Portfolio Optimization Using Fundamental Indicators Based on Multi-Objective EA

António Silva
Dep. Engineering and Management Instituto Superior Técnico
Lisbon, Portugal
danieltony2@gmail.com

Abstract—**This work presents a new approach to portfolio composition in the stock market. It incorporates a fundamental approach using financial ratios and technical indicators with a Multi-Objective Evolutionary Algorithms to choose the portfolio composition with two objectives the return and the risk. Two different chromosomes are used for representing different investment models with real constraints equivalents to the ones faced by managers of mutual funds, hedge funds, and pension funds. To validate the present solution two case studies are presented for the SP&500 for the period June 2010 until end of 2012. The simulations demonstrates that stock selection based on financial ratios is a combination that can be used to choose the best companies in operational terms, obtaining returns above the market average with low variances in their returns. In this case the optimizer found stocks with high return on investment in a conjunction with high rate of growth of the net income and a high profit margin. To obtain stocks with high valuation potential it is necessary to choose companies with a lower or average market capitalization, low PER, high rates of revenue growth and high operating leverage.**

Keywords—evolutionary algorithms, fundamental analysis, technical analysis, stock investments

I. INTRODUCTION

There are several ways to invest in the stock market namely technical analysis, value investing and the random walk theory. Technical analysis, studies the market patterns, and the demand and supply of stocks shares [1]. Value investing studies the financial information of the industries sector where the company operates to find the intrinsic value of a stock. The Random walk theory defends that the market discounts all future developments so that the investor cannot expect to outperform the general market [2].

Capital allocation is the set of decisions that an investor needs to take to solve the portfolio optimization problem, it has to divide his capital in a number of assets to maximize the expected return and minimize the risk. This is a complex problem, depending on the correlation of the assets, budget constraints and preferable industries by the investor.

This type of problems are complex and practically impossible to solve by deterministically techniques, this is the reason why researchers developed heurists like local search (LS), Tabu Search (TS), Simulated Annealing (SA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), and Evolutionary Algorithms (EA) to solve them[3].

EA are based in the process of biologic evolution, where the solutions of the problem are encoded in chromosomes and evaluated using a fitness function for selecting the best candidates to generate new populations. The repeating of the process of evaluation and reproduction trains the algorithm in finding solutions closest to the global optimal solution of the problem. The heuristic allow encoding in the chromosomes of different data structures that is a way of solving the problem with constraints [4].

The research done in the field proves that it is a heurist capable of solving and finding the pareto set, because EA are capable of processing a set of solutions in parallel and finding a good approximation in a single run [5].

EA solve high dimensional spaces problems because they combine features from random search and Monte Carlo methods with some powerful heuristics borrowed from natural evolution [6].

In this paper section II presents the state of art of the portfolio model optimization and the technical systems for active portfolio management. Section III discusses the proposed approach using EA to optimize the investment models developed. The experiments and results are present in section IV. Finally the conclusions appear in section V.

II. RELATED WORK

A. *State of the Art of Portfolio Optimization*

Some real-world problems involve simultaneous optimization of several incommensurable and often competing objectives. Normally there isn't only a single optimal solution, but a set of optimal solutions.

Multi-objective Optimization (MO) is a method to solve the problem of finding the best solutions when optimizing two or more objectives that are in conflict with each other, subjected to certain constrains [7].

There are many measures of risk and return to evaluate the performance of a portfolio, and these measures can be used as the objectives to be optimized by the EA. The most popular in portfolio management are the Compound annual growth rate

CAGR%, Managed Account Report (MAR) ratio, Sharpe Ratio, Value at Risk (VaR), the Mean (Portfolio Expected Return), and the variance [3].

Tettamanzi & Loraschi in 1993 describe a Multi-Objective Evolutionary Algorithm (MOEA) using the Markowitz model [6], but the measure of risk used is the lower partial moments or downside risk introduced by Harlow [8]. This objective takes into account the down-side part of the distribution of returns. The research proves that downside risk make the use of quadratic optimization techniques impossible, because the shape of the objective function is non-convex.

Cesarone, Scozzari, & Tardella used an algorithm based approach that starts from a pair of assets in the portfolio and tries to add one each time for the MVCCPO model. The simulations proved this model of investment has better performance than the Markowitz classical model [9].

In the work done by Chang, Meade, & Sharaiha in 2000 tree heuristics TS, GA and SA are used to solve the portfolio optimization problem. They also study the problem of finding the efficient frontier using the MVCCPO model. The results prove the existence of cardinality constraint affects in the shape of the efficient frontier, causing discontinuities in the curve [10].

In the work Lin & Wang in 2001, NSGA-II is used in the Markowitz´s Model with constraints of fixed cost and minimum lots. The results show the efficiency of the GA is undermined without the fitness scaling, and the transaction costs dislocates the pareto curve in the vertical axis [11].

Schaerf in 2002 uses MVCCPO model to compare and combine different neighborhood relations in the Pareto front, with local search strategies to find it [12].

Schyns & Crama in 2003 [13] describes the application of simulated annealing (SA) for solution of the Classic Markowitz model with more realistic constraints, the quantity constraint, cardinality constraint, turnover constraints and trading constraints. The advantage of using SA over other heuristic methods is the ability to avoid getting trapped in optimal local points, its flexibility and ability to approach global optimality. The important conclusions of this paper are the introduction of trading constraints is difficult to handle, and there is a trade-off between the quality of the solutions and the time of the simulations to find them.

