Abstract – This work is focused on the study of switching in optical networks. It begins by the analysis of the optical switching in the linear regime. The equation that governs the propagation of pulse in the linear regime is introduced and the behavior of the pulse along the fiber is investigated. In the case of the nonlinear regime, the nonlinear Schrödinger equation (NLS), which describes the pulse propagation, leads to the existence of solitons. Split Step Fourier Method was used to simulate the behavior of solitons along the fiber. Using a modified version of the split-step Fourier method, which includes intermodal dispersion, we show that this effect cannot be neglected in the switching of picosecond solitons, at different wavelengths. Finally, the study of controlled switching of solitons is addressed, where two types of switching is considered: self-switching and locally controlled switching.

Index Terms – Optical fiber, optical switching, GVD, linear regime, nonlinear regime, solitons, optical couplers, self-switching, locally controlled switching.

1. INTRODUCTION

Telecommunications is an exact science, which development depended strongly on the scientific discoveries and advances in mathematics that have taken place in Europe during the XIX century. Discoveries were made in the area of electromagnetism, which created the conditions for the appearance of the first telecommunications system based on electricity: the telegraph [1].

Any communication network depends heavily able to properly switch the traffic to your target devices. Accordingly, the electronics include a number of different devices for both traffic electronic level but also for traffic in optical fibers, where the optical-electrical conversion is a constant need, which limits the traffic, causing the commutation completely optical to be the desired solution. Accordingly, various devices have been used as supports for the study of coupling energy between channels. Of which are highlighted Bragg networks, birefringent fibers, couplers dual core among others. It has also been experienced couplers for new materials such as polymers and semiconductors [2].

The study of these devices began with beams of electromagnetic waves, which highlights the study of the nonlinear behavior of a directional coupler by Jensen [3]. Due to the necessity of switching pulses to transmit information and power even to poor performance compared to these beams led to the introduction of soliton switching, thus allowing the interconnection of these devices with long distance transmission systems based on soliton in a global perspective of communication networks. The first switching solitons was performed on an interferometer Segnac experimentally demonstrating the viability to use solitons in switching devices [2]. Currently, solution based on MEMS (micro-electromechanical systems) is mostly used to implement optical switching. MEMS switches consist of miniature movable mirrors with dimensions of the order of hundreds of microns [4].

2. SWITCHING IN LINEAR REGIME

2.1 Propagation equation in the linear regime

Due to group velocity dispersion (DVG), pulses that propagate in an optical fiber in the linear regime suffers a time broadening that can cause intersymbol interference [5]. The equation that governs the propagation of pulse in the linear regime is given by

$$\frac{\partial A}{\partial \zeta} + i \frac{1}{2} \text{sgn}(\beta_z) \frac{\partial^2 A}{\partial \tau^2} - \kappa \frac{\partial A}{\partial \tau} = 0$$

where

$$\beta_z = |\beta_z| \text{sgn}(\beta_z), \quad \kappa = -\frac{\beta_z}{6|\beta_z|\tau_0}$$

$$\frac{\partial A}{\partial \zeta} + \beta_z \frac{\partial A}{\partial \tau} + \frac{1}{2} \beta_z \frac{\partial^2 A}{\partial \tau^2} + \frac{\alpha}{2} A = 0.$$
2.1.1 Numerical simulation

For the numerical simulation of the propagation of pulse in an optical fiber in the linear regime there that use the FFT (Fast Fourier Transform) and IFFT (inverse fast Fourier transform). Considers the following initial impulse

\( A_i(\tau) = A(0, \tau) = \sec b(\tau) \) (4)

\[ \frac{\partial A_i}{\partial \xi} + \beta \frac{\partial A_i}{\partial \tau} + i \frac{1}{2} \left[ \sgn(\beta_1) \frac{\partial^2 A_i}{\partial \tau^2} + \mu \frac{\partial^2 A_i}{\partial \xi^2} \right] = i \kappa A_i \] (7)

Algebraically manipulating the equations obtained for the frequency domain

\[ \begin{bmatrix} \tilde{A}_i(\zeta, \xi) \\ \tilde{A}_o(\zeta, \xi) \end{bmatrix} = S \left[ \begin{bmatrix} \tilde{A}_i(0, \xi) \\ \tilde{A}_o(0, \xi) \end{bmatrix} \right] \] (8)

Wich

\[ S(\zeta, \xi) = \overline{\text{MR} (\zeta, \xi) \tilde{M}} \]

\[ = \exp \left[ -\frac{1}{2} \sgn(\beta, \xi^2 \zeta) \right] \left[ \cos \left[ \theta(\zeta, \xi) \right] \right. \left. \sin \left[ \theta(\zeta, \xi) \right] \right] \cos \left[ \theta(\zeta, \xi) \right] \]

where

\[ \theta(\zeta, \xi) = b(\xi) \zeta = \left( \kappa + \xi \delta + \frac{1}{2} \xi^2 \mu \right) \zeta. \] (10)

