The role of energy in economic growth:

a two-sector model with useful work

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Resumo

Um modelo bisectorial de crescimento económico é desenvolvido, separando as actividades energéticas (sector energético) dos restantes processos economicos (sector não energético). O sector energético agrega todos os processos responsáveis pela conversão de exergia primária em trabalho útil. A sua inclusão no modelo permite simular a dinâmica de oferta de energia como factor de produção e compreender a sua importância para o crescimento. O modelo bisectorial é analisado em comparação com o modelo de Solow, adoptando muitas das suas simplificações. Demonstra-se que o progresso tecnológico total pode ser representado de forma puramente Harrod-neutral neste modelo, assumindo um desenvolvimento específico a longo prazo para as variáveis-chave (crescimento regular). Vários factos estilizados de crescimento económico a longo prazo são avaliados usando o modelo desenvolvido, incluindo tendências relacionadas especificamente com o consumo de exergia e trabalho útil, e a sua relação com o desenvolvimento económico. Estes factos estilizados são testados empiricamente para a economia Portuguesa e comparados com um facto estilizado proposto mais robusto, relacionado com a intensidade de trabalho útil na economia. Simples exercícios de contabilidade de crescimento são conduzidos para a economia Portuguesa, com o modelo de Solow e o modelo bisectorial. A capacidade explicativa de diversas variáveis energéticas em relação ao crescimento da productividade total dos factores e da produção económica é avaliada e discutida para ambos os modelos. Os resultados obtidos sugerem um papel importante da energia no crescimento económico, e variáveis energéticas mais próximas dos processos produtivos na economia demonstram melhor capacidade para justificar variações de PTF e PIB.

Palavras-chave: crescimento económico, exergia, trabalho útil, bisectorial.

Abstract

A two-sector abstract economic growth model is developed, with a separation between energy-related activities (Energy Sector) and the remaining economic processes (Non-energy Sector). The E-Sector aggregates all processes within the economy which perform the conversion of primary exergy into useful work. It is introduced in order to simulate the availability dynamics of energy as a factor of production and understand its importance for growth. The two-sector model is analyzed in comparison with the standard Solow growth model, adopting many of its simplifying assumptions. In order to perform steady-state analysis, it is shown that total technological progress can be written as purely Harrod-neutral in this framework, assuming a specific long-run growth path for the key variables (regular growth). Several relevant stylized facts of long-run economic growth are addressed with the developed two-sector economic system, including specific exergy and useful work related trends in economic development found in the empirical literature. This stylized facts are also empirically tested for the Portuguese economy and compared with a more robust proposed useful work intensity stylized fact. Simple growth accounting exercises are conducted for the Portuguese economy, with the Solow model and the two-sector model. The explanatory power of several energy-related variables in total factor productivity and economic output growth is evaluated and discussed for the whole economy and the NE-Sector specifically. Results suggest that energy has an important role in economic growth, and energy variables closer to the productive processes in the economy are better suited to account for TFP and GDP variations.

Keywords: economic growth, exergy, useful work, two-sector.

Contents

List of Tables

List of Figures

List of Notation

α_i: Share of a given factor of production i in total income.

μ_i: Initial value of the first order growth rate of variable i in a regular growth path.

γ: Fraction of energy sector useful work output (B^U) used up in non-energy sector production processes.

δ: Depreciation rate of capital stock.

- δ_{NE}: Depreciation rate of non-energy sector capital stock (K^{NE}).
- δ_E: Depreciation rate of energy sector capital stock (K^E).

ϵ: Second-law efficiency.

- ϵ^A: Conversion efficiency.
- ϵ^D: Transformation efficiency.

λ: Total technological progress (A^L) growth rate.

σ: Fraction of investment (I) allocated to the non-energy sector.

ω_i: Damping coefficient corresponding to variable i in a regular growth path.

A: Technological progress, or technical change.

- A^{NE}: Technological capacity of the non-energy sector.
- A^E: Technological capacity of the energy sector (related to exergy-to-useful work conversion efficiency).
- A^L: Total technological progress, for the whole economic system (labor-augmenting).

B: Exergy/useful work.

- B^P: Primary exergy supply (correspondent to primary energy supply, E^P).
- B^F: Final exergy consumption (correspondent to final energy consumption, E^F).
- B^U: Energy sector useful work output.

C: Total aggregate consumption.

- C^{NE}: Consumption of goods and services produced by the non-energy sector.
- C^E: Direct consumption of useful work output from the energy sector, in value terms.

E: Energy.

E^P: Primary energy supply (as defined in IEA balances, plus energy from food & feed energy carriers).

E^F: Final energy consumption (equivalent to total final energy consumption plus energy industries own uses in IEA balances[1].)

F: Production function.

F^{NE}: Non-energy sector production function.

F^E: Energy sector production function.

G: Functional form for the evolution of energy sector technical change A^E.

g_i: Growth rate of variable i.

H: Functional form for the evolution of non-energy sector technical change A^{NE}.

I: Investment.

I^{NE}: Investment in the non-energy sector.

I^E: Investment in the energy sector.

K: Capital stock.

K^{NE}: Non-energy sector capital stock.

K^E: Energy sector capital stock.

L: Labor inputs.

M: Imports.

n: Population growth rate.

p_{B^U}: Price of useful work.

R: Rental price of capital.

r: Interest rate.

s: Saving rate.

UW: Useful work corresponding to final energy (E^F) and final exergy (B^F) consumption (including energy industries own uses, but excluding non-energy uses.)

w: Wage rate, or price of labour.

X: Exports.

Y: Gross domestic product (GDP) or economic output.

Y^{NE}: Non-energy sector output.

Y^E: Energy sector output in value terms.

[1] Including non-energy uses and also considering food & feed energy carriers.

List of Acronyms

AMECO: Annual Macro-ECOnomic database (European Commission).

CE: Compensation of Employees.

CES: Constant Elasticity of Substitution.

CFC: Consumption of Fixed Capital.

CHP: Combined Heat and Power (Cogeneration).

COFOG: Classification Of the Functions Of Government.

COICOP: Classification Of Individual COnsumption by Purpose.

CPI: Consumer price index.

E: (superscript) Relative to the Energy sector.

EC: Energy Consumption.

EEC: European Economic Community (pre-European Union).

ELC: ELectricity Consumption.

EP: Electricity Production.

GA: Growth Accounting.

GCF: Gross Capital Formation.

GDP: Gross Domestic Product.

GFCF: Gross Fixed Capital Formation.

GVA: Gross Value Added.

GOS: Gross Operating Surplus.

ICE: Internal Combustion Engine.

IEA: International Energy Agency.

KLEMS: Capital (K), Labor, Energy, Materials and Services.

LINEX: LINear-EXponential.

NA: National Accounts/Standard National Accounts.

NE: (superscript) Relative to the Non-energy sector.

NPISH: Non-Profit Institutions Serving Households.

OECD: Organisation for Economic Co-operation and Development.

PCA: Primary Converting Activities.

PD: Price deflator.

PIGST: Portugal, Italy, Greece, Spain and Turkey.

REXS: Resource EXergy Services.

SCA: Secondary Converting Activities.

TFP: Total Factor Productivity.

UK: United Kingdom.

USA: United States of America.

VAR: Vector AutoRegression.

VECM: Vector Error Correction Model.

WS: Wage and Salary earners.

Chapter 1

Energy and economic growth

Over the past two centuries, there has been an unprecedented global growth in population and GDP per capita. This has been accompanied by a rapid growth in the demand for energy and an increase in energy efficiency, through technological development, in order to meet the consumption demands [66]. Not surprisingly, the relationship between energy use and economic growth has received increasing attention in recent years, and there has been a growing body of literature that deals with the energy topic. This chapter presents the main motivations behind the inclusion of an energy variable in economic models, such as the one developed in this thesis. It begins by weighting the importance of energy (and natural resources) in economic production and growth, from a theoretical point of view. Several approaches regarding the introduction of energy as a factor of production in economic models are reviewed next.

1.1 The role of energy in economic growth

The standard neoclassical theory of economic growth, formulated independently by Robert Solow [69] and Trevor Swan [72], expresses the production of goods and services as a function of capital and labor inputs, with the major contribution to growth being attributed to an unexplained exogenous driver: "technological progress" (or total factor productivity). According to this paradigm, every product is produced from other products within the self-organized system, and growth occurs without any inputs of energy or materials from outside. Economic activity is conceptualized as a closed loop between abstract production, consumption and investment.

The connection with real material objects, energy, and the physical world from which they are extracted, processed and made into products, was never considered to be a crucial aspect of mainstream neoclassical economics until 40 years ago [32] [37] [19] and even recently, the essentiality of energy as a factor of production is overlooked in most textbooks and research papers. However, all economic production involves the transformation or transportation of matter in some way, and all such processes require energy. There is, at first sight, no reason why natural resources and energy shouldn't be regarded as essential to economic production, since there is a number of services of nature that cannot, even in principle, be replaced by man-made capital or human labor. This is the essence of *strong sustainability* [38], which states that the products created by mankind cannot replace the natural capital found in ecosystems. Some mainstream categories of neoclassical growth

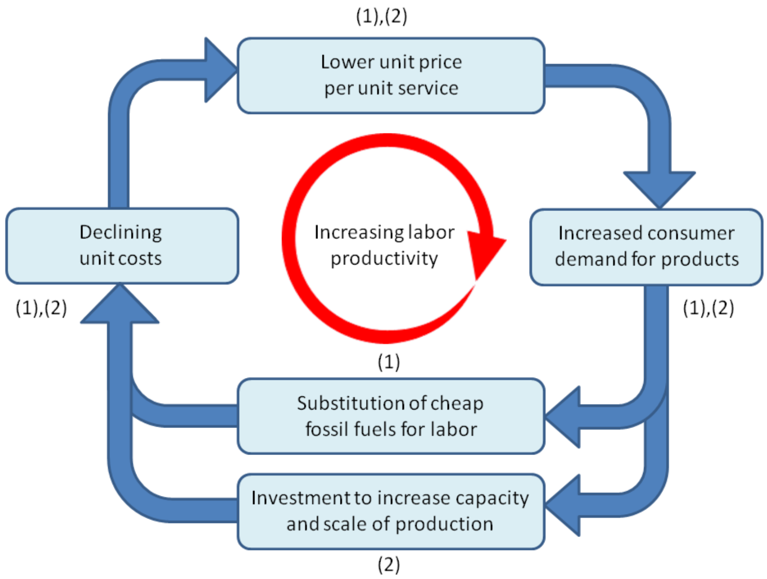


Figure 1.1: **Resource use (1) and scale-cum-learning (2) growth engines.** *Loop 1*: technological progress makes fossil fuels steadily and dramatically cheaper, which encourages the substitution of fossil fuel-derived energy and mechanical power for animal and human work. Cheaper fuels make it possible to construct better, cheaper, and more efficient machines, which in turn permits continuous and drastic further reductions in the costs of production and transportation of fossil fuels. *Loop 2*: economies of scale, standardization, division of labor by specialization, and "learning-by-doing" lead to reduction of costs and therefore increased capacity to further reduce production costs.

models focus, alongside technological change, on the consumption of natural capital[1] in determining sustainable growth[71]. In all these conventional approaches, the contribution of energy to economic activity is only considered relative to its cost within production, which suggests that decoupling economic growth from energy use is a reasonable possibility.

In contrast to this closed perpetual motion machine, the field of ecological economics proposes that the real economy behaves essentially like a large-scale materials processing system, powered by machines using mostly fossil-fuel energy[10]. In fact, economic growth since the 1780's has been driven to a large extent by the utilization of steam, combustion engines and electric motors, as a substitute and amplifier for human and animal work. One of the most important growth engines since the 1st Industrial Revolution has been the continuously declining real price of physical resources, especially energy and power delivered to the point of use. The increasing availability of energy from fossil fuels, and power from steam engines and internal combustion engines (ICE) has played a fundamental role in past economic growth [13]. The growth impetus due to fossil fuel discoveries and applications, as well as continuous cost reductions, lasted throughout the 19th and into the 20th century, with petroleum, internal combustion, and electrification. Another variant of this mechanism, which became increasingly important in the 19th century, relates to economies of scale, including standardization, division of labor and "learning-by-doing". This distinct feedback cycle is called the "Salter cycle", and combined with the resource cost-driven feedback cycle it has been the primary driver of economic growth in industrial countries until recently - Figure (1.1).

It is clear that the growth mechanism in Figure (1.1) must falter and eventually fail since fossil fuels and other extractive resources will inevitably become scarce, and the costs of finished

[1] Defined as renewable and non-renewable natural resources such as water and fossil fuels.

materials and services derived from them will rise. Indeed, some ecological economists and system ecologists have called attention to the fact that energy dependence means that near-exponential economic growth cannot be sustained indefinitely in a world of finite resources[36] [25]. Furthermore, according to Figure (1.1), as resource extraction and conversion costs fall due to economies of scale and "learning-by-doing", economic growth is stimulated, resulting in a further increase in the overall use of raw materials and fossil fuels. This is what is generally referred to as the *rebound effect* [61]. In this sense, the feedback mechanisms represented show that resource and energy consumption within the economy should be viewed as much as a driver of growth as a consequence, i.e., resource and energy flows are productive. This goes against the mainstream view of production theory, which defines capital and labor as *primary inputs*, existing at the beginning of the productive cycle and not consumed in production (although capital is subject to depreciation), while energy and other natural-resource based products are regarded as *intermediate inputs*, produced by the application of primary inputs and used up entirely in the production processes [70]. One of the reasons for neoclassical economists to distinguish labor and capital as primary inputs is their view of national accounts as having a fundamental role in production. It is easy to prove, for a single sector economy consisting of a large number of producers manufacturing a good using only capital and labor services, that factor payments and productivity are proportionally linked [47]. If labor and capital are the only factors of production, national accounts are set up to reflect payments to these factors in the form of rents (capital) and wages (labor), with gross domestic product being the sum of all such payments. Although energy and material services are not completely unpaid, the largest share of payments in the national accounts is attributed to labor ($\sim$ 70%) while capital receives most of the remaining share. Some energy and resource payments go to landowners and owners of financial capital, and others go to miners, oil riggers and workers in the extractive industries. However, since the share of direct payments to energy and natural resource owners in national accounts is only a few percent, they are not considered to be important factors of production, according to the perceived theory of income allocation.

The discrepancy between small factor payments directly attributable to physical resources - especially energy - and the apparent importance of energy inputs to economic production may be explained by under-payment to these inputs directly, in the form of depletion allowances and subsidies to consumers, or indirectly as exemptions from paying the costs of environmental damage caused by their activities, or both. Another hypothesis is that gains in energy efficiency are resulting in increased payments to capital and labor, instead of payments to energy inputs. The fact that the traditional theory of income allocation is proved under the over-simplifying assumption of a single sector model is also noteworthy. One alternative may be to view the economy as a materials processing chain system, introducing a multi-sector production process, where downstream value-added stages act as productivity multipliers. Under this multi-sector assumption, a factor receiving a very small share of the national income directly can contribute a much larger effective share of the value of aggregate production, i.e., be much more productive [11]. This hypothesis is further explored in Section 2.2 of this chapter, and is extremely relevant to the model developed in the following chapter.

The neoclassical worldview is also criticized by ecological economists for failing to ground economic activity in the physical reality (and limits) imposed by the laws of thermodynamics [52]. The first law of thermodynamics, often known as the "conservation law", states that the total energy of an isolated system cannot be created or destroyed, but only transformed from one form to another. Therefore, in order to obtain a given production output, with a certain embodied energy, equal or greater quantitites of energy must enter the production process. In the semi-closed global ecosystem, the only available source is solar energy, which can be used directly, or indirectly - embodied in fossil fuels. The by-products of the use of embodied solar energy are returned to the environment in the form of wastes (e.g. carbon dioxide emissions). The environmental impact of such waste products may compromise, irreversibly, the ecosystem services and functions upon which economic activity and human life rely on. These services cannot be substituted by any corresponding gains in man-made capital. The second law of thermodynamics ("the entropy law") implies that while energy and materials can be reused, they will increasingly reach a less useful state, i.e. their entropy will increase. It also implies that in order to transform one material to another, additional energy is required. Again, this means that there are limits to the extent to which energy can be substituted for by other inputs in the production processes.

Proeminent economists only began to express a serious interest in the relationship between economics and thermodynamics in the 1970s, mainly through the work of Georgescu-Roegen [33]. His fund-flow model describes production as a transformation process in which a flow of materials, energy, and information is transformed by two agents: human labor and manufactured capital. The central concern of this model is the environmental degradation caused by the throughput of energy from fossil fuels, due to entropy. Other ecological economists argue that, since the flow of energy and materials is what is being transformed, adding to the stock of manufactured capital without increasing these flows does not increase production[26]. Empirical studies on the substitution and complementarity of energy and man-made capital are few and produce varied results [21] [7]. Among other factors affecting the linkage between energy and growth are the various mechanisms through which energy use (and efficiency) can affect TFP [20]. Authors Robert Ayres and Benjamin Warr argue that the energy conversion efficiency can be used as a quantitative measure of the state of technology, either by function (such as transport or heating) or in the aggregate for the economy as a whole [14]. In terms of energy quality, Schurr and Netschert (1960) [63] were among the first to recognize its economic importance, noting that the general shift to higher quality fuels reduces the amount of energy required to produce a unit of GDP. Other works by Cleveland et al (1984) [24] and Kaufmann (1992) [45] explain much of the decline of US energy intensity in terms of shifts to higher quality fuels. The relation of exergy (defined as energy available to perform physical work) to energy content as a quality index now forms a common part of many industrial and ecological energy analyses [54]. The composition of energy inputs is also a factor affecting economic growth, since in the course of economic development, a countries' fuel mix tends to evolve as they move up the "energy ladder" [40]. Finally, shifts in the composition of output also influence the relationship between energy consumption and growth. In the earlier phases of development there is usually a shift from agriculture towards more industrial activities, while in the later stages there is a shift towards services and lighter manufacturing. It is often argued [55] that these transitions will result in an increase in energy used per unit of output in early stages of economic development, and a reduction of energy intensity in the later stages (a pattern similar to a Kuznets Curve [28]).

Theoretical and empirical evidence indicates that energy use and output are tightly coupled, with energy availability playing a key role in enabling growth. By only accounting for energy in terms of its relative cost within economic production, neoclassical economists have underestimated its importance for economic activity. The absence of energy from the neoclassical framework, and its failure to recognize the links between energy and growth, implies that this theory will not explore the dynamic relations between greater energy use and technical changes that are directed at harnessing growing supplies of energy. For ecological economists, energy is a fundamental factor enabling economic production and growth, and the two cannot be decoupled. This thesis is in line with the latter perspective.

This thesis will focus on the relation between energy consumption and economic growth. Related matters such as pollution and wastes are disregarded in the present work.

Thermodynamic concepts of exergy and useful work are introduced, and further developed in chapter 2 and appendix A. The conception of the economy as a materials-processing system with several sectors (or stages) of production is extremely important to the model's construction, since it seeks to solve the apparent inconsistency between small factor payments and the high correlation between energy inputs and economic output.

The two-sector model developed in light of theoretical insights provided mainly by Robert Ayres and Benjamin Warr is presented in chapter 2. Here, the energy and non-energy sectors are described in detail, and the model analysis is conducted in close comparison with the simple assumptions of the standard Solow growth model. In order to determine the existence and uniqueness of a steady-state, it is demonstrated that total technological progress can be written in a purely labor-augmenting form in the two-sector model, assuming a specific type of long-run growth (regular growth). This allows to perform comparative statics on the two-sector model, determining how variations in certain parameters affect the steady-state levels of output and capital per effective labor.

Chapter 3 addresses several long-term trends in economic development from empirical literature (stylized facts). Special relevance is given to textbook tendencies in growth reflected in the Kaldor facts and more recent empirical evidence relating energy use and prices to economic growth. The empirical analysis conducted in chapter 4 aims to provide insights on some of the stylized facts stated and analyzed. Kaldor's facts are empirically tested for the Portuguese economy, and compared with the proposed stylized fact regarding useful work intensity. Finally, the explanatory power of an additional energy-related variable (*energy*, *exergy* or *useful work*) in total factor productivity and output growth is estimated, assuming the economy as a single sector Solow model or as a two-sector growth model with a separation between the energy and non-energy related activities.

The main goal of this thesis is then to address the role of energy in economic growth from both a physical and economic perspective, and obtain evidence that links the consumption of energy converted to useful work (and respective efficiency) within production with the output generated by the economic system.

Chapter 2

The two-sector model

Throughout this chapter the abstract two-sector model is exposed and analyzed in detail. The aim is to develop a simple framework that enables the understanding of the proximate causes and mechanisms behind economic growth, under the assumption that energy (or an energy-related proxy) is an essential factor of production. This framework will be used to conduct steady-state analysis and comparative statics on several parameters, thereby gaining an understanding of what features are relevant to higher levels of income per capita and accelerating economic growth.

The Solow model is the starting point for almost all analyses of growth. Even models that depart fundamentally from Solow's are often best understood through comparison with this framework.

The proposed two-sector model analysis, based on the Solow approach, is introduced and explained in the first section of this chapter. After carefully characterizing and defining both sectors, Uzawa's steady-state theorem is invoked, and it is shown that, if the key variables for the two-sector model follow a specific type of long-run growth, defined as regular growth, total technological progress can be written in a purely labor-augmenting form. Finally, steady state equilibrium is determined and comparative statics are obtained for several parameters.

2.1 Economic growth and useful work - two-sector model

The point of departure for the model developed throughout the rest of this chapter is the neoclassical theory of growth originated by Solow. This theory has been developed over the last years with important contributions to questions related to energy, the environment, and economic growth, as well as natural resource extraction, environmental quality and income levels.

The model exposed here is based on the semi-empirical endogenous growth theory proposed by Ayres & Warr (2003,2005) [12] [13]. Their economic framework is modelled as a two-stage materials/energy processing system. Growth is simulated by a neoclassical production function with labor, capital and *useful work*. This last factor is determined from primary energy inputs multiplied by an empirically estimated average energy conversion efficiency, which is a function of changing technology over time. The two sectors of this model can be defined as: a *primary* sector that produces intermediate products, and a *secondary* sector that produces all final goods and services. The primary sector requires raw materials and fossil fuels taken from the environment, also denoted as physical resource inputs. This sector also uses labor and capital inputs in production. The secondary sector utilizes the outputs of the primary sector, which can be interpreted as processed materials and fuels, as well as labor and capital inputs. The output of the primary sector is a factor of production for the secondary sector's production function.

The two-sector model proposed here differs from the one presented by Ayres & Warr (2003,2005) in several aspects. The economy is described as a two-stage process with a separation between the energy-related activities and the remaining economic activities - Figure 2.1. The *Energy Sector* (E-Sector) is responsible for supplying the economic system with useful work, which can act as an intermediate or be directly consumed by households, government and NPISH[1]. The *Non-energy Sector* (NE-Sector) produces all kinds of final investment and non-energy related consumption goods and services in the economy.

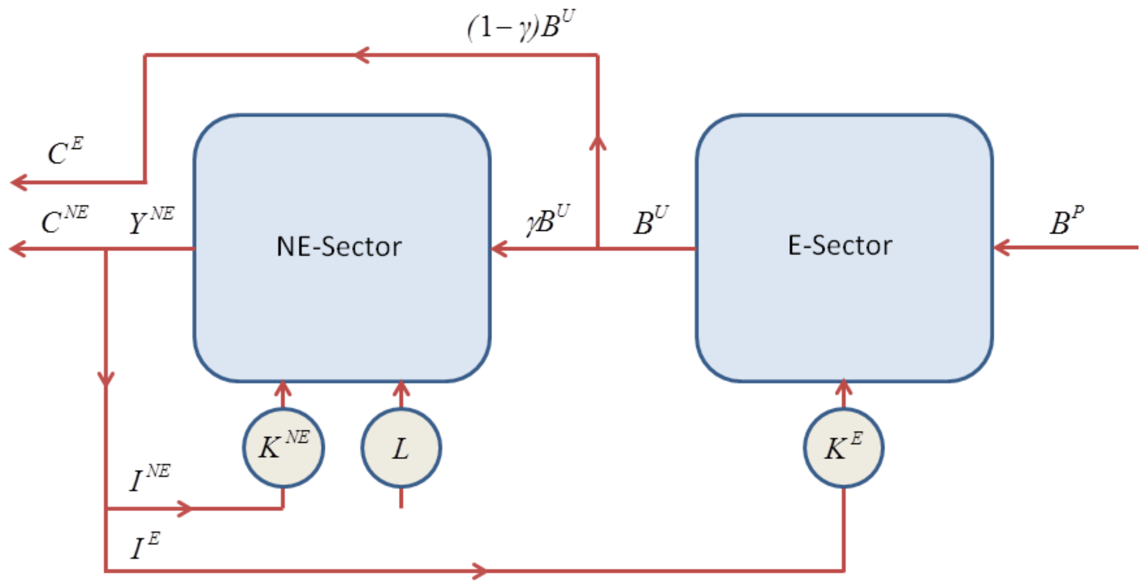


Figure 2.1: Diagram for the developed two-sector model of the economy.

Energy enters the economy through the E-Sector as primary exergy inputs from the environment, $B^P(t)$. Here, they will be converted into useful work, $B^U(t)$, in function of the physical capital invested in this sector, $K^E(t)$. A constant fraction of this useful work ($0 < \gamma < 1$) will be used as a factor of production for the NE-Sector, while the rest will be directly consumed by households, government, and NPISH, $C^E(t)$. The NE-Sector will use the outputs of the E-Sector not attributed to direct consumption, as well as its own capital inputs, $K^{NE}(t)$, and labor inputs, $L(t)$, to produce non-energy related consumption goods, $C^{NE}(t)$, and investment for both sectors, $I(t)$, through a neoclassical production function $Y^{NE}(t)$[2].

It is assumed that this is a closed economy, running in continuous time, and populated only by consumers (households, government and NPISH) and firms. Factors as imports/exports, taxes and subsidies, as well as capital transfers and net lending/borrowing are disregarded in this simple analysis. Markets are assumed as perfectly competitive, so that economic agents take prices as given. The total GDP of the economy is the sum of the NE-Sector output and the monetary value of final direct useful work consumption. There are, as usual, three different approaches to measuring GDP [51]: the *production*, or *value-added approach*; the *income approach*; and the *expenditure approach*.

The production approach sums the outputs of every class of enterprise and deducts intermediate

[1]Non-Profitable Institutions Serving Households.

[2]Consumption, investment, physical capital stock and output are defined in monetary terms (euros). Exergy and useful work variables B^P and B^U are defined in energy units (Joule). Labor inputs can be measured in terms of number of individuals or hours worked.

consumption (the cost of material, supplies, and services used in production) from this value to obtain GDP.

$$Y = Y^{NE} + (1-\gamma)p_{B^U}B^U; \tag{2.1}$$

The gross output for the system in Figure (2.1) is $Y^{NE} + p_{B^U}B^U$, where p_{B^U} is the relative price of directly consumed useful work, in respect to the price paid for useful work used in NE-Sector production. Therefore, $\gamma p_{B^U}B^U$ represents the total monetary value of intermediate inputs used up in NE-Sector production.

The income approach assumes that the incomes of the productive factors must equal the value of their respective product. GDP is measured by adding incomes that firms pay consumers for factors of production they hire. Assuming a full market equilibrium, there will be no profits, and GDP is:

$$Y = RK + wL - p_{B^P}B^P; \tag{2.2}$$

Here R and w correspond, as in the Solow model, to the rental price of physical capital and the wage rate paid to employees, respectively. The price paid by firms for primary exergy B^P is p_{B^P}.

The expenditure approach measures GDP by the total amount of money spent buying goods and services. In a closed economy, this translates as:

$$Y = C + I = C^{NE} + C^E + I^{NE} + I^E; \tag{2.3}$$

The three different approaches to calculate GDP should, in principle, yield the same result. The two separate sectors that form this economic framework, as well as their respective associated variables, are characterized in detail in the next section.