Lozano & Armañanzas in 2005 uses the heuristics greedy search (GS), SA, and the ACO. For the simulations they used data from five different market indexes. They varied the number of assets (K) in the portfolio for each simulation. The simulations show, that fewer assets in a portfolio can represent a higher expected return but it is obtained with a higher variance of the return. They concluded that ACO is a better heurist than the others to obtain portfolios solutions with higher risk more close to the true pareto front, and the SA fits better for lower risk values [14].

Clack & Patel in 2007 compared the performance of a standard EA against an Age-Layered Population Structure EA (ALPS EA). They use in the portfolio a basket of 82 stocks of the 100 available. The simulations performed showed that ALPS EA reduces the premature convergence, providing better fittest solutions than the EA [15].

Ghang, Yang, & Chang in 2009 [16], tested different risk measures in substitution of the mean-variance one of them is the variance with skewness, developed based on the theory that portfolio return may not be a symmetrical distribution, this means that the distribution of return of individual assets tend to exhibit a higher probability of extreme values, like it has been suggested first by Samuelson in 1958 [17]. The results show that MOEO is capable of finding a wider spread of solutions than the others algorithms, and is capable of competing with NSGA-II, SPEA 2 and PAES in portfolio optimization problems.

Krink & Paterlini proposed a new MOEA the Differential evolutionary for multiobjective Portfolio Optimization (DEMPO), that they compared with the QP and NSGA-II. The advantage of DEMPO is the ability to solve the portfolio optimization problem with real-world constrains with satisfying results in reasonable runtime [18].

Hirabayashi, Aranha, Hitoshi in 2009 proposed a GA to generate trading rules based in technical indicators (RSI, MA, percent difference from moving Average). The algorithm after entering in a position, will exit it based on the following genes stop loss or take profit optimized. They used this system to trade in the forex market (FX) [19].

Golmakani & Fazel in 2011 [20], used an extended model of Markowitz, with four constraints (minimum transaction lots, sector capitalization lots, cardinality, and quantity constraints). The authors proposed a heuristic called CBIPSO (combination of binary PSO with improved PSO) to solve the portfolio optimization problem of Markowitz. They compared their heuristic against the GA proposed by Soleimani in 2007 [21]. In the simulation they tested different portfolio sizes and expected returns, and they conclude that the CBIPSO outperforms Genetic Algorithms (GA), can achieve better solutions in less amount of time.

Hassan & Clack in 2009 tested the combinations of two techniques mating restriction and diversity enhancement in the algorithm SPEA2, to improve the robustness and the diversity of the solutions. To evaluate the quality of the solutions they used the Sharpe ratio [22].

Casanova in 2010 used a Learning Classifier System (LCS) in a dynamic learning system to select the stocks to invest based in technical and intuition analysis, the revaluation period RP, Average Revaluation Period(ARP), RSI, MA, DMA are the indicators used for ranking the best stocks for trade, considering the genes parameters (Days; Minimum value selection of the parameter, Variation allowed of the best stock, type of price) for each indicators; with a system for tactical asset allocation call Tradinnova-LCS, simulates the intelligent behaviour of an investor in a continuous market to form the portfolio. The system tested outperforms all the investment funds analysed by the INVERCO in the periods of simulation [23].

Gorgulho, Neves, & Horta in 2011 implemented an expert technical trading system, describing the system architecture and the investment simulator, and used GA to found the solutions. They tested the system against B&H strategy, and random selection, to prove the superiority of the GA system based in technical signals [24].

The approach of Kaucic presented in 2012 [25] is a trading system based in technical analysis, where an investment module is used to manage a portfolio with long and short positions to generate the so-called long-plus-short portfolio. A technical module is used for detecting overbought/oversold conditions and short-term changes in relative value in contrast to long-term trough a learning mechanism using EA that manages the information derived from the technical indicators incorporated.

Pandari, Azar, & Shavazi in 2012 [26] developed a MOEA model with six objectives to optimize, and tested it against the classical model of Markowitz. The conclusion that they arrive, is that their model use less risk, due to the higher number of objectives optimized by the algorithm.

Table I presents a summary of the different solutions related to the optimization of portfolios using several parameters to describe their main characteristics.

III. SYSTEM ARCHITECTURE

A. *Algorithm architecture*

The system architecture is presented in Fig. 1 constituted by three main modules, the Investor Simulator, the Optimizer and the Data. The main blocks of the architecture are the Data block that is accessed by the investment simulator to test the strategies, and the optimization block to implement the MOEA.

The Optimization Block uses the results Return of Investment (ROI) and Variance, obtained in the investor block to calculate the fitness function and evaluate the strategies. It also selects them for reproduction and applies the methods of crossover and mutation to create news chromosomes.