If the pulse does not exist in the second fiber obtained

\[ A_i(\zeta, \xi) = S_{21} A_i(0, \xi) \]

\[ A_o(\zeta, \xi) = S_{12} A_i(0, \xi) \]

where

\[ S_{21} = \exp \left[ -\frac{1}{2} \sgn(\beta, \xi^2 \zeta) \right] \cos \left[ \theta(\zeta, \xi) \right] \]

\[ S_{12} = \exp \left[ -\frac{1}{2} \sgn(\beta, \xi^2 \zeta) \right] \sin \left[ \theta(\zeta, \xi) \right] \]

Assuming \( A_i(0, \xi) = 0 \) we define the following transfer functions

\[ \tau_i(\zeta, \xi) = \left( \frac{\tilde{A}_i(\zeta, \xi)}{\tilde{A}_i(0, \xi)} \right) = \cos^2 \left[ \left( \kappa + \xi \delta + \frac{1}{2} \xi^2 \mu \right) \zeta \right] \] (15)

\[ \tau_o(\zeta, \xi) = \left( \frac{\tilde{A}_o(\zeta, \xi)}{\tilde{A}_o(0, \xi)} \right) = \sin^2 \left[ \left( \kappa + \xi \delta + \frac{1}{2} \xi^2 \mu \right) \zeta \right]. \] (16)

For the pulse that propagates along the two optical fibers, one uses the following process

i. Calculate \( \tilde{A}_i(0, \xi) = \text{FFT} \left[ A_i(0, \tau) \right] \);

ii. Calculate \( \tilde{A}_i(\zeta, \xi) \) e \( \tilde{A}_o(\zeta, \xi) \);

iii. Calculate \( A_i(\zeta, \tau) = \text{IFFT} \left[ \tilde{A}_i(\zeta, \xi) \right] \) e \( A_o(\zeta, \tau) = \text{IFFT} \left[ \tilde{A}_o(\zeta, \xi) \right] \).

Considering the anomalous dispersion region \( \left( \sgn(\beta) = -1 \right) \), \( \kappa = 1 \), \( \delta = -0.0156 \) and \( \mu = 2.4322 \times 10^4 \)

obtain the following graphs of pulse propagation along the two fibers.

For this case the pulse is insert into one fiber

\[ A_i(0, \tau) = \exp \left( -\frac{\tau^2}{2} \right) \quad \text{e} \quad A_o(0, \tau) = 0. \] (5)

The equations of the linear coupling between the two fibers is given by:

\[ \frac{\partial A_i}{\partial \xi} + \beta \frac{\partial A_o}{\partial \tau} + i \frac{1}{2} \left[ \sgn(\beta_1) \frac{\partial^2 A_i}{\partial \tau^2} + \mu \frac{\partial^2 A_o}{\partial \xi^2} \right] = i \kappa A_i \] (6)

In Figure 1 representing the input pulse and the output pulse. As could be expected the effect of the DVG is predominant. There is an enlargement and reduction of the amplitude of the output pulse along the fiber. This is due to the facts of different spectral components traveling at different velocities along the fiber. The same phenomenon is observed in 3D(See Figure 2).

2.2 Linear Coupling

Two identical fibers in a common cladding interacting, thus originating a phenomenon known as the intermodal dispersion caused by the existence of two super modis is considered. The dispersion will cause the passage of electromagnetic energy from one fiber to another and thus the pitch into a fiber will influence the field in the other, and thus the signal propagation in the fiber depends on the existing signal to propagate in another.

For this case the pulse is insert into one fiber

\[ A_i(0, \tau) = \exp \left( -\frac{\tau^2}{2} \right) \quad \text{e} \quad A_o(0, \tau) = 0. \] (5)
Observing the Figures 3 and 4 we conclude that there is a periodic coupling along the fiber which, when the amplitude of the signal fiber is a maximum in the other reaches zero. Can also conclude that it is the presence of a perfect switching where the boost for each engagement, remains constant in each energy transfer. Therefore, we can say that there is a perfect pulse switching between the two fibers.

2.3 Optics Fiber Arrays

2.3.1 Coupling between two fibers

The crosstalk caused by the coupling between optical fibers can be a serious problem in optical communication systems. Two parallel and identical cores radius \( a \) operating in linear regime are embedded in a common cladding. The separation between the axes is \( \rho \geq 2a \).