Energy sector

The energy sector (E-Sector) introduced here is innovatively defined, when compared to general economic models and accounts. The energy sector is usually associated with the energy industries, i.e. those involved in the production and sale of energy, including fuel extraction, manufacturing, refining, and distribution. Specifically, this sector usually includes:

- the petroleum industry, including oil companies, petroleum refiners, fuel transport and end-user sales at gas stations;
- the gas industry, including natural gas extraction and coal gas manufacture, as well as distribution and sales;
- the electrical power industry, including electricity generation and electric power distribution and sales;
- the coal industry;
- the nuclear power industry;
- the renewable energy industry, comprising alternative energy and sustainable energy companies, including those involved in hydroelectric power, wind power, and solar power generation, and the manufacture, distribution, and sale of alternative fuels;

- traditional energy industry based on the collection and distribution of firewood, the use of which, for cooking and heating, is particularly common in poorer countries;

In the model presented here, the E-Sector aggregates every single process that performs the conversion of primary exergy (extracted from the environment) into actual useful work performed. Any device, machine, or process that performs this conversion (final-use devices) is included in this E-Sector. Therefore, such goods as appliances or vehicles, used by firms or households, are part of the E-Sector, and their production is considered investment in the form of *energy capital*, K^E. This will be a crucial assumption of this model. A detailed examination of the constituents of energy capital is presented in chapter 4.

As mentioned in the previous chapter (and further explored in appendix A), energy consumption is really the increase of anergy at the expense of useful exergy, through entropy-increasing conversion processes. Energy can only be converted into useful physical work if there is a gradient. For example, it is the temperature difference between two reservoirs that determines the amount of work that can be extracted by a so-called heat engine [22]. On the other hand, virtually none of the heat energy present in ocean water can be used to perform useful work. Useful work, like exergy, is quantifiable, and can be measured with acceptable accuracy. This allows for the construction of aggregate measures of all resource flows into the economic system, as well as an aggregate measure of all processed intermediate flows.

From the second law of thermodynamics, as resource exergy is dissipated and destroyed in all transformation processes, it is theoretically possible to estimate a second law efficiency (appendix A) for each exergy consuming process, whose value is determined on a unique scale (bounded between zero and one) defined relative to a minimum necessary exergy requirement to achieve a given task [**?**]:

$$\epsilon(t) = \frac{desired\ exergy\ transfer}{relevant\ exergy\ input},\ with\ \ 0 \leq \epsilon(t) \leq 1; \tag{2.4}$$

This second-law efficiency depends on the end-use and technological level. It is widely accepted as a figure of merit for energy use. It measures, for each process, the distance from its theoretical ideal, and therefore the quality of a given energy use. Alternatively, the second-law efficiency (2.4) can be defined as

$$\epsilon(t) = \frac{\begin{array}{c} minimum\ amount\ of\ work \\ required\ to\ produce\ the\ desired\ energy\ transfer \end{array}}{\begin{array}{c} maximum\ amount\ of\ work \\ that\ could\ be\ produced\ from\ the\ relevant\ energy\ input \end{array}}; \tag{2.5}$$

In addition to characterizing efficiency trends in individual technologies, it is also possible to combine efficiencies within and across activities to characterize their aggregate efficiency. The exergy-based definition provides a unified framework to combine efficiencies of many different technologies. For example, the aggregate exergy efficiency of a mix of technologies is simply the total work performed divided by the natural resource exergy input. This is very useful when there is a need to measure and study the performance of energy uses throughout a country or an economy.

The generation of a useful work supply from natural resource exergy involves transformation and conversion losses - Figure (2.2). Transformation losses depend on the efficiency of the energy transformation sector (e.g. the transformation of fossil fuels to electricity or crude oil to products from crude oil). These transformations take place within the energy industries listed above. Conversion

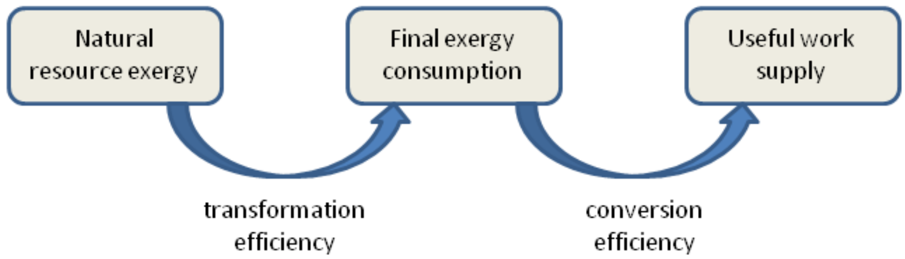


Figure 2.2: Primary-to-useful exergy flow.

losses refer to the efficiency of energy use equipment such as furnaces, boilers, ICE, and end-use devices that convert final exergy into some form of useful work (e.g. automobiles, appliances, etc.). The second-law efficiency can then be separated between the efficiency related to the allocation of exergy to each category of useful work, or *transformation efficiency* $\epsilon^D(t)$, and the efficiency related to the useful work supply extracted from final exergy, or *conversion efficiency* $\epsilon^A(t)$:

$$\epsilon = \epsilon^D \cdot \epsilon^A, \ with \ \ 0 \leq \epsilon^i \leq 1 \ \ and \ \ i = D, A; \tag{2.6}$$

Natural resource exergy to useful work conversion in the two-sector model presented here takes place entirely within the E-Sector, and its efficiency is assumed to be a function of this sector's technological capacity. The designation $A^E(t)$ is adopted from here on when referring to the total conversion efficiency from natural resource exergy to useful work in the E-Sector, $\epsilon(t)$. This is also the variable defining technological change for this sector. The rival factors of production entering the E-Sector are primary exergy inputs, B^P, and energy capital, K^E. Useful work is produced as a function of these factors and A^E:

$$B^U = F^E[K^E, B^P, A^E]; \tag{2.7}$$

Instead of introducing both K^E and B^P as factors of production, it will be assumed that useful work supply B^U depends on K^E alone. For simplicity, K^E and B^P are considered perfect complements of each other. This is a reasonable assumption since they must be consumed together to satisfy E-Sector demand. Physical goods such as energy capital are inherently scarce by definition, therefore K^E is considered a scarce good. Primary exergy is taken as a free good, meaning that it is available without limits in this framework. Due to their perfect complementarity, this means that in order to increase the input of B^P it suffices to increase K^E. The total primary exergy used in production for the E-Sector is only determined after the total output for the model is known. Under the proposed assumption of perfect complements the production function for the E-Sector (2.7) assumes a Leontief form:

$$F^E[K^E, B^P] = min(aK^E, bB^P), \ with \ a, b = constant > 0; \tag{2.8}$$

Choosing $K^E(t) = K^E$ to be a well defined at a given moment, there are three possible cases for (2.8). These are:

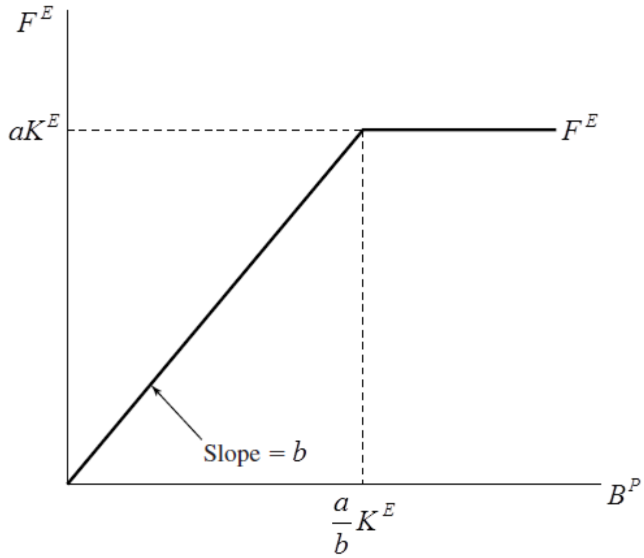


Figure 2.3: Energy sector output Leontief production function.

$$\begin{cases} If\ aK^E > bB^P,\ then\ F^E = bB^P; \\ \\ If\ aK^E = bB^P,\ then\ F^E = aK^E = bB^P; \\ \\ If\ aK^E < bB^P,\ then\ F^E = aK^E; \end{cases} \tag{2.9}$$

E-Sector production is assumed to be maximized (that is, F^E is at its maximum point). The Leontief production function is not differentiable, but exhibits a maximum when $aK^E = bB^P$ - Figure (2.3). This means that, at the point where E-Sector production is maximized, there is:

$$B^P = \frac{a}{b}K^E \Rightarrow B^P \propto K^E; \tag{2.10}$$

From (2.10) it results that the E-Sector production function can be written as depending only on physical energy capital K^E and technological progress A^E, which assumes a energy capital augmenting form. In order to further simplify the analysis, F^E takes the form of an AK production function, becoming:

$$B^U = F^E[A^E K^E] = A^E K^E; \tag{2.11}$$

In this simplified form, useful work output from the E-Sector is merely a function of the technological capacity of that sector and the physical capital invested in it. In value terms, output for the E-Sector will be given by:

$$Y^E = p_{B^U} B^U = \tilde{B}^U; \tag{2.12}$$

A constant positive fraction of the total useful work supply, γB^U, is used as a factor of production in the NE-Sector production function. The remaining fraction of B^U is directly consumed by households, government and NPISH. In monetary terms, this is written as:

$$\begin{cases} Y^E = C^E + \gamma \tilde{B}^U; \\ \\ C^E = (1-\gamma)\tilde{B}^U; \end{cases} \tag{2.13}$$

This concludes the description of the two-sector model's E-Sector. The NE-Sector is presented next.

Non-energy sector

The sector responsible for the economic activities not related to exergy-to-useful-work conversion is the NE-Sector. This sector is defined by a production function which uses physical capital, labor, and useful work inputs as rival factors of production in order to generate consumption goods for households, government and NPISH, as well as investment for both economic sectors. The production function also depends on the level of technology of the NE-Sector production processes. Output for this sector can be written as:

$$Y^{NE} = F^{NE}[K^{NE}, \gamma B^U, L, A^{NE}]; \tag{2.14}$$

In (2.14), Y^{NE} represents the flow of output from the NE-Sector and L is the measure of labor, which can be the number of hours worked or the total number of workers. The input γB^U is the fraction of total useful work output from the E-Sector used in NE-Sector production processes. Physical capital used in the production of non-energy goods and services is defined as *non-energy capital*, K^{NE}. Technological progress for the NE-Sector, A^{NE}, is exogenously given.For now, it is important to note that (2.14) has the form of a neoclassical production function and therefore will verify the same properties. Namely:

- **Constant returns to scale and Euler's theorem:** The production function (2.14) is homogeneous of degree one in K^{NE}, B^U, and L. That is, it verifies constant returns to scale on all three rival factors of production:

  $$F^{NE}\left[\lambda K^E, \lambda\gamma B^U, \lambda L, A^{NE}\right] = \lambda F^{NE}\left[K^E, \gamma B^U, L, A^{NE}\right],\ \forall \lambda > 0; \tag{2.15}$$

  Euler's theorem states that:

  $$F^{NE}\left[K^E, \gamma B^U, L, A^{NE}\right] = F^{NE}_{K^{NE}} K^{NE} + F^{NE}_{B^U} B^U + F^{NE}_L L,\ with\ F^{NE}_i = \frac{\partial F^{NE}}{\partial i}; \tag{2.16}$$

- **Positive and diminishing marginal products:** Holding constant the levels of technology, labor, and useful work, each additional unit of non-energy capital increases output. As capital rises, the increase of output is progressively smaller. The same is true for increments of labor and useful work, holding all other factors of production constant.

  $$\begin{aligned} &\frac{\partial F^{NE}}{\partial K^{NE}} > 0,\ \ \frac{\partial^2 F^{NE}}{\partial K^{NE^2}} < 0; \\ &\frac{\partial F^{NE}}{\partial B^U} > 0,\ \ \frac{\partial^2 F^{NE}}{\partial B^{U^2}} < 0; \\ &\frac{\partial F^{NE}}{\partial L} > 0,\ \ \frac{\partial^2 F^{NE}}{\partial L^2} < 0; \end{aligned} \tag{2.17}$$

- **Essentiality:** All three rival factors of production are essential to produce goods and services. Moreover, if any rival input to production is infinitely supplied, output produced will be infinite.

$$\begin{aligned} F^{NE}\left[0, \gamma B^U, L, A^E\right] = F^{NE}\left[K^{NE}, 0, L, A^E\right] = F^{NE}\left[K^{NE}, \gamma B^U, 0, A^E\right] = 0; \\ F^{NE}\left[\infty, \gamma B^U, L, A^E\right] = F^{NE}\left[K^{NE}, \infty, L, A^E\right] = F^{NE}\left[K^{NE}, \gamma B^U, \infty, A^E\right] = \infty; \end{aligned} \tag{2.18}$$

- **Inada conditions:** From (2.7) useful work output is assumed to be dependent on investment in physical capital in the E-Sector. One can assume the same standard conditions that apply to K^{NE} also apply to K^E. Therefore, the Inada conditions are:

$$\begin{gathered} \lim_{K^{NE}\to\infty}\left(\frac{\partial F^{NE}}{\partial K^{NE}}\right) = 0,\ \ \lim_{K^{NE}\to 0}\left(\frac{\partial F^{NE}}{\partial K^{NE}}\right) = \infty,\ \ for\ all\ K^E, L > 0; \\ \lim_{K^{E}\to\infty}\left(\frac{\partial F^{NE}}{\partial K^{E}}\right) = 0,\ \ \lim_{K^{E}\to 0}\left(\frac{\partial F^{NE}}{\partial K^{E}}\right) = \infty,\ \ for\ all\ K^{NE}, L > 0; \\ \lim_{L\to\infty}\left(\frac{\partial F^{NE}}{\partial L}\right) = 0,\ \ \lim_{L\to\infty}\left(\frac{\partial F^{NE}}{\partial L}\right) = \infty,\ \ for\ all\ K^{NE}, K^E > 0; \end{gathered} \tag{2.19}$$

  The marginal product of each of the three rival factors of production, assuming the remaining factors remain positive, approaches zero as that factor's value tends to infinity, and vice-versa.

As previously stated, the economy in question is assumed to be closed. All NE-Sector production is alocated either to consumption of non-energy goods and services, $C^{NE}(t)$, or investment in the form of physical capital for the whole economy, $I(t)$. Investment is further divided between investment in the form of physical energy capital alocated to the E-Sector $I^E(t)$, and physical non-energy capital alocated to the production processes in the NE-Sector $I^{NE}(t)$:

$$\begin{cases} Y^{NE} = C^{NE} + I; \\ \ \ I = I^E + I^{NE}; \end{cases} \tag{2.20}$$

In line with the Solow approach adopted here, investment in this framework is also equivalent to an exogenous constant fraction of output[3], the saving rate s. The remaining fraction of NE-Sector output is attributed to consumption of non-energy goods and services:

$$\begin{cases} I = s \cdot Y^{NE} \\ C^{NE} = (1-s) \cdot Y^{NE} \end{cases},\ 0 \le s \le 1; \tag{2.21}$$

Investment alocated to the NE-Sector is equivalent to an exogenous constant fraction of total investment I, denoted σ, and assuming values between zero and unity, $0 \le \sigma \le 1$. Investment alocated to the E-Sector corresponds to the remaining fraction, $1-\sigma$, of total investment:

$$\begin{cases} I^{NE} = \sigma \cdot I = \sigma \cdot s \cdot Y^{NE}; \\ I^E = (1-\sigma) \cdot I = (1-\sigma) \cdot s \cdot Y^{NE}; \end{cases} \tag{2.22}$$

[3]In this case, simply the output correspondent to NE-Sector production, Y^{NE}.

Both types of investment, I^E and I^{NE}, add to the capital stocks K^E and K^{NE}, respectively. These depreciate and lose value at distinct rates, assumed here to be constant but not necessarily equal. The laws of motion for capital stock related to both sectors of the economy are:

$$\begin{cases} \dot{K}^{NE} = I^{NE} - \delta_{NE} K^{NE}; \\ \\ \dot{K}^{E} = I^{E} - \delta_{E} K^{E}; \end{cases} \tag{2.23}$$

NE-Sector labor inputs, L, are related to exponential population growth, given exogenously by a constant growth rate, n, as in the Solow model.

$$L = L_0 e^{nt}, \ with \ n = constant > 0; \tag{2.24}$$

The characteristic technological level associated with the production processes in the NE-Sector is assumed to be labor-augmenting in nature:

$$F^{NE}[K^{NE}, \gamma B^U, L, A^{NE}] = F^{NE}[K^{NE}, \gamma B^U, A^{NE} L]; \tag{2.25}$$

This concludes the description of the NE-Sector for this economy. In order to establish a comparison with the Solow growth model, the steady-state and equilibrium conditions for the two-sector model proposed above are determined next.

Two-sector model dynamical system

The previous section dealt with the main description of the two-sector growth model, the definition of variables and their interrelationships. These are now summarized as a dynamical system of equations. The laws of motion for both types of physical capital, K^E and K^{NE}, are given by (2.23). The fractions of NE-Sector output alocated to non-energy and energy capital formation are $\sigma \cdot s$ and $(1 - \sigma) \cdot s$, respectively, with $0 \leq \sigma, s \leq 1$. The remaining fraction of output is consumed by households, government and NPISH (2.21). Capital stocks depreciate at distinct constant rates: δ_{NE} for K^{NE}, and δ_E for K^E. The NE-Sector production function depends on the technological level of its production processes, characterized in (2.25), as well as physical non-energy capital, labor inputs, and a fraction γ of the useful work provided by the E-Sector. Labor inputs grow exponentially according to (2.24) and useful work supplied is dependent on the stock of physical energy capital and the technological capacity associated with the conversion efficiency of primary exergy inputs into useful work (2.11). The functional form for the E-Sector technological energy efficiency A^E is still unknown but it is assumed that its variation in time $\dot{A}^E$ depends on the current level of technology. The same applies to the technological capacity of the NE-Sector, A^{NE} (instead of having technological progress grow at a constant exponential rate, as in the Solow model). Under the above conditions, the framework developed for this model can be described by the following dynamical system of equations:

$$
\begin{cases}
\dot{K}^{NE} = \sigma \cdot sF^{NE}\left[K^{NE}, \gamma F^{E}(A^{E}K^{E}), A^{NE}L\right] - \delta_{NE}K^{NE}; \\
\dot{K}^{E} = (1-\sigma) \cdot sF^{NE}\left[K^{NE}, \gamma F^{E}(A^{E}K^{E}), A^{NE}L\right] - \delta_{E}K^{E}; \\
\dot{L} = n \cdot L; \\
\dot{A}^{E} = G(A^{E}); \\
\dot{A}^{NE} = H(A^{NE});
\end{cases} \tag{2.26}
$$

From the system in (2.26), given the initial endowments for K^E, K^{NE}, L, and knowing the functional form for A^E and A^{NE}, it is possible to obtain the levels of consumption for both sectors of the economy[4], C^{NE} and C^E.

2.2 Uzawa's theorem and regular growth

Technical change, in this model, is present in two distinct forms. These are the energy capital augmenting technological progress associated with primary exergy-to-useful work conversion in the E-Sector, A^E, and the labor augmenting technical change characterizing the production processes in the NE-Sector, A^{NE}. In practice, technological change can be a mixture of different kinds of factor augmenting techniques [1], and can be expressed as a vector valued index of technology. In this case,

$$
\mathbf{A}(t) = \left(A^{E}(t), A^{NE}(t)\right); \tag{2.27}
$$

The production function as expressed in (2.14) is too general to perform growth analysis. Under this general structure, there may not be balanced growth for the two-sector model developed. In general economics literature, balanced growth means a path along which output, consumption and capital stock are positive and grow at constant rates. A general result on balanced growth is that, given the fundamental law of motion for capital, and indenpendently of how saving is determined and how labor and technology change, if there is balanced growth with positive gross saving, then the ratios output-capital (Y/K) and consumption-output (C/Y) are constant. Another way of putting it would be: as long as gross saving is positive, constancy of the Y/K and C/Y ratios is sufficient to ensure balanced growth. This is the notion of balanced growth adopted in the following analysis.

In reality, economic growth has many non-balanced characteristics. The share of different institutional sectors, for example, changes systematically over the growth process, with agriculture shrinking and manufacturing first increasing and then shrinking. A balanced growth path is much easier to handle when performing model analysis, due to the laws of motion for the economy being represented by difference or differential equations with well-defined steady states. Balanced growth, however, is not compatible with every configuration for technological progress in a neoclassical production function framework. In fact, although all forms of technological change look equally

[4] The growth rate n, saving rate s, depreciation rates δ_E and δ_{NE}, and fractions γ and σ are defined outside the model.

plausible ex ante, the condition of balanced growth implies that all technological progress must be labor augmenting or Harrod-neutral.

Uzawa was the first to prove this in his steady-state growth theorem [75], although his work is primarily concerned with showing the equivalence of Harrod neutral and labor augmenting technical change, formalizing the graphical analysis by Robinson (1938) [58]. Alternative statements and proofs for this theorem appear in the relevant literature: Barro & Sala-i-Martin (1995) [18] present a more restrictive version, claiming that if technical change is factor augmenting at a constant exponential rate, then steady-state growth requires it to be labor augmenting. McCallum (1996) [48] provides a proof very similar, however unintuitive, to Uzawa's approach. Russell (2011) [60] exploits some methods from physics to construct a mathematical proof of the theorem, through the use of a class of partial differential equations called advective equations. A more clear statement of the theorem is found in Jones & Scrimgeour (2008) [43] which says that if a neoclassical growth model exhibits steady state growth (balanced growth), then technical change must be labor augmenting, at least in the steady state. It fails, however, to provide a reason as to why technology would take this labor augmenting form, an omission common to every approach found in the literature.

A stronger version of the result first stated and proved by Uzawa is given by Schlicht (2006) [62] and adapted by Acemoglu (2008) [1]. In contrast to the classical statement and the more recent version by Jones & Scrimgeour (2008), this proof does not involve assumptions about factor pricing, such as marginal productivity theory, or savings behavior. The proof proposed by Schlicht (2006) establishes that exponential growth implies Harrod-neutral technical progress. It starts by considering an economy with a neoclassical production function F that exhibits, at any point in time, constant returns to scale, and inputs of labor, L, and capital K, as factors of production. Investment is identical to savings, and the capital stock is augmented over time by investment and reduced by depreciation, like in (2.23). It then proceeds to claim that, if the system possesses a solution in which output, consumption and capital are nonnegative and grow at constant rates, the production function can be written with technological progress as purely labor augmenting. The propositions in Schlicht (2006) and Acemoglu (2008) do not make any reference to equilibrium behavior or market clearing, and it is not stated for a balanced growth path, but simply an asymptotic path with constant rates of output, capital, and consumption growth. Since the proof only exploits the definition of asymptotic growth paths, the constant returns to scale nature of the aggregate production function, and the resource constraint, its result is a very powerful one. Moreover, it does not state that technological change must be labor augmenting all the time, but instead requires that the production function must be able to assume a purely labor-augmenting technological progress form asymptotically.

Within the two-sector framework developed here, the proof given in Schlicht (2006) and Acemoglu (2008) will be applied to a more general regularity concept than that of exponential growth. The purpose is to show that the production function (2.14) can be written with purely labor-augmenting technical progress and that a balanced growth path is possible for the proposed model. In most growth theories, the concept of balanced growth is generally synonymous with exponential growth, if only for its convenient simplicity. However, some models may feature a different pattern of growth. For example, the Ramsey model with AK technology features arithmetic GDP per capita growth [57], and authors such as Mitra (1983) [49], Pezzey (2004) [56] and Asheim et al (2005) [8] consider growth paths of the form $x(t) = x_0(1 + \mu t)^{\omega}, \mu, \omega > 0$, which can be referred to as quasi-arithmetic growth. In their 2006 paper, Christian Groth et al (2010) [35] define the rationale for a broader concept of regular long-run growth, which subsumes the whole range between exponential growth and stagnation, including arithmetic growth and some kind of less-than-arithmetic growth. This broader concept is simply labelled *regular growth*, and is defined in the next pages. First, considering the variable $x(t)$ to be a positively-valued differentiable function of time t, and writing

Table 2.1: Long-run growth paths subsumed in regular growth, including limiting cases.

Label	Coefficients	Time path
Exponential growth	$\omega = 0$, $\mu > 0$	$x(t) = x_0 e^{\mu t}$, $\mu > 0$
More-than-arithmetic growth	$0 < \omega < 1$, $\mu > 0$	$x(t) = x_0(1 + \mu\omega t)^{1/\omega}$, $\mu, \omega > 0$
Arithmetic growth	$\omega = 1$, $\mu > 0$	$x(t) = x_0(1 + \mu t)$, $\mu > 0$
Less-than-arithmetic growth	$1 < \omega < \infty$, $\mu > 0$	$x(t) = x_0(1 + \mu\omega t)^{1/\omega}$, $\mu, \omega > 0$
Stagnation	$\omega = \infty$, $\mu > 0$	$x(t) = x_0$

its growth rate as:

$$g_1(t) := \frac{\dot{x}(t)}{x(t)}; \tag{2.28}$$

The variable $g_1(t)$ is the first-order growth rate of $x(t)$. Since a more general concept than exponential growth is intended, g_1 is allowed to be time variant. Regularity here relates to the way the growth rates change over time. Pressuposing g_1 to be strictly positive within the time range considered, $g_2(t)$ denotes the second-order growth rate of $x(t)$ at time t:

$$g_2(t) := \frac{\dot{g}_1(t)}{g_1(t)} = -\omega g_1(t), \quad \forall t > 0; \tag{2.29}$$

The second equality in (2.29) is the criterion for defining regular growth, according to Groth et al (2010). The second-order growth rate is proportional to the first-order growth rate with a factor $\omega \geq 0$, which is called the *damping coefficient* and indicates the rate of damping in the growth process.

Now, letting x_0 and μ denote the initial time values $x(0) > 0$ and $g_1(0) > 0$, the unique solution of the second-order differential equation in (2.29) is:

$$x(t) = x_0(1 + \mu\omega t)^{1/\omega}, \ \mu, \omega \geq 0; \tag{2.30}$$

This simple formula describes a family of growth paths, which are indexed by the damping coefficient ω. There are three well-known special cases of (2.30). For $\omega = 0$, the first-order growth rate will be $g_1(t) = \mu$, a positive constant. This is the case of exponential growth[5]. At the other extreme is the limiting case of $\omega \to \infty$, which leads to the constant path of complete stagnation, $x(t) = x_0$[6]. The third special case, $\omega = 1$, represents arithmetic growth, $\dot{x}(t) = \mu$, $\forall t \geq 0$. These three special cases and the intermediate ranges are listed in Table (2.1) and represented graphically in Figure (2.4).

In the context of the two-sector model proposed, each of the key sector-specific variables is assumed to follow an asymptotic path similar to (2.30). That is, the output from the NE-Sector Y^{NE}, labor L, energy and non-energy physical capital stocks K^E and K^{NE}, and consumption for both sectors C^E and C^{NE} assume the functional form:

[5] It can be shown that $\lim_{\omega \to 0} x_0(1+\mu\omega t)^{1/\omega} = x_0 e^{\mu t}$, using l'Hôpital's rule for 0/0 on $ln(x(t)) = ln(x_0) + \frac{1}{\omega} ln(1+\mu\omega t)$.

[6] Using l'Hôpital's rule for ∞/∞ on $ln(x(t))$.