TABLE I. Overview over different approaches to portfolio optimization

Reference	Period of Simulation	Algorithms Utilized	Markets tested	Fitness functions	Constraints	Portfolio analysis	Results Obtained
Chang, Meade, & Sharaiha [10]	Mar 1992 to Sep of 1997	GA TS SA	Hang Seng DAX, FTSE S&P, Nikkei	Mean, Variance	minimum lots	Markowitz´s Model	Best results obtained with the GA Heuristic
Lin & Wang, [11]	Mar 1992 to Sep of 1997	GA based on NSGA –II and Genocop	Hang Seng index	Mean, Variance	fixed transaction costs minimum lots	Markowitz´s Model	The proposal GA solve the portfolio selection problem efficiently
Schaerf, [12]	NA	TS SA LS	Hang Seng, DAX,FTSE,S&P, Nikkei	Average percentage loss	cardinality quantity	Markowitz´s Model	The Tabu search is the heuristic that achieves the best Pareto Curve.
Schyns & Crama, [13]	6 Jan of 1988 to 9 Apr 1997	SA	151 US Stocks	Mean, Variance	Floor ceiling, turnover, trading and quantity	Markowitz´s Model with	The SA is able to handle more classes of constrains than other heuristics
Lozano & Armañanzas, 2005 [14]	Mar 1992 to Sep 1997	GS SA ACO	Hang Seng DAX ,FTSE S&P ,Nikkei	Mean, Variance	cardinality quantity	Markowitz´s Model	They obtained a portfolio with a return of 3 and risk 0.1
Clack & Patel, [15]	31 of May of 1999 to 31 of Dec 2005	ALPS[1] system incorporated in GP	FTSE 100	Sharpe Ratio		Non-Linear Model	The ALPS GP obtained a return of 50%,and the standard GP a return of 33%
Ghang, Yang, & Chang, [16]	From Jan2004 to Dec 2006	GA	Hang SENG FTSE S&P	Mean, variance, Semi-variance, Variance with Skewness		Markowitz´s Model	The higher return obtained for S&P was 0.0023 with a risk 0.0008
T. Krink, S. Paterlini[17]	From Oct 2001 to Oct 2006		SPMIB.I	Montly Return, and Variance	Quantity, change of weigh, limits of class asset weigh, limited asset allocation turnover rate	Markowitz´s Model	The DeMPO outperfoirmed the NSGA-II
Chen, Weng, & Li, [27]	Mar1992 to Sep 1997	MOEO algorithm	Hang Seng DAX ,FTSE S&P ,Nikkei	Mean, Variance	cardinality quantity	Markowitz´s Model	Best performance obtained with a MOEO with a Return of 0.00859 and risk 0.000417
Anagnostopoulus & Mamanis, [28]	Mar1992 to Sep 1997	NPGA2,NSGA-II PESA, SPEA2, e-MOEA	Hang Seng DAX, FTSE, S&P, Nikkei	Mean, Variance	cardinality quantity	Markowitz´s Model	The SPEA2 is superiority than others MOEA
Casanova, [23]	2005 to 2009	LCS model	IBEX 35	ROI		Technical Analysis	15,3%
Gorgulho, Neves, & Horta,[24]	06 Jan 2003 to 06 Jan 2009	GA	DJI	ROI		Technical trading system	60%
Kaucic[25]	25 Jan2006 to 19 Jul 2011	EA	DJI	Information Ratio, Omega, Sortino ratio		Technical trading system	70% 125% 119%
Pandari, Azar, & Shavazi, [26]	Mar2002 to Mar2008	MOEA	Tehran Stock	Cumulative Return, Mean Return		GA model using Sharpe Ratio, Markowitz´s Model	600% 350%

The Data Block uses the financial statements and stock quotes to calculate the ratios used by the investor simulator. The Investor Simulator Block tests the strategies obtained in the optimization block, depending on the investment model and using the inputs given by the user, for getting the data for testing. It evaluates in each day the stocks, calculate the ROI of the portfolio from the beginning of the simulation. With the recorded values for each day of the ROI, it is then calculated the monthly variance of the return.

Each strategy is tested in the simulator using the retrieved time series, with a window that slides one day at each increment. The selected financial ratios are evaluated, and the computations are performed for doing the trading decisions of the day. After finishing the time frame of each period of training, it is recorded the results, and performed the reproduction of population. This process is repeated to achieve the number of interaction that signals the end of the training. After the conclusion of the training of the population the external file with the best solutions is used in the real test.

The system uses the following parameters, the period of training, and the period of real test, also transactions costs are included at to 2% of the stock value.

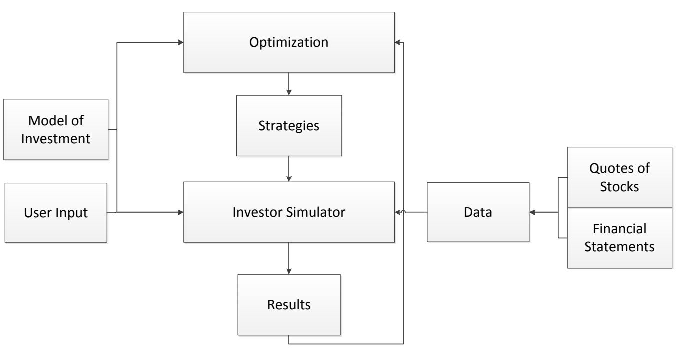

Fig.1. System architecture

B. Fundamental Ratios

From the financial statements information is calculated the financial ratios that are quantitative measures that allow analyzing a company in terms of profitability, liquidity, debt and growth. They are used to compare companies inside the same industry and to draw conclusions about the best companies to investment.

Next it will be presented the financial ratios used to evaluate the companies and the technical indicators used for trading.

Debt Ratio (DR) is used to measure the level of debt, of the company. Companies that have high debt compared to its assets are at greater risk of going bankrupt in the occurrence of an adverse economic cycle, or by suffer a reduce in their profits by increasing interest rates of their debts. Usually companies with low *Debt Ratio* are in extremely competitive industries with constant need for innovation and updates of its products and production processes, which are carried out using external financing.

$$DR = \frac{Total\ Debt}{Total\ Assets} \tag{1}$$

The Return of Equity (ROE) measures the performance of the company to generate profits using the company equity. It is obtained by three factors, the operational efficiency, the efficient use of assets, and the financial leverage. This ratio is used by investors to select companies that maximize the investment made in them, this means for every dollar invested are capable of creating a net profit greater in terms of percentage of capital invested.

$$ROE = \frac{Net\ Income}{Total\ Equity} \tag{2}$$

The Profit margin ratio (PM) measures the profitability of the company, calculating the percentage of the revenue retained after paying the costs of operating, administrative, financial and taxes.

$$PM = \frac{Net\ Income}{Revenue} \tag{3}$$

When analyzing the evolution of the *Revenue* with the *Profit Margin* it is possible to make valid predictions about the future of the company.

