MODEL AND DYNAMIC SIMULATION PROGRAM FOR VEHICLE ANALYSIS ACCOUNTING SUSPENSION COMPLIANCE

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ABSTRACT

The goal of the present work was to develop a dynamic model to provide the engineer with the possibility to incorporate the elastic deformation of some suspension components into vehicle simulations. This work will focus on a particular type of vehicle: a Formula Student prototype.

An introduction to the Formula Student competition and the team from Instituto Superior Técnico was the first focus of this work. An evaluation of the current state of simulations including the effect of compliance in the dynamics of the vehicle was done next. The model that allows the engineer to evaluate the vehicle’s performance was developed next. This model uses the different subsystems of the vehicle to return the main performance variables like the accelerations, velocities, trajectory angles and the vehicle’s trajectory. The driver inputs (steering wheel and pedals) are also the main inputs of the model. Next, a decision was taken in regard to the approach to include compliant components along with the parameters that would be affected. The compliant components were chosen and presented along with a revision of their design to obtain the deformations.

Finally these compliant parts were incorporated in the model and simulations were run to verify the influence of compliant components.

Keywords: Dynamic modelling of vehicles, suspension, compliance, dynamic simulation of vehicles

1 INTRODUCTION

Formula Student is an engineering competition that allows students to put in practice the engineering and management practices they learn along their scholar path. The students are encouraged to design, build and race a formula prototype. The design decisions involved in the project and the performance of the vehicle are evaluated during events all over the world where the students showcase their skills to judges respected in the engineering sector.

Projecto FST Novabase is the formula student team from Instituto Superior Técnico. Founded in 2001, it is the longest-active team in Portugal and the most successful. The fifth prototype of the team, FST 05e, will be the modeled car in this work. With the development of earlier prototypes, the knowledge in vehicle dynamics grew within the team and that knowledge has been passed through different generations of members. The FST 04e was the first prototype to be carefully analysed using advanced vehicle dynamics knowledge. Such analysis can be seen in (Neves 2012).

Component’s compliance should be in mind of every designer and vehicle performance analyser and its effects are often overlooked. It was made popular in the Formula Student community that the majority of the cars exhibit excessive compliance because of the low awareness and the ambitious designs of the students.

This work intends to further develop the dynamic model of the vehicle in order to improve the vehicle dynamics understanding of the team and to help in the design of the next prototypes.

2 STATE OF THE ART IN SIMULATIONS CONTEMPLATING COMPLIANCE

Usually in a mechanical model that does not contemplate body interactions, the inclusion of compliant components is very restricted and usually ignored. In multibody dynamics simulation this occurs more often, the modeling of each individual component provides the freedom to introduce flexible bodies and measure their effect in vehicle performance and suspension kinematics. This multibody simulation is not easy to achieve in a Formula Student team and commercial software is usually used for reliability. The most famous software must be the MSC ADAMS. This software has also a vehicle dynamics dedicated environment, making it a favorite not only of Formula Student teams but also some recognized automotive brands.

In regards to measuring the effects of compliance in vehicle kinematics the most common and accepted way to do it is by performing a full vehicle Kinematics and Compliance (K & C) test. These machines provide the engineers with the capability of applying relative displacements between the suspension and the chassis, forces at the contact patch and combined situations to better recreate vehicle steady-state conditions to evaluate kinematic relationships for more accurate vehicle modeling. By performing such tests the engineer is able to build the vehicle model with the real kinematic relationships contemplating compliance
instead of the perfectly kinematic relationships obtained from kinematic multibody simulation. Kinematics and Compliance testing is an expensive effort for a Formula Student team, and there is no capable machine in Portugal to perform said tests.

This work positions itself in the middle of the two dynamic modeling approaches. It models the vehicle behavior by directly using laws of motion equations to describe the vehicle motion, kinematic relationships to model wheel orientations, solid mechanics and even physical testing to obtain the components displacements.

3 VEHICLE DYNAMICS CONCEPTS

3.1 COORDINATE SYSTEMS

In this work, two main coordinate systems and other two for modelling of specific subsystems are used. All of these coordinate systems are in accordance with (SAE 2008). The first coordinate system is the vehicle coordinate system and in this work it is attached to the body centre of gravity (xyz). The equations of motion of the vehicle are expressed in this coordinate system. Secondly, a ground coordinate system (XYZ) is used to track the trajectory evolution of the vehicle. The third coordinate system is used to specify the pickup points of the suspension in the chassis and at the wheel (xsyz). The fourth coordinate system used is the tire coordinate system (xtytz). The tire coordinate system is attached to the contact patch of each tire and accompanies the rotation of the z axis of the wheel.