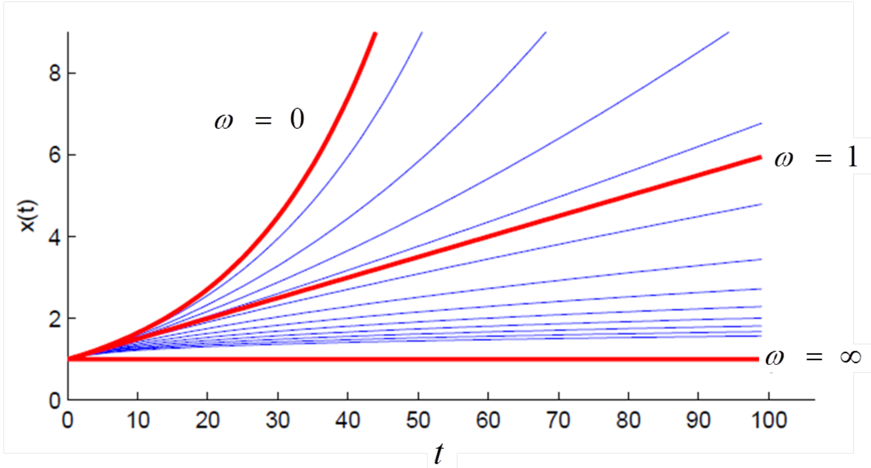


Figure 2.4: This figure is based on $\mu = 0.05$ and $x_0 = 1$. In this case, the time paths do not intersect. Intersections occur for $x_0 < 1$. For large t however, the graphical representation is always as shown here.

$$\begin{cases} i = i_0\left(1+\mu_i\omega_i t\right)^{1/\omega_i} \\ \\ g_i(0) = \mu_i \\ \\ \dfrac{\dot{g}_i}{g_i} = -\omega_i g_i \end{cases} , \; with \; i = Y^{NE}, C^E, C^{NE}, K^E, K^{NE}, L; \tag{2.31}$$

Like before, i_0, μ_i and ω_i are constant positive values, distinct for each variable in the model. The time-varying growth rate g_i is positively defined for all variables as $(\partial i/\partial t)/i$.

The sum of NE-Sector output, Y^{NE}, and the monetary value of direct useful work consumption, C^E, constitutes the total output for the economy, Y. Total consumption C is the sum of non-energy related goods and services consumption, C^{NE}, and direct useful work consumption, C^E, while total capital K represents the sum of physical non-energy capital K^{NE}, and energy capital K^E. The next step is to show that, assuming the sector-specific key variables of the model (Y^{NE}, C^E, C^{NE}, K^{NE}, K^E, L) behave according to a regular growth path as in (2.31), then in order for the total aggregate variables (Y, C, K) to follow a similar growth path, they must grow at the same rate as the respective disagregate variables[7]. This is demonstrated in detail in appendix B.

The variable C^E is a component of both total output and total consumption. From the previous result that $g_Y = g_{Y^{NE}} = g_{C^E} = g_{C^{NE}} = g_C$, the growth rates of total output and total consumption will be equal. Furthermore, it is possible to show, from the laws of motion for capital and resource constraints - (2.26) - that the growth rates of total output and total capital are also identical - $g_Y = g_K$. This is also demonstrated in appendix B. The most important results derived from the previous calculations are the correspondence between sector-specific key variables and the respective aggregate variables:

$$g_{Y^{NE}} = g_{C^E} = g_Y; \tag{2.32}$$

[7] e.g. total capital K grows at the same rate as energy capital K^E and non-energy capital K^{NE}: $g_K = g_{K^E} = g_{K^{NE}}$.

And the correspondence between the growth rates of total output, consumption, and capital for the two-sector model:

$$g_Y = g_C = g_K; \tag{2.33}$$

The technology-related factors in the NE-Sector production function can be represented by the vector valued index in (2.27). Consequently, the NE-Sector production function can be written as:

$$Y^{NE} = F^{NE}\left[K^{NE}, \gamma K^E, L, \mathbf{A}\right]; \tag{2.34}$$

Assuming that the vector valued index $\mathbf{A}(t)$ is normalized to unity for $t = 0$, the production function (2.34), recalling the regular growth paths assigned to the key variables (2.31), is:

$$\frac{Y^{NE}}{(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{\frac{1}{\omega_{Y^{NE}}}}} = F^{NE}\left[\frac{K^{NE}}{(1+\mu_{K^{NE}}\omega_{K^{NE}}t)^{\frac{1}{\omega_{K^{NE}}}}}, \frac{\gamma K^E}{(1+\mu_{K^E}\omega_{K^E}t)^{\frac{1}{\omega_{K^E}}}}, \frac{L}{(1+\mu_L\omega_L t)^{\frac{1}{\omega_L}}}, 1\right] \tag{2.35}$$

Multiplying both sides of (2.35) by the quantity $(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{1/\omega_{Y^{NE}}}$, and remembering the constant returns to scale property of neoclassical production functions results in:

$$Y^{NE} = F^{NE}\left[\frac{(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{\frac{1}{\omega_{Y^{NE}}}}}{(1+\mu_{K^{NE}}\omega_{K^{NE}}t)^{\frac{1}{\omega_{K^{NE}}}}}K^{NE}, \frac{(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{\frac{1}{\omega_{Y^{NE}}}}}{(1+\mu_{K^E}\omega_{K^E}t)^{\frac{1}{\omega_{K^E}}}}\gamma K^E, \frac{(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{\frac{1}{\omega_{Y^{NE}}}}}{(1+\mu_L\omega_L t)^{\frac{1}{\omega_L}}}L\right] \tag{2.36}$$

The calculations presented in appendix B show that $\mu_{Y^{NE}} = \mu_Y$, $\mu_{K^{NE}} = \mu_{K^E} = \mu_K$ and $\mu_Y = \mu_K$. Likewise, $\omega_{Y^{NE}} = \omega_Y$, $\omega_{K^{NE}} = \omega_{K^E} = \omega_K$ and $\omega_Y = \omega_K$. Therefore, the ratios inside (2.35) which appear multiplied by the first two factors of production, K^{NE} and K^E, will be equal to unity.

The only remaining time-varying multiplier inside the NE-Sector production function will be associated with the production factor labor, L. This is the labor-augmenting total technological progress characteristic of production in this economy, which will be defined as $A^L(t)$. The growth rate of $A^L(t)$ will be the difference between the growth rates of the NE-Sector output (or total output) and labor.

$$A^L(t) = \frac{(1+\mu_{Y^{NE}}\omega_{Y^{NE}}t)^{1/\omega_{Y^{NE}}}}{(1+\mu_L\omega_L t)^{1/\omega_L}} \Rightarrow \frac{\dot{A}^L(t)}{A^L(t)} = g_{Y^{NE}}(t) - g_L(t) = \lambda(t); \tag{2.37}$$

Thus, it is shown that total technological change can be written in a purely Harrod-neutral form, under the assumption of regular growth for the key variables in the proposed two-sector model. The demonstration, like Schlicht's (2006), only exploits the definitions of asymptotic paths and regular growth, the constant returns to scale nature of the production function and the resource constraints. This result implies that it is possible to consider a balanced growth path for the two-sector model presented in this chapter. There is no reason provided as to why technological change should assume this Harrod-neutral form. The steady state theorem simply indicates that if technology did not take this form, balanced growth would not be possible. It will be shown in the next section how this result will allow for the determination of steady-state equilibrium and comparative statics in the proposed model.

2.3 Steady-state equilibrium and comparative statics

In light of the results obtained in the previous section, the dynamical system in (2.26) can be simplified and rewritten as:

$$\begin{cases} \dot{K}^{NE} = \sigma \cdot s \cdot F^{NE}\left(K^{NE}, \gamma K^{E}, A^{L}L\right) - \delta_{NE}K^{NE}; \\ \\ \dot{K}^{E} = (1-\sigma) \cdot s \cdot F^{NE}\left(K^{NE}, \gamma K^{E}, A^{L}L\right) - \delta_{E}K^{E}; \\ \\ \dot{L} = n \cdot L; \\ \\ \dot{A}^{L} = \lambda \cdot A^{L}; \end{cases} \tag{2.38}$$

The simplest way of analyzing this economy is to express all key variables in terms of a normalized variable. Since efficiency units of labor are now given by $A^L L$ in (2.38), and F^{NE} exhibits constant returns to scale in its three rival factors (K^{NE}, K^E, L), an *effective capital-labor* ratio can be defined for both types of capital, as:

$$k^{NE} = \frac{K^{NE}}{A^{L}L}, \;\; k^{E} = \frac{K^{E}}{A^{L}L}; \tag{2.39}$$

NE-Sector output can consequently be rewritten as:

$$y^{NE} = \frac{Y^{NE}}{A^{L}L} = F^{NE}\left(\frac{K^{NE}}{A^{L}L}, \frac{\gamma K^{E}}{A^{L}L}, 1\right) = f^{NE}\left(k^{NE}, \gamma k^{E}\right); \tag{2.40}$$

As for the laws of motion for K^E and K^{NE}, they can be obtained for these new variables by differentiating (2.39) with respect to time. What results is the new dynamical system describing the two-sector model, where the four equations in (2.38) are reduced to two equations:

$$\begin{cases} \dot{k}^{NE} = \sigma \cdot s \cdot f^{NE}\left(k^{NE}, \gamma k^{E}\right) - \left(\delta_{NE} + n + \lambda\right)k^{NE}; \\ \\ \dot{k}^{E} = (1-\sigma) \cdot s \cdot f^{NE}\left(k^{NE}, \gamma k^{E}\right) - \left(\delta_{E} + n + \lambda\right)k^{E}; \end{cases} \tag{2.41}$$

New energy (non-energy) capital formation depends on the difference between investment in the E-Sector (NE-Sector) and the effects of depreciation, population growth and labor augmenting technological change. Investment $s \cdot f^{NE}\left(k^{NE}, \gamma k^{E}\right)$ is used to replenish the effective capital-labor ratio for several reasons: depreciation of capital stocks δ^{NE} and δ^{E} and the fact that the capital stock of the economy has to increase as population grows at a rate n and technology grows at rate λ, in order to maintain the effective capital-labor ratio constant.

Given the system (2.41) and assuming constant returns to scale and the Inada conditions hold for the NE-Sector output, there will exist a unique steady-state equilibrium where the effective capital-labor ratios $k^{NE^*}, k^{E^*} \in (0, \infty)$ are given by:

$$\begin{cases} \dfrac{f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right)}{k^{NE}} = \dfrac{\left(\delta_{NE} + n + \lambda\right)}{s \cdot \sigma}; \\ \\ \dfrac{f^{NE^*}\left(k^{NE^*}, \gamma k^{E^*}\right)}{k^{E^*}} = \dfrac{\left(\delta_{E} + n + \lambda\right)}{s \cdot (1-\sigma)}; \end{cases} \tag{2.42}$$

In this case, NE-Sector output and consumption per effective labor are given, respectively, by:

$$\begin{aligned} y^{NE^*} &= f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right); \\ c^{NE^*} &= (1-s)\cdot f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right); \end{aligned} \tag{2.43}$$

The steady state equilibrium is defined in terms of k^E and k^{NE}. A trivial steady state exists when $f^{NE}(0,0)$ (with $k^{NE^*} = k^{E^*} = 0$). However, it is relevant to focus on a steady state equilibria with $k^{NE^*} > 0$ and $k^{E^*} > 0$. The existence and uniqueness of this steady state is easily determined, and the respective calculations are summarized in appendix B.

The simplified model achieved in (2.41) abstracts from many features of real economies in order to focus on the relevant parameters. An understanding of how differences in certain parameters translate into differences in growth rates or output levels is essential for any model analysis. This is done through comparative statics. The steady-state levels of k^E and k^{NE} can be denoted as depending on the relevant parameters, such as:

$$\begin{aligned} &k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right); \\ &k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right); \end{aligned} \tag{2.44}$$

The detailed calculation of comparative statics is presented in appendix B. The conclusions are as follows:

$$\begin{aligned}
&\frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial s} > 0; \quad \frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \sigma} > 0; \quad \frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \gamma} > 0; \\
&\frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \delta^{NE,E}} < 0; \quad \frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \lambda} < 0; \quad \frac{\partial k^{NE^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial n} < 0; \\
&\frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial s} > 0; \quad \frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \sigma} < 0; \quad \frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \gamma} > 0; \\
&\frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \delta^{NE,E}} < 0; \quad \frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial \lambda} < 0; \quad \frac{\partial k^{E^*}\left(s, \sigma, \delta^E, \delta^{NE}, \lambda, n, \gamma\right)}{\partial n} < 0;
\end{aligned} \tag{2.45}$$

A higher saving rate s leads to higher levels of both types of capital in the steady state, because a higher s indicates a larger portion of NE-Sector output devoted to capital investment. Likewise, higher σ means that there is a larger portion of total investment allocated to non-energy capital, and consequently less investment allocated to energy capital. Increases in the depreciation rates δ^{NE} and δ^E will translate in a lowering of the steady state level of non-energy and energy capital, respectively. A higher fraction of useful work γ used as a factor of production in the NE-Sector will result in higher levels of output for this sector and higher investment in capital. Finally, increases in the growth rates of both labor and total technological change will have a negative effect on the steady state levels of k^E and k^{NE}.

Comparative statics are also performed for the non-energy sector steady state level of output, $y^{NE^*} = f^{NE}(k^{NE^*}, \gamma k^{E^*})$. The results are (see appendix B):

$$\frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial s} > 0; \quad \frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial \delta^E} < 0; \quad \frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial \delta^{NE}} < 0;$$
$$\frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial \lambda} < 0; \quad \frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial n} < 0; \quad \frac{\partial y^{NE^*}(s,\delta^E,\delta^{NE},\lambda,n,\gamma)}{\partial \gamma} > 0; \tag{2.46}$$

Here, higher s and γ will affect NE-Sector output in the steady-state positively, while δ_E and δ_{NE}, population growth or technological change will have a negative effect on y^{NE^*}. The fraction of savings allocated to investment in the NE-Sector, σ, will also affect the level of output in the steady state. However, in this case three possibilities arise:

$$\begin{cases} If \ \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| > \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right|, \ then \ \dfrac{\partial y^{NE^*}}{\partial \sigma} > 0; \\ If \ \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| = \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right|, \ then \ \dfrac{\partial y^{NE^*}}{\partial \sigma} = 0; \\ If \ \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| < \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right|, \ then \ \dfrac{\partial y^{NE^*}}{\partial \sigma} < 0; \end{cases} \tag{2.47}$$

Depending on the values of $\partial k^{NE^*}/\partial \sigma$, $\partial k^{E^*}/\partial \sigma$, and the marginal productivities of the NE-Sector output for both types of capital, the fraction of investment allocated to the NE-Sector will have a positive, neutral, or negative effect on the steady state level of output per effective labor. The analysis of the transitions presented in (2.47) should prove interesting in future developments of the work presented here.

The comparative statics related to the fractions σ and γ, the growth rates n and λ, and the depreciation rates δ^{NE} and δ^E in (2.46) will immediatly apply to NE-Sector consumption per effective labor steady state level, c^{NE^*}. However, this steady-state level will not be monotone regarding the saving rate s:

$$\frac{\partial c^{NE^*}(s)}{\partial s} = \left[f^{NE}_{k^{NE^*}} - (\delta^{NE} + n + \lambda)\right] \frac{\partial k^{NE^*}(s)}{\partial s} + \left[f^{NE}_{k^{E^*}} - (\delta^{E} + n + \lambda)\right] \frac{\partial k^{E^*}(s)}{\partial s}; \tag{2.48}$$

The value for the saving rate that will maximize the level of c^{NE^*}, s_{gold}, will be such that $\partial c^{NE^*}(s_{gold})/\partial s = 0$.

This concludes the exposition of the proposed two-sector abstract growth model, with Solow-like simplifying assumptions.

2.4 Overview

This chapter laid the theoretical foundations of the two-sector economic model proposed in this thesis, as well as the purely labor augmenting form for total technological progress (under two-sector model's assumptions and regular growth paths) and the determination of steady-state equilibrium and comparative statics. The Solow model is taken as a starting point and recurrent frame of comparison for the proposed model.

Section 1.1 states the general framework of the two-sector model - Figure (2.1) - and elaborates on the definitions of the Energy and Non-energy sectors in which it is divided. Equations (2.11), (2.12) and (2.13) resume the useful work production and allocation in the E-Sector, while the NE-Sector is summarized by equations (2.14), (2.21), and (2.22), with the laws of motion for capital given by (2.23). Finally, the dynamical system describing the whole model is given by (2.26).

Section 1.2 introduces the notion of regular growth for the key variables in the model (2.31) and succefully extends Schlicht's (2006) and Acemoglu's (2008) demonstration to the two-sector model, with regular growth paths adopted for its variables. Technical change in the NE-Sector production function is written in a purely labor-augmenting form - Equation (2.37). The simplified dynamical system is presented in Section 1.3 by (2.38) and rewritten in terms of effective labor variables (2.39). Comparative statics are determined for the steady state levels of energy and non-energy capital - (2.45) - and NE-Sector output - (2.46) and (2.47).

Chapter 3

Stylized facts

This chapter takes the two-sector model developed in the previous chapter as a standpoint to address several stylized facts of economic growth, with special attention to those which are more influential in the literature, and the ones relevant to the relationship between energy use and growth.

A stylized fact is a term used in social sciences, especially economics, to name a simplified representation of an empirical finding. It often summarizes complicated statistical calculations in the form of a broad generalization which, although essentially true, may have innacuracies in the details. Stylized facts are widely used in economics to motivate the construction of complex models and/or validate their assumptions in a focussed way. The comparative advantages of one model over another can be set in clear perspective through the referencing of the stylized facts that the respective models can explain.

This chapter deals first with the validation of the Kaldor's facts of long-run economic growth, within the developed two-sector model. It then applies the same treatment to other recently proposed stylized facts of economic growth collected from the relevant literature. The final section enunciates the stylized fact suggested by the useful work consumption accounting study, undertaken by André Serrenho for Portugal, between 1856 and 2009 [64]. This last stylized fact is a major motivation of the work developed in this thesis.

3.1 Kaldor's facts

Nicholas Kaldor introduced the term stylized fact in the context of economic growth theory in 1957 [44]. Criticizing contemporary neoclassical growth models, Kaldor argued that theory construction should begin with a summary of the relevant facts, as recorded by statisticians. He summarized the statistical properties of long-term economic growth[1] and pointed out a total of six historical constancies revealed by empirical investigation: the statements now known in the literature as Kaldor's facts.

1. Output per worker grows at a roughly constant rate that does not diminish over time.

2. Capital per worker exhibits continuing growth over time.

3. Capital per output ratio is roughly constant.

[1] Derived from U.S. and U.K. data.

4. The rate of return on capital is constant.

5. The shares of capital and labor in net income are nearly constant.

6. The real wage grows over time.

These statements aimed to reflect what economists had learned about growth trends in the 20th century, and have since become the empirical evidence that almost every growth model needs to confirm or reject. They are true for many countries, though not for all. Although these statements are far from being free from controversy they provide, in general, an accurate picture of growth in industrialized countries.

In order to address Kaldor's facts within the framework of the two-sector model developed in chapter 2, the type of long-run growth introduced in the previous chapter (regular growth) will be considered for the disaggregate sector-specific economic variables.

Kaldor's facts in the two-sector model

This section seeks to answer the questions: (a) which of Kaldor's stylized facts are compatible with the two-sector model developed in this thesis; (b) under what conditions are these stylized facts valid for the two-sector model, assuming regular long-run paths. Before addressing the validity of Kaldor's facts however, it is necessary to write these statements in a mathematical form adapted to the variables used in the two-sector model analysis. This is done systematically for each of Kaldor's facts.

- **Kaldor's fact 1:** *Output per worker grows at a roughly constant rate that does not diminish over time.*

Output per worker is equivalent to total output (Y) divided by labor inputs (L). Any scenario of long-run growth must verify, in order for Kaldor's fact 1 to be true:

$$\frac{\partial}{\partial t}\left(\frac{Y}{L}\right) / \left(\frac{Y}{L}\right) = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = g_Y - g_L = constant > 0; \tag{3.1}$$

Assuming regular growth paths, the growth rate of purely labor-augmenting technological progress A^L is the difference between the growth rate of Y and the growth rate of L: $\lambda = g_Y - g_L$. However, since the first-order growth rates for total output and labor are time-varying under regular growth, it cannot be determined from the model's assumptions alone whether this difference is constant and/or positive in value. Hence the first of Kaldor's facts for long-run growth can only be verified, within the framework of the two-sector model under study, by imposing the additional external condition:

$$Condition\ 1 : \lambda = g_Y - g_L = constant,\ with\ g_Y > g_L; \tag{3.2}$$

In short, in order for the first of Kaldor's facts to be valid in the framework of the two-sector model developed, total technological progress must grow at a positive constant rate.

- **Kaldor's fact 2:** *Capital per worker grows over time.*

Capital per worker is total stock of physical capital (K) divided by labor inputs L. If this ratio grows over time, its 1st-order time derivative must be positive. This implies that K must grow faster than labor:

$$\frac{\partial}{\partial t}\left(\frac{K}{L}\right) > 0 \Leftrightarrow g_K - g_L > 0 \Leftrightarrow g_K > g_L; \tag{3.3}$$

Again, this relationship is not evident from the two-sector model's assumptions alone, assuming regular growth. However, it was shown that the model's sector-specific variables can be aggregated in terms of total variables (see chapter 2 and appendix B) and that the growth rates of K and Y are identical.

$$g_K = g_Y; \tag{3.4}$$

From (3.4), imposing on the model the same external necessary condition adopted when addressing Kaldor's fact 1, (3.2), the second of Kaldor's facts is confirmed under the two-sector framework, for a regular growth scenario. That is, if Kaldor's fact 1 is true within this framework, then Kaldor's fact 2 is also true.

$$g_K = g_Y > g_L; \tag{3.5}$$

- **Kaldor's fact 3:** *Capital per output ratio is roughly constant.*

Capital per output ratio is K/Y. This ratio is constant if its 1st-order time derivative is zero. That is:

$$\frac{\partial}{\partial t}\left(\frac{K}{Y}\right) = 0 \Rightarrow \frac{\dot{K}}{K} - \frac{\dot{Y}}{Y} = 0 \Rightarrow g_K = g_Y; \tag{3.6}$$

This stylized fact is immediatly confirmed from the two-sector model's analysis, assuming a regular growth dynamic. As mentioned in the assessment of Kaldor's fact 2 above, the assumption that long-term growth behaves according to a regular growth path results in the growth rates for K and Y being identical (3.4), thus confirming Kaldor's fact 3.

- **Kaldor's fact 4:** *The rate of return to capital is constant.*

In the two-sector model developed, the income approach to determining GDP is given by:

$$Y = RK + wL + p_{B^P}B^P; \tag{3.7}$$

The rental rate is assumed to be the same for both types of physical capital: $RK = R(K^{NE} + K^E)$. By definition [1], the rental rate on capital is:

$$R(t) = \frac{\partial Y}{\partial K}; \tag{3.8}$$

Total output and total capital for the two-sector model, under regular growth, are:

$$\begin{cases} Y(t) = Y_0\left(1 + \mu_Y\omega_Y t\right)^{1/\omega_Y}; \\ K(t) = K_0\left(1 + \mu_K\omega_K t\right)^{1/\omega_K}; \end{cases} \tag{3.9}$$

It was shown, in the previous chapter that the resource constraints will imply that $\mu_Y = \mu_K$ and $\omega_Y = \omega_K$, which in turn means that Y and K behave according to the same dynamics: $(1 + \mu\omega t)^{1/\omega} = (1 + \mu_Y\omega_Y t)^{1/\omega_Y} = (1 + \mu_K\omega_K t)^{1/\omega_K}$. Therefore, it can be shown that:

$$\begin{cases} Y(t) = \dfrac{Y_0}{K_0} K(t); \\ \\ \dfrac{K(t)}{K_0} = (1 + \mu\omega t)^{1/\omega}; \end{cases} \tag{3.10}$$

And consequently the rental rate can be determined. This confirms Kaldor's fact 4 for regular long-run growth, based purely on the model's assumptions, with no additional conditions.

- **Kaldor's fact 5:** *The share of capital and labor in net income is nearly constant.*

This stylized Kaldor's fact can be subdivided into two separate statements. These are, from the definitions of capital and labor shares in total income:

Kaldor's fact 5(a): *The share of capital in net income is nearly constant.*

$$\frac{R(t)K(t)}{Y(t)} = constant; \tag{3.11}$$

Kaldor's fact 5(b): *The share of labor in net income is nearly constant.*

$$\frac{w(t)L(t)}{Y(t)} = constant; \tag{3.12}$$

Kaldor's fact 5(a) requires the 1st-order time derivative of (3.11) to be zero. The growth rate of the rental rate $R(t)$ over time will be equal to the difference between the growth rate of total output, g_Y, and the growth rate of total capital, g_K. From the two-sector model analysis, and as expressed in Kaldor's fact 3, the growth rates g_Y and g_K are identical - Equation (3.4). Therefore, in order for Kaldor's fact 5(a) to be confirmed for the developed model, the growth rate of $R(t)$ must be zero, i.e. the rental rate must be constant.

$$\frac{\partial}{\partial t}\left(\frac{R(t)K(t)}{Y(t)}\right) = 0 \Leftrightarrow \frac{\dot{R}(t)}{R(t)} + \frac{\dot{K}(t)}{K(t)} - \frac{\dot{Y}(t)}{Y(t)} = 0 \Leftrightarrow \frac{\dot{R}(t)}{R(t)} = 0; \tag{3.13}$$

As shown above for Kaldor's fact 4, $R(t)$ is constant without any additional conditions imposed on the two-sector model. Since Kaldor's facts 3 and 4 are both confirmed from the model's assumptions alone, it follows that Kaldor's fact 5(a) is also validated without further external conditions.

Under ordinary single-sector Solow models assumptions, it would suffice to show that $R(t)K(t)/Y(t)$ is constant in order for the other component of the income approach to GDP to be constant, since:

$$Y(t) = R(t)K(t) + w(t)L(t) \Rightarrow 1 = \frac{R(t)K(t)}{Y(t)} + \frac{w(t)L(t)}{Y(t)}; \tag{3.14}$$

However, the income approach GDP for the two-sector model developed takes into account the payments to primary exergy inputs, as expressed in (3.7). Therefore it is not possible to conclude straightforwardly that the $w(t)L(t)/Y(t)$ component will also be constant.

For Kaldor's fact 5(b), its validity requires the 1st-order time derivative of (3.12) to be zero. The wage rate, $g_w = \dot{w}/w$, must then be equal to the difference between the growth rate of total output and the growth rate of labor.

$$\frac{\partial}{\partial t}\left(\frac{w(t)L(t)}{Y(t)}\right) = 0 \Leftrightarrow \frac{\dot{w}(t)}{w(t)} + \frac{\dot{L}(t)}{L(t)} - \frac{\dot{Y}(t)}{Y(t)} = 0 \Leftrightarrow g_w + g_K - g_Y = 0; \tag{3.15}$$

This cannot be confirmed for the two-sector model from its assumptions alone. An additional condition must be imposed, as in Kaldor's fact 1. This condition is:

$$Condition\ 2 : g_w = g_Y - g_L; \tag{3.16}$$

Only by imposing this condition on the original two-sector model's assumptions will Kaldor's fact 5(b) be confirmed for the developed framework.

- **Kaldor's fact 6:** *Real wage grows over time.*

In order for real wage $w(t)$ to grow over time, its 1st-order time derivative must be positive:

$$\frac{\partial w}{\partial t} > 0 \Leftrightarrow \frac{\dot{w}}{w} = g_w > 0; \tag{3.17}$$

Imposing the condition (3.2) on the two-sector model implies that Y/L grows at a positive constant rate, i.e. $g_Y - g_L > 0$. The other condition imposed on the model (3.16) states that g_w is equal to the difference $g_Y - g_L$. Admitting Kaldor's facts 1 and 5(b) to be true for the two-sector framework[2], it results that Kaldor's fact 6 must also be true for the two-sector model.

$$g_w = g_Y - g_L = \lambda = constant > 0; \tag{3.18}$$

Specifically, the wage rate grows at the same positive constant rate as total technological progress.

This concludes the assessment of Kaldor's stylized facts of long-term economic growth for the two-sector model. The interrelations between Kaldor's facts and the imposed external conditions on the model are represented in Figure (3.1), for a scenario of regular growth as defined in this thesis.

3.2 Useful work intensity for Portugal

Stylized facts can be extracted from energy production and consumption patterns. Studies focused on the Portuguese economy present a set of aggregated stylized facts, including key indicators like energy dependence and energy intensity [2]. The data comes from the available series provided by the International Energy Agency (IEA). The structure of final energy consumption by sector is the reflex of the structure of the economy and its level of development. Energy intensity[3] is a typical variable when energy stylized facts are analyzed. The path of energy intensity usually depends on a complex interaction between structural factors and cyclical developments. Chima (2011) [23] presents a list of literature references regarding the determinants of energy intensity.

