Gravitoelectromagnetism : The Lense-Thirring Effect

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Abstract

General Relativity is the major theorical framework that explaind many gravitational phenomena in our Universe, and several experiments was planed and executed to verify the main prediction of this theory.

One of lastest tests involve the measure of the Lense-Thirring effect, predicted by GR and explained naturally with a special theorical framework which use a formal analogy of electrodynamics. Using the so called GEM (Gravitoelectromagnetism) framework, it's possible ao ease the derivation and calculation of Lense-Thirring effect. Using the lastest data from Gravity Probe B, so far, the theorical prediction in GR/GEM agrees the experimental data obtained from this probe.

Keywords: Gravitoelectromagnetism, Gravity Probe B, Lense-Thirring Effect.

1.Introduction:

In this little introductory paper will introduce the basic mathematical formulation of Gravitoelectromagnetism (GEM) Model, which is a formal analogy of Electromagnetism when applied to the Linearized Solution of General Relativity, as described by Mashhoon [1].

GEM framework is usefull to explain the geodesic effects recently observed by Gravity Probe B [2], using a nice analogy of an analog of magnetic field around a curly source.

The inertial rotating source in primary will produce a so-called gravitomagnetic force, and any sattelite around the primary should be affected by this field, which acordly by RG is no more than the non-diagonal terms of metric.

Taking the linear approximation of RG field equations [3], we can define a newtonian potential as:

$$\overline{h_{00}}=-\frac{4\phi}{c^2} \qquad (1)$$

And the new gravitomagnetic potential as:

$$\overline{h_{0i}}=-\frac{2A_i}{c^2} \qquad (2)$$

It is easy to compute explicitily the GEM potencials in terms of primary mass (M) and their angular momentum (J). If we admit a round primary (like a star or a planet), then the newtonian potential will value:

$$\phi=-\frac{GM}{R} \qquad (3)$$

And the gravitomagnetic potential is just the effect of angular momentum of the primary:

$$\vec{A}=\frac{G[\vec{L}\times\vec{r}]}{R^3 c} \qquad (4)$$

Applying the equations (3) and (4) on Linear Einstein Field Equations [3], we can derive a gravitoelectric field (rougly a small correction of newtonian force):

$$\vec{E}=-\nabla\phi-\frac{1}{2c}\frac{\partial\vec{A}}{\partial t} \qquad (5)$$

And the gravitomagnetic field:

$$\vec{B}=\frac{1}{2}(\vec{\nabla}\times\vec{A}) \qquad (6)$$

With this two fields (5,6) we can evaluate their own version of Maxwell's Equations of GEM model:

$$\vec{\nabla}\cdot\vec{E}=-4\pi G\rho \qquad (7)$$

$$\vec{\nabla}\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \qquad (8)$$

$$\vec{\nabla}\cdot\vec{B}=0 \qquad (9)$$

$$\vec{\nabla}\times\vec{B}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}-\frac{4\pi G}{c}\vec{J} \qquad (10)$$

We can also dedrive the Lorentz Force of GEM from the geodesic equation from RG:

$$\vec{F}=m\vec{E}+2m(\frac{\vec{v}}{c}\times\vec{B}) \qquad (11)$$

2. Derivation and Explanation of the Geodesic Effect and Lense-Thirring Effect

The Geodesic Effects observed in RG can be understond in our GEM model as an simple concept from the Classic Electrodynamics, the Larmor Frequency derived in many physics fields such the plasma physics. [4]

From the Lorentz Force (11), we can split the movement of a satellite around the primary, with a tangential component which is the usual keplerian orbit, and a new axial origin vector derived from the gravitomagnetic force which will cause a Larmor-type precession of the keplerian orbit.

Returning to the RG formalism, this Larmor precession is due to the torsion of local metric due to angular momentum of the primary [1].

Any orbit around the primary will show a tiny variation of the keplerian parameters, and the rate of precession can be calculated using the RG, and the re-interpreted using the GEM terms.

Using the correct approuch, the derivation of geodesic effects of local metric are based by a propriety in RG [5] that it called the Fermi-Walker paralell transport, which constrains the conservation laws of physics to the geodesic path of the particles in the local manifold ruled by RG.

Since the gravity filed in RG is equal to a free particle in a curved spacetime, then we can assume the angular momentum of the satellite around the primary is a constant of motion:

$$S_\alpha S^\alpha=S_{(i)}S^{(i)}=const. \qquad (12)$$

And the covariant derivative of the angular momentum will obey the following evaluation:

$$\frac{DS_\alpha}{ds}=u_\alpha\left(\frac{Du^\sigma}{ds}S_\sigma\right) \qquad (13)$$

Which he angular momentum depends of the velocity and position of the satellite.