The Price Earnings Ratio (PER) is a valuation ratio of a company´s current share price compared to its per-share earnings; it is used to choose the companies that are more undervalue. PER tends to be lower for companies such as slow growers and higher for fast growers because in the share price is incorporated investor expectations regarding to the future. The best use of the ratio is to find the cheapest company in a sector, but can be used to compare companies in different business sectors. Allow to compare the historical record of PER in order to get a better perspective on the levels of overvaluation and undervaluation of each stock.

$$PER = \frac{Share\ Price}{EPS} \tag{4}$$

The percentage of revenue growth (RG) is an economic indicator that shows the evolution of the business, two factors are responsible for its increases, the company is a better competitor and is gaining market share to other competitors, or the company is inserted in a fast growing sector and are growing with it. Although this indicator is important to analyze the company from the growth point of view, not give any indication of the profitability of the company.

$$RG = \frac{Revenue_{Actual} - Revenue_{Last\ Year}}{Revenue_{Last\ Year}} \quad (5)$$

Common stock outstanding (CSO) represents the fundamental ownership hold of the corporation by the shareholders. When the company issues shares this number is added to the previous value in the *Balance sheet*, representing an increase of total number of shares and a distributing of the company's value by a greater number of shares. The reduction of the number of outstanding shares represents an increase in EPS and a decrease of PER ratio.

The objective when using this indicator is to find companies that in recent years have repurchased its shares and thus reduced the number of shares outstanding.

$$\Delta CSO = \frac{\mathrm{CSO}_{Actual} - \mathrm{CSO}_{Last\ year}}{\mathrm{CSO}_{Last\ year}} \quad (6)$$

A positive trend of net income (NI) shows that the company has increased its profits, the objective of this indicator is to choose the companies with higher growths. A positive trend of net income shows that the company has consistency and durability in the profits, and a good result obtained in one year, are not just a result of favourable economic growth of the GDP or a financial engineering. When comparing the trends of net income and stock price, divergences can be found between both, and can be an indication of undervaluation of the company, if the net income trend shows a positive tendency and the stock prices don´t show a similar behaviour.

$$\Delta\ \mathrm{NI} = \frac{\mathrm{NI}_{\mathrm{Actual}} - \mathrm{NI}_{\mathrm{Last\ Year}}}{\mathrm{NI}_{\mathrm{Last\ Year}}} \quad (7)$$

The Payout ratio measures the percentage of net income distributed by the investor has dividends. Companies with high PR are stable companies that are investing less in the growth of the company, and is stock price will valorize less than fast growth stocks. The objective is select companies with low payout ratios.

$$\text{Payout Ratio} = \frac{Dividends\ per\ Share}{Earnings\ Per\ Share} \quad (8)$$

Companies with increased capital expenditures (CE) where the net income not have the same behaviour is probable inserted in a competitive industry. The objective of this indicator is to avoid companies with higher positive variation of capital expenditures than the NI.

$$\Delta\ \mathrm{CE} = \frac{CE_{Actual} - CE_{Last\ Year}}{CE_{Last\ Year}} \quad (9)$$

Cash from operating activities is an indicated used to check the real efficiency of operations in generated money, because the companies can have in the income statement high operations net income, but can be an inefficient in the collection of tis profits. This indicator is used to select companies that have high net incomes produced by the operations.

$$\Delta\ \mathrm{CFOA} = \frac{CFOA_{Actual} - CFOA_{Last\ Year}}{CFOA_{Last\ Year}} \quad (10)$$

C. *Designing the trading strategy*

An automatic trading system is the set of parameters and indicators used that specifically define what the algorithm will do. The trading indicators and parameters are going to be presented next.

They are markets to invest, market timing, protection of capital, exit of a wining position, position size, and stock number.

Select the Markets to invest, considering the available budget, possible cost in transactions, the knowledge it has about the business of the companies to invest, the access to information, the experience. In this approach was selected the stocks that compose the index SP&500.

Entry or Market Timing is the exact price and the market conditions that need to be present to enter in a stock position, the objective is to define a set of rules that give an entry signal to improve the timing of buying and increase the reliably of the system[29]. There exist a great number of different types of moving averages, depending of the calculated method, but it functions and interpretations are the same.

Simple Moving average (SMA) is a technical indicator used to predict the price tendency, the intersection between the SMA and the price, defines the point to enter the market. In Fig. 2 is a trend price with a SMA of 50 days.

Fig.2. Trend of SP500 with SMA of 50 days

Stops Loss and Protection of capital are predetermined policies that reduce a portfolio exposure, by getting out of a losing position, not allowing one or more investments to continue to lose money, and protect the remaining capital available to continue the investing activity. The stop loss percentage, defines the maximum loss for each investment, by defining the exit point, when the price tendency does not follow as expected, the stop price is calculated using equation 8.

$$P_{Stop} = P_{Entry} \times (1 - StopLoss) \qquad (8)$$

For **Exit of a winning position** it is used the parameter take profit, it has the function of deciding the percentage profit obtained for the investment made. This will determine the sale value of successful investments based on the purchase price of the shares according to equation 9.

$$P_{profit} = P_{Entry} \times (1 + TakeProfit) \qquad (9)$$

The **position size** is the percentage of the current portfolio value to invest in each new stock this defines the level of concentration or diversification in the portfolio. For example an strategy that invest a low percentage in each investment, the portfolio can have a higher number of assets, if the value of position size is high, for example 20 % which is the maximum possible due to Quantity constraint, the Portfolio will be limited to 5 stocks.