One of the major motivations for the work developed in this thesis were the results obtained by Serrenho et al (2013) regarding a useful work accounting methodology applied to Portugal between 1856 and 2009 [64]. There is a clear distinction between the evolution path of final exergy consumption and useful work consumption, which is justified by a change in the aggregate second-law efficiency. This shows technical change at the energy use level. This study is based on an

[2]Through imposition of the two external conditions mentioned.

[3]The ratio between energy consumption and GDP.

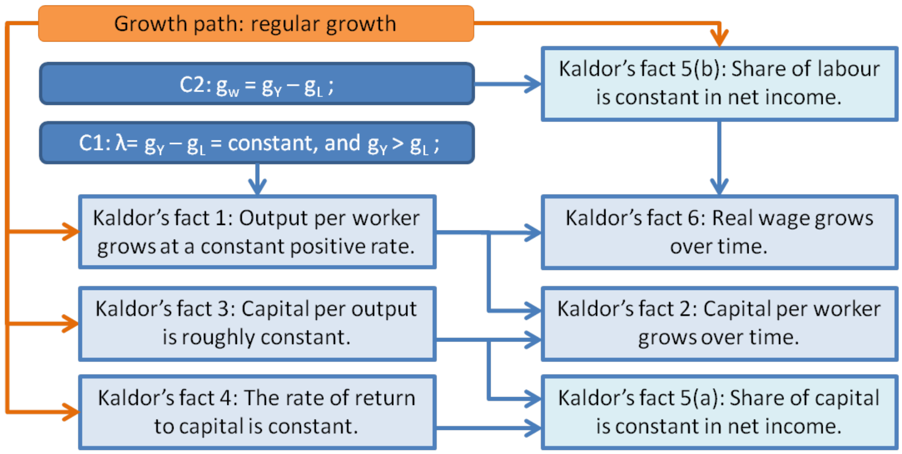


Figure 3.1: **Interrelations between Kaldor's facts of economic long-run growth, the proposed two-sector model (assuming regular growth paths), and the imposed external conditions.** Kaldor's facts 3 and 4 are confirmed solely based on the underlying assumptions of the two-sector model. Kaldor's facts 1 and 5(b) are valid by imposing additional conditions on the model, namely that $\lambda = constant > 0$ (C1) and $g_w = g_Y - g_L$ (C2). Kaldor's facts 2 and 5(a) are easily confirmed for the proposed framework by accepting the validity of Kaldor's facts 1 and 3 (for Kaldor's fact 2), and Kaldor's facts 3 and 4 (for Kaldor's fact 5(a)). Kaldor's fact 6 simply results from the two-sector model under the assumption that both of Kaldor's facts 1 and 5(b) are valid.

improved version of the methodology applied by Benjamin Warr for a societal exergy analysis [**?**], focusing on final-to-useful processes. A five-step methodology is adopted for each energy carrier [15]:

1. Conversion of existing final energy data to exergy data;
2. Allocation of each final exergy consumption to one of the useful work categories defined (heat, mechanical drive, light, other electrical uses and muscle work);
3. Computation of final exergy consumption by category;
4. Definition of an aggregate second-law final-to-useful efficiency for each useful work category;
5. Determination of an aggregated useful work value for each category;

Five forms of useful work are considered according to final use: heat, light, mechanical drive, muscle work and electricity. As final energy/exergy uses and its efficiencies are different in nature, grouping them in distinct categories allows for the study of the evolution in the paradigms of energy uses and, consequently in efficiencies' structural changes.

- **Heat:** This includes all final energy uses where heat is used in a given process or device. There are three major subcategories, distinguished by temperature range. *High Temperature Heat* $(T > 600°C)$ includes industrial processes such as iron and steel manufacturing, cement production and glass making, as well as petroleum refining (furnaces, calcinating equipment, etc). *Medium Temperature Heat* $(T < 600°C)$ is usually used as process heat in the industrial sectors (circulation heaters, immersion heaters). Finally, *Low Temperature Heat* $(T < 100°C)$ includes water and space heating uses (boilers, domestic heaters, refrigeration).

- **Mechanical drive:** In this category, all mechanical work end uses are included. This means internal combustion engines, electric engines and animal-drawn vehicles (cars, aircrafts and motorized vehicles in general).

- **Light:** All lighting end uses regardless of energy source (all kinds of lamps and lightbulbs, lanterns and candles).

- **Other electrical uses:** All electricity final uses that do not provide heat, mechanical drive or light (electronic equipment, telecommunications).

- **Muscle work:** Useful work from feed and food energy uses.

The work of Serrenho et al (2013) found that useful work consumption in Portugal experienced a 26-fold increase throughout the time period studied. During the same time period, final exergy consumption increased by 9-fold and population increased by 3-fold. This illustrates the different behaviors of these quantitites due to efficiency improvements and transformations in intensities.

The structure of useful work also changed towards a higher dependency on mechanical drive services, as well as other electric uses and higher temperature heat uses. These changes are a consequence of the industrialization process and increasing mobility needs.

Figure (3.2) represents final exergy and useful work intensities in Portugal from 1856 and 2009. Until 1920, both curves present a somewhat decreasing trend, caused by efficiency remaining almost constant throughout this period. From 1920 to 1950 there was a slight improvement in these efficiencies, which led to an accelerated decrease in final exergy intensity and a stable trend in useful work intensity. Overall, there was a reduction in the demand for exergy resources per unit of GDP.

The period of fast improvements that occured between 1950 and 1960 led to a sharp reduction in exergy intensity. After 1960 this decreasing path was interrupted as a consequence of the most significant industrialization in Portugal. Useful work intensity also decreased slightly until the 1970, but then exhibits a sharp increase during this decade. Serrenho et al (2013) attribute this behavior to two factors: a vast increase in the population, as a consequence of the decolonization process; and a significant increase in the use of electricity-powered goods and the popularization of individual automobiles, intensifying the domestic (non-productive) energy uses.

From 1980 onwards both final exergy and useful work intensity series remained almost stationary, whereas the aggregate efficiency increased only slightly.

The most interesting result from this useful work methodology is the overall stability of useful work intensity over the 154 year period in focus. Useful work intensity does not exhibit a significant time trend throughout this period (it varies no more than 20% below and above its average value). When compared with useful work intensities of other countries[4], the results obtained for Portugal stand out due to their stability. Generally, useful work intensities grew significantly after World War II, peaking almost simultaneously around 1970 [?]. On the other hand, Portuguese useful work intensity declined between 1961-1974 due to the war economy of the colonial war, similarly to what had occured in the U.K. and U.S.A. during World War II.

The need for reducing the level of resource utilization per unit of GDP represents, on the energy side, a reduction of the primary energy (exergy) intensity. This may be achieved, theoretically, by three ways: an increase in primary-to-final efficiency, an increase in final-to-useful intensity, or by decreasing useful work intensity.

[4] Austria, Japan, U.K., U.S.A.

$$\frac{Primary\ exergy}{GDP} = \frac{Primary\ exergy}{Final\ exergy} \frac{Final\ exergy}{Useful\ work} \frac{Useful\ work}{GDP}; \tag{3.19}$$

The results and trends obtained by Serrenho et al (2013) concerning useful work intensity for Portugal suggest that further reduction in primary energy (exergy) intensity in the Portuguese economy may only be achieved by increasing primary-to-final efficiency and/or final-to-useful efficiency. There are limits to these efficiencies, imposed by thermodynamics, and so reductions in primary energy (exergy) intensity may only occur with structural changes that reduce useful work intensity.

Mathematically, under the assumptions of the two-sector model presented in chapter 2, the tendency for a constant useful work intensity can be expressed as[5]:

$$\frac{\partial}{\partial t}\left(\frac{B^U}{Y}\right) = 0 \Leftrightarrow \frac{\dot{B}^U}{B^U} = \frac{\dot{Y}}{Y} \Leftrightarrow B^U \propto Y; \tag{3.20}$$

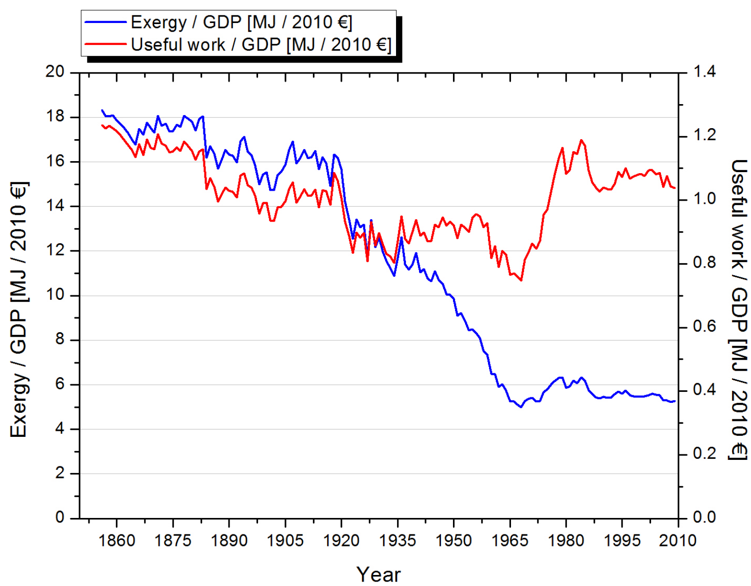


Figure 3.2: Exergy intensity and useful work intensity for Portugal between 1856 and 2009. Source: Serrenho et al (2013).

In the two-sector model developed total output is the sum of NE-Sector output Y^{NE} and the monetary value of E-Sector-supplied useful work directly consumed, C^E. The value attributed to B^U (measured in energy units - Joule) is given by a price p_{B^U}.

$$Y = Y^{NE} + C^E = Y^{NE} + p_{B^U}(1-\gamma) B^U; \tag{3.21}$$

The useful work output from the E-Sector can then be written as:

[5]Considering useful work as just the output of the E-Sector B^U and GDP as just the sum of consumption and investment Y (without imports or exports). This does not affect the proportionality relationship between the two variables.

$$C^E = p_{B^U}(1-\gamma)B^U \Rightarrow B^U = \frac{C^E}{p_{B^U}(1-\gamma)}; \tag{3.22}$$

The first-order derivative of B^U then becomes:

$$\frac{\dot{B}^U}{B^U} = \frac{\dot{C}^E p_{B^U}(1-\gamma) - \dot{p}_{B^U}(1-\gamma)C^E}{p_{B^U}(1-\gamma)C^E}; \tag{3.23}$$

In a first approximation of the two-sector model, the price associated with direct consumption of useful work may be assumed to be constant. In that case, the growth of B^U mimics that of direct consumption C^E. Hence:

$$\frac{\dot{B}^U}{B^U} = \frac{\dot{C}^E}{C^E} = \frac{\dot{Y}}{Y}; \tag{3.24}$$

The last equality is true assuming a regular growth path for all sector-specific variables of the two-sector model, as was shown in chapter 2 and appendix B. Therefore the simplified two-sector model with regular growth paths verifies the proposed stylized fact for a stable useful work intensity in the Portuguese economy, assuming that the price of direct useful work consumed has not changed. This is clearly an unrealistic assumption, and one that deserves future treatment in additional research to be performed on this abstract growth model.

In the next chapter, empirical data obtained from Portuguese economic and energy consumption accounts will be shown to corroborate some of the assumptions made when addressing the more relevant stylized facts and other simplifications introduced earlier in chapter 2. The evidence from national statistics presented next and the brief analysis performed in this chapter indicate that there is enough reason to support the development of a two-sector model as the one constructed in chapter 2 of this thesis to confirm historical economic trends and more recent stylized facts.

3.3 Overview

The present chapter seeks to validate some of the assumptions made when developing the two-sector model in chapter 2. This is done by confronting the two-sector model with a selection of stylized facts from statistical analysis of real economies over long time periods.

Section (4.1) begins by addressing the neoclassical long-term trends of economic development known as Kaldor's facts, within the two-sector model framework developed. This is done under the assumptions of regular growth paths and an additional definition of growth paths (x-growth), which are assumed for all the key disaggregate variables in the model.

Section (4.3) gives special attention to a relevant suggested stylized fact concerning the relationship between useful work consumption and GDP growth for Portugal (1856-2009). This interesting result was one of the motivations behind the development of the present dissertation. It will be addressed again in the following empirical analysis.

Chapter 4

Empirical analysis

This chapter establishes a bridge between the abstract two-sector model developed in chapter 2 and empirical data collected from economic and energy accounts. Certain assumptions from the two-sector model can be validated by comparison with statistical time series. For the purposes of this work, the empirical analysis is restricted to a single country: Portugal. Future work may extend the methodology presented here for other countries or groups of countries. The economic data used throughout this chapter, spanning the period between 1960 and 2012, was collected from the European Commission's annual macro-economic database (AMECO) [6] and Eurostat [29], and compared with additional data collected from the Portuguese National Statistics Institute (INE) [42], the Bank of Portugal [17] and the Groningen Growth and Development Center (GGDC) [34]. The data concerning energy consumption was retrieved from the International Energy Agency (IEA) database [41] and complemented with additional data from Henriques (2011)[39]. Data on useful work consumption is estimated from the useful work accounting study of André Serrenho for Portugal [64].

First, some of the most relevant stylized facts presented in chapter 3 are empirically tested using statistical data for the Portuguese economy[1]. These classically established trends are then compared with the stylized fact regarding the constancy of useful work intensity, resulting from Serrenho et al (2013).

In order to compare theoretical predictions of the two-sector model with available data, the latter must be expressed in terms of the variables defined in chapter 2. The second section in this chapter presents a detailed breakdown of Portuguese annual national accounts for consumption and investment, and a proposed aggregation of these accounts under the new variables: direct useful work consumption (C^E), consumption of non-energy related goods and services (C^{NE}), investment in the Energy Sector (I^E) and investment in the Non-energy Sector (I^{NE}). Time series for capital stocks are constructed after accounting for saving and depreciation in each sector. Labor inputs are accounted from statistics on total employment and number of hours worked.

The third section analyzes energy consumption accounts and estimates the total consumption of exergy by the E-Sector (B^P), as well as useful work consumed by the NE-Sector's productive processes, γB^U, and directly consumed by households, government and NPISH, $(1-\gamma)B^U$. The division of energy accounts by institutional sector, energy carrier and useful work end-uses allows for an estimate of the fraction of useful work allocated to production of goods and services in the NE-Sector and the fraction directly consumed by households, government, and NPISH.

[1] Treating the Portuguese economy as a single-sector system.

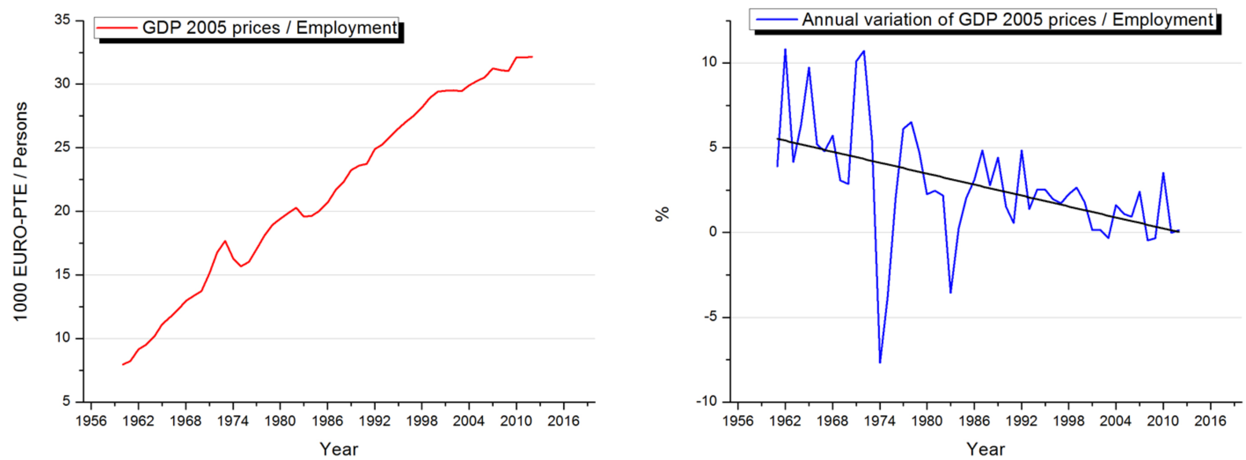


Figure 4.1: **Left graph:** Gross Domestic Product at 2005 prices (AMECO) per number of workers (employment, total economy - GGDC). **Right graph**: Annual variation of output per worker.

The final section of this chapter addresses the estimation of total factor productivity for the Portuguese economy, and seeks to determine the weight of an additional factor of production (energy, exergy or useful work) in this residual. TFP is estimated for the Portuguese economy by applying a growth accounting approach [1] to statistical data collected from National Accounts. The results obtained are compared with TFP estimates determined by AMECO, using a different, Cobb-Douglas based, methodology. Regression analysis is performed at the NE-Sector level and on the economy as a whole, in order to evaluate the explanatory power of each factor of production to output and TFP growth. The results obtained are discussed, and justification for the inclusion of an energy-related input in economic production is debated.

4.1 Empirical testing of stylized facts

This section seeks to empirically test some of the relevant stylized facts of long-term economic growth presented in chapter 3. Namely, it intends to compare and discuss the validity of several classically established stylized facts (Kaldor's facts) with the discovered statistical trend regarding the stability of useful work intensity over the focused time period, for Portugal.

Kaldor's facts for the Portuguese economy

The data available from the AMECO and GGDC databases permits an empirical and systematical testing of each of the Kaldor's facts as presented in the previous chapter. The tendencies observed from the economic variables obtained are related with several events from the Portuguese economic history. The interpretation of statistical results is made by resorting to relevant studies, namely from authors Lains & Ferreira da Silva (2005) [46] and Amaral (2010) [5].

Figure (4.1) shows the time series for output (GDP at 2005 prices, taken from AMECO data) divided by the number of workers (employment for the total economy, taken from the GGDC database). According to Kaldor's facts, output per worker should grow at a roughly constant rate, that does not diminish over time. The graph on the left shows that the GDP-employment ratio

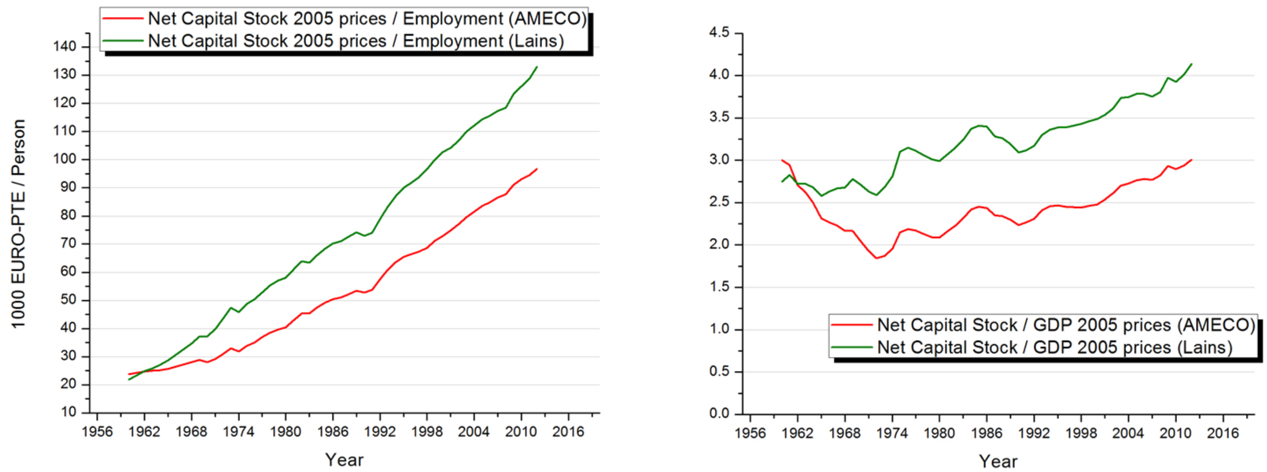


Figure 4.2: **Left graph:** Net capital stock at 2005 prices (AMECO) over total employment (GGDC) versus net capital stock (Lains & Ferreira da Silva (2005)) over total employment (GGDC). **Right graph:** Net capital stock over GDP at 2005 prices, considering net capital stock series from AMECO and Lains & Ferreira da Silva (2005).

does indeed grow for the time period considered. However, observing the annual variation, it is patent that this growth has been less and less pronounced over time.

From 1960 to 1973, the annual variation was high, corresponding to the last stretch of the period known as the ”golden age” of the Portuguese economy, which registered the highest growth in its history. This golden age was ended by the 1974 Democratic Revolution, and the oil crisis of late-1973. Afterwards the Portuguese economy experienced expansion periods (1976-1978 and 1980-1983) alternated with contration periods (1978-1980 and 1983-1985), following the interventions of the IMF in the country. In 1985 Portugal joined the European Economic Community. The period of 1986 to 1990 marks the best economic growth of the Portuguese democracy. This was followed by a decrease in growth from 1992 to 1994, and a long period of slow growth since the implementation of the euro currency (1999).

Other statements from Kaldor's facts suggest that capital per worker grows over time, and that the capital-output ratio remains constant in the long-run. Figure (4.2) shows, on the left, two series for capital stock per worker. This is due to the discrepancy in net capital stock series obtained from the AMECO database and the ones presented in the chapter dedicated to capital (authored by Miguel Lebre de Freitas) in work of Lains & Ferreira da Silva (2005) [46]. The series given by AMECO are overall lower, but both show a continuing growth over the time period considered, confirming Kaldor's statement that capital per worker grows over time.

The graph on the right of Figure (4.2) shows the capital-output ratio series obtained using AMECO and Lains & Ferreira da Silva (2005) capital stock series. The difference here is substantial, especially before 1974, when the two series exhibit different behaviors. The AMECO capital-output ratio decreases rapidly, while the Lains ratio remains more or less stable. After 1974, both series have the same tendencies, with the Lains ratio having overall higher values (increasing by approximately 50% of its 1960 value by 2012). In any of these series, Kaldor's statement that the capital-output ratio should remain stable in the long-run is not verified. Thus this Kaldor fact cannot be empirically confirmed for the Portuguese economy. It is worth mentioning that growth models different from the neoclassical approach can have capital deepening, where capital grows

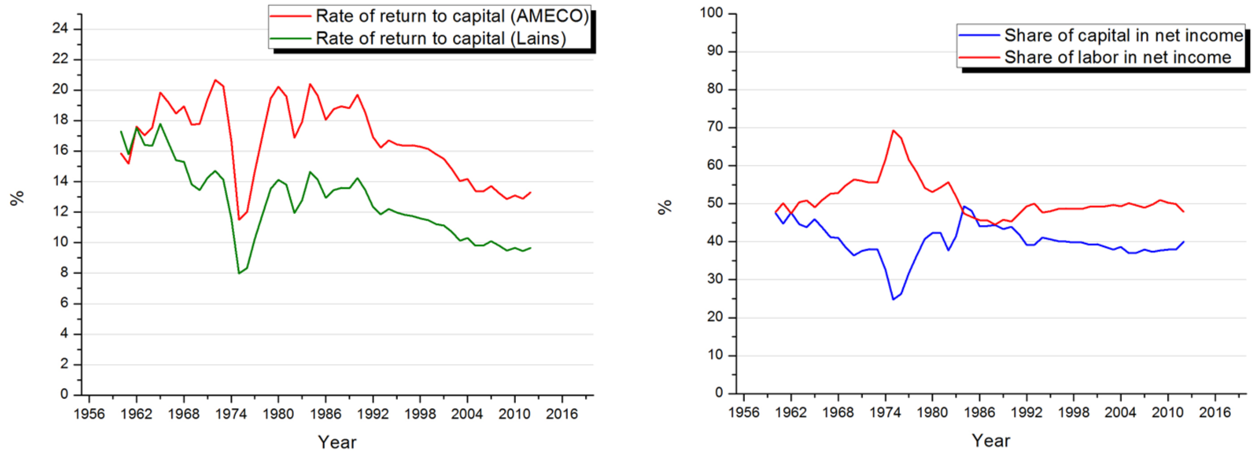


Figure 4.3: **Left graph:** Rate of return to capital determined from AMECO and Lains & Ferreira da Silva (2005) capital stock series. **Right graph:** Capital and labor shares in total net income, obtained from AMECO data.

faster than output, and still have a steady state and balanced growth. It is understood that the capital-output ratio has been relatively stable for much of the 20th century. However, it has been increasing steadily for the past 30 years [1]. This is due mostly to price adjustments of new capital goods, of higher quality, which have just recently been incorporated in the capital-output ratio calculations.

Figure (4.3), on the left, presents the estimated rate of returns to capital, obtained by dividing payments to capital (corresponding to gross operating surplus in the AMECO database) by capital stock provided by AMECO and Lains & Ferreira da Silva (2005), respectively. The two series show again a disagreement between 1960 and 1974, exhibiting opposite trends (the AMECO rate of return increases, while the Lains & Ferreira da Silva series decreases). After the sharp decrease on the rate of return in 1974, both series show the same behavior, remaining fairly stable from 1980 to 1992 and gradually decreasing since then.

Kaldor's statement regarding the stability of real rates of return appears to be heavily influenced by the experience of the United Kingdom, where this rate seems to have no long-run trend [18]. For the United States and other fast-growing countries (e.g. South Korea, Singapore), the long-term data suggests a decline in the real rate of returns. Kaldor's hypothesis of roughly constant rate of return may need to be replaced by a tendancy for returns to fall over some range as an economy develops.

The graph on the right of Figure (4.3) shows the factor shares in net income corresponding to capital and labor, each obtained by dividing the payments to the respective factor[2] by GDP. The labor share grows gradually until 1974, when it suffers a severe increase, brought by the increase in the unit cost of labor caused by the Democratic Revolution, which saw an increase in wages as well as the implementation of labor policies such as the minimum wage.

Kaldor's statements regarding the constancy of the rate of return to capital and of the factor shares corresponding to capital and labor in the long-run are not verified for the Portuguese economy. Even disregarding the special conditions associated with the 1974 revolution, the rate of return clearly shows an overall decreasing trend, while the factor shares show few signs of stability.

[2]Gross operating surplus in the case of capital and compensation of employees in the case of labor.

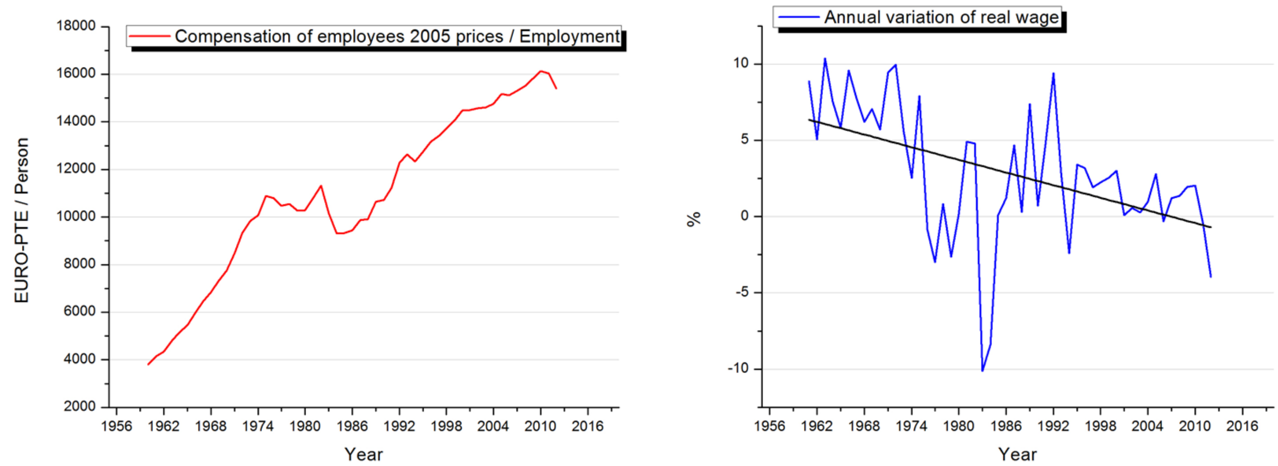


Figure 4.4: **Left graph:** Real wage series for Portugal. **Right graph:** Annual variation of the real wage between 1960 and 2012.