Taking explicity the (13) according to the local metric, we obtain an equation of motion similar to the gyroscope [5]:

$$\frac{d\vec{S}}{ds}=\dot{\vec{\Omega}}\times\vec{S} \qquad (14)$$

Where the derivative of the Larmor-type

frequency of the (14) is equal to:

$$\dot{\vec{\Omega}}=-\frac{1}{2}\vec{v}\times\frac{d\vec{v}}{dt}+\vec{v}\times\nabla\phi-\frac{1}{c^2}(\nabla\times\vec{A}) \quad (15)$$

By splitting (15) into three terms, we obtain the Thomas Precession (due to kinematics), the De Sitter or geodesic precession and the Lense-Thirring precession. [5]

Since the Thomas Precession is irrelevant in many non-relativistic bound systems (inertial frames, for example), the other two factors of (15) can be simplified in our toy model:

Let's assume the source's primary gravitational filed have a spherical symmetry with some multi-polar expantion corrections.

The multi-polar expantion is needed to some precision tests like the Graviy Probe B [2], since it's orbit is a low altitude type, and the quadrupolar terms of gravitiational field cannot be ignored.

Taking this constraints seriously, the De Sitter Precession rate of a low-orbit satelitte will be:

$$\vec{\Omega}_G=\frac{3G}{2r^3}\left[\left(M-\frac{3}{r^2}I\right)(\vec{r}\times\vec{v})+\frac{3}{r^2}(I\vec{r}\times\vec{v})\right](16)$$

And the Lense-Thirring Precession will be:

$$\vec{\Omega}_{\mathrm{LT}}=\frac{2GI}{r^3}\left(-\vec{\omega}+\frac{3}{r^2}(\vec{\omega}\cdot\vec{r})\vec{r}\right) \qquad (17)$$

The “I” term in (16) and (17) is the inertial momentum of the primary. Ignoring oblate factors of the curvate of the Earth, the inertial momentum of a sphere is:

$$I=\frac{2}{5}Mr^2 \qquad (18)$$

This means that it's possible to derive a theorical evalution that should be agree if the RG is correct at the GEM model approximation.

For a low orbit satelitte, the average value of geodesic precession (De Sitter) would be:

$$\langle\Omega_G\rangle=\frac{3(GM)^{3/2}}{2c^2r^{5/2}}\left[1-\frac{9}{8}J_2\left(\frac{R}{r}\right)^2\right] \quad (19)$$

And the Lense-Thirring precession would be:

$$\langle\Omega_{\mathrm{LT}}\rangle=\frac{2\pi GMR^2}{5c^2r^3T}\left[1-\frac{219}{392}J_2\left(\frac{R}{r}\right)^2\right] \quad (20)$$

Taking the numerical orbital values of GP-B [2] into (19,20), we get:

$$\langle\Omega_G\rangle=6{,}606\, arc/\,yr \qquad (21)$$

$$\langle\Omega_{\mathrm{LT}}\rangle=39{,}2\, marc/\,yr \qquad (22)$$

We talk the experimental results after later in next setion, to introduce first the GP-B setup.

3. The Gravity Probe B Setup and Brief Experimental Results

The Gravity Probe B experiment can be traced back to early 1960 when two Ph.D. in Physics publish a paper, “Possible New Experimental Test of General Relativity” in “Physical Review Letters”, which propose the use of a gyroscope in space to measure the geodesic effects predicted by RG.

The project gain momentum when it's sponsered by NASA in the transition form 1960's to 1970's, specially in the brand new space technology and better high precision manufacturing technics.

The main prototype of GP-B was designed in 1984, which consists a highlly round sphere (to reduce oblatness), a telescope, a helium cooling device and the measurements hardware support.

To avoid complications in harmonics analysis, the gyro's sphere should be perfectly smooth (and the final sphere was a oblatness less that 40 nm in 10 cm of sphere radius!), and this achivement was awarded by the Guiness Book of Records! [2]

The satellite telescope are used to maintain the satellite aligned to a distant star, whose relative movement in celestial sphere can be ignored without jeopardize the measurements. (This means the distant star is an “perfect” inertial frame in the short period of the GP-B mission when collects data.)

After several years of internal development, the GP-B was finally launched in 2004, and the main mission occured between 2005 and 2007, however the data analisys will take several years to account.

In 2012 [6], the experimental results of geodesic effect and frame-dragging was publish by NASA, and the values are:

$$\langle \Omega_G \rangle_{\exp} = 6{,}6018 \pm 0{,}0183\, arc/yr \quad (23)$$

$$\langle \Omega_{LT} \rangle_{\exp} = 37{,}2 \pm 7{,}2\, marc/yr \quad (24)$$

This means the first high precison test of RG show that the gravity is a space-time metric effect, and the results appart of statistical flutuations, are consistent with the prediction of RG.

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