \right] \\
+ \frac{d}{dt} L_m \left[ (\sigma_s + 1) \vec{i}_s + \vec{i}_{sr} \right] \\
0 = i_s R_s + j (\omega_s - \omega_r) L_m \left[ \vec{i}_s + (\sigma_r + 1) \vec{i}_{sr} \right] \\
+ \frac{d}{dt} L_m \left[ \vec{i}_s + (\sigma_r + 1) \vec{i}_{sr} \right]
\] (63)

With the objective of controlling the machine, a fictitious space vector current \( \vec{i}_{mr} = i_{mr} e^{j\rho} \) is introduced and a change of variable \( \vec{i}_{mr} = \vec{i}_s + (\sigma_r + 1) \vec{i}_{sr} \) occurs, according to [12]. Solving in order to \( \vec{i}_s \), results:
\[
\vec{i}_s = \frac{\vec{i}_{mr} - \vec{i}_s}{\sigma_r + 1}
\] (64)

Substituting (62) and (64) in (4), the electromagnetic torque is given by:
\[
T_e = \text{Im} \left\{ L_m \left( \frac{\vec{i}_{mr} - \vec{i}_s}{\sigma_r + 1} \right) \vec{i}_{sq} \right\}
\] (65)

Expanding (65) and taking into consideration that \( \text{Im} \left\{ \vec{i}_s e^{-j\rho} \right\} = i_{sq} \), we get:
\[
T_e = \frac{L_m}{\sigma_r + 1} \hat{i}_{mr} i_{sq}
\] (66)

Now, substituting (64) in the second equation of (63), it is obtained:
\[
0 = \frac{\vec{i}_{mr}}{\sigma_r + 1} - \frac{\vec{i}_s}{\sigma_r + 1} R_r + j (\omega_s - \omega_r) L_m \vec{i}_{mr} \\
+ \frac{d}{dt} \left[ L_m \vec{i}_{mr} \right]
\] (67)

By defining:
\[
\tau_r = \frac{L_m (\sigma_r + 1)}{R_r}
\] (68)

And decomposing (67) into its real and imaginary components, it is obtained:
\[
\tau_r \frac{d}{dt} \vec{i}_{mr} = -\vec{i}_{mr} + i_{sd} \\
\omega = \omega_r + \frac{i_{sq}}{\tau_r \vec{i}_{mr}}
\] (69) (70)

Since we are working now with SI units, the equation of the mechanical motion is given by:
\[
\frac{d\omega_r}{dt} = \frac{1}{J} (T_e - T_i)
\] (71)

with \( J = \frac{2 h S_h}{\omega_q^2} \)

So with that form, the model of machine studied in chapter III, was changed in order to include \( i_{mr} \).

It is now necessary to look for the controller of the machine. Firstly, we have to define the reference currents. According to [12] and looking for (66) the following schematic resumes how the reference currents are obtained:

![Schematic diagram of currents control](image)

According to [12], the transfer function of Fig. 9 is:
\[
K(s) = \frac{1}{s} (\frac{(s + 0.625)}{s})
\] (72)

Then, we have to control the modulation indexes in order to obtain the correct stator voltage. Substituting (64) in the first equation of (63) we obtain:
\[
\vec{V}_s = \vec{i}_s R_s + j \omega L_m \left[ (\sigma_s + 1) \vec{i}_s + \frac{\vec{i}_{mr} - \vec{i}_s}{\sigma_r + 1} \right] \\
+ L_m \frac{d}{dt} \left[ (\sigma_s + 1) \vec{i}_s + \frac{\vec{i}_{mr} - \vec{i}_s}{\sigma_r + 1} \right]
\] (73)

According to [12], developing (73) it is possible to deduce:
\[
\frac{V_{sd} + V_{sq}}{R_s} = (i_{sd} + i_{sq}) + j \omega \tau_s \sigma (i_{sd} + i_{sq}) \]
\[
+ j \omega \tau_s (1 - \sigma) \hat{i}_{mr} + \tau_s \frac{d}{dt} [i_{sd} + i_{sq}] + \tau_s \left[ 1 - \sigma \right] \frac{d}{dt} \hat{i}_{mr}
\] (74)
Assuming that the complex variables of (74) are solved in their real dq components according to (37):

\[
\tau_s \frac{di_{sd}}{dt} + i_{sd} = \omega \tau_s i_{sq} - (1 - \sigma) \tau_s \frac{di_{mr}}{dt} + \frac{V_{sd}}{R_s}
\]

(75)

\[
\tau_s \frac{di_{sq}}{dt} + i_{sq} = -\omega \tau_s i_{sd} - \omega (1 - \sigma) \tau_s \hat{i}_{mr} + \frac{V_{sq}}{R_s}
\]

(76)

Defining two new control variables:

\[
u_d = \tau_s \sigma \frac{di_{sd}}{dt} + i_{sd}
\]

(77)

\[
u_q = \tau_s \sigma \frac{di_{sq}}{dt} + i_{sq}
\]

Substituting (76) in (75):

\[
u_d = \omega \sigma \tau_s i_{sq} - (1 - \sigma) \tau_s \frac{di_{mr}}{dt} + \frac{V_{sd}}{R_s}
\]

(78)

\[
u_q = -\omega \sigma \tau_s i_{sd} - \omega (1 - \sigma) \tau_s \hat{i}_{mr} + \frac{V_{sq}}{R_s}
\]

Rewriting (77) in order to obtain the stator voltage we deduce:

\[
V_{sd} = R_s \left[ \nu_d - \omega \sigma \tau_s i_{sq} \right]
\]

(79)

\[
V_{sq} = R_s \left[ \nu_q + \omega \sigma \tau_s i_{sd} \right]
\]

(80)

According to (79), it is possible to define the modulation indexes of the inverter from the next schematic:

According to [12], the transfer function in Fig. 10 is given by:

\[
k(s) = \frac{k_p s + k_i}{s}
\]

(80)

with \( k_p = \frac{\sigma \tau_s}{\tau_i} \) and \( k_i = \frac{1}{\tau_i} \).