Finally, Figure (4.4) exhibits the evolution of the real wage in Portugal between 1960 and 2012, obtained from AMECO and GGDC data. This series is determined by dividing the compensation of employees component of GDP (AMECO) by the total employment series (GGDC). Overall the real wage grows over the time period considered, in accordance with Kaldor's facts. However, there are notable periods for which the real wage remained approximately stagnant (1975-1980, a period during which there was a devalorization of the Portuguese currency) or dropped (1982-1984, coinciding with the IMF intervention in the country, following an unsustainable expansionist policy implemented by the government in functions).

In conclusion, the stylized facts proposed by Kaldor seem to only partially apply to the Portuguese economy. Output per worker grows over time, although not at a constant rate. Capital per worker and real wage increase in the long-run, but the capital-output ratio, as well as the rate of return to capital and the factor shares in net income, have not remained stable over the 50-year period studied here.

The trends exhibited by the Portuguese economy for capital stock per worker, output per worker, returns to capital and wage rate can be related to a behavior of transitional dynamics. In a Cobb-Douglas case, the growth of output per worker mimics that of capital per worker. It can be shown, in this case, and for a Solow framework, that the growth rate of output per worker necessarily falls as capital per worker rises. Furthermore, the rate of return to capital depends on the marginal product of capital, and moves during the transition as capital changes. The neoclassical production function exhibits diminishing returns to capital, and so the rate of return to capital declines as capital grows. The wage rate, following a competitive markets assumption, increases as the capital stock grows. These effects are observed in the empirical analysis above. The Cobb-Douglas case implies that the share of rental income on capital in total income, the capital share, must be constant. Although that is not observed for the whole time period empirically studied, one can argue that the perturbation in factor shares during the 1970's for Portugal was due to extraordinary circumstances, and that both the labor and capital shares have since tended to a somewhat constant level.

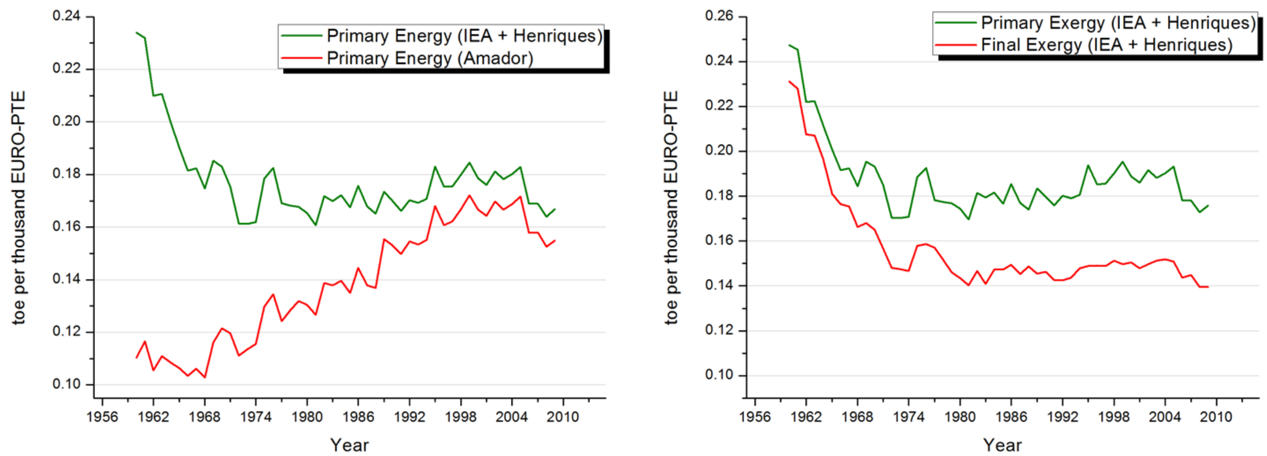


Figure 4.5: **Left graph:** Primary energy intensity for Portugal between 1960 and 2009, considering IEA data alone (Amador) and IEA data complemented with additional data from Henriques (2011). **Right graph:** Primary and final exergy intensities for Portugal (1960-2009), determined from IEA data complemented with data from Henriques (2011).

Energy, exergy and useful work intensities

Many variables affect energy intensity (energy consumption over GDP) in an economy. This indicator, however, is often used as a proxy for energy efficiency. Empirical assessments of this ratio are available, for example, from the Bank of Portugal Economic Bulletin - Amador (2010)[2]. Here, the primary energy intensity is charted for Portugal from 1960 to 2008. Primary energy intensity has recorded an ascending trend until the 90's, followed by a period of relative stabilization and then a decline in the last years of the analysis - Figure (4.5), left graph. Energy consumption is measured in terms of toe (tonne of oil equivalent)[3] and GDP is represented in 1000 EURO-PTE, at constant 2005 prices.

Primary energy data used by Amador (2010) corresponds to gross inland energy consumption in IEA accounts, which covers primary production, net imports, variation of stocks and subtracts energy provided to international aviation or maritime bunkers. This data corresponds to energy carriers such as: coal & coal products, oil & oil products, combustible renewables, natural gas, electricity and CHP heat. By taking into account energy inputs from carriers not figured in conventional energy accounting statistics, such as food & feed for animals and humans, and non-conventional sources (wind and water streams for mechanical drive uses in boats, mills and wells, for example) the energy intensity picture changes dramatically, as can be seen in the left graph of Figure (4.5). Here, additional data from energy carriers of food & feed and other non-conventional sources was obtained from the work of Henriques (2011)[39], which also reestimated the values of energy extracted from combustible renewable energy carriers prior to 1990. With this additional data, the primary energy intensity shows a large drop[4] from the beggining of the time period until 1974, after which it remains fairly stable until early 1990's, where it assumes a similar behavior to the series obtained by Amador (2010).

[3]The amount of energy released by burning one tonne of crude oil: 1 TJ = 23.885 toe, approximately.

[4]Energy inputs from food & feed carries are more relevant in the past than in recent years, due to their relationship with the agricultural sector.

The conversion of energy consumption figures to series reflecting the exergy consumption of the economy is fairly straightforward, following the methods developed in Serrenho et al (2013)[64]. In the context of most energy calculations, energy is displayed in the form of fuel, electricity, heat, or non-energy products. The exergy content of several types of energy flows is presented in appendix A. An exergy factor is commonly defined as the ratio of exergy over energy content. Kinetic, potential, mechanical and electric energy can be completely converted to work, thus they have an exergy factor of one[5]. Different energy carriers have different exergy factors, from which exergy data can be obtained for each group of carriers.

Primary exergy inputs can be determined immediatly from IEA (and additional sources) primary energy balances, by applying the relevant exergy factors. The resulting time series per unit of GDP is presented in the right graph in Figure (4.5). The shape of the graph is similar to the corresponding primary energy intensity, shown on the left. The two curves represented in this graph exhibit an increasing gap between them from 1960 to 2012. This is due primarily to changes in electricity supply. Around 1960 electricity was mainly generated from hydroelectric processes, with high efficiency. Later, not only electricity consumption grew significantly, but also a large share of it generated from heat processes, with much lower efficiencies.

Final energy consumption corresponds to total energy consumed by end users, such as households, industry and agriculture. These series are obtained by subtracting losses associated with the energy transformation sector, and can be converted into exergy figures by using the same methodology as in the primary energy/exergy case. The final exergy consumption intensity series obtained for Portugal (again, from IEA data complemented with data from Henriques (2011)) is represented alongside with primary exergy intensity in the right graph of Figure (4.5), for comparison. The final exergy intensity exhibits the same decreasing trend from 1960 to mid-1970's. This period corresponds to a moment of fast efficiency improvements in the Portuguese production, motivated by the electrification process that occurred firstly in industries, and a pervasive use of electric mechanical drive. The increasing high temperature heat uses caused by the second industrialization era in Portugal also contributed to this rise in efficiency. After 1980, final exergy intensity remained almost constant, in fact more so than primary exergy intensity.

Finally, useful work data for the Portuguese economy is available from Serrenho et al (2013). This data is obtained through the application of a detailed methodology, as explained in the previous chapter. Useful work intensity is determined in Serrenho et al (2013) by dividing the useful work consumption figures by the GDP at constant prices for the Portuguese economy. The constant price GDP is determined from the application of a consumer price index (CPI) to the nominal (current) values of GDP obtained from the AMECO database. Alternatively, GDP at constant prices for a given base year[6] can be determined by applying a price deflator (PD). Both the CPI and the PD are measures of price inflation/deflation with respect to the base year. However, while the CPI is based on a fixed basket of consumer goods and services, the PD "basket" is allowed to change with people's consumption and investment patterns. That way, new expenditure patterns show up in the deflator as people respond to changing prices. The GDP price deflator therefore reflects up-to-date expenditure patterns, unlike the consumer price index.

Useful work intensity in Serrenho et al (2013), calculated for a 154-year period between 1856 and 2009 does not exhibit a significant trend for Portugal, remaining fairly stable. This constitutes a possible stylized fact regarding energy use in Portugal. The main variations in useful work intensity, according to Serrenho et al (2013) occur during the 1970's due to an increase in population[7] and

[5]The exergy content of an electric energy flow equals its energy.

[6]In this case, and throughout the entire empirical analysis, the base year is 2005.

[7]Caused by the decolonization process and the consequent flux of Portuguese citizens to the country.

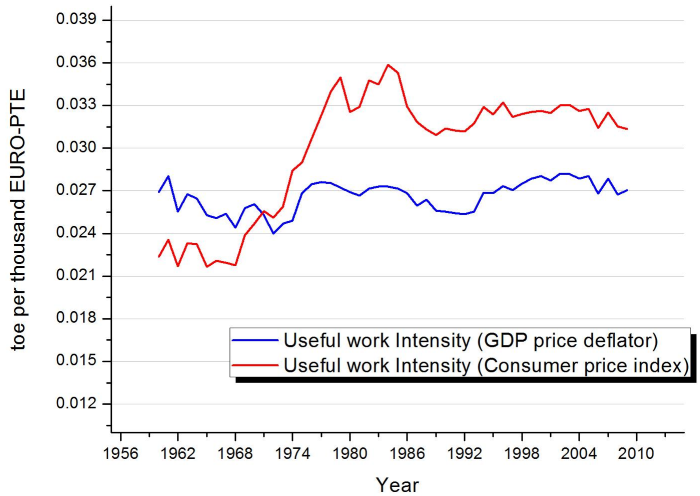


Figure 4.6: Useful work intensity for the Portuguese economy, from 1960 to 2009, using GDP at 2005 prices determined through a consumer price index and through a price deflator.

a significant increase in the use of electricity-powered goods, as well as individual cars. However, when calculating the useful work intensity with a price deflator GDP at 2005 prices, these variations disappear from the time series - Figure (4.6). The result is a much more stable intensity for useful work consumption in Portugal, from 1960 to 2009. This reinforces the proposed stylized fact of a constant useful work intensity for the Portuguese economy. Comparing the intensities presented in Figures (4.5) and (4.6), despite the difference in scale, it is also visible that useful work, the energy variable closest to economic productive processes, exhibits the most stable behavior of all energy consumption variables considered here. From 1961 to 1974, the Portuguese economy adopted a war economy that promoted the cutback on all non-productive energy uses. This led to the slight decrease in useful work intensity verified for this period in Figure (4.6). During the 1980's, the deindustrialization process in Portugal saw a decrease in the use of high-temperature heat useful work in favor of an increase of useful work consumption in transportation and other electrical uses. This transition to a service economy caused a decrease in useful work intensity, as a consequence of replacing high temperature heat uses (with high useful work intensities) for individual cars and lower heat uses, with lower useful work intensities [64].

Considering the empirically tested Kaldor's facts for the Portuguese economy presented in Figures (4.1), (4.2), (4.3) and (4.4), and weighting these trends with the proposed stylized fact presented in Figure (4.6), one can infer that the useful work intensity stylized fact is empirically more robust for the Portuguese economy, while classically defined Kaldor's facts of long-run growth exhibit, at best, a partial connection with Portuguese economic data. This constitutes an extremely interesting result.

4.2 Decomposition and aggregation of National Accounts

National accounts (NA) are based on the implementation of complete and consistent accounting techniques for measuring the economic activity of a nation. These include detailed underlying measures that rely on double-entry accounting. National income and product accounts provide estimates for the value of income and output per year, including GDP. The expenditure approach focuses on estimating total output by measuring the amount of money spent by its agents.

$$GDP = C + I + (X - M); \tag{4.1}$$

The variable C corresponds to total final consumption expenditure: the sum of private expenditure by households, NPISH, and general government expenditure. Variable I corresponds to gross private domestic investment [8] while X and M stand for gross exports and imports, respectively.

As determined in chapter 2, due to the separation of the economy in two distinct sectors and the redefinition of variables under the quantities C^{NE}, C^E, I^{NE} and I^E, total final consumption expenditure and GCF, *as defined in the two-sector model*, will not correspond exactly to the homonymous components in NA. Specifically, some goods usually accounted for as consumption will be redefined as E-Sector investment in the form of capital, due to their role in the conversion of exergy into useful work. Therefore, under the two-sector model's definitions, total final consumption expenditure ($\hat{C}$), and GCF ($\hat{I}$), will be:

$$\begin{cases} \hat{C} = C^{NE} + C^E < C; \\ \hat{I} = I^{NE} + I^E > I; \\ \hat{C} + \hat{I} = C + I; \end{cases} \tag{4.2}$$

Expenditure approach GDP (4.1) becomes, for the two-sector model:

$$GDP = \hat{C} + \hat{I} + (X - M); \tag{4.3}$$

The following sections deal with the decomposition of C and I time series in the national accounts according to final use and asset type, respectively. Each category is analyzed and allocated to the four disaggregate key variables of the two-sector model. The allocation is made according to some simplifying criteria, which should not, in principle, affect the general accuracy of the results.

Consumption

Total final consumption expenditure in NA comprises the expenditure on goods and services used for direct satisfaction of individual needs (*individual consumption*) or collective needs of members of the community (*collective consumption*). Final consumption expenditure is composed of:

- Final consumption expenditure of households and NPISH (*Private final consumption expenditure*);
- Final consumption expenditure of general government;

The most important part of final consumption expenditure comes from households, including all durable and non-durable consumed goods. Exceptions are the purchases for own-construction or improvements of residential housing, which are treated as part of GCF [67].

The Classification of Individual COnsumption according to Purpose (COICOP) is a Reference Classification published by the United Nations Statistics Division. It divides the purpose of individual consumption expenditures incurred by households, NPISH, and general government. The classification units are transactions. There are three structure levels:

[8] Gross capital formation - GCF.

Code	Definition
01	Food and non-alcoholic beverages
02	Alcoholic beverages, tobacco and narcotics
03	Clothing and footwear
04	Housing, water, electricity, gas and other fuels
05	Furnishings, household equipment and routine household maintenance
06	Health
07	Transport
08	Communication
09	Recreation and culture
10	Education
11	Restaurants and hotels
12	Miscellaneous goods and services

Table 4.1: Household expenditure COICOP, Structure Level 1: Divisions (two-digit).

- Structure Level 1: Divisions (two-digit)
- Structure Level 2: Groups (three-digit)
- Structure Level 3: Classes (four-digit)

Divisions 01 through 12 refer to the consumption expenditure of households. Divisions 13 and 14 concern, respectively, consumption expenditure of NPISH and general government. For the purposes of this analysis, only Divisions 01 through 12 will be considered for private final consumption expenditure (disregarding NPISH), while general government expenditure will be dealt with further on. The first structure level is presented in Table (4.1).

The INE and Eurostat databases provide statistical data on the final consumption expenditure of households by COICOP divisions and groups, but not classes. This means that the aggregation of the datasets in terms of new variables must be performed at a group level, with some loss of accuracy.

Some divisions and groups can be immediatly allocated to one of the new variables, while others include elements that should be allocated to different key variables (in any case, no more than two). When the latter is the case, the criteria used is to split the group in half and attribute each fraction to each of the corresponding variables. The aggregation of COICOP Divisions and groups under the model's variables is presented in Table (4.2), where $C^{NE(H)}$, $C^{E(H)}$, and $I^{E(H)}$ refer, respectively, to the fraction of household's final consumption expenditure ($C^{(H)}$) alocated to C^{NE}, C^{E}, and I^{E}. The weight of each division and group on total consumption expenditure C is also represented.

It is important to notice that the redefinition of certain consumption goods as investment, characteristic to our two-sector model, carries some concerns regarding consistency in accounting. The concept of imputed rents applies to any capital good[9] and it must be taken into consideration when redefining automobiles and appliances, for example, as components of investment. The full cost of using a fixed asset in production is also measured by the actual or imputed rental on the asset, and not by depreciation alone. Although imputed rents are not explicitly estimated in this analysis for each new product redefined as capital investment, it is expected that the effect of these imputed rents will later be "caught" when estimating consumption of fixed capital for sector specific investment in the two-sector model.

[9] It is most commonly used in reference to home ownership.

Table 4.2: COICOP categories allocated to the disaggregate variables $C^{NE(H)}$, $C^{E(H)}$ and $I^{E(H)}$.

Division	Group	% of NA C	Variable
01		12.3-18%	$C^{E(H)}$
02		2.2-3.6%	$C^{E(H)}$
03		4.4-7.4%	$C^{NE(H)}$
04	04.1 Actual rentals for housing	0.6-1.5%	$C^{NE(H)}$
	04.2 Imputed rentals for housing	4.2-6.2%	$C^{NE(H)}$
	04.3 Maintenance and repair of the dwelling	0.2-0.3%	$C^{NE(H)}$
	04.4 Water supply and miscellaneous services relating to the dwelling	0.4-1.3%	$C^{NE(H)}$
	04.5 Electricity, gas and other fuels	1.8-2.7%	$C^{E(H)}$
05	05.1 Furniture and furnishings, carpets and other floor coverings	1.1-1.8%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	05.2 Household textiles	0.3-0.5%	$C^{NE(H)}$
	05.3 Household appliances	0.7-0.8%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	05.4 Glassware, tableware and household utensils	0.4-1.1%	$C^{NE(H)}$
	05.5 Tools and equipment for house and garden	0.1-0.2%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	05.6 Goods and services for routine household maintenance	1.3-1.9%	$C^{NE(H)}$
06	06.1 Medical products, appliances and equipment	0.9-2%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	06.2 Outpatient services	1.7-4.6%	$C^{NE(H)}$
	06.3 Hospital services	0-0.3%	$C^{NE(H)}$
07	07.1 Purchase of vehicles	2.9-5.9%	$I^{E(H)}$
	07.2 Operation of personal transport equipment	4.8-6.4%	$C^{E(H)}$
	07.3 Transport services	0.8-1.6%	$C^{NE(H)}$
08	08.1 Postal services	0.1-0.2%	$C^{NE(H)}$
	08.2 Telephone and telefax equipment	$\sim$ 0%	$I^{E(H)}$
	08.3 Telephone and telefax services	1.4-2.4%	$C^{NE(H)}$
09	09.1 Audio-visual, photographic and information processing equipment	0.7-1.3%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	09.2 Other major durables for recreation and culture	$\sim$ 0.1%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	09.3 Other recreational items and equipment, gardens and pets	0.8-1.1%	$C^{NE(H)}$
	09.4 Recreational and cultural services	1-2.2%	$C^{NE(H)}$
	09.5 Newspapers, books and stationery	0.8-1.4%	$C^{NE(H)}$
	09.6 Package holidays	0.3-0.6%	$C^{NE(H)}$
10		0.8-1.1%	$C^{NE(H)}$
11		6.9-8.5%	$C^{NE(H)}$
12	12.1 Personal care	1.1-2.1%	50% $C^{NE(H)}$ 50% $I^{E(H)}$
	12.2 Prostitution	0%	$C^{NE(H)}$
	12.3 Personal effects	0.5-0.9%	$C^{NE(H)}$
	12.4 Social protection	0.7-1.2%	$C^{NE(H)}$
	12.5 Insurance	0.7-1.9%	$C^{NE(H)}$
	12.6 Financial services n.e.c.	0.3-3.3%	$C^{NE(H)}$
	12.7 Other services n.e.c.	0.8-1.4%	$C^{NE(H)}$

Final consumption of households includes expenditures on services such as electricity, gas and heat supply. These and other expenditures (such as purchase of fuels) are considered as payments for the exergy contained in those products, and indirectly, for the useful work provided by that exergy (see chapter 2). The groups Electricity, gas and other fuels (04.5) and Operation of personal transport equipment (which includes some fuels) are allocated to direct useful work consumption by households, $C^{E(H)}$. The first 2 COICOP divisions can also be immediatly allocated to $C^{E(H)}$. These divisions concern payments to food products, which can ultimately be equated with payments for the useful work (muscle work) extracted from those products. It is assumed that all exergy contained in the food products is in fact converted into muscle work.

Due to the innovative definition adopted for the E-Sector, all products that perform the conversion of final exergy to useful work are considered as E-Sector physical capital investment. This means that goods such as motorized vehicles and electric appliances, usually considered as final consumption goods, will be redefined as capital investment I^E. The groups Purchase of vahicles (07.1) and Telephone and telefax equipment (08.2) are the only ones completely allocated to I^E, despite also including repair of such equipment, which is negligible.

Most remaining individual groups and divisions can be completely allocated to non-energy related consumption $C^{NE(H)}$. This is the case of Education (10), Restaurants and hotels (11) and any group composed uniquely of services (e.g. Outpatient services (06.2)). Among those groups with elements corresponding to two key variables, the majority consists of categories attributable to $C^{NE(H)}$ or $I^{E(H)}$. One example is the group Audio-visual, photographic and information processing equipment (09.1), which includes physical capital responsible for the conversion of exergy into useful work for other electrical uses (television sets, stereo systems, personal computers, etc) and non-energy related consumption goods (compact discs, lenses, software, etc). For these cases, the consumption series are split between the two key variables.

The INE and Eurostat statistical databases provide datasets on Portuguese annual final consumption expenditure of households by COICOP divisions and groups, for the period 1988-2011. The Eurostat database also provides annual percentages for COICOP divisions and groups in total household consumption. The AMECO datasets, on the other hand, provide statistics on Portuguese annual private final consumption expenditure for the period between 1960 and 2012, but fail to discriminate between COICOP Divisions.

In order to obtain datasets for final household consumption according to COICOP divisions and groups, from 1960 to 2012, the total percentages of final consumption attributed to $C^{NE(H)}$, $C^{E(H)}$ and $I^{E(H)}$ were estimated for the period made available by the Eurostat and INE databases (1988-2011) - Table (4.3).

The percentages for $I^{E(H)}$ exhibit a trendless behavior for the time period in focus, and so it is assumed that this behavior is extended for the remaining periods (1960-1987) and (2011-2012). The percentage values attributed to these time periods is the individual average of the annual percentages for the known time period (1988-2011). Consumption $C^{NE(H)}$ exhibits a steady growth from 1988 to 2011. The variation of $C^{NE(H)}$ is almost linear for the considered time period, and so its values for 1960-1987 and 2011-2012 can be estimated assuming the extension of this trend for those years[10]. Consumption $C^{E(H)}$ decreases between 1988 and 2011, and its share of household consumption expenditure can be determined from the other two shares. Combining these shares with the datasets for annual private consumption expenditure from the AMECO database, it is possible to obtain a sufficiently accurate time series for consumption accounts under the new variables. A similar

[10]This is a crude estimation, especially considering the critical behavior of economic variables around the Democratic Revolution (1974).

Year	$C^{NE(H)}$ (%)	$C^{E(H)}$ (%)	$I^{E(H)}$ (%)
1988	51.10	37.30	11.70
1989	51.90	38.00	10.20
1990	51.75	38.20	9.95
1991	53.45	36.70	9.85
1992	53.20	35.90	10.90
1993	53.25	36.70	9.95
1994	53.10	37.00	10.00
1995	57.10	33.30	9.70
1996	57.10	32.70	10.20
1997	57.60	31.60	10.70
1998	57.25	31.50	11.05
1999	57.50	30.90	11.70
2000	58.05	29.60	12.15
2001	58.45	30.20	11.45
2002	59.05	30.10	10.95
2003	59.15	30.50	10.55
2004	59.25	30.30	10.45
2005	59.35	30.20	10.45
2006	59.35	30.30	10.35
2007	60.30	29.60	10.20
2008	60.35	29.80	9.85
2009	61.35	29.90	8.75
2010	60.40	30.00	9.50
2011	61.50	30.10	8.30
Average	-	-	10.37

Table 4.3: Shares of $C^{NE(H)}$, $C^{E(H)}$ and $I^{E(H)}$ in private final consumption expenditure C^H (1988-2011).

simplification will be made for the final consumption expenditure of general goverment categories (COFOG).

Final consumption expenditure of general government consists of expenditure incurred by government in its production of non-market final goods and services[11] and market goods and services provided as social transfers in kind. Included in the final consumption expenditure of general government are:

- Non-market output other than own-account capital formation, which is measured by production costs less incidental sales of government output;
- Expenditure on market goods and services that are supplied without transformation and free of charge to households (social transfers in kind);

Similarly to private final consumption expenditure, the United Nations Statistics Division classifies government expenditure data from NA according to the purpose for which the funds are used. This is done through the Classification of the Functions of Government (COFOG), which allocates government expenditure for specific uses. It corresponds to the 14th division in COICOP classification and exhibits a similar structure level, with transactions as units. The COFOG divisions are presented in Table (4.4).

[11] Except Gross Fixed Capital Formation - GFCF.

Code	Definition
01	General public services
02	Defence
03	Public order and safety
04	Economic affairs
05	Environmental protection
06	Housing and community amenities
07	Health
08	Recreation, culture and religion
09	Education
10	Social protection

Table 4.4: Government expenditure COFOG, Structure Level 1: Divisions (two-digit).

Similarly to COICOP, there is available data from the INE and Eurostat databases regarding the COFOG's divisions and groups, but no discrimination according to classes. The aggregation under the two-sector model's key variables must again be performed at the group level. Table (4.5) shows the allocation of COFOG's divisions and groups to the variables $C^{NE(G)}$, $C^{E(G)}$ and $I^{E(G)}$ (representing the portions of general government consumption expenditure, C^G, attributed to C^{NE}, C^E and I^E, respectively). The shares of each division and group on total consumption C are also represented.

The first 3 COFOG divisions, plus Environmental protection (05), Education (09) and Social protection (10) can be fully allocated to the variable $C^{NE(G)}$, as well as most of the remaining groups. The exceptions are the groups Transport (04.5), Communication (04.6), Street lighting (06.4) and Medical products, appliances and equipment (07.1), all of which including elements that correspond to different key variables. Their total value is split equally between the variables $I^{E(G)}$ and $C^{NE(G)}$. The group Fuel and Energy (04.3) is entirely allocated to direct consumption of useful work by government, $C^{E(G)}$. Again, the price paid by government for fuels can be equated to the price paid for the useful work extracted from those fuels.

The methodology used to obtain time series for final general government expenditure according to COFOG divisions and groups from the Eurostat, INE and AMECO databases is analogous to the one used for the COICOP divisions and groups for final consumption expenditure of households. Eurostat provides percentage datasets for every COFOG group, in relation to total government consumption, for the period between 1995 and 2011. The total percentages corresponding to the aggregation of variables $C^{NE(G)}$ and $C^{E(G)}$ appear to be trendless for this period of time, so their average values are determined and extended to the time periods between 1960-1994 and 2011-2012, for which data on total general government consumption expenditure is available from the AMECO database. The percentages attributed to $I^{E(G)}$ are estimated from the other shares, for each year. The Eurostat percentages and average values are presented in Table (4.6).

Total final consumption in NA is the sum of private final consumption expenditure and general government consumption expenditure. The sum of the aggregate variables corresponding to consumption of non-energy goods and services by households and government ($C^{NE(H)}$ and $C^{NE(G)}$ respectively) with the direct consumption of useful work by households and government ($C^{E(H)}$ and $C^{E(G)}$) constitutes total final consumption under the new definitions of the two-sector model developed:

$$\hat{C} = C^{NE} + C^{E} = C^{NE(H)} + C^{NE(G)} + C^{E(H)} + C^{E(G)}; \tag{4.4}$$

Table 4.5: COFOG categories allocated to the disaggregate variables $C^{NE(G)}$, $C^{E(G)}$ and $I^{E(G)}$.