3.2 SLIP QUANTITIES

The first slip quantity is the side-slip angle, \( \beta \). This is the angle between the vector velocity and the x axis of the vehicle coordinate system.

The second slip quantity is the Slip Angle, SA. This is the angle measured between the direction in which the wheel is heading and the direction of the velocity of that wheel projected onto the xYt plane.

The other slip quantity is not an angle and is a percentage of the slip of the wheel in the xzt plane. The tire longitudinal slip ratio, SL, is defined as a percentage by dividing the tire longitudinal slip velocity by the reference wheel-spin velocity:

\[
SL = \frac{\omega_{wheel} - \omega_0}{\omega_0}
\]  

3.3 VEHICLE GENERAL CHARACTERISTICS

The wheelbase, \( l \), is the distance measured parallel the x direction of the vehicle between the front and rear contact patch on the same side. The track, \( t \), is the distance measured parallel to the y direction of the vehicle between the left and right contact patch of the tire.

In vehicle dynamics, several mass properties are defined and in this work the follow will be used:

- Vehicle operating mass – The mass of the car plus a 68 kg driver;
- Unsprung mass (\( m_{us} \)) – The mass of the car that is not carried by the suspension, being directly supported by the tire
- Sprung mass (\( m_s \)) – The mass of the car that is carried by the suspension being calculated as the subtraction between the vehicle operating mass and the total of the unsprung masses.

3.4 SUSPENSION & STEERING PROPERTIES

The steering angle, \( \delta \), is the angle defined for each wheel as the angle between the x axis of the vehicle and the wheel plane measured about the z axis of the vehicle.

The camber angle, \( \gamma \), is the angle between the z axis of the vehicle and the wheel plane measured about the x axis of the vehicle. The convention is that it is positive when the top of the wheel leans toward the vehicle. An important angle in tire dynamics is the inclination angle, IA, which is similar to the camber angle. The difference is that the inclination angle is the angle measured with respect to the axis perpendicular to the ground plane instead of the z axis of the vehicle and is positive when the top of the wheel is leaning towards the right.
The steering axis is defined by the pickup points at the wheel of the upper and lower wishbone for a double-wishbone suspension. The two important angles are the kingpin inclination angle, KPI, and the caster angle, \(\nu\). These angles contribute to the change of camber angle when steering the wheel.

![Figure 4 - KPI and Caster angles](image)

An important steering characteristic for the modeling approach taken for the steering system is the \(c\)-factor of the steering system. The definition of the \(c\)-factor for a linear steering box is the rack displacement per revolution of the input shaft.

A relevant point in the design process and simulation is the instantaneous center of rotation of the suspension in 2D. This point is defined for each wheel. The angles defined below are used to compute jacking forces that contribute to anti-roll, anti-dive and anti-squat and anti-lift behavior.

![Figure 5 - IC Definition (front view)](image)

![Figure 6 - IC definition (side view)](image)

### 3.5 Suspension Springs and Dampers

In order to know the change in vertical load applied at the wheel center in function of the displacement of the wheel one needs to easily translate the wheel movement to spring movement. The spring displacement in function of the displacement of the wheel can be calculated. This function is called the Motion Ratio, MR. This ratio is linear for the FST 05e.

\[
MR = \frac{\text{Spring Travel}}{\text{Wheel Center Travel}} 
\]  
(2)

With this function the effect of the springs installed can be evaluated by calculating an equivalent stiffness of a spring mounted between the wheel center and the tire, \(K_s\).

\[
K_s = k_s \times MR^2 
\]  
(3)

The same methodology can be assumed for the effective damper mounted between the wheel center and the tire, \(C_s\).