In the simulation, \( \tau_i = 0.1 \) s.

VIII. SIMULATION

After deducing the entire model of machine speed control some simulations were carried out to prove the veracity of this model.

Firstly, the start-up of the system is presented. To make the system work, it’s necessary to divide the start-up in three stages. The rectifier must be turned on with the objective of setting a constant DC voltage:

![Fig. 11 – DC Voltage during start-up](image)

After \( V_{DC} \) is set to the reference voltage, it is necessary to establish the excitation current. For this purpose, the control signal \( \hat{i}_{mr-ref} \) is linked. So \( i_{sd} \) and \( i_{mr} \) will tend to \( \hat{i}_{mr-ref} \).

![Fig. 12 – Stator current and reference current during start-up](image)
After the excitation current is established, $T_{ref}$ control can be finally activated, and the machine starts and the speed grows up:

If we consider that the machine is in a steady state at nominal speed and we lower the $P_{ref}$ signal control, the speed of the machine decreases:

We verify that the machine was not significantly affected by the voltage dip because rectifier control detects the voltage dip and increments the current in order to maintain the DC voltage almost constant.

**IX. CONCLUSION**

In this paper, the double-cage induction model was reduced to a fifth order model to simplify the digital simulation. Posteriorly, this model was approximated by a single-cage induction model in order to implement the controllers. This approximation is not a good one because, during the start-up process, the model of a double-cage induction machine is much faster than the single cage one. This was already expected because the outer cage of a double cage exists precisely to increase the start-up torque. However, the final result and the variations are the same in the two models. The
control of the machine takes into consideration the model of the single-cage induction machine.

To control the machine there are two converters, a rectifier and an inverter. The rectifier establishes a determined DC voltage to allow the machine to work and provides the active and reactive power to the machine. The rectifier, through a power signal control, regulates the speed of the machine.

Furthermore, the system is prepared to deal with voltage dips and frequency oscillations. In these cases, the PLL acts to keep \( V_n \) equal to zero in order to minimize the perturbations in the system.

APPENDIX

\[ V_n \rightarrow \text{Voltage of stator windings.} \]
\[ \omega_s \rightarrow \text{Angular frequency of power supply.} \]
\[ R_{s1}, R_{s2}, R_{r1}, R_{r2} \rightarrow \text{Resistance of stator, inner and outer cage windings, respectively.} \]
\[ L_{s1}, L_{s2}, L_{r1}, L_{r2} \rightarrow \text{Leakage inductances of stator, inner and outer cage windings, respectively.} \]
\[ \varphi_{s1}, \varphi_{s2}, \varphi_{r1}, \varphi_{r2} \rightarrow \text{Flux-linkage of stator, inner and outer cage windings, respectively.} \]
\[ i_{s1}, i_{s2}, i_{r1}, i_{r2} \rightarrow \text{Current of stator, inner and outer cage windings, respectively.} \]
\[ i_m \rightarrow \text{Magnetizing current.} \]
\[ V_m \rightarrow \text{Voltage in AC-side of the rectifier.} \]
\[ i \rightarrow \text{Input current of the grid.} \]
\[ \rho \rightarrow \text{Angle for the dq-frame transformations in the machine side.} \]
\[ \tau_s, \tau_r \rightarrow \text{Stator and rotor time constant.} \]
\[ L \rightarrow \text{Grid-concetion Inductance.} \]
\[ R \rightarrow \text{Grid connection Resistance.} \]
\[ R_{on} \rightarrow \text{Resistance of semi-conductors.} \]
\[ V_{DC} \rightarrow \text{DC Voltage.} \]

\[ V_{DC\,\text{ref}} \rightarrow \text{Reference DC Voltage.} \]
\[ P_{\text{ref}} \rightarrow \text{Active power of the machine.} \]
\[ C \rightarrow \text{Compensator.} \]
\[ P_{\text{loss}} \rightarrow \text{Power losses.} \]
\[ P_{\text{DC}} \rightarrow \text{Power on the DC-side of the rectifier.} \]
\[ P_t \rightarrow \text{Power on the AC-side of the rectifier.} \]
\[ m_d, m_q \rightarrow \text{Modulation indexes of the inverter in d and q coordinates, respectively.} \]
\[ T_{\text{ref}} \rightarrow \text{Reference Torque} \]
\[ i_{mr\,\text{ref}} \rightarrow \text{Excitation reference current} \]

The motor used in this simulation has 4700 hp, 2970 rpm, 6.6 KV, 50 Hz and has the following parameters in per unit, taken from [1]: \( \{ S_p = 4.173 \, \text{MVA}, V_n = 6.6 \, \text{kV} \} \):

\[ \begin{align*}
    R_1 &= 7.4 \times 10^{-3} \\
    R_2 &= 7.27 \times 10^{-3} \\
    R_3 &= 1.36 \times 10^{-1} \\
    L_1 &= 3.47 \times 10^{-4} \\
    L_2 &= 3.58 \times 10^{-4} \\
    L_3 &= 2.63 \times 10^{-4} \\
    L_m &= 0.0115
\end{align*} \]

REFERENCES