Division	Group	% of NA C	Variable
01		3.3-6.2%	$C^{NE(G)}$
02		0.6-1.1%	$C^{NE(G)}$
03		0.9-1.3%	$C^{NE(G)}$
04	04.1 General economic, commercial and labour affairs	0-0.5%	$C^{NE(G)}$
	04.2 Agriculture, forestry, fishing and hunting	0-0.5%	$C^{NE(G)}$
	04.3 Fuel and energy	0-0.2%	$C^{NE(G)}$
	04.4 Mining, manufacturing and construction	0.1-0.4%	$C^{NE(G)}$
	04.5 Transport	1.2-1.7%	50% $C^{NE(G)}$ 50% $I^{E(G)}$
	04.6 Communication	$\sim$ 0%	50% $C^{NE(G)}$ 50% $I^{E(G)}$
	04.7 Other industries	$\sim$ 0.2%	$C^{NE(G)}$
	04.8 R&D Economic affairs	$\sim$ 0.1%	$C^{NE(G)}$
	04.9 Economic affairs n.e.c.	0-0.1%	$C^{NE(G)}$
05		0.2-0.4%	$C^{NE(G)}$
06	06.1 Housing development	0-0.2%	$C^{NE(G)}$
	06.2 Community development	0.1-0.3%	$C^{NE(G)}$
	06.3 Water supply	0.1-0.2%	$C^{NE(G)}$
	06.4 Street lighting	$\sim$ 0%	$C^{NE(G)}$
	06.5 R&D Housing and community amenities	$\sim$ 0%	$C^{NE(G)}$
	06.6 Housing and community amenities n.e.c.	$\sim$ 0%	$C^{NE(G)}$
07	07.1 Medical products, appliances and equipment	0.5-1.1%	50% $C^{NE(G)}$ 50% $I^{E(G)}$
	07.2 Outpatient services	0.8-2.4%	$C^{NE(G)}$
	07.3 Hospital services	0.2-1.9%	$C^{NE(G)}$
	07.4 Public health services	$\sim$ 0%	$C^{NE(G)}$
	07.5 R&D Health	$\sim$ 0%	$C^{NE(G)}$
	07.6 Health n.e.c.	0-0.1%	$C^{NE(G)}$
08	08.1 Recreational and sporting services	0.2-0.3%	$C^{NE(G)}$
	08.2 Cultural services	0.2-0.3%	$C^{NE(G)}$
	08.3 Broadcasting and publishing services	0.1-0.2%	$C^{NE(G)}$
	08.4 Religious and other community services	$\sim$ 0%	$C^{NE(G)}$
	08.5 R&D Recreation, culture and religion	$\sim$ 0%	$C^{NE(G)}$
	08.6 Recreation, culture and religion n.e.c.	$\sim$ 0%	$C^{NE(G)}$
09		2.9-3.8%	$C^{NE(G)}$
10		5.4-8.9%	$C^{NE(G)}$

Year	$C^{NE(G)}$ (%)	$C^{E(G)}$ (%)	$I^{E(G)}$ (%)
1995	95.88	0.07	4.05
1996	95.28	0.08	4.63
1997	95.82	0.14	4.03
1998	95.13	0.74	4.13
1999	96.12	0.15	4.03
2000	95.84	0.07	4.09
2001	94.51	0.02	5.47
2002	94.75	0.02	5.23
2003	94.69	0.03	5.27
2004	95.92	0.42	4.51
2005	95.02	0.12	4.85
2006	95.84	0.04	4.12
2007	95.83	0.09	4.09
2008	94.85	0.17	4.98
2009	94.91	0.15	4.94
2010	95.55	0.08	4.36
Average	95.37	-	4.55

Table 4.6: Shares of $C^{NE(G)}$, $C^{E(G)}$ and $I^{E(G)}$ in general government consumption expenditure $C^{(G)}$ (1995-2010).

A comparison between total consumption expenditure according to NA and as defined in the two-sector model is presented in Figure (4.7), along with the percentual shares of $\hat{C}$ attributed to C^{NE} and C^{E}.

The annual variation for total consumption as defined in the two-sector model is very similar to the annual variation for NA total consumption. Both show the same trends throughout the given time period, including slower growth periods around 1980-1985 and 1992-95. After 2008, consumption growth slows down (and is eventually inverted in 2010), possibly due to the negative influence of the recent economic crysis (and austerity policy measures adopted in its response) on consumption patterns.

Consumption of NE-Sector produced goods and services C^{NE} seems to have increased at the expense of C^{E} for the considered time period. According to the data, this is justified mainly by a decrease in the C^{E} divisions of Food and non-alcoholic beverages (01) and Alcoholic beverages, tobacco and narcotics (02), accompanied by a combined increase in C^{NE} categories such as Imputed rentals for housing (04.2) and Restaurants and hotels (11).

Investment

Gross capital formation in NA is equivalent to investment in capital goods. It includes produced capital goods (e.g. machinery, buildings and roads) and improvements to non-produced assets. GCF measures the additions to the capital stock of buildings, equipment and inventories, i.e., the addition to the capacity to produce more goods and income in the future. The components of GCF are:

- Gross fixed capital formation (GFCF);
- Changes in inventories;
- Acquisition less disposals of valuables;

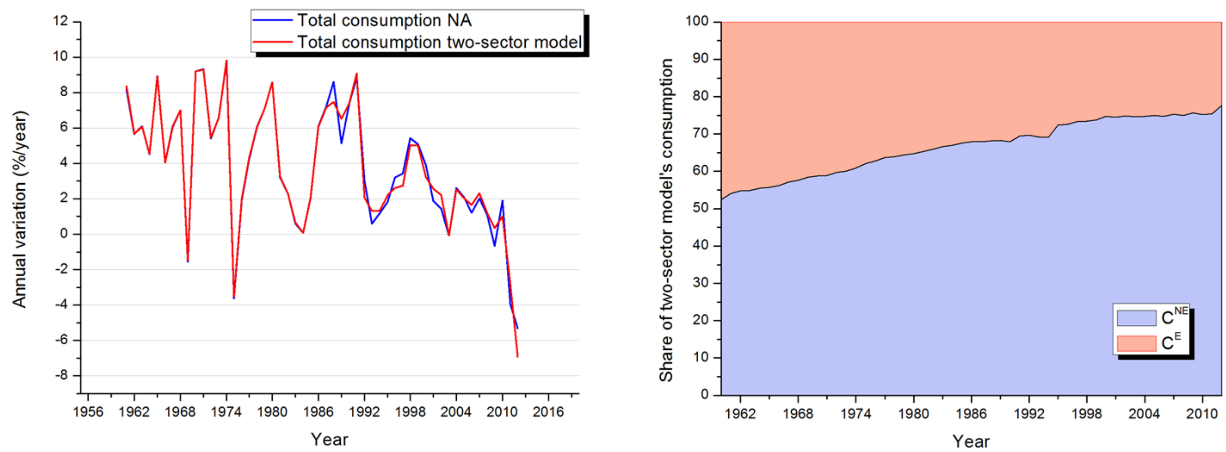


Figure 4.7: **Left graph:** Annual variation of C and $\hat{C}$ between 1960 and 2012 (Mrd EURO-PTE at 2005 constant prices). **Right graph:** percentages of C^{NE} and C^{E} in relation to $\hat{C}$, as defined in the two-sector model. (1 Mrd = 1,000,000,000)

For the purposes of the analysis in this chapter, only the major component in GCF - gross fixed capital formation (GFCF) - will be considered.

GFCF is a component of GDP expenditure defined by the United Nations System of National Accounts (SNA) and the IMF Balance of Payments system, which illustrates how much of the new value added in the economy is invested rather than consumed. It is, like consumption, a flow value[12]. The range of fixed assets included in statistical measurement is defined by their useful purpose. Vehicles constitute fixed assets, but are included in GFCF accounts only if their use is related to work activities, that is, within the scope of production. An automobile purchased for personal use does not constitute GFCF, but appears in the final consumption expenditure of households, as seen in Table (4.2). However, under the innovative definition of consumption and investment introduced here, automobiles participate in the conversion of exergy into useful work, and therefore will always constitute capital investment in the E-Sector of the economy, whether they are used for production or for leisure.

Non-produced assets such as land, mineral reserves and natural resources (water, primary forests, etc.), as well as repair work and purchases of household durable equipment, are excluded from the official measure of GFCF. Detailed breakdowns of GFCF are available by type of asset (plant, machinery, land improvements, buildings, vehicles, etc.).

For the analysis performed in this section, GFCF will be decomposed in terms of asset type and allocated under the two-sector model defined variables for capital investment in the E-Sector ($I^{E(C)}$) and capital investment in the NE-Sector ($I^{NE(C)}$). Investment $I^{E(C)}$ is then summed to the I^{E} components from COICOP and COFOG consumption expenditure accounts obtained earlier ($I^{E(H)}$ and $I^{E(G)}$, respectively) to form total capital investment alocated to the E-Sector.

The AMECO database provides datasets on GFCF by types of asset, at constant 2005 market prices, from 1960 to 2012. The types of asset covered are: construction (dwellings, non-residential construction and civil engineering) and equipment (metal products, machinery and transport equipment). According to the definitions of the European System of Accounts (ESA95), only produced fixed non-financial assets constitute GFCF - Figure (4.8).

[12]Measured over an interval of time, as opposed to a stock value, which is measured at a specific moment.

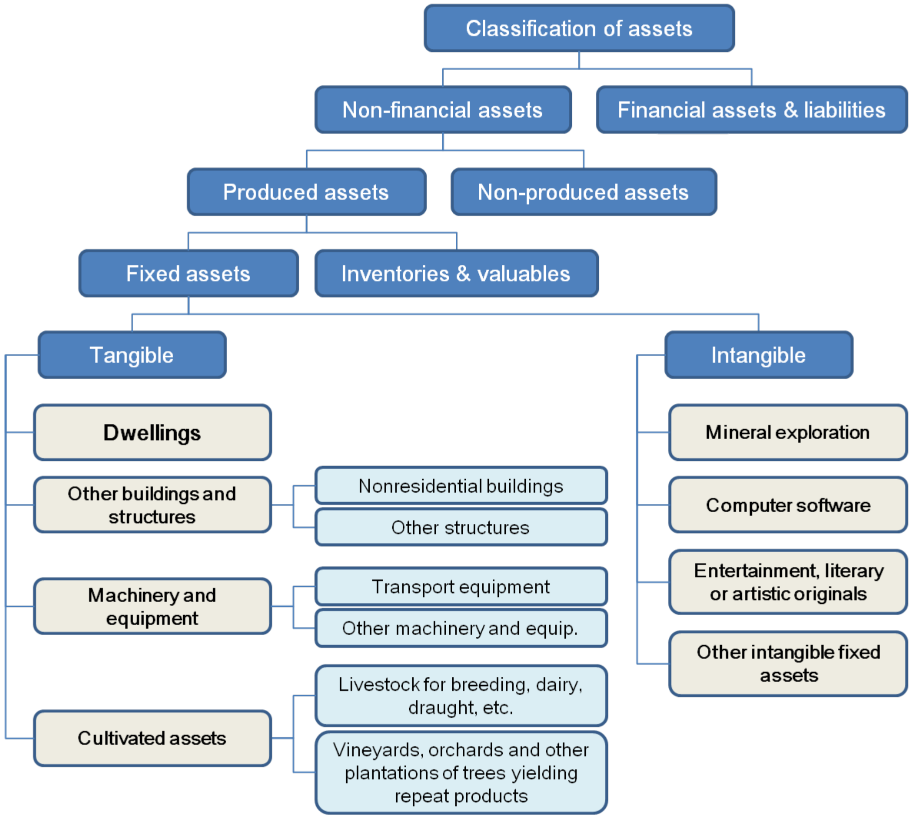


Figure 4.8: Classification of fixed assets according to the European System of Accounts (ESA95). Breakdown of non-financial produced fixed assets by type of asset.

The definition of GFCF according to ESA95 includes cultivated assets, as well as intangible fixed assets. However, the AMECO database does not provide time series regarding these types of assets for Portugal, at constant prices. The Eurostat and INE databases only provide data at current prices, and only between 1995-2012. According to these sources, cultivated assets account for only approximately 0.9-1.4% of total GFCF during this 17-year period, and therefore can be disregarded from the present analysis, in a first approach. If accounted for, these assets would constitute investment in the E-Sector.

Intangible fixed assets time series can be estimated from the remaining data with some accuracy, at least for the period between 1990 and 2012. Before that, the share attributed to these assets in total GFCF is very small. These assets constitute, under this analysis, investment in the NE-Sector. The GFCF categories by asset type are allocated under the new variables $I^{NE(C)}$ and $I^{E(C)}$, as shown in Table (4.7).

The asset categories of construction are entirely allocated as NE-Sector investment $I^{NE(C)}$. The bulk of the dwellings category refers to buildings and structures used as residences. Some types of residences included, such as mobile homes and caravans, could be considered $I^{E(C)}$, but are negligible. Non-residential construction and civil engineering refers to warehouse, industrial and commercial buildings, mostly. Other structures such as roads, streets, tunnels, harbours and airfield runways are also included. The categories of cultivated and intangible fixed assets are also entirely allocated as $I^{NE(C)}$. Cultivated assets include livestock for breeding, dairy, draught, vineyards, orchards and other plantations of trees yielding repeat products. Intangible fixed assets consists mainly of mineral exploration, computer software, entertainment and literary/artistic originals.

Category	Type of asset	% of NA GFCF	Variable
Construction	Dwellings	13.9-62.2%	$I^{NE(C)}$
	Non-residential construction & civil engineering	23.6-40.9%	$I^{NE(C)}$
Equipment	Transport equipment	3.4-11.5%	$I^{E(C)}$
	Metal products and machinery	6.3-31.4%	50% $I^{NE(C)}$ 50% $I^{E(C)}$
Intangible fixed assets		0-8.7%	$I^{NE(C)}$

Table 4.7: GFCF by asset type allocated to the disaggregate variables $I^{NE(C)}$ and $I^{E(C)}$.

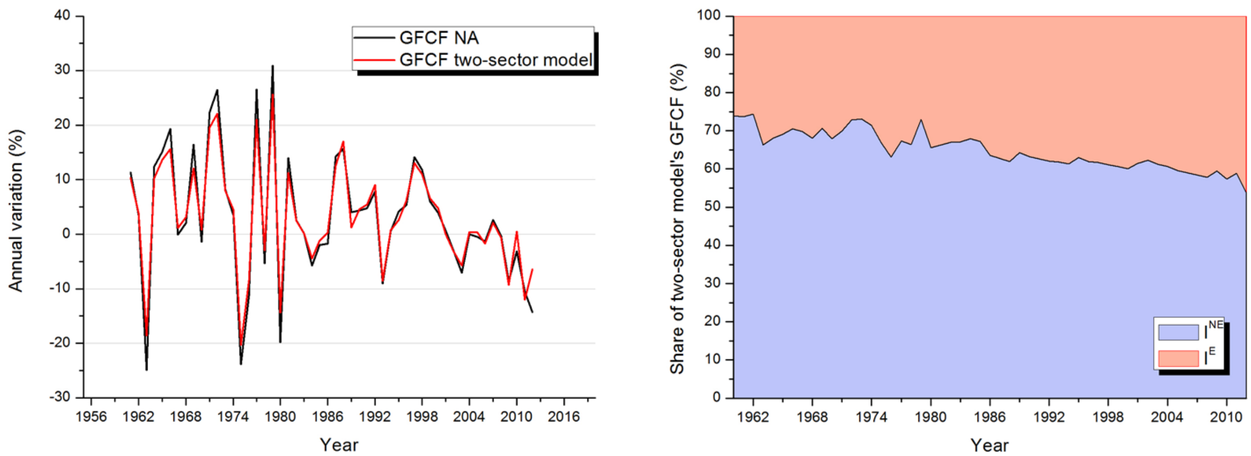


Figure 4.9: **Left graph**: annual variation for I and $\hat{I}$, between 1960 and 2012 at constant 2005 market prices. **Right graph:** percentages of I^{NE} and I^{E} constituting $\hat{I}$ in the two-sector model.

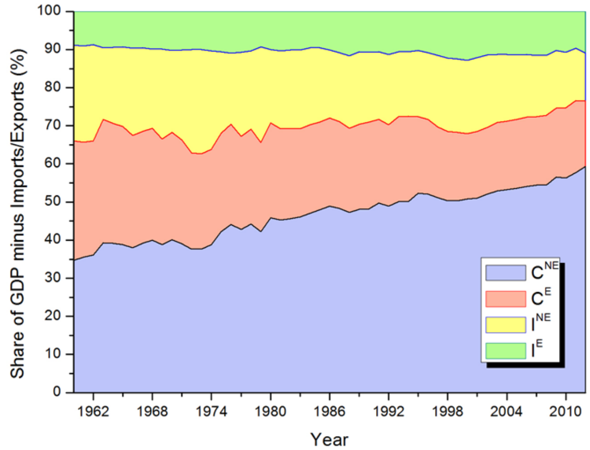
Transport equipment is entirely allocated as E-Sector investment $I^{E(C)}$. This category includes motor vehicles, motorcycles, railway and tramway locomotives and aircrafts, all of which perform a conversion of final exergy into useful work (namely in the form of mechanical drive).

The Metal products and machinery category includes assets responsible for converting exergy into useful work (office machinery, communication equipment, agricultural machinery, etc.) and assets which constitute NE-Sector investment. Analogous to the assumptions adopted when working with the COICOP and COFOG accounts, this category was equally split between the variables $I^{E(C)}$ and $I^{NE(C)}$.

The fraction of GFCF allocated as NE-Sector investment constitutes the totality of investment in this sector: $I^{NE} = I^{NE(C)}$. Investment in the E-Sector is the sum of the fraction of GFCF allocated to that end, $I^{E(C)}$, with the components obtained from consumption accounts for households and government, $I^{E(H)} + I^{E(G)}$. Total investment in the two-sector model is:

$$\hat{I} = I^{NE} + I^{E} = I^{NE(C)} + I^{E(C)} + I^{E(H)} + I^{E(G)}; \tag{4.5}$$

The comparison between NA investment and investment as defined in the two-sector model is presented in Figure (4.9), along with the percentual share of I^{NE} and I^{E} that constitute $\hat{I}$. As described in (4.2), two-sector model's investment assumes higher values than NA defined investment throughout the entire time period under study. This is due to goods usually defined as consumption in NA being redefined as E-Sector investment I^{E}.



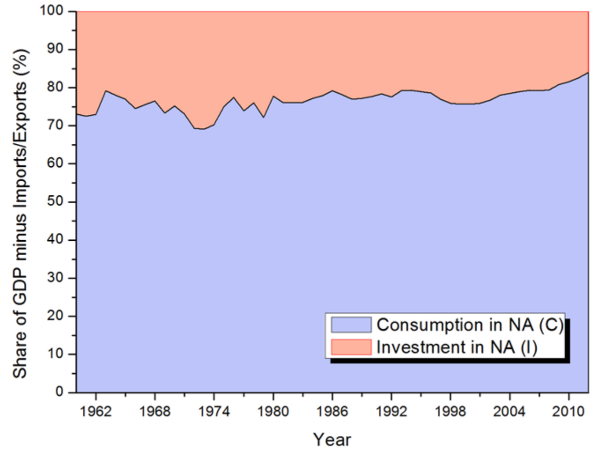


Figure 4.10: **Left graph:** Percentages of $GDP - (X - M)$ attributed to each two-sector model defined variable: C^{NE}, C^{E}, I^{NE} and I^{E}. **Right graph:** Percentages of GDP (minus Imports/Exports) attributed to total consumption expenditure and GFCF as defined in NA.

The graph on the right in (4.9) shows an increasing share of I^E in total investment. The main contributor to this percentual growth is the Metal & Machinery category in NA GFCF ($I^{E(C)}$), which increases significantly throughout this period. The Medical products, appliances and equipment group in COICOP also contributes to the increase in I^E.

Portugal has shown a high propensity to invest in the second half of the 20th century, as confirmed by historical analysis of economic development [46]. The beginning of the 1960s marks a profound transformation in the Portuguese economy, with higher competition and permeability with respect to external technological innovations. Investment shows a slight drop in 1962-63, due to the uncertainty generated with the start of the colonial war. After that, investment rises, reaching a peak just before 1974, the year of the democratic revolution. The post-revolutionary period (1975-76) was characterized by institutional and political instability, accounting for a considerable drop in GFCF. The first decade of democracy was accompanied by uncertainty, generated by a difficult international conjucture and slow institutional consolidation. A GFCF drop of conjectural nature occurs during the stabilization programme of 1983-85. After joining the EEC, Portugal beneficted from greater political stability.

Consumption and investment according to the two-sector model

Combining the results obtained from the previous sections on the reallocation of consumption and investment NA figures under the defined variables for the two-sector model (C^{NE}, C^{E}, I^{NE} and I^{E}), it is possible to obtain a comparison of the composition of GDP (minus Imports/Exports) for each approach. This is presented in Figure (4.10).

The first observation that can be made is that the GDP shares from E-Sector related variables (C^{E} and I^{E}) exhibit a less pronounced variation than the NE-Sector related variables C^{NE} and I^{NE}. The graph on the right suggests a trend towards an increase in total consumption expenditure over the last 50 years for Portugal, at the expense of a complementary decrease of investment[13]. The graph on the left illustrates that these trends seem to be mostly due to the increase in consumption of NE-Sector produced goods and services (C^{NE}) and a corresponding decrease of investment in the

[13] In the form of GFCF.

NE-Sector (I^{NE}). The share of directly consumed useful work (C^E) decreases approximately 12% throughout this 50 year period. Meanwhile, the share of investment in the E-Sector (I^E) increases slightly in the same period, starting at approximately 8% in 1960 and reaching shares of up to 10% in recent years.

Capital stock and depreciation

In order to estimate the time series for capital stock pertaining to each sector on an annual basis (i.e. K^{NE} and K^E) it is necessary to first determine the annual time series for the consumption of fixed capital in each sector (CFC^{NE} and CFC^E), and the initial values[14] for the capital stock series, $K^{NE}(1960)$ and $K^E(1960)$. The respective annual capital stock series can then be obtained by using the perpetual inventory method[15]:

$$\begin{cases} K^{NE}(t+1) = K^{NE}(t) - CFC^{NE}(t) + I^{NE}(t); \\ \\ K^{E}(t+1) = K^{E}(t) - CFC^{E}(t) + I^{E}(t); \end{cases} \tag{4.6}$$

Consumption of fixed capital is a term used in business and national accounts to measure the amount of fixed capital that is used up, each year, in the process of generating new output. It may also include other expenses incurred in using or installing fixed assets beyond actual depreciation charges. The AMECO database provides time series for CFC at current prices for Portugal, between 1960 and 2012. These values can be converted into constant 2005 prices by using a price deflator[16].

CFC estimated in this way relates to investment according to the Portuguese NA. Using simple association, the CFC series corresponding to I^{NE} and I^E are determined by:

$$\begin{cases} CFC^{NE} = \dfrac{I^{NE}}{GFCF} \times CFC; \\ \\ CFC^{E} = \dfrac{I^{E}}{GFCF} \times CFC; \end{cases} \tag{4.7}$$

The initial values for the capital stock time series $K^{NE}(1960)$ and $K^E(1960)$ are also determined by simple association. The AMECO database provides data on the total capital stock by year (K), determined from GFCF and CFC values expressed in these accounts. The initial values for E-Sector capital stock and NE-Sector capital stock are given by:

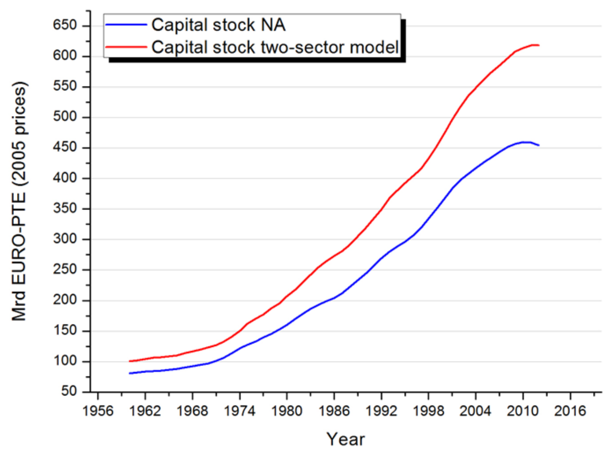
$$\begin{cases} K^{NE}(1960) = \dfrac{I^{NE}(1960)}{GFCF(1960)} \times K(1960); \\ \\ K^{E}(1960) = \dfrac{I^{E}(1960)}{GFCF(1960)} \times K(1960); \end{cases} \tag{4.8}$$

The annual series for K^{NE} and K^E can then be determined, by (4.6). Total capital K annual series according to the two-sector model's definitions is given by the sum of the K^{NE} and K^E series. Figure (4.11) shows the comparison between NA and model's total capital stock, and the share of K^{NE} and K^E that constitute $\hat{K}$.

[14] Within the time period considered, 1960 to 2012.

[15] This method tracks the existing stock of fixed assets by estimating the amount of fixed assets installed, as a result of GFCF undertaken in previous years, that have survived to the current period.

[16] The price deflator used was that of GFCF, which is provided by the AMECO database.



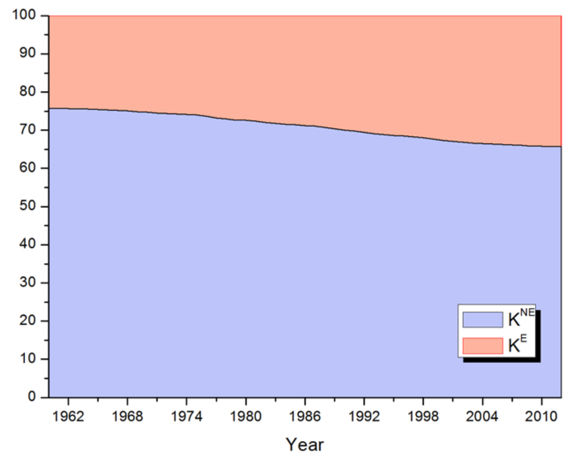


Figure 4.11: **Left graph:** Capital stock time series for Portugal (1960-2012) according to NA assumptions (K) and two-sector model's assumptions ($\hat{K}$), at constant 2005 prices. **Right graph:** Shares of K^{NE} and K^{E} in $\hat{K}$ at constant 2005 prices.

It should be noted that both CFC and capital stock series for the two-sector model are being determined under simplifying assumptions, in order to facilitate the simple analysis intended in this dissertation. In reality, and since various goods usually assigned to consumption expenditure are being redefined as investment/capital goods, a more detailed study of each type of good should be undertaken to account for its depreciation over time. The initial values of K^{NE} and K^{E} used in the perpetual inventory method are also the result of simplifying assumptions adopted in the present analysis. The growth accounting methodology introduced below deals mostly with the growth rates of variables, therefore these simplifications should not, in principle, harm the results.

One interesting result that can be obtained from the decomposition of capital stock between the two sectors of this framework is related to the relationships between all three input factors of production to the NE-Sector: Useful work (B^U), Labor (L) and capital stock (K^{NE}) - Figure (4.12).

The variation of the ratio between useful work and labor inputs to the NE-Sector grows approximately by five-fold from 1960 to 2009. The variation of the ratio between capital stock attributed to the NE-Sector and labor inputs is also very pronounced, increasing by approximately four-fold throughout this time period.

However, the ratio between useful work inputs and capital stock K^{NE} remains fairly constant for whole 50-year period represented here, when compared with the other ratios. This is an indication that possibly the capital stock used as a factor of production in this sector, and the useful work inputs used in NE-Sector production, are complementary factors. That is, the decrease in price in one of them (e.g. useful work) results in an increase in the demand for the other (capital K^{NE}). This is not unnatural, since capital stock in production is activated by useful work, and useful work without capital stock has no role in economic production.

The large variation of the ratios between useful work and labor inputs, and capital stock and labor inputs, is probably due to the fact that these factors are substitutes, in the sense that an increase in the price of one of them (e.g. labor) leads to an increase in the demand for the other (useful work, or capital K^{NE}).