Regarding roll behavior, usually some race cars are equipped with anti-roll bars (ARB), which is the case of the FST 05e. To calculate the equivalent spring between the wheel center and the tire that represents the ARB one needs to first traduce the equivalent torsional stiffness of the bar to a linear stiffness, \(k_{\text{ARB}}\), and then apply the motion ratio of the wheel/ARB couple to get the equivalent stiffness, \(K_{\text{ARB}}\).

\[
k_{\text{ARB}} = \frac{K_{\text{BAR}}}{\text{actuating arm}^2} 
\]  
(4)

\[
K_{\text{ARB}} = k_{\text{ARB}} \times MR^2 
\]  
(5)

In (4), \(K_{\text{BAR}}\) represents the torsional stiffness of a bar as in (Beer, Jr. et al. 2011).

### 4 Model Development – Without Compliance

#### 4.1 Driver Inputs

Since the FST 05e is a fully-electric powered vehicle and the gearbox has a single gear ratio the only inputs considered from the driver will be the pedals (% of pedal defelction for throttle pedal and kg of force for brake pedal) and the steering wheel (input degrees).

#### 4.2 Tire Modeling

In order to be able to simulate the vehicle behavior one must have a way to obtain the tire forces and moments to calculate the car’s accelerations. One way to do this is to use a function that defines the tire. This function must accept certain inputs and return the output forces and moments presents in the contact patch. Hans Pacejka was able to evaluate tire data and trace the typical behavior of a tire with the variation of certain conditions of operation. With this evaluation he was able to create is well-reputed Magic Formula where each output is a function of the operating conditions. This formulation is described in (Pacejka 2005). This formulation makes use of the following operating conditions: \(F_z\) and \(IA\). The variables of the formulations are SA, SL or both.

Tire testing is one of the single most important things for vehicle dynamics. Fortunately, FSAE teams can have access to tire data because of the TTC (Tire Test Consortium). Combining the raw data made available by the TTC contributors and the implementation of the Pacejka functions one can arrive at a tire model:
4.3 AERODYNAMICS

To treat the aerodynamic forces, the approach taken in this work requires the knowledge of the drag and lift coefficients, the area of the wings and also the position of the aerodynamic devices with respect to the center of gravity. In order to translate the aerodynamic forces to the wheels the approach taken in this work is to define an aerodynamic reference point to which all the aerodynamic forces are translated. This results in a point that has all the drag and lift forces and no moment:

\[
\sum M_{RP} = 0 \Leftrightarrow D_y \cdot (-z_{yw}) + L_y \cdot (x_{yw} - x_{RP}) - D_x \cdot (Z_{yw}) - L_x \cdot (-x_{yw} - x_{RP}) = 0
\]  

(7)

With this reference point defined one can add the acceleration caused by the drag force to the acceleration caused by the tire forces and distribute the lift force between the four wheels according to the \( x_{RP} \) position. This defines the lift distribution for use in the vibrational model:

\[
LD = WD - \frac{x_{RP}}{l}
\]  

(8)

4.4 VEHICLE VIBRATIONAL MODEL

In order to calculate the vertical forces in the wheels a vibrational model of the cars suspension will be implemented due to the fact that it provides the possibility to contemplate the transient part of weight transfer, include migrating instant centers and better represent the anti-features.

The equation to be solved is then:

\[
[M] \{\ddot{z}\} + [C] \{\dot{z}\} + [K] \{z\} = \{Q\}
\]  

(9)

In the above equations, \( z \), represents the vector of displacements of the sprung and unsprung masses of each corner and the roll angle of the chassis. \( M, K \) and \( C \) are the mass, stiffness and damping matrices.
\[
\Delta F_z = \frac{m \dot{z} \cos \beta}{l} 
\]
\[
\Delta F_z = F_s \tan \eta (Braking \rightarrow Front \ axles) 
\]
\[
\Delta F_z = -F_s \tan \eta (Braking \rightarrow Rear \ axles) 
\]
\[
\Delta F_z = -F_s \tan \sigma (Driving \rightarrow Rear \ axles) 
\]
For the aerodynamic lift:
\[
\Delta F_z = L(1 - LD) \rightarrow Rear \ Axle 
\]
\[
\Delta F_z = L(LD) \rightarrow Front \ Axle 
\]

### 4.5 Dynamics of a Particle in Non-Uniform Circular Motion

A vehicle moving around a track can be described as an object with a certain linear velocity of its center of gravity and also an angular velocity around this same point. As described in (Beer, Johnston et al. 2006) one can define an instantaneous center of curvature for this vehicle in the following way:

\[
\text{Figure 12 - Vehicle motion}
\]