The longitudinal electric field in each “fiber” (with \( n = 1, 2 \)) is

\[
E_{\alpha n} (x, y, z, t) = F_{\alpha n} (x, y) B_{n z} (z, t) \tag{17}
\]

where \( F_{\alpha n} (x, y) \) are the basic modal functions associated to \( LP_{\alpha n} \) mode in each core. Considering the scalar theory of modal coupling fibers, the longitudinal electric field is given by

\[
E_{\alpha} (x, y, z, t) = B_{1 z} (z, t) F_{\alpha} (x, y) + B_{2 z} (z, t) F_{\alpha} (x, y). \tag{18}
\]

When the distance between the two cores is finite \( (\rho \geq a) \) we have

\[
\frac{\partial}{\partial z} \tilde{B}_1 (z, \omega) = i \beta (\omega - C (\omega)) \tilde{B}_1 (z, \omega) \tag{19}
\]

Where the coupling coefficient \( C \) was introduced as well as the longitudinal propagation constant \( \beta = \beta_n + \Delta \beta \), where \( \Delta \beta \) is the disturbance due to electromagnetic interaction between the two cores. It is possible to write the equation (20) as follows

\[
\frac{\partial}{\partial z} \Phi (z, \omega) = i \overline{\Lambda} (\omega) \cdot \Phi (z, \omega) \tag{20}
\]

where

\[
\overline{\Lambda} (\omega) = \overline{M} - \overline{K} (\omega) \cdot \overline{M} \tag{23}
\]

The relationship between the field intensity along the fiber to input and can be expressed as follows

\[
t (L_c, \omega) = \frac{\tilde{B}_1 (L_c, \omega)}{\tilde{B}_1 (0, \omega)} \cdot \overline{V} = \cos [\overline{M} (\omega)] \cdot \tilde{B}_0 (0, \omega) \cdot \overline{V} \tag{24}
\]

Considering a coupler in the optical domain with length \( L_c \), in that \( \tilde{B}_1 (0, \omega) = 0 \), we define de following transfer functions

\[
t (L_c, \omega) = \frac{\tilde{B}_1 (L_c, \omega)}{\tilde{B}_1 (0, \omega)} \cdot \overline{V} = \cos [\overline{M} (\omega)] \cdot \tilde{B}_0 (0, \omega) \cdot \overline{V} \tag{24}
\]

\[
l_n = \frac{\pi}{2C (a_k)} \Rightarrow t (L_n, \omega) = 0, \quad t, (L_n, \omega) = 1. \tag{26}
\]

Also, for \( L_c = L_n \) (full-beat length), as

\[
L_n = \frac{\pi}{C (a_k)} \Rightarrow t (L_n, \omega) = 0, \quad t, (L_n, \omega) = 1. \tag{27}
\]

In the following figure you can check the progress of the transmission coefficient for a half-beat coupler.
Figure 1 – Transfer functions for the twin-core fiber full-beat coupler for the carrier frequency $\omega = \omega_0$.

Considering $s = \rho a$, with $s \geq 2$ and $\rho = \sqrt{1 - b}$, where $w = \sqrt{b}$ and $\Delta = (n_i^2 - n_f^2)/2n_i^2$, the coupling coefficient is given by

$$C = \frac{\sqrt{2\Delta} u^2 K_n(3w)}{a \sqrt{v^2 - K_i(w)^2}}$$  \hspace{1cm} (28)

where $v = n_i k_a \sqrt{2\Delta}$ is the normalized frequency, $k_a = 2\pi/\lambda$ and $K_n$ and $K_i$ are the zero and first modified Bessel functions of the first kind, respectively; the normalized propagation constant $b$ can be calculated by the Rudolph-Neumann equation

$$b = \left(1.1428 - \frac{0.9960}{v}\right)^2.$$ \hspace{1cm} (29)

Considering the half-beat coupler for the central nominal wavelength $\lambda_0 = 1.55 \mu m$ with $a = 4.5 \mu m$, $n_i = 1.5$, $\Delta = 0.3\%$, and varying value of $s$, we obtain the results of Figure 6.

2.3.2 Asynchronous coupling

So far, we always consider that the transmission coefficients of the two cores are identical, i.e., there is always a synchronous coupling between the two cores. However, this is not always true, there are cases where the propagation constant of a core differs from other. In these cases, we are faced to an asynchronous coupling [6]- [7].

If, for $s = 0$, the input power is only in core (1), i.e., $B_i(0) = 1$ and $B_f(0) = 0$, we have

$$\begin{align*}
\begin{bmatrix}
\bar{B}_i(z) \\
\bar{B}_f(z)
\end{bmatrix} &= \begin{bmatrix}
\cos(\psi z) - i \Delta \sin(\psi z) \\
i C_{2i} \sin(\psi z)
\end{bmatrix} \exp[i(\beta_i + \beta_f) z / 2 z] \\
&= \begin{bmatrix}
\cos(\psi z) - i \Delta \sin(\psi z) \\
\sin(\psi z)
\end{bmatrix} \exp[i(\beta_i + \beta_f) z / 2 z] \quad \text{if } s = 0
\end{align*}$$  \hspace{1cm} (29)

where

$$\psi = \sqrt{\Delta^2 + C_{2i} C_{2i}} \quad \text{and} \quad \Delta = \frac{\beta_i - \beta_f}{2}.$$ \hspace{1cm} (30)

Assuming the codirectional coupling, i.e., that the pulses propagate in the same direction, we have $C_{2i}(\omega) = C_{2i}$.