The **stock number** parameter defines the number of stocks that the portfolios need to have.

D. Chromosome structure

The representation of the chromosomes is similar for the two investments models, where each individual of the population is composed of a sequence of values called genes.

The Chromosome is divided in two parts, the first group is the financial ratios weights and the second one is the trading parameters.

- First Model of the Chromosome

This model only invests in companies that have some potential for a good return, using the first parameter of trading, the gene of Min value for Portfolio optimized, to filter the candidates to enter in the portfolio. This gene defines the minimum global value of the stock that a candidate needs to have to enter in the portfolio.

The first seven genes are financial ratios weighs. The remaining genes define the behavior of the trading systems, in Fig. 3 is represented the model of the chromosome. The seven weights are the DR, ROE, PM, PER, GR, Variation of Commons Stock outstanding, and Growth in Net income.

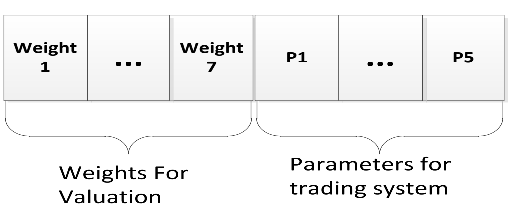

Fig.3. Chomossome that represent the first model

The **Global value** of each stock is a valuation done daily using the trimestral financial ratios, and the daily PER that is actualized using the adj. close price, to ranking the stocks for the algorithm to make a trading decision, this values are calculated using the vector of weights of ratios of the chromosome multiplied by the respective ratios of the company, as demonstrated by Eq. 10.

$$Global_{value} = \sum_{i=1}^{n} weight_i * Ratio_i \qquad (10)$$

It uses five parameters of a technical trading system the stop loss, take profit to determine the exit points, the number of days to use in a SMA for trigger the entry point, and a size position to define the percentage to allocate to each stock. In Fig. 4 is represented the part of chromosome that refers to the trading parameters.

Min value for Portfolio	Stop Loss	Take profit	Days of MA	Size of the Position

Fig.4. Trading Parameters for first Model

- *Second Model of Chromosome*

This model was developed to simulate a portfolio that always keeps a determined number of stocks in it. It has the first seven genes equal to the previous model, and the gene of the position size, and adds a new one, the limit number of stocks in the Portfolio.

The model operation is to maintain the stocks in the portfolio which have highest valuations, when one stock outside of the portfolio has a better valuation that the worst in the portfolio, this will be replaced by the first. In Fig. 5 is represented the chromosome.

Weight 1	...	Weight 7	Stocks Number	Size of the Position

Fig.5. Chromossome representing the second model

- *Third Model of Chromosome*

This model was developed to improve the efficiency of the MOEA with the increase of fundamental indicators used in the valuation of stocks. It is introduced to the previous valuation stocks models three new fundamental indicators, the payout ratio, the Growths of CFOA and CE. The trading parameters are equal to the Fig. 6.

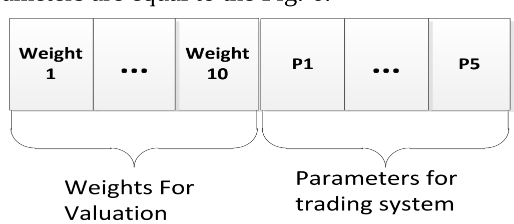

Fig.6. Chromossome representing the second model

E. Constraints

For the simulations of the portfolios to be more realistic as possible in the design of the models was included some real constraints. For the first model it was considered the following four, while for the second model is excluded the transaction costs constraint.

Cardinality constraint limited the portfolios to a maximum of 20 stocks, the limit was chosen based on obtaining the benefits of a diversified portfolio, and studies on mathematical models that show maintaining a very diverse portfolio of stocks about 30 presents a higher level of risk reduction [30].

The limit allows greater economic monitoring of the selected companies, like the philosophy of Warren Buffett, in which it should be possible to focus more attention on a smaller set of companies instead of making a great diversification, because the investor have better knowledge about the company, its products, competitors, debt levels, and future long-term economic prospects of the business [31].

Quantity constraint was used to limit the gene position size, by putting a maximum and minimum value for the size of the position. The minimum limit is set to 5% of the portfolio value at the time of the transaction, and a maximum value is 20%. The lower limit is to avoid positions practically insignificant to the performance of the portfolio, and the upper limit is to avoid too much exposure or weight for any stock.

Long only constraint signifies that it is not allowed to perform short selling operations, meanings the weight invested in any stock is always positive.

It is a restriction used by a great number of value investors and institutional funds because the risks associated with the short selling it is considered higher than with long positions.

Transaction costs for the first model is charged commission of 2% of the value of each transaction for simulating real costs. For the second model the transaction costs are ignored to allow the algorithm to change the assets without restriction and affecting the return.

F. Optimization strategy

For optimizing the investment models, it was used MOEA proposed by Zitzler and Thiele in 1998 the SPEA II [5]. The crossover method selects from the mating pool four chromosomes randomly, to provide their genes for a new chromosome. The mutation performed selects randomly genes, from one to four, and mutate them using a rate limited by an interval. For evaluating the solutions it was used the measures of ROI and Variance.