In this way one can define the accelerations of the vehicle in the vehicle coordinate system.
\[
a_x = \dot{v} \cos \beta - \omega v \sin \beta = \dot{v}_x - \omega v_y 
\]
\[
a_y = \dot{v} \sin \beta + \omega v \cos \beta = \dot{v}_y + \omega v_x 
\]

### 4.6 Vehicle Longitudinal Dynamics

The FST 05e is powered by two electric AC motors and this work makes use of their power curve. The transmission of the vehicle is a gearbox with only one input to output ratio, this is called the transmission ratio, TR. The torque available from the motor is calculated according to the power curve and the delivered torque found from the pedal deflection and the transmission ratio.
\[
T_{st} = f(\omega_m) 
\]
\[
T_{input} = P_o \times T_{st} 
\]
\[
T_{wheel} = T_{input} \times TR 
\]

The brake system is modeled to receive the input force of the driver’s foot and translate it to torque applied to the wheel through use of the several parameters that define the components of the brake system. This braking torque is then reacted at the contact patch by the tire longitudinal force which generates a deceleration in order to slow the vehicle’s speed.

To calculate the longitudinal force at the contact patch one needs to calculate the longitudinal slip ratio. This is done by analyzing the rotational dynamics of the wheel:
\[
\delta_{\text{wheel}} = \frac{T_{\text{wheel}} - F_s \cdot R_{\text{wheel}}}{I_{\text{wheel}}} 
\]

By calculating the angular acceleration of the wheel when subjected to the moments described above, one can once again use the Simulink environment advantages of looped systems, differentiation and integration. By differentiating the angular acceleration one obtains the wheel angular velocity, which serves as an input to the longitudinal slip ratio calculation as described in chapter 2.

### 4.7 Vehicle Lateral Dynamics

The steering wheel input of the driver is translated to steering rack displacement using the c-factor explained earlier:
\[
\text{rack displacement} = (c_{\text{factor}}) \cdot \delta_{SW} 
\]

This rack displacement is then the input to a Simmechanics model of the steering system that outputs the steering angle of each wheel.

\[
\text{Figure 13 - Steering system in Simmechanics}
\]

In order to calculate the slip-angle of each wheel, the sideslip angle of each wheel must also be known.
\[
\beta_i = \tan^{-1} \left( \frac{v_i}{v_o} \right) 
\]
\[
SA_i = \beta_i - \delta_i 
\]

The inclination angle is calculated in the following way:
The inclination angle variation with the vertical wheel movement is obtained using the 3-point method along a normalized wheel travel distance.

\[
\begin{align*}
IA_{FR} &= IA_{static,FR} + \frac{\partial IA}{\partial \delta_{FR}} (z_{wheel,FR} - z_{static}) + \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) \\
IA_{FL} &= IA_{static,FL} - \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) + \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) \\
IA_{FR} &= IA_{static,FR} + \frac{\partial IA}{\partial \delta_{FR}} (z_{wheel,FR} - z_{static}) + \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) \\
IA_{FL} &= IA_{static,FL} - \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) + \frac{\partial IA}{\partial \delta_{FL}} (z_{wheel,FL} - z_{static}) \\
\end{align*}
\]

The inclination angle gain with vertical motion is considered in static conditions equal to the camber gain, and also to the variation of the KPI angle, giving.

\[
\frac{\partial IA}{\partial \delta_{wheel}} = \frac{\Delta KPI}{\Delta z_{wheel}} = \frac{[KPI]_{z_{static}} - [KPI]_{z_{static} + 10mm}}{10} 
\]

The influence of the steering angle of the wheels is taken into account in the following way:

\[
\begin{align*}
\frac{\partial IA}{\partial \delta_{FR}} &= KPI \cdot (1 - \cos \delta_{FR}) - \nu \cdot \sin \delta_{FR} \\
\frac{\partial IA}{\partial \delta_{FL}} &= -KPI \cdot (1 - \cos \delta_{FL}) - \nu \cdot \sin \delta_{FL}
\end{align*}
\]