Still considering a coupler with length $L_c$, we can define the two power transfers functions

$$t(z) = \left| \frac{\bar{B}_i(z)}{\bar{B}_i(0)} \right|^2 = \frac{C_{2i} C_{2i}}{\psi^2} \cos^2(\psi z) + \frac{L_c^2}{\psi^2} \cos^2(\psi z) + \frac{\Delta^2}{\psi^2}$$  \hspace{1cm} (31)

and

$$t_f(z) = \left| \frac{\bar{B}_f(z)}{\bar{B}_f(0)} \right|^2 = \frac{C_{2i} C_{2i}}{\psi^2} \sin^2(\psi z).$$  \hspace{1cm} (32)

As can be seen in the Figure 7 and 8, in the asynchronous coupling, $\beta_i \neq \beta_f$, the power transfer from one fiber to another is never complete. It occurs periodically with a maximum at $L_c = \pi/2\psi$ (half-beat length) [7]. It is only possible to completely transfer the power when the phase mismatch is $\Delta = 0$, that is, for $\beta_i = \beta_f$.

Figure 6- Transfer functions for the twin-core fiber half-beat coupler: a) for $s = 9$ and b) for $s = 6$

Figure 7- Transfer function for an asynchronous coupler with different values of phase mismatch condition: $\Delta = 0.8$
3. Nonlinear Switching

3.1 Propagation Equation in Nonlinear Regime

Because of the optical Kerr effect, the propagation of pulses in a single mode optical fiber for nonlinear dispersive regime (RNLD) is governed simultaneously by DVG and the self-modulation phase as well as other higher order effects [8]. Under nonlinear equation propagating pulse is given by

\[ i \frac{\partial u}{\partial \xi} - \frac{1}{2} \text{sgn}(\beta_J) \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = -i \frac{\Gamma}{2} u + i\kappa \frac{\partial^3 u}{\partial \tau^3}. \] (34)

Demonstration of pulse propagation in nonlinear equation system can be found at [8]. Must emphasize that by not considering the nonlinear effects of higher order (such as the Raman effect or self steepening). These equations are not applicable to ultra-short pulses with a duration that is of the order of femtoseconds or subpicoseconds.

Neglecting losses \((\Gamma = 0)\) and the dispersion of higher order \((\kappa = 0)\), equation (34) can be written as

\[ i \frac{\partial u}{\partial \xi} - \frac{1}{2} \text{sgn}(\beta_J) \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = 0. \] (35)

Equation (35) is known as nonlinear equation nonlinear Schrödinger (NLS) and belongs to a special class of nonlinear differential equation that can be solved using the inverse method Dispersion or IST (inverse scattering transform). Considering the anomalous dispersion zone \(\text{sgn}(\beta_J) = -1\) where light solitons may occur equation (35) can be written as

\[ u(\zeta, \tau) = \eta \text{sech} \left[ \eta (\tau - q_0) \right] \exp \left( i \frac{\Gamma}{2} \eta^2 \zeta \right) \] (36)

where the coefficient

\[ \eta = \sqrt{2} \eta_0. \] (37)

The parameter \(\eta\) establishes both the amplitude and pulse width. \(q_0\) defines the center of the pulse with respect to \(\zeta = \tau = 0\) and the parameter \(\eta_0\) sets the stage for \(\zeta = \tau = 0\). The solution presented in equations (36) is called the fundamental soliton.

Using the IST, it is possible to show that any incident pulse \(u_0(\tau) = N \sec h(\tau), N^2 = L_0 / L_{NL}\) (39) where \(L_0\) the dispersion length and \(L_{NL}\) the nonlinear length. The parameter \(N\) is integer and represents the order of the soliton thus leading to the propagation of a soliton order \(N\). However, with the exception of the fundamental soliton \((N=1)\), the soliton with \(N \geq 2\) not keep their shape, instead, present a periodic evolution with time \(\zeta = \pi / 2\).

\[ u(\zeta, \tau) = \text{sech}(\tau) \exp \left( i \frac{\zeta}{2} \right) \] (38)

In Figure 10 and 11 is represented the evolution of the soliton along the fiber. The third-order soliton to propagate initially suffers a contraction in which the effects of SPM compared to predominate DVG for later split into several components to come back later join thereby recovering its original shape when the time is distance. This behavior is observed for all solitons except soliton of the first order (see Figure 10), to which is propagated without undergoing distortion. Due to their behavior in terms of communication the first order soliton is the most interesting [9].
3.2 Coupling Equation in the Nonlinear Regime