IV. RESULTS

A. First model using Multi-objective

The simulation for the first model of the chromosome was trained from 2010-06-17 to 2012-01-03, using a MOEA with two objectives comulative return and variance. In Fig 7 it is presented the results of the real test executed between 04.01.2012 and 07.06.2013, with the population obtained in the training. Also in this figure the training of the algorithm for this same period of test is shown. The trained pareto curve is more ideal, but the test points (with values that came from the training in the previous period) also show that they represent a pareto curve with diverse and well-spaced points.

The performance of the index represented as triangle in the figure for the same period proves the use of this MOEA is a better investment technique than just following the index.

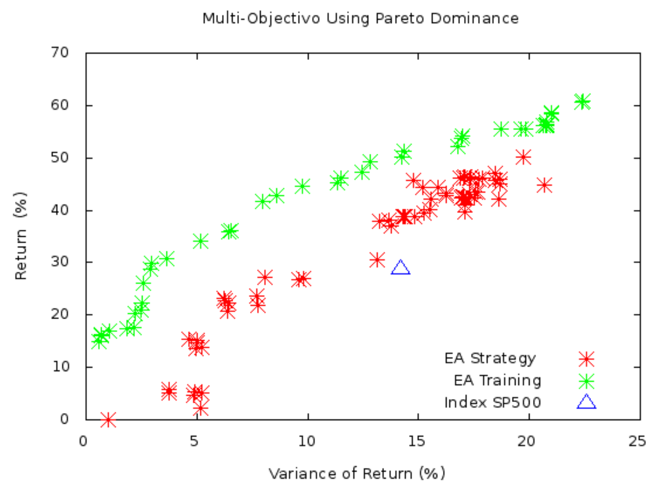

Fig.7. Results from real test using the objectives return and variance of return.

From this simulation were selected five results with different variances to analyze the chromosomes and get a better insight on the type of solutions found by the algorithm. Table II shows the results for the two objectives of the chromosomes selected.

TABLE II. Results from the test

	Return (%)	Variance (%)
SP&500	28.69	14,20
Chromosome ID		
1	50.24	19.71
27	45.83	17.70
36	38.81	14.28
51	13.66	4.94
54	22.73	6.25

In Table III is presented the value of the genes for the same chromosomes. When analyzing the genes of the first chromosome it can be seen it has a preference for companies with high profit margins, earnings growth, and uses little diversification. The chromosome 54 is a strategy that allows greater diversification, it allocates 5% on each stock, the ratios with more importance are the ROE, and the rate of growth of net income. It is a more conservative strategy where it invests in established businesses with monopoly characteristics. The chromosome 51 is the strategy that has the higher frequency of trading, but obtained a lower return, due to the transactions costs. The analyses of the genes in the chromosome, demonstrates the higher returns are achieved by a better selection of companies with a higher concentration of the investment.

TABLE III. Genes of the Chromosomes

Genes	C1	C27	C36	C51	C54
Debt ratio	0,53	0,53	0,28	0,28	0,28
ROE	0,64	0,12	2,24	0,12	2,24
Profit Margin	3,01	0,61	0,49	0,96	1,01
PER	0,54	0,40	0,23	0,23	0,27
Δ of Rev.	0,34	1,13	0,62	0,65	0,57
Δ common Stock out	1,33	1,08	1,04	0,97	1,25
Δ net income	1,79	2,07	1,24	1,70	1,95
Global Value	0,00	0,00	0,00	0,00	0,00
Stop Loss	1,11	0,88	1,98	0,88	0,82
Take Profit	1,89	2,46	1,18	0,12	1,36
Days of MMA	1,00	5,00	4,00	1,00	1,00
Position Size	0,20	0,20	0,20	0,05	0,05

B. Second Model

The population in this simulation was trained using the MOEA during the period 2010-06-17 to 2012-06-11. The real test is executed from 2012-06-12 to 2013-06-11. The results in Fig. 9 are solutions with lower risk in relation to the index, where it is verified the effect of the efficiency in the use of risk for the obtained returns.

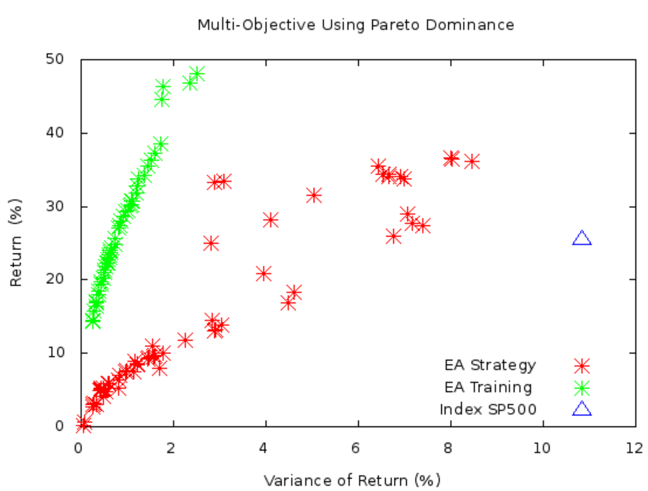

Fig.9. Result of real test using the second model

Five chromosomes were selected from the population based in the results that they obtained in the real test, they are shown in Table IV.

TABLE IV. Results of the real test

	Return (%)	Variance (%)
SP&500	25.55	10.82
Chromosome ID		
2	36.4	8.02
10	27.59	7.18
23	18.31	4.58
29	11.84	2.25
40	8.89	1.16

By analyzing the genes in Table VI the difference in the results by the chromosome 2 in relation to the others are justified by more concentration of the investments, and the selection of companies with better profit margin and net income. The lower return of chromosome 40 is justified by the lower importance to the level of debt and Profit margin of the companies with lower percentages invested in each stock.