4.8 VEHICLE ACCELERATIONS

The tire horizontal forces must be transformed from the wheel coordinate system to the vehicle coordinate system:

\[
\begin{align*}
F_{y} &= F_y \cdot \cos \delta_{Y} + F_y \cdot \sin \delta_{Y} \\
F_{x} &= -F_y \cdot \sin \delta_{Y} + F_y \cdot \cos \delta_{Y}
\end{align*}
\]

To calculate the accelerations of the vehicle the following calculation loop is used:

\[
\begin{align*}
\sum F_x &= m \cdot (v_x - \omega \cdot v_y) \\
\sum F_y &= m \cdot (v_y + \omega \cdot v_x) \\
\sum M_z &= I_z \cdot \ddot{\theta}
\end{align*}
\]

To obtain the vehicle velocity, the magnitude variation of the velocity vector is integrated. The longitudinal, lateral and angular velocities can then be used in the calculation of the longitudinal slip ratio and slip angle as explained earlier.

5 APPROACH FOR INCLUDING COMPLIANT COMPONENTS

5.1 PARAMETERS INFLUENCING VEHICLE BEHAVIOR

There are several vehicle characteristics that can affect vehicle behavior as can be seen from the model description in chapter four. The predominant system in a formula student car influencing the dynamic of the vehicle is the suspension system, and this will be the target of this work. As shown in chapter four, the most relevant parameters used to formulate a semi-empirical model of the tire used in this work are:

- Vertical load;
- Slip angle;
- Inclination angle;
• Longitudinal slip ratio.

Changes in these parameters result in a difference in tire horizontal forces and tire moments. Thus this work attempts to quantify deformations in suspension components that can influence these parameters and have an effect on tire forces and moments.

The slip angle is a parameter directly influenced by the steering system. The deformations in the steering system from the steering wheel to the tie-rod influence the slip angle compliance.

By modeling the wishbones as deformable components, one can calculate new KPI angle gains that directly influence the inclination angle.

5.2 LOAD CASES CALCULATION

The loads on the suspension components arise from direct reactions to what happens at the contact patches of the tires. To calculate the loads on wishbones and tie-rods a Simulink model of the suspension is built. By introducing the contact patch forces and moments the model is equipped with joint sensors that calculate the load in each link of the suspension. The tie-rod links forces are used to calculate the force in the steering rack and then the torque presented in the steering pinion is computed:

\[
T_{\text{steering}} = F_{\text{rack}} \cdot \frac{c_{\text{joint}}}{2\pi} \quad (35)
\]

5.3 DEFORMATION ANALYSIS

The steering shaft is designed as a constant section component in the FST 05e.

Using basic solid mechanics one can calculate the twist angle, \( \psi \), of a shaft subjected to a known torque:

\[
T_{\text{steering}} = J \cdot \frac{G}{l_{\text{steering-shaft}}} \psi \quad (36)
\]

In the case of the wishbones and steering rods, in the case of the FST 05e, they are attached to both the chassis and the upright by means of spherical bearings:

Being mounted with spherical bearings makes the loading of the wishbone arms easy to predict since it will only withstand axial loads – compression or tension. These loads are calculated as described in 5.2.

In the case of steel or aluminum wishbones one can simply use Hooke’s Law to compute the extension and consequent elongation of the links:

\[
\left\{ \begin{array}{l}
\varepsilon = \frac{F}{E \times A} \\
\Delta l = \varepsilon \times l_0 
\end{array} \right. \quad (37)
\]

In the case of CFRP suspension links like the ones present in the FST 05e, one needs to find a way to compute this elongation. The CFRP link is composed by aluminum inserts (containing the spherical bearings) bonded to the carbon fiber tube.

To calculate the deformation of the adhesive one needs to compute the stresses in the adhesive, which are not constant along the length of the adhesive joint as in (Adams, Comyn et al. 1996). This behavior was first approached by Volkersen in 1938 and consists on one of the oldest theories to analyze single-lap joints taking into account the stiffness of the adherends. Due to its simplicity, vast literature sources and the available adhesive properties it was the technique used by the team to design the bonded joints.