In this section the nonlinear equations that govern the coupling switching pulses in optical fiber couplers with Kerr nonlinearity appears. To this effect will only be considered with two parallel and identical core are embedded in a common cladding. Considering the zone of anomalous dispersion coupled NLS equations are given by

\[
i \frac{\partial u_1}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u_1}{\partial \tau^2} + |u_1|^2 u_1 + \kappa u_2 = 0 \tag{40}
\]

\[
i \frac{\partial u_2}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + |u_2|^2 u_2 + \kappa u_1 = 0 \tag{41}
\]

Adapting the Taylor expansion and Assuming \( \beta_1 \) and \( \beta_2 \) Numbers represent the longitudinal wave for the odd supermodos and the optical coupler pairs of respectively two fibers of the same core, the equations (40) and (41) can be rewritten as follows

\[
i \frac{\partial A_1}{\partial \zeta} + \beta \frac{\partial A_1}{\partial \tau} + C_1 \frac{\partial A_1}{\partial t} - \beta \frac{\partial^2 A_1}{\partial \tau^2} - C_1 \frac{\partial^2 A_1}{\partial \tau^2} + \gamma |A|^2 A_1 + C_2 A_1 = 0 \tag{42}
\]

\[
i \frac{\partial A_2}{\partial \zeta} + \beta \frac{\partial A_2}{\partial \tau} + C_1 \frac{\partial A_2}{\partial t} - \beta \frac{\partial^2 A_2}{\partial \tau^2} - C_1 \frac{\partial^2 A_2}{\partial \tau^2} + \gamma |A|^2 A_2 + C_2 A_2 = 0 \tag{43}
\]

Considering again the normalized variables \( \zeta \) e \( \tau \), and also normalized amplitudes \( u_i(\zeta, \tau) \), Equations (42) and (43) can be rewritten in the following form

\[
i \frac{\partial u_1}{\partial \zeta} + \delta \frac{\partial u_1}{\partial \tau} + \frac{1}{2} \left( \frac{\partial^2 u_1}{\partial \tau^2} - \mu \frac{\partial^2 u_1}{\partial \tau^2} \right) + |u_1|^2 u_1 + \kappa u_2 = 0 \tag{44}
\]

\[
i \frac{\partial u_2}{\partial \zeta} + \delta \frac{\partial u_2}{\partial \tau} + \frac{1}{2} \left( \frac{\partial^2 u_2}{\partial \tau^2} - \mu \frac{\partial^2 u_2}{\partial \tau^2} \right) + |u_2|^2 u_2 + \kappa u_1 = 0 \tag{45}
\]

where

\[
\delta = \frac{C_1 L_0}{\tau_0} \quad \mu = \frac{C_1 L_0}{\tau_0}. \tag{46}
\]

the coefficients \( \delta \) e \( \mu \) coupling coefficients are first and second order respectively.

3.3 Intermodal dispersion in switching soliton at different wavelengths

To solve the equations (47) and (48) we have to develop a new version of the SSFM. This method allows to compute \( u_i \) and \( u_\ell \) at \( \zeta + h \) from their values at \( \zeta \), where \( h \) is the step size. This interactive procedure is repeated from \( \zeta = 0 \) to \( \zeta = \zeta_s = L/L_0 \). Usually, in the SSFM, the evolution along \( \zeta \) is analyzed in two consecutive steps to separate the linear from the nonlinear terms [10].

\[
i \left( \frac{\partial u_1}{\partial \zeta} + \delta \frac{\partial u_1}{\partial \tau} + \frac{1}{2} \left( \frac{\partial^2 u_1}{\partial \tau^2} - \mu \frac{\partial^2 u_1}{\partial \tau^2} \right) \right) + |u_1|^2 u_1 + \kappa u_2 = 0 \tag{47}
\]

\[
i \left( \frac{\partial u_2}{\partial \zeta} + \delta \frac{\partial u_2}{\partial \tau} + \frac{1}{2} \left( \frac{\partial^2 u_2}{\partial \tau^2} - \mu \frac{\partial^2 u_2}{\partial \tau^2} \right) \right) + |u_2|^2 u_2 + \kappa u_1 = 0 \tag{48}
\]

In the first step, only the nonlinear terms are considered. Let us introduce the new amplitudes \( v_1 \) and \( v_2 \) such that, (with \( n = 1, 2 \))

\[
v_1(\zeta, \tau) = u_1(\zeta, \tau) \exp \left( \frac{i h}{\tau_0} \right) \tag{49}
\]

In the second step, the nonlinear terms are disregarded. We define \( \bar{v}_1(\zeta, \xi) \) and \( \bar{v}_1(\xi, \xi) \) as the Fourier transforms of \( v_1(\zeta, \tau) \) and \( v_2(\zeta, \tau) \), respectively. Then we have

\[
\bar{v}_1(\zeta + h, \xi) = \exp \left( \frac{-i h \xi^2}{2} \right)
\]

\[
\bar{v}_1(\zeta + h, \xi) = \cos \left( \theta(\tau, h) \right) \sin \left( \theta(\tau, h) \right) \left[ \bar{v}_1(\zeta, \xi) \right] + i \sin \left( \theta(\tau, h) \right) \cos \left( \theta(\tau, h) \right) \left[ \bar{v}_1(\zeta, \xi) \right]
\]

where

\[
\xi = \Omega \tau, \quad \theta(\tau, h) = \left( \kappa + \xi \delta + \frac{\xi \mu}{2} \right) h. \tag{51}
\]