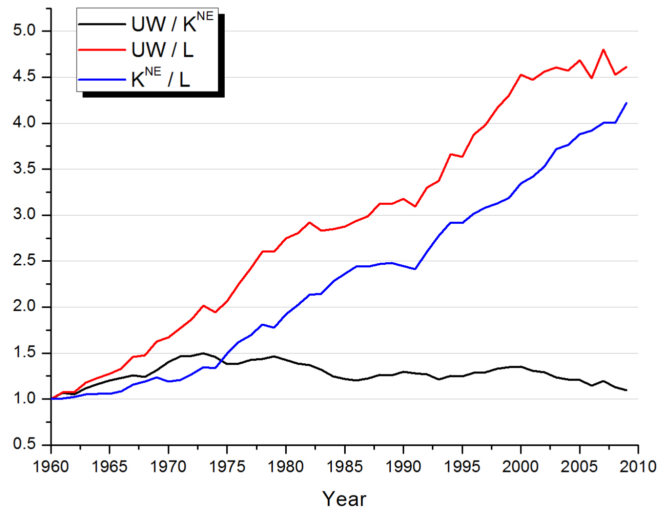


Figure 4.12: Relationships between input factors to the NE-Sector: Useful work (B^U), Labor (L) and capital stock (K^{NE}). Series represented normalized to the respective initial value (1960).

Labor inputs

The factor of production concerning labor inputs includes the various productive services provided by human beings. Labor can be measured in different ways, such as total hours worked or number of employees. Under the two-sector model proposed in chapter 2, labor inputs are entirely allocated to NE-Sector production. Throughout the rest of the empirical analysis performed here, labor inputs will be measured in terms of total annual hours worked, for the total economy. The total annual hours worked series will be estimated by multiplying employment data (in number of persons) by data concerning annual hours of work produced per worker.

Eurostat's AMECO provides data series concerning total employment, but not number of hours worked, for the time period between 1960 and 2012. Furthermore, the AMECO database is in contradiction with another major economic database (GGDC - Groningen Growth and Development Center [34]) when it comes to employment data for Portugal, especially concerning the crucial period between the golden age of the 1960s and the slowdown of the mid-1970s. The main reason pointed for this divergence is the low quality of the official Portuguese demographic statistics, particularly the 1970 census [4]. Figure (4.13) shows the difference between AMECO and GGDC employment series for the Portuguese economy.

It is unclear how exactly the AMECO employment series was determined. GGDC employment data comes from various issues of OECD's Labour Force Statistics [53]. Given the shape of the two graphical representations, the GGDC data seems to better suit the historical and political events in Portugal which affected the employment rate. Between 1960 and 1970 there was a strong emigration movement in Portugal, as well as a large deployment of soldiers to the African Colonial Wars. These movements produced a combined effect which led to a decline in population and, consequently, employment[17]. After the 1974 revolution, population increased greatly, resulting from the return of the colonists living in Africa.

[17] Both movements involved mostly men of an active age.

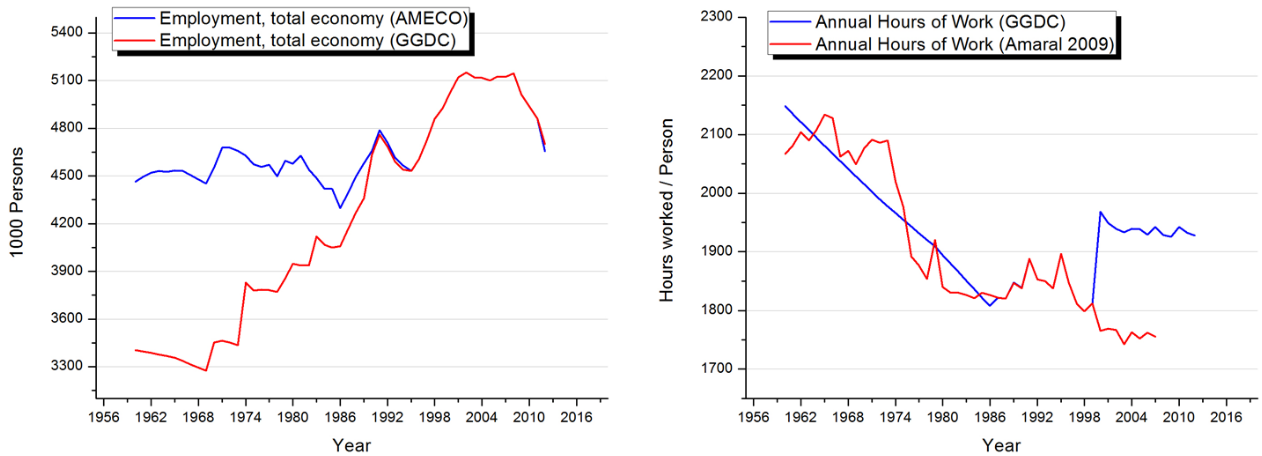


Figure 4.13: **Left graph:** Comparison between AMECO and GGDC total employment data. In 1960, the difference between time series is more than 10% of the Portuguese population that year. **Right graph:** Comparison between GGDC annual hours worked per person and the alternative series provided by Amaral (2009). The results prior to 1990 are clearly different from GGDC data.

The GGDC employment time series accurately depicts these movements, while the AMECO employment data fails to account for these historical tendencies. After 1990, the two series converge. The GGDC employment data is therefore used throughout the rest of this analysis, instead of the AMECO series.

Unlike AMECO, the GGDC database provides annual hours of work series. Until 1987, these series are built through a mixture of linear interpolations, extrapolations and direct information from Eurostat and OECD sources. Amaral (2009) offers an alternative series, grounded in more direct data [4]. This alternative assumes the weekly working hours series in manufacturing[18] to be representative of all economic sectors, and proceeds to splice this data with the GGDC series. The results concerning the period between 1960 and 1990 are clearly different from GGDC data, as shown in the right graph of Figure (4.13). The series provided by Amaral (2009) will be used along with GGDC employment data to construct an annual time series of total hours worked for the whole economy - equation (4.9). This series will correspond to labor supply for the remaining of the empirical analysis presented in this chapter - Figure (4.14).

$$Total\ hours\ worked\ (L) = Annual\ hours\ worked\ per\ person\ \times\ Employment; \tag{4.9}$$

4.3 Energy and useful work accounts

The two-sector model's variables defining the amount of exergy from natural resources entering the E-Sector (B^P) and the useful work output of this sector (B^U)[19] are still to be addressed. Moreover, it is important to know the fraction of B^U that is used in the production processes of the NE-Sector (γB^U) and the fraction that is directly consumed - $(1-\gamma)B^U$. These figures can be determined by

[18] Available from 1956 to 1990 from INE.
[19] In energy units - Joule.

Figure 4.14: Annual hours worked for the whole Portuguese economy (1960-2012).

analyzing the energy balances for the Portuguese economy, and applying the methodology developed in Serrenho et al (2013) [64].

Decomposition of energy consumption

Energy balances for Portugal are obtained from the IEA databases. These databases provide numbers on primary energy supply (E^P), gross energy consumption, energy industry own-use, and final energy consumption. The data is organized by energy carrier (oil & oil products, coal & coal products, natural gas, combustible renewables, electricity & CHP heat) and by institutional sector (industry, transport, other, non-energy uses and energy industries own-uses) - Figure (4.15). Additional energy from food & feed and non-conventional carriers comes from Henriques (2011). The energy balances must be converted to exergy and useful work series to obtain, for the whole economy, a primary exergy supply (B^P) and a total useful work supply (UW, which aggregates useful work supplied to the energy industries and E-Sector useful work output B^U). The exergy inputs to the E-Sector are linked with primary energy supply in IEA energy balances. E-Sector useful work output, on the other hand, is associated with total final exergy consumption (B^F - which discounts consumption by the transformation sector) minus consumption of exergy for energy industries own-uses.

Analyzing the IEA energy balances decomposition by institutional sector (and further subdivisions) it is possible to estimate the total final energy (minus energy industries own-uses) directly consumed by households, government and NPISH (corresponding to $(1-\gamma)B^U$) and energy used in NE-Sector production (corresponding to γB^U). The aggregation of energy balances under these two categories is illustrated in Table (4.8).

All energy consumption by the industry sector is linked to NE-Sector production. The majority of subdivisions in transport and other institutional sectors are also assigned to the variable γB^U. The exceptions are the subdivisions of road, which includes fuels consumed by private and government owned vehicles, and the residential subdivision, which corresponds to the energy consumed by households. The road subsection is split in half, similarly to what was done for the consumption and investment accounts in the previous sections. Non-energy uses are not considered in this analysis, since the primary goal of these uses is not energetic, but material-related. They are not converted

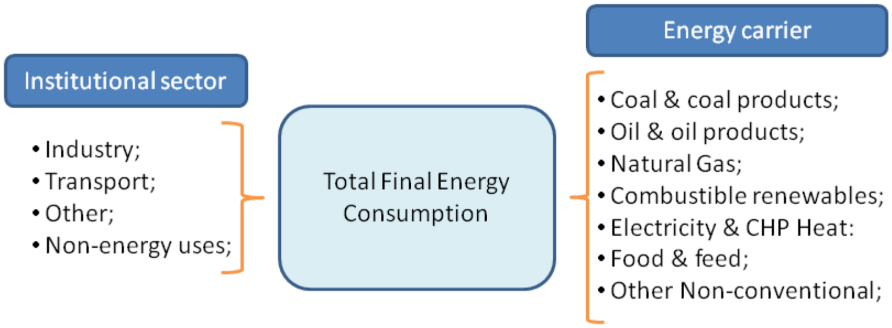


Figure 4.15: Decomposition of total final energy consumption by institutional sector (left) and by energy carrier (right).

Institutional sector		Variable
Industry		γB^U
Transport	World aviation bunkers	γB^U
	Domestic aviation	γB^U
	Road	50% γB^U
		50% (1-γ)B^U
	Rail	γB^U
	Pipeline transport	γB^U
	World marine bunkers	γB^U
	Domestic navigation	γB^U
	Non-specified (transport)	γB^U
Other	Residential	(1-γ)B^U
	Commercial & public services	γB^U
	Agriculture/forestry	γB^U
	Fishing	γB^U
	Non-specified (other)	γB^U

Table 4.8: Balances for total final energy consumption (minus energy industries own uses) allocated between direct consumption, γ, and NE-Sector production, $1-\gamma$.

into useful work, since they are not involved in the energy-related exploitation of a resource[20].

Energy balances for Portugal between 1960-2009 are obtained from IEA databases concerning the energy carriers of coal & coal products, oil & oil products, natural gas, combustible renewables, and electricity & CHP heat, for all institutional sectors. Data for energy carriers of food & feed is obtained from Henriques (2011) [39]. Since energy from food & feed carriers in recent decades concerns mostly food for humans, it is entirely assigned to direct consumption γ. Non-conventional carriers form a very small percentage of total final energy consumption, and are therefore disregarded in the present analysis.

Useful work consumption

From the useful work data determined in Serrenho et al (2013) and the variable aggregation in Table (4.8), it is possible to estimate the fractions γ and $(1-\gamma)$ of useful work E-Sector output B^U.

[20]To assess the overall exergy flow in a country, material flow analysis would be need in order to account for the total chemical exergy of the materials used. For more on this subject see [64] and [15].

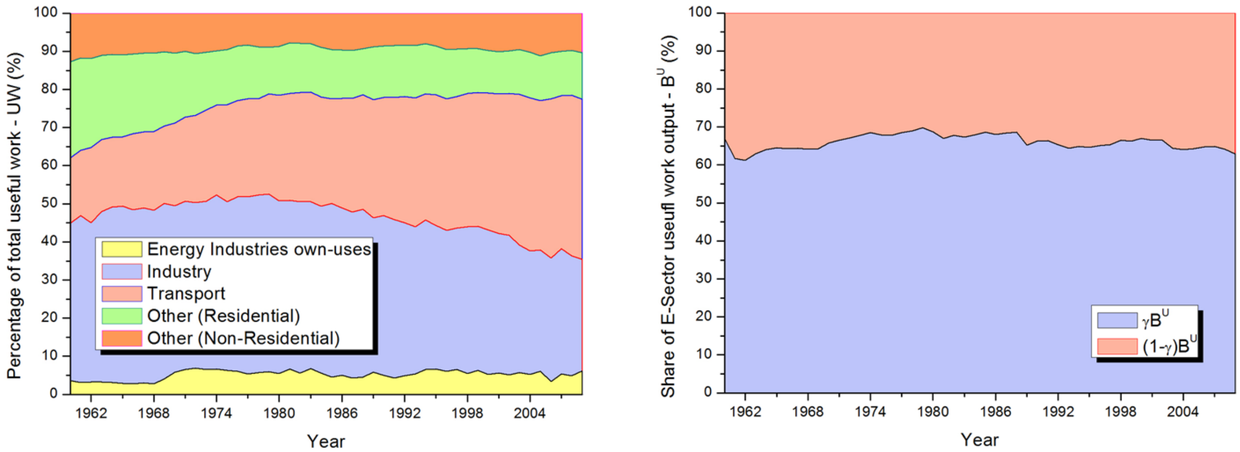


Figure 4.16: **Left graph:** Useful work consumption by institutional sector for the Portuguese economy (1960-2009). **Right graph:** Shares of B^U assigned to direct consumption $(1-\gamma)$ and NE-Sector production (γ).

This is done by allocating the exergy consumed in a given institutional sector, supplied by a given energy carrier (e.g. the energy extracted from coal & coal products used in iron & steel industry), to the relevant useful work end-uses[21] (as presented in chapter 3). Serrenho et al (2013) provides a suggested correspondence between economic sectors, energy carriers and useful work categories. In this approach, food & feed carriers can be straightforwardly allocated to muscle work. The allocation of the remaining carriers is performed by estimating the main energy end-uses in the different economic sectors. For example, coal products in the industrial sector are mostly used for high and medium heat generation. The carrier corresponding to electricity is the most difficult to allocate, given its many applications. The electricity uses in the Portuguese economy are estimated from the evolution of shares of different electricity uses in the USA and the UK [16] [31].

The right graph in Figure (4.16) shows the shares of B^U allocated to direct consumption and NE-Sector production. The assumptions made in chapter 2 regarding the constancy of the fraction γ allocating B^U to direct consumption of NE-Sector production are justified by Figure (4.16). The percentual shares of γB^U and $(1-\gamma)B^U$ appear to be relatively constant from 1960 to 2009. The largest share corresponds to useful work consumed by the productive processes in the NE-Sector, which is not surprising. This share assumes values between 60% and 73%. The smallest share, corresponding to useful work directly consumed by households, government and NPISH, oscillates between percentual shares of 27% and 40%. The left graph in Figure (4.16) show the share of total useful work UW allocated to each institutional sector considered. Initially, the largest share corresponds to industry, but by the end of the time period, the transport sector took its place as the largest useful work consuming sector. This can be attributed to an increase in the use of electricity for mechanical drive, as well as an increase in automobile uses. Meanwhile, the residential useful work consumption decreased since 1960 (although remaining fairly stable since 1980) while non-residential uses (commercial, agriculture, etc) and energy industries own-uses varied very little.

Comparing the decomposition of National Accounts in Figure (4.10) (right graph) with the useful work decomposition in Figure (4.16) (right graph), an interesting observation can be made. Namely, the NA decomposition according to two-sector variables shows that the direct consumption

[21] Heat (high, medium and low), mechanical drive, light, other electrical uses and muscle work.

of useful work by households, government and institutions has decreased throughout the 50-year period presented.

Recalling that this direct useful work consumption C^E is given by (see chapter 2):

$$C^E = p_{B^U}(1-\gamma)B^U; \tag{4.10}$$

Where p_{B^U} represents the relative price paid for directly consumed useful work, in respect to the price paid for useful work used in NE-Sector production. Dividing both sides of equation (4.10) by total output:

$$\frac{C^E}{Y} = p_{B^U}(1-\gamma)\frac{B^U}{Y}; \tag{4.11}$$

In the r.h.s. of equation (4.11), there are two terms which are approximately constant, from the empirical evidence obtained throughout this chapter: the fraction of useful work attributed to direct consumption $(1-\gamma)$, and the useful work intensity B^U/Y[22]. Consequently, since C^E/Y decreases, the relative price p_{B^U} also decreases from 1960 to 2009.

This decreasing of p_{B^U} may be due to an evolution in the preferences of consumers, attributing a greater value to useful work incorporated in produced goods and services of the NE-Sector, than useful work directly consumed.

4.4 Total factor productivity

Total factor productivity (TFP) in economics accounts for effects in total output not explained by traditionally measured inputs of labor and capital stock. Generally, assuming all inputs as accounted for, this variable can be taken as a measure of long-term technological change, or dynamism. TFP cannot be measured directly, but rather assumes the form of a residual[23]. In spite of this, TFP is often seen as the real driver of growth within an economy. Several studies reveal that, while labor and investment are important contributors, TFP may account for more than half of output growth within economies. This section analyses the TFP for the Portuguese economy, and seeks to subtract some of its weight as a driver of growth by introducing an energy-related factor of production, alongside labor and capital stock.

Determination of TFP according to AMECO

The AMECO database provides a time series for TFP, expressed in terms of 2005 values, for Portugal between 1960 and 2012. This series is determined from other data available on the AMECO database, using a methodology that is presented here for comparison with the growth accounting methodology adopted further on in this thesis. The details regarding this methodology were kindly provided by the AMECO database contact, Antonis Avdoulos[24].

The methodology used by AMECO is based on a Cobb-Douglas production function, with constant output elasticities for labor and capital stock. TFP appears as an exogenous multiplier A:

[22] B^U is not exactly equal to UW since it discounts useful work used in the E-Sector. However, the difference between the two is minimal, and the same trends that were observed for UW still apply to B^U.

[23] Its rate of change is also called the Solow residual [1].

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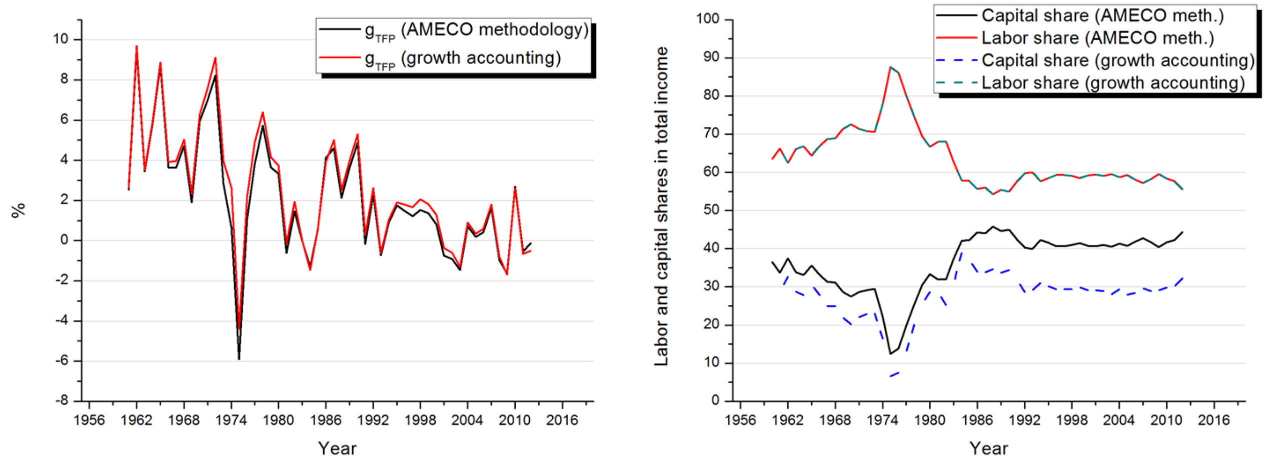

Figure 4.17: **Left graph:** Growth rate of Total Factor Productivity according to the methodology used in the AMECO database and the growth accounting approach (using AMECO Employment series). **Right graph:** Capital and labor shares in total income according to the AMECO methodology and the growth accounting aproach.

$$Y = A(L)^{\alpha}(K)^{1-\alpha}; \tag{4.12}$$

The time series used when determining TFP in this way are, for the total economy:

- Total employment, domestic (L - Persons);
- Wage & salary earners, domestic (WS - Persons);
- Compensation of employees at current market prices (CE - EURO-PTE);
- GDP at current market prices (Y_{curr} - EURO-PTE);
- GDP at 2005 market prices (Y_{2005} - EURO-PTE);
- Net capital stock at 2005 market prices (K - EURO-PTE);

Output elasticity of labor (α) is equivalent to the real unit cost of labor[25]. It is determined by multiplying the compensation of employees (CE) by the ratio of total employment (L) to the number of wage & salary earners (WS). This number is then divided by Y_{curr} in order to obtain the labor share in total output:

$$Real\ unit\ labor\ cost = \alpha = \frac{CE}{Y_{curr}} \times \frac{L}{WS}; \tag{4.13}$$

The Cobb-Douglas framework admits constant returns to scale if the sum of capital and labor output elasticities equals unity. An average value of labor output elasticity ($\hat{\alpha}$) is calculated for the time period between 1960 and 2012. Capital stock output elasticity is assumed to be complementary to labor elasticity (i.e. $1 - \hat{\alpha}$). TFP is then determined by:

[25] i.e. the cost of labor per unit of output produced.

$$TFP = \frac{I_{Y_{2005}}}{(I_L)^{\hat{\alpha}}(I_K)^{1-\hat{\alpha}}}; \tag{4.14}$$

Where $I_{Y_{2005}}$, I_L, and I_K are, respectively, the indexes of GDP at 2005 levels, total employment, and net capital stock at 2005 levels, evaluated in comparison with the initial year, 1960:

$$I_X(t) = \frac{X(t)}{X(1960)}, \ with\ X = Y_{2005}, L, K; \tag{4.15}$$

The annual TFP series obtained with (4.14) can be expressed in 2005 levels by setting the 2005 corresponding value equal to 100, and rewriting the remaining annual figures in proportion to this year. The TFP growth rate obtained through this methodology (*TFP growth rate AMECO*) for Portugal, between 1960 and 2012, is shown in Figure (4.17).

The "golden period" of Portuguese growth, in which a more sustained and intense TFP growth was verified, corresponds to the period 1958-1975. In the post-revolutionary period after 1975 there is a severe drop in TFP growth, followed by a considerable recovery. The year 1986 marks the joining of the European Economic Community (EEC, later renamed European Union). As a result of EEC/EU structural and cohesion funds, the Portuguese economy progressed significantly after this, which is patent in the TFP growth throughout this time period. The decade following the year 2000 has been named "the lost decade" due to the slow growth of the Portuguese economy by any standard. Sources for this slowdown suggest the initial efficiency gains resulting from entering the European Communities in 1986 were exhausted by that time. This decade is characterized by a low TFP growth rate in figure (4.17).

Growth accounting

Growth accounting is a procedure introduced by Robert Solow in 1957 [69], generally used in economics to measure the contribution of different factors to economic growth, and to indirectly compute the residual corresponding to TFP. Growth accounting has become one of the most common macroeconomic tools. This methodology will be used throughout the remainder of this chapter to obtain estimates of TFP, with and without an additional energy-related factor of production, for the Portuguese economy. In its simplest version, the growth accounting framework as proposed by Solow considers a continuous-time economy and an aggregate production function, of the sort:

$$Y(t) = F\left[K(t), L(t), A(t)\right]; \tag{4.16}$$

Where, as usual, Y, K, L, and A correspond, respectively, to output, capital stock, labor inputs, and technological level. Dropping the time dependence and denoting the partial derivatives of F with respect to its arguments by F_A, F_K, and F_L yields:

$$\frac{\dot{Y}}{Y} = \frac{F_A A}{Y}\frac{\dot{A}}{A} + \frac{F_K K}{Y}\frac{\dot{K}}{K} + \frac{F_L L}{Y}\frac{\dot{L}}{L}; \tag{4.17}$$

The growth rates of output, capital stock, and labor are denoted by $g_Y = \dot{Y}/Y$, $g_K = \dot{K}/K$, and $g_L = \dot{L}/L$, respectively. The contribution of technology to growth is defined as:

$$x = \frac{F_A A}{Y}\frac{\dot{A}}{A}; \tag{4.18}$$

The Solow growth model's assumptions of competitive factor markets imply that the partial derivatives of F with respect to capital and labor inputs correspond to the rental rate R and wage rate w, respectively. The factor shares for capital stock (α_K) and labor (α_L) are defined as:

$$\alpha_K = \frac{RK}{Y}, \;\; \alpha_L = \frac{wL}{Y}; \tag{4.19}$$

Taking all this into consideration, equation (4.17) can be rewritten as the fundamental growth accounting equation:

$$x = g_Y - \alpha_K g_K - \alpha_L g_L; \tag{4.20}$$

This identity allows the estimation of the contribution of technological progress to economic growth using available data on factor shares, output growth, labor force growth, and capital stock growth. This contribution associated with technology corresponds to TFP.

The equation in (4.20) is exact in continuous time, because it is defined in terms of instantaneous changes, i.e. derivatives. In practice, when applying the growth accounting framework to empirical data, the values are expressed as changes over discrete time intervals. These intervals can correspond to a year or, in case of better data, over a quarter or a month. This raises the question that factor shares can change within a discrete time interval, so using beginning-of-period or end-of-period values might lead to biased estimates of the contribution of TFP to output growth. The most common way of dealing with this problem, besides using as high-frequency data as possible, is to use factor shares calculated as the average of the beginning-of-period and end-of-period values. Therefore, in discrete time, equation (4.20) becomes:

$$\hat{x}_{t,t+1} = g_{Y,t,t+1} - \hat{\alpha}_{K,t,t+1} g_{K,t,t+1} - \hat{\alpha}_{L,t,t+1} g_{L,t,t+1}; \tag{4.21}$$

Where $g_{i,t,t+1}$ corresponds to the growth rate of factor $i = Y, K, L$ between time t and $t+1$. The average factor shares between t and $t+1$ are:

$$\hat{\alpha}_{K,,t,t+1} = \frac{\alpha_K(t) + \alpha_K(t+1)}{2}; \;\; \hat{\alpha}_{L,t,t+1} = \frac{\alpha_L(t) + \alpha_L(t+1)}{2}; \tag{4.22}$$

Equation (4.21) is a good approximation to (4.20) when the difference between t and $t+1$ is small and the capital-labor ratio does not change dramatically during this time interval. Since the data used for the growth accounting procedures presented next is annual, the growth accounting equation used will have the form of (4.21), and annual factor shares will be determined according to (4.22).

In order to establish the equivalence of both methods of TFP estimation enunciated above, their results are compared in Figure (4.17). The TFP growth rate obtained from the growth accounting methodology (*TFP growth rate GA*) is determined from (4.21). The time series used to account for K and L are the same as in the AMECO case. The growth rate g_Y is calculated from GDP at 2005 market prices time series. The labor share is determined by dividing the compensation of employees at current prices (CE) by GDP at current prices (Y_{curr}). Capital share in total output is determined by the quocient between gross operating surplus (obtained from the AMECO database) minus the estimated compensation of self-employed persons[26], and total output, at current prices: $\alpha_K = GOS/Y$.

Regression analysis, total economy

The average shares of capital and labor inputs in total income, determined from both methods introduced above, exhibit the values: $\hat{\alpha}_K \simeq 0.367$ and $\hat{\alpha}_L \simeq 0.633$ (from the AMECO methodology);

[26]The unit cost of labor in (4.13) takes into account the compensation of self-employed, so this value must be subtracted from GOS to avoid double counting.

No. of Observations: 52

i	α_i	R^2	τ
g_{TFP}	1.09	0.84	0.01
	(0.01)		(0.00)

Table 4.9: Simple linear regression between g_Y and g_{TFP}, using just AMECO data for the Portuguese economy. Errors represented in parenthesis.

$\hat{\alpha}_K \simeq 0.277$ and $\hat{\alpha}_L \simeq 0.637$ (from the GA methodology) - Figure (4.17). Despite some variation, the values are consistent with usual estimates for these shares, which attribute about 1/3 of national income to capital and the remaining 2/3 to labor [1]. Performing a simple linear regression between the growth of TFP and the variation of total output for the AMECO methodology/growth accounting approach introduced above, one obtains that approximately 80% of the variation in total output can be explained by the variation in TFP, determined according to these methods - Table (4.9).