TABLE V. Genes of the Chromosomes

Genes	C2	C10	C23	C29	C0
Debt ratio	0,000006	0,000002	0,017	0,0003	0,000002
ROE	0,00015	0,00015	0,31	0,0029	0,31
Profit Margin	0,3578	0,0059	0,02617	0,2156	0,00286
PER	1,11	1,11	2,099	0,8637	2.099
Δ of Rev.	0,0739	0,0739	0.8549	1.0786	0,01089
Δ common Stock out	1,3989	1.3983	1.9047	1.0967	1.5
Δ net income	2.9453	1,8516	0.331	0,043	0,638
Stocks Number	8	7	9	6	4
Position Size	0,18	0,18	0,063855	0,0638	0,0638

C. Third Model

The population in this simulation was trained using the MOEA during the period 2012-01-02 to 2012-12-31. The real test is executed from 2013-01-02 to 2014-02-07. The results in Fig. 10 are more concentrated that the previous experiences meaning that with more fundamental indicators it is possible to focus more the type of portfolio management to obtain.

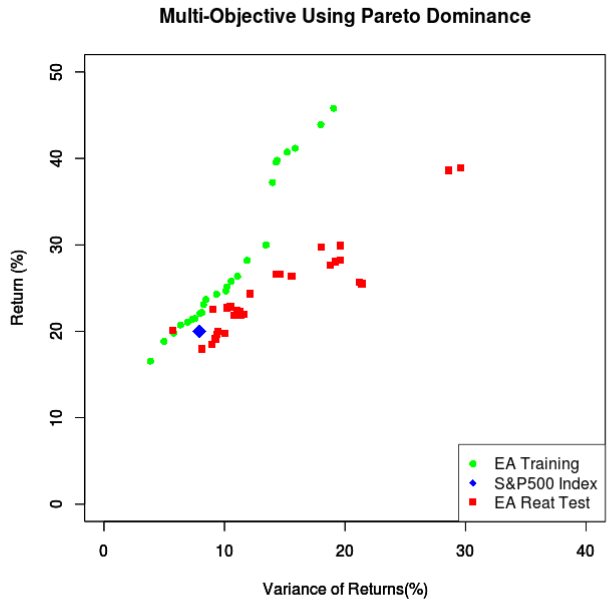

Fig.10. Result of real test using the second model

The Fig.11 represent the results obtained in the training period 2012-01-02 to 2012-12-31for the first and third model. It is verified an higher diversity of results for the first model, with higher divergence of results.

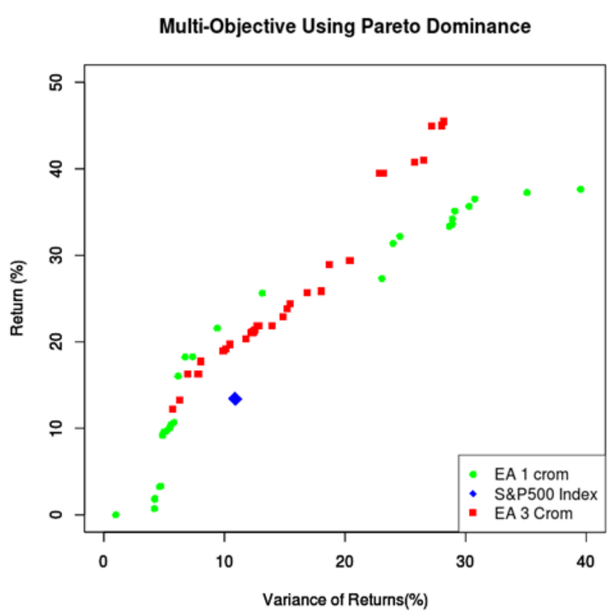

Fig.11. Training of first and third models

In the Fig. 12 is represented for the same period of training the real test for the Model1 and Model 3 in the period 2013-01-02 to 2014-02-07. The analyze of results show the first model of chromosomes have a higher despersion of results than the third model. It is concluded that the introduction of other variables to evaluate the stocks improve the MOEA.

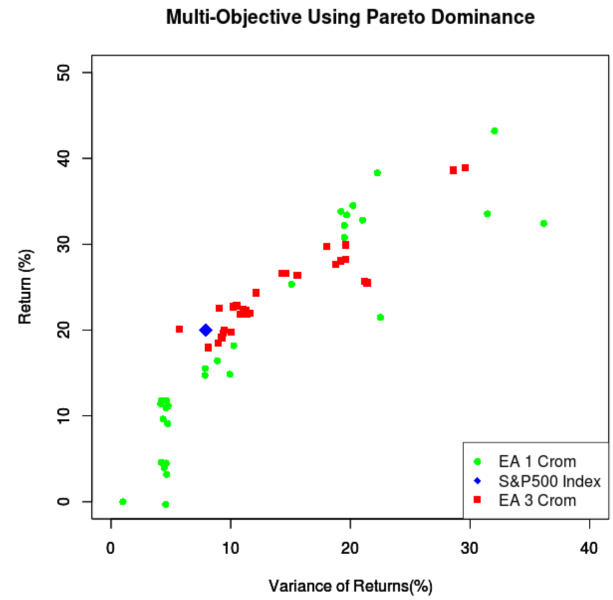

Fig.12. Result of real test using the first and third model

V. CONCLUSIONS

This work proposes a multi-objective GA to efficiently manage portfolio compositions.

This approach can be applied to the management of portfolios with great results due to the optimization of the strategies and the ability to analyze hundreds of companies in a few seconds by the algorithms.