Finally, to obtain \( u_1(\zeta + h, \tau) \) and \( u_2(\zeta + h, \tau) \) we merely have to find the inverse Fourier transform of \( \bar{v}_1(\zeta + h, \xi) \) and \( \bar{v}_1(\zeta + h, \xi) \), respectively. Obviously, to perform direct or inverse Fourier transforms we use the fast-Fourier-transform (FFT) [10].

3.3.1 Soliton switching with different wavelengths

Definindo coeficiente de transmissão

\[
T = \frac{1}{Q} \int |u_1(\zeta, \tau)|^2 d\tau \tag{52}
\]

where \( Q \) represents the total energy given by

\[
Q = \int |u_1(\xi, \tau)|^2 + |u_2(\xi, \tau)|^2 d\tau. \tag{53}
\]

To analyze the effect of IMD on the soliton with different wavelength switching, we study the evolution of the transmissivity as a function of input peak power normalized (\( p \)) for \( \lambda = \lambda_0, \lambda = \lambda_0 \pm \Delta \lambda \), with \( \Delta \lambda = 30 \text{nm} \) [10].
Figure 1 – Transmission coefficient $T$ as a function of the normalized peak power $p$ for different wavelengths.

The result of Figure 12 corresponds to an input

$$u_i(0, \tau) = \sqrt{p} \sec h\left(\sqrt{p} \tau\right) \exp(-i \xi h)$$

$$u_x(0, \tau) = 0$$

To study the effect of IMD on the soliton switching in WDM (Wavelength-Division Multiplexing) systems, we analyzed the behavior of switching to the next input

$$u_i(0, \tau) = \sqrt{p} \sec h\left(\sqrt{p} \tau\right) \sum_{\xi_1}^{\xi_2} \exp(-i \xi h)$$

with $u_i(0, \tau) = 0$, $\xi_1 = \xi(\lambda = 1.52 \mu m)$, $\xi_2 = \xi(\lambda = 1.58 \mu m)$. We considered the peak power normalized input $p = 9$, for all inputs.

Figure 2 – Output signal in linear regime without IMD; a) parallel output; b) cross output.

Figure 3 – Output signal in linear regime with IMD; a) parallel output; b) cross output.

According to Figure 13 we can see that when we do not have the effect of IMD there is a perfect switching between the two fibers, since there are no signs of the output signal obtained from the first fiber-to-fiber output signal is exactly introduced the entry to the fiber 1.

Figure 4 – Output signal in nonlinear regime without IMD; a) parallel output; b) cross output.

Figure 5 – Output signal in nonlinear regime with IMD; a) the parallel output; b) the cross output.

As the frequency variation lies within the anomalous-dispersion region, the leftmost pulse in Figs. 11, 12, 13 and 14 corresponds to the lowest wavelength, i.e., to $\lambda = 1.52 \mu m$. When IMD accounts for the effect of the switching in the soliton effect but IMD is significant than in the linear regime. There is a considerable difference between the results obtained with and without accounting for the IMD. So in WDM systems is always advisable accounting for IMD. Once lies within the region of anomalous dispersion, the impulse that is leftmost corresponds to a shorter wavelength, or $\lambda = 1.52 \mu m$.

### 3.4 Cross-phase modulation

The effective index of refraction for each mode of propagation depends not only on optical intensity of the optical beam itself (Kerr effect), but also to other optical intensity of the optical beams that propagate simultaneously in the fiber. This non-linear phenomenon is called cross-phase modulation (XPM).

Just as SPM, XPM causes spectral broadening of the pulse and can also alter the temporal shape of the pulse to interact with the dispersion.

When not considered losses in the fiber dispersion and third order equations coupling NLS reduce

$$\frac{\partial A_i}{\partial z} + \frac{i}{2} \beta_{2i} \frac{\partial^2 A_i}{\partial T^2} = i \gamma_i (|A_j|^2 + 2|A_i|^2) A_i$$

$$\frac{\partial A_i}{\partial z} + d \frac{\partial A_i}{\partial T} + \frac{i}{2} \beta_{2i} \frac{\partial^2 A_i}{\partial T^2} = i \gamma_i (|A_j|^2 + 2|A_i|^2) A_i.$$
Defining $T_o$ as the pulse width related to $\lambda$, defines the walk off length $L_w$, as the fiber length during which two overlapping pulses are separated from each other due to the difference in group velocity

$$L_w = T_o |d|.$$  \hfill (59)

Similarly defines the length of the dispersion such that

$$L_D = T_o^2 |\beta_2|.$$  \hfill (60)

Considering the initial amplitude given by

$$A_1(0,T) = \sqrt{P_1} \exp \left( -\frac{T^2}{2T_o^2} \right)$$  \hfill (64)

$$A_2(0,T) = \sqrt{P_2} \exp \left( -\frac{(T-T_o)^2}{2T_o^2} \right)$$  \hfill (65)

where $P_1$ e $P_2$ represents the peak power $T_o$ is the initial time delay between the two pulses.