For a single-lap joint as presented above, using Volkersen’s theory one arrives at:

\[
\left\{ \begin{array}{l}
l_{\text{adhesive}}(x) = \frac{PW}{2\pi \sinh \left( \frac{W}{l_{\text{adhesive}}} x \right)} - \frac{PW}{2\pi \cosh \left( \frac{W}{l_{\text{adhesive}}} x \right)} \cos(Wx) + w \\
M = \frac{E_l \cdot E_t}{E_l + E_t} \left( \frac{W}{l_{\text{adhesive}}} x \right) \\
W = \frac{E_l \cdot E_t}{E_l + E_t} \left( \frac{W}{l_{\text{adhesive}}} x \right) \\
\end{array} \right. \quad (38)
\]

In these equations, \( E_i \) represents the Young’s Modulus of each adherend and \( E_{\text{adhesive}} \) is the shear modulus of the adhesive.
To use this theory for the design of the suspension links the approach taken was to use the cylindrical shape of the bonding area and treat it as a single-lap joint with the following characteristics:

\[
w = 2\pi \left( \frac{TD}{2} - \frac{h_{\text{adhesive}}}{2} \right) \quad (39)
\]

\[
t_z = \frac{TD}{2} - h_{\text{adhesive}} - \frac{ID}{2} \quad (40)
\]

Using once again solid mechanics one can correlate the shear strain with the shear stress and compute the displacement of the bonded joint.

\[
\Delta l_{\text{adhesive}} = \tan(\chi) \cdot h_{\text{adhesive}} \quad (41)
\]

\[
\tan(\chi) = \frac{r_{\text{adhesive}}}{G_{\text{adhesive}}} \quad (42)
\]

The stress is taken at the extremities of the bonded joint because that’s the relevant deformation for elongation analysis of the suspension link. For the computation of the CFRP tube elongation one can use equation (37) making sure to use the equivalent longitudinal elasticity modulus of the fiber used. This said one finally computes the suspension link elongation in the following way:

\[
\Delta l_{\text{link}} = \Delta l_{\text{CFRP}} + 2 \cdot \Delta l_{\text{adhesive}} \quad (43)
\]

Since for the design of the FST 05e physical testing was done to ensure that the adhesive joints were able to sustain the expected loads, these tests were analyzed in this work in order to establish a better deformation model.

As can be seen the error is large and conclusions can be tough to draw. In part because only the displacement of the machine’s crosshead and the load exerted are measured. This makes it impossible to separate the deformation of the carbon tube from the deformation of the bonded joint. Volkersen’s theory is fairly old but can be hold accountable to some extent to treat bonded joints. The problem with this and almost every theory is that surface condition can’t be accounted for when doing the calculations. Also because of the manufacturing process of the specimens, the specific bondline thickness cannot be totally guaranteed, in part because of machining imprecisions, the tube manufacturing which presents an imperfect inner surface and also the increase in thickness due to manual sanding of the surfaces during the surface preparation stage. All this contributes to the increase of bondline thickness that can alter the results of the physical test. Looking at (41), one can see that an increase in bondline thickness increases the distortion angle. Even though the stresses are lower, this fact can’t prevent the distortion from rising. Since the error is calculated to the full extent of the specimen and the deformation of the tube and the adhesive cannot be separated the decision was to take the error and apply it to the theoretical value, this means considering the error as a multiplication factor to (43).

\[
E_R = \frac{\Delta l_{\text{exp}} - \Delta l_{\text{theoretical}}}{\Delta l_{\text{exp}}} \quad (44)
\]

\[
\Delta l_{\text{link}} = \frac{\left( \Delta l_{\text{CFRP}} + 2 \cdot \Delta l_{\text{adhesive}} \right)}{1 - E_R} \quad (45)
\]

### Table 1 - Results comparison

<table>
<thead>
<tr>
<th>Tube ID</th>
<th>Load (kN)</th>
<th>Strain</th>
<th>Exp Δl (mm)</th>
<th>The Δl (mm)</th>
<th>Rel. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 mm</td>
<td>1</td>
<td>0.000245</td>
<td>0.017125382</td>
<td>0.01904</td>
<td>65%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.000734</td>
<td>0.051376147</td>
<td>0.05783</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.001223</td>
<td>0.085626911</td>
<td>0.09271</td>
<td></td>
</tr>
<tr>
<td>10 mm</td>
<td>1</td>
<td>0.000365</td>
<td>0.025541852</td>
<td>0.01198</td>
<td>53%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.001095</td>
<td>0.076625556</td>
<td>0.03593</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.001824</td>
<td>0.127709261</td>
<td>0.05989</td>
<td></td>
</tr>
</tbody>
</table>