The EA using fundamental analysis and trading parameters to stock investment finds solutions that in general outperformed the Index.

The solutions with higher returns and with less variance in the portfolio are due to higher concentration of the investments in a few stocks, and better selection of the companies based in the financial ratios ROE, debt ratio, and growth rate of net income.

REFERENCES

[1] S. Achelis, *Technical analysis from A-to-Z.* Vision Books 2000.

[2] J. Murphy, *Technical analysis of the financial markets:A comprehensive guide to trading methods and Aplications.* Prentice Hall Press 1999.

[3] K. Metaxiots, K. Liagkouras, "*Multiobjectve Evolutionary Algrithms for Portfolio Management:A comprehensive literature review*" Expert Systems with Applications, Vol. 39, pp. 11685- 14085, 2012.

[4] F. Streichert, H. Ulmer, A. Zell, "*Evaluating a hybrid encoding and three crossover operators on the constrained portfolio selection problem",* Evolutionary Computation, CEC, 2004.

[5] E. Zitzler, L. Thiele, "*An Evolutionary Algorithm for Multiobjective optimization: The Strength Pareto Approach",* TIK-Report, NO. 43, MAY, 1998.

[6] A. Tettamanzi, A. Loraschi, "*An evolutionary algorithm for portfolio selection in a downside risk framework",* 1993.

[7] C. Fonseca,, & P. Fleming, P. (1993). Genetic Algorithms for Multiobjective Optimization:Formulation, discussion and generalization page 416--423. San Francisco, CA, USA, Morgan Kaufmann Publishers Inc, 1993.

[8] H. *Harlow, Asset allocation in a downside-risk,* Financial Analysts Journal, *Vol. 47, No. 5, pp. 28-40 1991.*

[9] F. Cesarone, A. Scozzari, F. Tardella, "*Efficient Algorithms for Mean-Variance Portfolio Optimization With Hard-World Constraints",* 2009.

[10] T. Chang, N. Meade, Y Sharaiha, *Heuristics for cardinality constrained portfolio optimisation"*, Computer & Operations Research, No 27, pp 1271-1302, 2000.

[11] D. Lin, S. Wang, "*A multiobjective genetic algorithm for portfolio selection problem",* 2001.

[12] A. Schaerf, "*Local Search Techniques for Constrained Portfolio Selection Problems*", Computational Economics, pp. 177-190, 2002.

[13] M. Schyns, Y Crama, "*Simulated annealing for complex portfolio selection problems*", European Jornal of Operational research, 150, pp.546-571, 2003.

[14] J. Lozano, R. Armañanzas, *A Multiobjective Approach to the Portfolio Optimization Problem, 2005.*

[15] C. Clack, S. Patel, "*ALPS evaluation in Financial Portfolio Optimization", IEEE CEC, Sep 2007.*

[16] T. J. Ghang, S. C. Yang, K. J. Chang, "*Portfolio optimization problems in different risk meusures using genetic algorithm.*", Expert Systems with Applications, Vol. 36, pp. 10529- 10537, 2009.

[17] P. Samuelson, The fundamental approximation theorem of portfolio analysis in term of means variances and higher moments. *Review of Economic Studies*, 65-86, 1958.

[18] T. Krink, S. Paterlini, "Multiobjective optimization using differential evolution for real-world *Portfolio optimization.*", Computational Management Science, Vol. 8, pp. 157- 179, 2009.

[19] A. Hirabayashi, C. Aranha, I. Hitoshi. "Optimization of the trading Rule in Foreign Exchange using Genetic Algorithm." Genetic and Evolutionary Computation, GECCO, 2009.

[20] H. R. Golmakani, M. Fazel, "*Constrained Portfolio Selection using Particle Swarm Optimization"* Expert Systems with Applications", Vol. 38, pp. 8327-8335, 2011.

[21] Soleimani, H. (2007). *Portfolio selection using genetic algorithm.*

[22] G. Hassan, C. Clack, "Robustness of Multiple Objective GP Stock-Picking in Unstable Financial Markets." Genetic and Evolutionary Computation, GECCO, 2009.

[23] I. J. Casanova, "Trading-LCS:Dynamic stock portfolio decision-making assistance model with based machine learning", pp. 1-8, 2010.

[24] A. Gorgulho, R. Neves, N. Horta, "Appying a GA kernel on optimizing technical analysis rules for stock picking and portfolio composition", Expert Systems with Applications, Vol. 38, No. 11, pp. 14072 - 14085 , October, 2011.

[25] M. Kaucic, Portfolio Management Using Artificial Trading Systems Based on technical Analysis, 2012.

[26] A. R. Pandari, A. Azar, A. R. Shavazi, "Genetic algorithms for portfolio selection problems with non-linear objectives. *African Journal of Business Management*, Vol 6, pp 6209-6216, 2012.

[27] Chen, M. R., Weng, J., & Li, X. (2009). Multiobjective Extremal Optimization for Portfolio Optimization Problem. *IEEE*, 552-556.

[28] K. Anagnostopoulos, G. Mamanis, "The mean-varaince cardinality constrained portfolio optimization problem:An experimental evaluation of five multiobjectives evolutionary algorithms", Expert Systems with Applications, Vol. 38, pp. 14208-14217, 2011.

[29] V. Tharp, *Trade to Financial Freedom.*McG-Hill, 2007.

[30] D. Hillier, S. Ross, W. Randolph, J. Jeffrey, B. Jordan, *Corporate Finance* (First European Edition ed.). McGraw-Hill, 2010.

[31] L. Cunningham, *The essays of Warren Buffett: Lessons for corporate America, The Cunningham Group 1998.*