In the following example the following values were used:

$P_1 = 500 mW$, $P_2 = 250 mW$, $\gamma_1 = \gamma_2 = 10 W^{-1} km^{-1}$, $L_w = 3 km$, $T_o = 15 ps$, $T_o = 0$ e $\delta = 5$.

![Figure 12](image_url)

**Figure 12-** XPM effect in the spectrum of the two pulses ($L = 0$)

![Figure 13](image_url)

**Figure 13-** Spectral broadening and asymmetry caused by XPM in two impulses spectrum ($L = L_w$)

Figure 12 illustrates the effect of walk off. Initially the pulses are overlapped, from the moment that increases the impulses separated due to difference in group velocity. This effect is only significant for length longer than the walk off length fiber.

In Figure 13, the most notable feature is the spectral asymmetry that is solely due to XPM. The spectrum of the pulse 2 is more skewed because the contribution of XPM is higher ($P_2 = 2P_1$).

4. CONTROLLED OPTICAL SWITCHING

4.1 Self-routing switching

Self-switching can be defined as the capability of a pulse switch where channel to another channel based on its peak power. In other words self switching control systems has entered the impulses, since the switching depends on the energy of each pulse. So two impulses deferent energy can be diverted into different channels [11]. (see Figure 27). Sets are thus two switch: parallel-state corresponding to the initial permanency energy channel and cross-state corresponding to the transition of power to the opposite channel.

![Figure 14](image_url)

**Figure 14-** Self-routing diagram.

The results in the next section correspond to an input as

$$u(\zeta = 0,\tau) = \sqrt{P_{in}} \sec h \left( \sqrt{P_{in}} \tau \right)$$  \hfill (66)

$$v(\zeta = 0,\tau) = 0,$$  \hfill (67)

where $P_{in}$ is the input power. The total energy of the system is given by

$$Q(\zeta) = \int_{-\infty}^{\infty} |u(\zeta,\tau)|^2 + |v(\zeta,\tau)|^2 d\tau.$$  \hfill (68)

4.1.1 Transmission

The transmission coefficient $T$ represents the total energy that remains in the channel $u$ at the point $\zeta$. It can be calculated using the following equation (52)

Figure 15 depicts the transmission coefficient for half-beat and full-beat couplers. For the half-beat couplers and for the range $p < 4$, there is a transfer of energy to the channel $v$ of at least 90%. For $p > 9$ the energy remains in the original channel (channel $u$). For full-beat couplers, there is no range of power values that allow the transfer to channel $v$. So this coupler is excluded due to inability to control the target pulse [11].

![Figure 15](image_url)
To avoid intersymbol interference, it is necessary to determine the minimum distance between the two pulses. According to [11] the minimum distance is

\[ \Delta_{\text{min}} = 6 \],

for the following input

\[
\begin{align*}
    u(\zeta = 0, \tau) &= p_u \sec h \left[ \sqrt{p_u} (\tau - \Delta /2) \right] + \sqrt{p_u} \sec h \left[ \sqrt{p_u} (\tau + \Delta /2) \right] \\
    v(\zeta = 0, \tau) &= 0, 
\end{align*}
\] (69) (70)

Figure 17 depicts the input and output absolute values of the two pulse described above, with a time separation \( \Delta = 10 \) and an input powers \( p_u = 4 \) and \( p_v = 10 \).

\[ \begin{align*}
    u(\zeta = 0, \tau) &= \sqrt{p_u} \sec h \left[ \sqrt{p_u} (\tau - \Delta /2) \right] + \sqrt{p_u} \sec h \left[ \sqrt{p_u} (\tau + \Delta /2) \right] \\
    v(\zeta = 0, \tau) &= 0.
\end{align*} \] (3) (4)

where \( \sigma \) is the XPM coefficient. As we can see, in Figure 18, as \( \sigma \) increases, we need greater power to get the same value of \( T \).

\[ \begin{align*}
    \frac{\partial u}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + \left( |u|^2 + |\sigma| |v|^2 \right) u + \kappa v &= 0 \quad (1) \\
    \frac{\partial v}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 v}{\partial \tau^2} + \left( |v|^2 + |\sigma| |u|^2 \right) v + \kappa u &= 0 \quad (2)
\end{align*} \]

4.2 Phase-controlled switching

The controlled switching is performed through the use of a control pulse in the unused coupler input \( v \). We can control the switch because there is a phase difference between the control pulse and the data pulse (Fig. 20) [11].