### 6 MODEL PRESENTATION – WITH COMPLIANCE

#### 6.1 MODEL SCHEMATIC MODIFICATIONS

In order to include compliance in the model developed in chapter four, one must find a way to employ the knowledge of chapter five.
The deformation calculation employs the models of the compliant components described earlier and has the computation progress shown below:

In the case of the steering mechanism, the twist angle of the steering shaft is subtracted to the steering input:

\[ \delta_{SW}^* = \delta_{SW} - \psi \]  
(46)

Simmechanics was once again used for the computation of the steering angle of the wheels by using a sliding joint between two subcomponents of the tie-rod. This joint was actuated by the previously calculated elongation of the tie-rod.

To compute the inclination angle modifications, one can see the advantage of using the 3-point method. By calculating the elongation of the suspension links one can perform the calculations of equation (28) by adding this elongation to the static length of the suspension links.

### 6.2 SKID PAD SIMULATION

The Skid Pad is the formula student event that aims to measure the vehicle’s cornering capability. By performing turns around a similar corner radius the vehicles are compared. The measured time is the average time between the right circumference and the left circumference. Each run contemplates two laps around the right circumference and two laps around the left circumference.

To simulate the vehicle in the Skid Pad and extract a time around one circumference only one lap is completed.

When steering the wheels, the driver imposes a deformation on the tire, causing a lateral force (and consequently slip angle) to appear. The forces involved come to the driver through the steering components. Those components compliant behavior can be seen below:

As one can see, the steering shaft twist angle is not very noticeable. On the other side, the most loaded wheel in the front axle forces its steering rod to stretch 0.15 mm. This causes the steering angle of that wheel to decrease by 0.2º.

The forces and moments on the wheels also load the wishbones, causing deflections in these components. Below, the links compliances from the most loaded wheel (front right wheel) are shown:
As one can see, the displacement of the most loaded link is around 0.45 mm in compression. These compliances cause the inclination angle of the wheels to change from the nominal values of the rigid simulation. Below, is shown the inclination angles of all wheels:

![Graph showing inclination angles of all wheels](image)

Figure 26 - IA during Skid Pad simulation

As seen in the figures above, the trajectory suffers a considerable change from the rigid model simulation and the vehicle’s acceleration is also reduced, even though it begins to increase as the simulation goes on.

![Graph showing path radius and radial acceleration](image)

Figure 27 - Path radius and radial acceleration

<table>
<thead>
<tr>
<th>Rigid Model</th>
<th>Compliant Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.796 G</td>
<td>Maximum radial acceleration 1.790 G</td>
</tr>
<tr>
<td>4.722 s</td>
<td>Elapsed Time 4.735</td>
</tr>
</tbody>
</table>

All the changes shown above capitalize into a slower time around the Skid Pad even if by not a big difference. In Table 2 the comparisons of the most notable variables between the two simulations can be seen.

7 CONCLUSIONS

The objective of building a vehicle model contemplating compliant components was fulfilled. The vehicle model was developed and the methodology for the inclusion of compliant components was exposed. As can be seen from the results in Table 4, the result of the inclusion of compliant components was a small difference in elapsed time due to small changes in trajectory. This difference is not very significant and the conclusion is that the modeled compliant components may not have a very preponderant influence in the FST 05e behavior around the Skid Pad. This may not be true for other simulations or with the future inclusion of more compliant components, but this is not the mainly intent of this work.

One must notice that the rigid model simulation of the Skid Pad is a best case scenario in terms of elapsed time. This event requires more driving skills than the acceleration event and the inputs in the model are ideally set at the start of the simulation. One should expect a lower performance from the real FST 05e, depending on the driver and track conditions but for comparison purposes the simulation is valid. It must be noted from the simulations that even though the driver inputs were held constant for purposes of comparison, the small changes in trajectory and velocity could cause the driver to change the inputs so the elapsed times could have a larger difference if a Skid Pad simulation with a real driver was performed.

Validation of results with further physical testing, preferably with an instrumented FST 05e, would have been helpful and it is certainly the missing piece of this work. Besides that fact, this work can contribute to the design process of the next prototypes for Projecto FST Novabase and serve as a building ground for further improvements in vehicle dynamics simulation performed by the team.

8 REFERENCES