Unlike the self-switching, the switching controlled target pulse is dynamically controlled by the network.

\[ \begin{align*}
    u(\zeta = 0, \tau) &= \sqrt{P_m} \sec h \left( \sqrt{P_m} \tau \right) \\
    v(\zeta = 0, \tau) &= \frac{\sqrt{P_m}}{\sqrt{r}} \sec h \left( \sqrt{P_m} \tau \right) \exp(i\phi).
\end{align*} \] (3) (4)

where \( P_m \) determines the coupler's operating point, \( r \) is the relation between the power of the two pulses and \( \phi \) is the phase difference between them.

4.2.1 Transmission Optimization

The transmission curves as a function of the relative phase between the two pulses, \( \phi \), are obtained for a given pair of values \( (r, P_m) \). In [11] we can found an
optimization method that allows us to get best pair \((r, p_{\pi})\) for a half-beat coupler. According to [11] the best pair is \(r = 15\) and \(p_{\pi} = p_{\text{opt}} = 7.5\) (see Fig. 21).

As the self-switching in controlled switching is necessary to determine the minimum temporal distance between the two pulses so there is no interference between them. According [11] \(\Delta_{\text{min}} = 4\). The Figure. 20 we can see the absolute value of the input and the control pulse, with \(\Delta = 25\), \(p_{\text{opt}} = 7.5\) and \(r_{\text{opt}} = 15\).

Fig. 23 presents the evolution along \(\zeta\) for two pulses introduced at the channel \(u\) input with \(p = 7.5\), synchronized with two pulses in channel \(v\) with relative phase \(\phi = 0\) (parallel-state) and \(\phi = 180^\circ\) (cross-state).

As the self-switching in controlled switching is necessary to determine the minimum temporal distance between the two pulses so there is no interference between them. According [11] \(\Delta_{\text{min}} = 4\). The Figure. 20 we can see the absolute value of the input and the control pulse, with \(\Delta = 25\), \(p_{\text{opt}} = 7.5\) and \(r_{\text{opt}} = 15\).

4.2.2 Cross-phase modulation effects

In this section we analyze the influence of XPM effect on controlled switching. In Figure 24 is shown the transmission curve for the optimized parameters of the coupler. It can be seen that for values of \(\sigma < 0.2\) there is an increase of the margin phase \(\Delta\phi_{1}\) and reducing the margin \(\Delta\phi_{2}\) for values \(0.2 < \sigma < 0.9\) does not exist \(\Delta\phi_{1}\) and there is an increased \(\Delta\phi_{2}\).

Normally the XPM effects in twin-core fiber couplers are neglected however, in Figure 25 we can see a small increase in the value of \(\sigma\) completely influence the switching pulse.
5. CONCLUSION

It was shown that linear regime is governed by group velocity dispersion, which causes interference between symbols and therefore decreases the maximum possible transmission rate. Considering the optical coupling in the linear regime, it was concluded that switching is periodic i.e. when the signal is at a maximum core, the other is zero. It was also shown that there is no distortion pulse propagation, where the optical intensity remains constant after each energy transfer cycle. When the coupling is completely synchronous, i.e. when the two cores have the same propagation constant, energy transfer between cores is periodic and complete. However, when the coupling is asynchronous, the energy transfer is periodic but not complete.

Analyzing the behavior of the pulse in the non-linear regime, we conclude that the possibility of having solitons in the optical fiber is the result of a balance between the group velocity dispersion and self-phase modulation. Acting separately both effects limit the performance of systems. However, in the anomalous dispersion regime it is possible that the SPM effect is such that fully compensates the broadening of pulses induced dispersion. The pulses in this situation will propagate unchanged, maintaining its shape. It was also found that no loss of the fundamental soliton maintains its shape along the fiber thus making its use attractive in the telecommunication system.

We have shown that the coupled nonlinear equations that govern the switching dynamics of fiber couplers should always take IMD into account, because the variation of the coupling coefficient with the frequency is not negligible. When considering the nonlinear effects in the switching of solitons, the IMD influence in nonlinear regime is sharper than in the linear regime.

In the study of self-switching, we have defined two states: parallel state and cross state. Half-beat and full-beat couplers were considered. The transmission curves for these two types of couplers were obtained. It was concluded that, unlike the half-beat coupler, the full-beat coupler does not have a well defined characteristic for power transmission from channel to another. So this coupler was excluded because of inability to control the destination of pulse.

Analyzing the XPM effect on the self-switching, it was concluded that, when $\sigma$ increases, more power is required at the entry, for the same value $T$. It is also necessary to choose two levels of power further afield, as $T$ grows slower. In locally controlled switching we conclude that for $\sigma > 0.2$ the effect of XPM begins to influence the switching pulses.

6. REFERENCES