The two-sector model's assumptions introduced in chapter 2 imply that capital stock will be higher in this framework, when compared with the NA figures. The time series for capital stock under the two-sector model's assumptions was calculated in (4.6). In order to perform growth accounting with the two-sector model, the value of payments to K^{NE} (GOS^{NE}) and K^E (GOS^E) must be determined. This is done by assuming the ratio of payments to capital stock is constant, that is[27]:

$$i = \frac{GOS}{K} = \frac{GOS^{NE}}{K^{NE}} = \frac{GOS^E}{K^E}; \tag{4.23}$$

The sum of GOS^{NE} and GOS^E forms the payments to capital stock according to the two-sector model, $G\hat{O}S$. The average share of capital in national income, for Portugal, under this two-sector framework is then: $G\hat{O}S/GDP = \hat{\alpha}_K \simeq 0.386$. This value is higher than the average shares obtained assuming a standard economy, based on NA - Figure (4.18). Labor inputs will now be given by the employment series provided by GGDC, multiplied by the average hours of work provided by Amaral (2009) - equation (4.9). The growth rate of TFP associated with the total economy under two-sector model's assumptions is represented in Figure (4.18), in comparison with g_{TFP} under standard NA assumptions. The TFP growth rates are estimated for both GA1 and GA2, through (4.20). Through simple linear regressions, as before, one concludes that TFP variation can explain approximately 60% of total output growth in either of these scenarios - Table (4.10).

The next step is to introduce an energy variable in the total economic framework as an additional input to production, in the same way as capital and labor. The relevant candidates to represent energy inputs to the economy as a whole are primary energy/exergy supply (E^P and B^P, respectively), total final energy/exergy consumption (E^F and B^F, respectively) and total useful work supply (which discounts non-energy uses) UW. These energy-related variables will be included in a growth accounting framework with NA and two-sector model's assumptions, for comparison. The first will only use data from NA, and will be named GA1. The second assumes higher values for capital stock and capital share, resultant from the two-sector model's assumptions, and will be denoted as GA2.

The contribution of each production factor to economic growth is measured, according to the growth accounting procedure in (4.21). In order to compare the contribution of factors and TFP

[27] GOS is expressed in units of $euro/year$, while K is in $euro$ units. The quotient between the two is in $year^{-1}$ units.

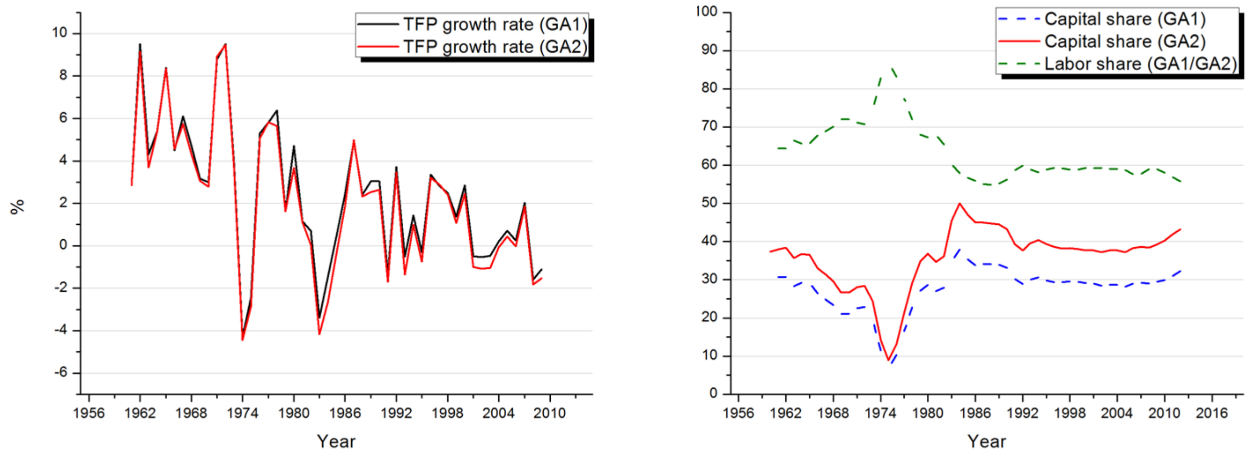

Figure 4.18: **Left graph:** Solow residuals for the total Portuguese economy under standard NA assumptions (GA1) and two-sector model's assumptions (GA2). **Right graph:** Capital and labor shares according to NA assumptions (GA1) and two-sector model's assumptions (GA2).

No. of Observations: 52

i	α_i	R^2	τ
g_{TFP}(GA1)	0.83	0.64	0.02
	(0.09)		(0.01)
g_{TFP}(GA2)	0.81	0.66	0.02
	(0.02)		(0.01)

Table 4.10: Simple linear regression between g_Y and g_{TFP} under GA1 and GA2 growth accounting scenarios, for the Portuguese economy. Errors represented in parenthesis.

to total output growth throughout the studied time period, 7 distinct 7-year periods are defined, spanning the period between 1961 and 2009. The contribution for the whole period (1960-2009) is also determined, for each growth accounting scenario - Table (4.11). The average growth rate over the 7-year periods and for the whole period 1960-2009 is calculated according to the formula[28]

$$growth\ rate = \left(\frac{first\ year\ value}{last\ year\ value}\right)^{\frac{1}{n}} - 1,\ with\ n = number\ of\ years; \qquad (4.24)$$

The results obtained are in line with other growth accounting procedures performed for the Portuguese economy [3] [27]. The major contribution to GDP growth after 1982 was capital deepening, while TFP dominated before that period. Labor is the factor that contributes less throughout most of the 50-year period. Considering the entire time period 1960-2009, TFP seems to contribute overall to 60% of output growth, in agreement with the result obtained from linear regression - Table (4.10). The GA2 scenario exhibits a larger contribution of capital accumulation to growth, and therefore a slightly smaller TFP.

In order to determine the weight of an energy-related factor input in TFP growth for both cases, a simple linear regression is performed considering each of the five energy inputs mentioned. Linear

[28] Where first and last year refer, respectively, to the first year in a given time period and the last year of the same time period.

	GA1				GA2			
Years	$\alpha_K g_K$	$\alpha_L g_L$	g_Y	TFP	$\alpha_K g_K$	$\alpha_L g_L$	g_Y	TFP
1961-1967	0.39	-0.31	5.45	5.38	0.56	-0.31	5.45	5.20
1968-1974	0.82	1.32	5.59	3.45	0.91	1.32	5.59	3.36
1975-1981	0.82	-0.38	4.04	3.61	1.07	-0.38	4.04	3.36
1982-1988	1.10	0.63	2.51	0.78	1.54	0.63	2.51	0.34
1989-1995	1.09	0.54	2.46	0.83	1.48	0.54	2.46	0.44
1996-2002	1.11	0.57	2.88	1.21	1.36	0.57	2.88	0.95
2003-2009	0.47	-0.09	0.45	0.07	0.70	-0.09	0.45	-0.17
1960-2009	0.96	0.29	3.56	2.31	1.29	0.29	3.56	2.09

Table 4.11: Decomposition of GDP growth, according to growth accounting, for the Portuguese economy, for GA1 and GA2 scenarios. All values are in percentage (%).

regression is the predominant empirical tool in economics. It can be used to identify the relationship between a single predictor variable and the response variable, when all other predictor variables are "held fixed". Throughout this analysis linear regressions will be used to estimate how well the variation in different factors correlates with TFP and output growth. The simplifying assumption that the relationship between these variables is linear comes from the fundamental equation of growth accounting - equation (4.21).

In this case, the purpose is to estimate how much of TFP growth for Portugal can be really explained by growth in the energy variable chosen. That is:

$$g_{TFP} = \alpha_i g_i + \tau, \ with \ i = E^P, B^P, E^F, B^F, UW; \tag{4.25}$$

The share of the energy-related factor in national income α_i, corresponding to the slope of the curve in (g_i, g_{TFP}) space, is determined through (4.25) where g_{TFP} and g_i are, respectively, the dependent and independent variables. The disturbance term τ captures all other factors which influence g_{TFP}, besides g_i. The simple linear regression results obtained through this method are presented in Table (4.12).

A first observation seems to suggest that there is a very low correlation between TFP growth and variations in primary energy and primary exergy inputs. The coefficient of determination (R^2) for both the GA1 and GA2 scenarios is approximately 3%. These linear regressions estimate regression coefficients of E^P (B^P) in the order of 0.15 (0.14). The inclusion of E^F or B^F as the appropriate energy consumption variable returns higher values of R^2 and corresponding regression coefficient estimates for both cenarios, indicating that these variables are better suited to explain growth in TFP. In this case, the shares of E^P (B^P) are in the order of 0.41 (0.4) for GA1 and 0.42 for GA2 (with R^2 of 12%). Finally, the inclusion of UW as a factor of production in GA1 returns an estimate for the regression coefficient of 0.47 and the highest coefficient of determination (21%). For the GA2 scenario, the inclusion of UW suggests a slightly higher correlation between growth of UW and of TFP, with an estimated regression coefficient of 0.50 and a coefficient of determination of 23%. The UW factor included in the GA2 scenario shows a marginally more significant fit with the TFP growth data. Useful work consumption also exhibits the highest share of payments α_i of all energy consumption variables considered, and the smallest residual τ.

Simple linear regressions can also be performed by adopting the growth rate of total output g_Y as the dependent variable in (4.25). The explanatory power of each variable input in total output growth can thus be independently evaluated and later compared with multiple linear regression

No. of Observations: 49

	GA1			GA2		
i	α_i	R^2	τ	α_i	R^2	τ
E^P	0.15	0.03	0.02	0.15	0.03	0.02
	(0.12)		(0.01)	(0.13)		(0.01)
B^P	0.14	0.03	0.02	0.14	0.03	0.02
	(0.12)		(0.01)	(0.13)		(0.01)
E^F	0.41	0.12	0.01	0.42	0.12	0.01
	(0.16)		(0.01)	(0.17)		(0.01)
B^F	0.40	0.12	0.01	0.42	0.12	0.01
	(0.16)		(0.01)	(0.16)		(0.01)
UW	0.47	0.21	0.01	0.50	0.23	0.01
	(0.13)		(0.01)	(0.13)		(0.01)

Table 4.12: Simple linear regressions of g_{TFP} with five distinct energy-related inputs, for GA1 and GA2, with $Y = C + I + (X - M)$ as measured in Standard National Accounts. The corresponding errors are represented in parenthesis.

No. of Observations: 49

	K(GA1)	$\hat{K}$(GA2)	L	E^P	B^P	E^F	B^F	UW
α_i	0.52	-0.07	0.46	0.27	0.26	0.56	0.55	0.58
	(0.27)	(0.29)	(0.16)	(0.13)	(0.12)	(0.16)	(0.16)	(0.13)
τ	0.02	0.04	0.03	0.03	0.03	0.02	0.02	0.02
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
R^2	0.07	0.00	0.15	0.09	0.09	0.20	0.20	0.29

Table 4.13: Simple linear regressions of g_Y for GA1 and GA2 cenarios, with $Y = C+I+(X-M)$ as measured in Standard National Accounts. The corresponding errors are represented in parenthesis.

analysis. The outcomes of simple linear regressions performed in this way are presented in Table (4.13).

The single variable that appears to explain most of total output growth is total useful work inputs UW ($\sim$29%), followed by final energy and exergy inputs, with coefficients of determination of the order of 20%. Both primary energy and exergy inputs account for approximately 9% of output growth. Labor accounts for 15% and capital inputs according to the GA1 scenario seem to account for only a very small percentage of output growth ($\sim$7%). In the GA2 scenario, the simple regression returns a very small coefficient of determination ($\sim$0%). Also in the GA1 scenario, this coefficient is very small ($\sim$5%), which contrasts with the results obtained from growth accounting decomposition for the entire period 1960-2009. This may indicate, as Robert Ayres defends [9], that factors of production are not necessarily paid according to their productivity. In this case, capital stock shows clear signs of being overpayed, according to these regression results.

Multiple linear regression is a generalization of linear regression by considering more than one independent variable. Multiple regressions are here performed for both growth accounting scenarios (GA1 and GA2), first considering only capital and labor inputs, and then with an additional energy-related input. This is done in order to estimate how much of output growth g_Y can be explained by growth in the different inputs to the whole economy: capital (g_K), labor (g_L) and the additional energy-related input. The multiple linear regressions performed with capital and labor inputs only are of the form:

No. of Observations: 49

	GA1						GA2					
i	α_i	α_K	α_L	R^2	Adj. R^2	τ	α_i	α_K	α_L	R^2	Adj. R^2	τ
-	-	0.01	0.46	0.15	0.11	0.03	-	-0.52	0.48	0.20	0.17	0.05
		(0.31)	(0.17)			(0.01)		(0.28)	(0.16)			(0.01)
E^P	0.21	-0.10	0.42	0.20	0.14	0.03	0.26	-0.63	0.41	0.28	0.23	0.05
	(0.13)	(0.31)	(0.17)			(0.01)	(0.12)	(0.28)	(0.16)			(0.01)
B^P	0.20	-0.10	0.42	0.19	0.14	0.03	0.25	-0.64	0.40	0.28	0.23	0.05
	(0.12)	(0.31)	(0.17)			(0.01)	(0.12)	(0.28)	(0.16)			(0.01)
E^F	0.57	-0.37	0.44	0.33	0.28	0.03	0.59	-0.73	0.40	0.42	0.38	0.05
	(0.17)	(0.29)	(0.15)			(0.01)	(0.15)	(0.25)	(0.15)			(0.01)
B^F	0.57	-0.37	0.44	0.33	0.28	0.03	0.58	-0.73	0.40	0.42	0.38	0.05
	(0.16)	(0.29)	(0.16)			(0.01)	(0.15)	(0.25)	(0.14)			(0.01)
UW	0.53	-0.18	0.39	0.39	0.34	0.02	0.52	-0.51	0.38	0.44	0.40	0.04
	(0.13)	(0.27)	(0.15)			(0.01)	(0.12)	(0.24)	(0.14)			(0.01)

Table 4.14: Multiple linear regressions of g_Y for GA1 and GA2 scenarios, with $Y = C+I+(X-M)$, as measured in Standard National Accounts. Errors represented in parenthesis.

$$g_Y = \alpha_K g_K + \alpha_L g_L + \tau; \tag{4.26}$$

And, with an additional energy-related factor of production:

$$g_Y = \alpha_K g_K + \alpha_L g_L + \alpha_i g_i + \tau,\ with\ i = E^P, B^P, E^F, B^F, UW; \tag{4.27}$$

The factor shares α_K, α_L and α_i represent the parameters to be estimated. The term τ corresponds to the independent identically distributed normal error. The results obtained for both GA1 and GA2 using this methodology are presented in Table (4.14). The first row shows multi-linear regressions without additional energy-related inputs for GA1 and GA2.

In the case of simple linear regressions, the coefficient of determination (R^2) corresponds to the squared correlation between the outcomes and the values of the single regressor used for prediction. In the case of multiple linear regressions, one must take into account that, since the predictors are calculated by ordinary least squares regression, R^2 increases with the number of variables added to the model[29]. Therefore, there is a chance that, when adding a variable, R^2 will increase due to chance alone. In order to adjust for the number of explanatory terms in a model, relative to the number of data points, one can resort to the adjusted R^2. Unlike R^2, the adjusted R^2 only increases when a new explanator is included only if the new explanator improves R^2 more than would be expected in the absence of any explanatory value being added by the new explanator [74]. In the multiple regression results shown below, both R^2 and Adj. R^2 will be determined.

For the GA1 scenario, multiple regression without an additional energy input justifies approximately 11% of growth in output. Inclusion of either primary energy or exergy inputs returns a better fit, with 14% of output growth explained. Inclusion of final energy or exergy inputs doubles the coefficient of determination to 28%, while inclusion of useful work inputs explain approximately 34% of output growth, the highest coefficient for this scenario. For the GA2 scenario, the fits with and without any energy-related input are slightly better than in GA1. With the inclusion of UW inputs, the multiple regression explains approximately 40% of output growth.

The multiple regressions without any energy variables for both scenarios can be directly compared with the growth accounting procedure performed before - Table (4.11). As in the simple

[29] This is called "Kitchen sink regression".

regressions for total output, the role of capital stock is here very diminished. The TFP variation obtained with multiple regression is approximately 3.45%, a slightly larger value than the 2.31% resulting from growth accounting decomposition for the whole period 1960-2009.

Another interesting observation is the fact that adding any energy consumption variable to the multiple regressions brings the estimated share of capital in total payments to negative values. This is likely due to the fact that capital stock inputs, unlike labor inputs (total number of hours worked) do not accurately reflect effective capital (capital actually used productively in the economy). The energy variables act as a more accurate measure for this effective capital, and hence "rob" total capital K of some of its share in total payments.

4.5 Overview

This chapter forms a bridge between the theoretical analysis of the two-sector abstract growth model developed in chapter 2 with statistical data available on economic and energy variables for Portugal in the last 50 years.

Section (5.1) empirically tests some of the relevant stylized facts of long-term growth presented in chapter 3. The purpose is to compare the validity of classically established facts (i.e. Kaldor's facts) with the stylized fact proposed in the context of this thesis, regarding the stability of useful work intensity for the time period studied, in the Portuguese economy.

Section (5.2) decomposes Portuguese National Accounts in terms of the key variables defined in the two-sector model. Consumption is decomposed in terms of COICOP (private, institutions) and COFOG divisions (government). Some assumptions are made in order to account for consumption goods which perform exergy-to-useful work conversion. These are redefined as physical capital investment in the E-Sector. Investment is decomposed in terms of type of asset (GFCF categories). Capital stock series are obtained from GFCF and consumption of fixed capital annual time series. Labor inputs are estimated from total employment figures and number of hours worked.

Section (5.3) deals with the two-sector model's energy variables. Energy balances obtained from the IEA and Henriquer (2011) data are broken down in terms of institutional sector and energy carrier. Primary energy, exergy and useful work inputs, as well as useful work directly consumed by households, government and NPISH and useful work inputs to the NE-Sector are estimated from IEA data and the useful work accounting results presented in Serrenho et al (2013).

Section (5.4) introduces a growth accounting methodology to estimate total factor productivity in the Portuguese economy. The growth accounting approach is compared with the methodology used in the AMECO database. Simple and multiple linear regressions are performed with the inclusion of several possible energy-related variables as production inputs. The weight of these variables in explaining TFP and output growth is evaluated and discussed.

Chapter 5

Conclusions

This thesis attempts to address the role of energy in economic development, by constructing a two-sector abstract model of economic growth with an additional input to production, which reflects the energy consumption of the economic system, in qualitative terms. This is done by invoking the thermodynamic concepts of exergy, second-law efficiency and useful work.

Empirical analysis conducted for the Portuguese economy in the last 50 years using several energy consumption variables suggests that useful work consumption is the appropriate energy consumption variable to account for economic growth. Namely, of all the energy consumption variables considered throughout this analysis, useful work intensity in relation to output variation is the only one exhibiting an approximately constant and stable behavior throughout the entire period studied. This indicates that useful work consumption is the energy variable that best correlates with long-term economic growth for the Portuguese economy, in the last 50 years, and constitutes an argument for the inclusion of useful work consumption as the appropriate variable representing energy inputs in a growth model for the Portuguese case.

Assuming that the Portuguese economy can be described by a basic Solow growth model, further empirical analysis showed that the approximate constancy of useful work intensity for the time period studied constitutes a more empirically robust stylized fact for the Portuguese economy than most classically defined long-term stylized facts (Kaldor's facts). The development of growth models for the Portuguese economy, with energy inputs, should therefore consider a constant useful work intensity as a relevant long-term stylized fact in its analysis.

Further analysis using a growth accounting methodology uncovered several interesting observations. Namely:

- Total factor productivity was the major contributor to Portuguese economic growth between 1960 and 1980, being surpassed after that period by factor contributions, especially capital accumulation. Nevertheless, TFP variations can still account for up to 65% of total output variations between 1960 and 2009.

- Useful work consumption is the energy consumption variable studied that best correlates with both TFP and total output variations. Useful work can account for up to 21% of TFP growth in a simple linear regression and approximately 34% in a multiple linear regression, together with the primary factors of production labor and capital. The regression results improve with energy consumption variables closer to the productive processes, useful work being the closest.

- Multiple linear regressions performed without any assumptions on the factor shares of capital and labor in total payments indicate that these factors may be overpayed in the Portuguese economy, given their apparent productivity. Capital, in particular, appears to be severely overpayed. Multiple regressions also suggest that any energy consumption variable may constitute a better measure of effective capital stock than the total capital stock variable used in this analysis.

The development and analysis of a two-sector abstract growth model produced interesting insights, especially in relation to Uzawa's steady-state theorem. It was shown that, for the two-sector framework, assuming regular growth paths for the sector-specific variables, it is possible to write total technological progress in a purely labor-augmenting (Harrod-neutral) form. This demonstration can be, in principle, applied to any general multi-sector model similar to the framework developed here.

The two-sector model framework allowed to decompose and redefine National Accounts and energy balances according to the sector-specific variables of this framework. Several interesting observations were made from this decompositions. Namely:

- The observed decrease in the share of direct useful work consumption in total GDP (minus imports/exports) can be attributed to a decrease in the relative price of useful work directly consumed, in relation to the price paid for useful work used in NE-Sector production. This suggests that consumers preferences have evolved in the last 50 years, attributing a greater value to useful work embodied in NE-Sector goods and services than useful work directly consumed.

- An estimation on the capital stock attributed to each sector hints at the possibility that useful work and capital stock inputs to the NE-Sector are complementary goods, in contrast with capital stock and labor inputs to this sector, which are viewed as substitute goods. This is justified by the fact that capital stock need useful work to be activated, and useful work has no productive value by itself in this sector.

Overall, the results obtained with the empirical data collected from the Portuguese economy seem to support the main idea in this thesis: that energy has a considerably more important role in economic growth than standard theory suggests, and that useful work, as an energy-related variable closer to the productive processes of the economic system, is better suited to account for that growth. Regression analysis as performed in this work can only be used to argue for correlation between variables, and not causation. Future work will tackle this issue and attempt to conclude on the causality relationship between useful work consumption and economic growth.

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Appendix A

Exergy, second-law efficiency and useful work

The first law of thermodynamics states that the total energy of an isolated system is constant: it can be transformed, but never created or destroyed. Expressions adopted in everyday conversation and even environmental or economic debate, such as "energy consumption" or "energy conservation" are technically innacurate. A different thermodynamic concept from energy is needed to articulate what is actually consumed. When energy is used, it is not destroyed but merely converted to a less useful form (i.e. a form of less exergy).

Formally, the exergy of a system is the maximum amount of physical work that can theoretically be recovered as that system approaches equilibrium with its surrondings, reversibly. It is a combination property, depending on the state of both the system in question and the environment. A reference state must be defined for a given use of the concept of exergy. Generally, this reference state is defined to be the state of thermodynamic equilibrium characterized by the same temperature, pressure and chemical composition of the environment. A system that is in equilibrium with its surrondings has zero exergy and is said to be at the dead state.

Exergy is closely related to the thermodynamic concept of entropy. Exergy quantifies the potential of energy to disperse in the course of it's diffusion into the environment, while entropy quantifies the state of dispersion, the extent to which the energy in question is dispersed. In other words, exergy accounts for the irreversibility of a process due to increase in entropy (from the second law of thermodynamics). The amount of increased entropy is proportional to that of consumed exergy and the proportional constant is the ambient temperature in Kelvin scale [65].

$$B_{destroyed} = I = T_0 S_{gen}; \tag{A.1}$$

The product of entropy and environmental temperature is called *anergy*, and represents dispersed exergy. Energy then consists of two components: the dispersed part (anergy), and the part which has the potential to disperse (exergy). The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant. Destroyed exergy results from irreversibilities such as friction, mixing, chemical reactions, heat transfer through finite temperature difference, unrestrained expansion, non-quasi-equilibrium compression, or expansion. All these processes generate entropy. Letting B be the exergy or available work, decreasing with time, and S_{total} the entropy of the system and its reference environment enclosed together in a larger isolated system, increasing over time, then:

$$\frac{dB}{dt} \leq 0 \; is\ equivalent\ to \; \frac{dS_{total}}{dt} \geq 0; \tag{A.2}$$

These statements are both expressions of the second law of thermodynamics. Table (A.1) illustrates the exergy content of different energy flows.

There are various forms of exergy B [10].

- **Exergy of kinetic energy:** Kinetic energy is a form of mechanical energy and can be converted directly into work. Kinetic energy itself is the work potential (or exergy) of kinetic energy, independent of the temperature and pressure of the environment.

$$B_{kin} = E_{kin} = \frac{\vec{V}^2}{2}\ (kJ/kg); \tag{A.3}$$

  Where $\vec{V}$ is the velocty of the system in relation to the environment.

- **Exergy of potential energy:** Potential energy is a form of mechanical energy and can also be converted directly into work. Like kinetic energy, potential energy corresponds to its own exergy, independent of temperature and pressure of the environment.

$$B_{pot} = E_{pot} = g \cdot z\ (kJ/kg); \tag{A.4}$$

  Where z is the height of the system in relation to the environment.

- **Exergy of thermal energy:** The exergy of the thermal energy of thermal reservoirs is equivalent to the work output of a reversible Carnot heat engine operating between the reservoir and the environment. Figure (A.1) represents a piston-cylinder device, exchanging heat and work with the surronding environment. The work done by the gas expanding inside the piston-cylinder device is boundary work.

$$\delta W = PdV = (P - P_0)dV + P_0 dV = \delta W_{b,useful} + P_0 dV; \tag{A.5}$$

  The actual work done by the gas is:

$$W = W_{b,useful} + \int P_0 dV = W_{b,useful} + W_{surr}; \tag{A.6}$$

  The quantity W_{surr} corresponds to work performed on the surrondings, which cannot be used for other useful purposes. Any useful work delivered by a system such as the one in Figure (A.1) is due to the pressure above the atmosferic level.

  The exergy of a system as the one represented in Figure (A.1) may be determined by considering how much of its heat transfer is converted to work entirely. To assure the reversibility of the process, the relevant heat transfer is assumed ot occur through a reversible heat engine.

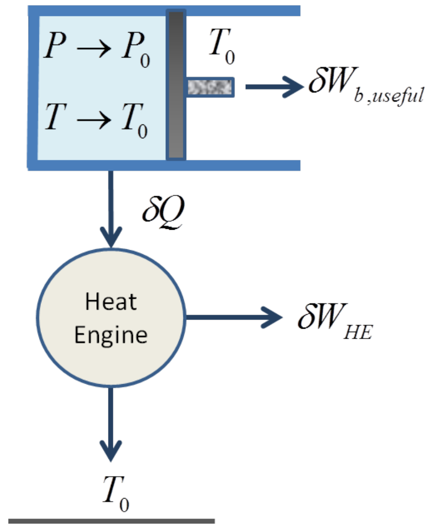

Figure A.1: Piston-cylinder thermodynamic device.

The heat transfer is taken to be from the system to the surroundings, and so the conservation of energy has the form:

$$\delta E_{in} - \delta E_{out} = dE_{sys} \Leftrightarrow 0 - \delta Q - \delta W = dU; \tag{A.7}$$

Integrating from the given state to the dead state[1], total useful work due to a system undergoing a reversible process like the one in Figure (A.1) is given by:

$$W_{total\ useful} = (U - U_0) + P_0(V - V_0) - T_0(S - S_0); \tag{A.8}$$

This is the definition of exergy. Including the kinetic and potential energy components, the *thermomechanical exergy* of a closed system is, formally:

$$B = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + m\frac{\vec{V}}{2} + mgz; \tag{A.9}$$

As a system undergoes a process between the specified initial and final states, there is a maximum amount of work that can be produced - *reversible work* W_{rev}. This is the useful work output obtained when the process between the initial and final states is executed in a totally reversible manner. The difference between reversible work and useful work $W_{b,useful}$ is due to the irreversibilities present during the process. For a totally reversible process, the useful and reversible work terms are identical and irreversibility is zero.

$$I = W_{rev} - W_{b,useful}; \tag{A.10}$$

[1] The given state represented by no subscript and the dead state represented by the *0* subscript.

Energy flow (E)	Exergy content (B)	Observations
Fuel	$B = -\Delta G = \|\Delta H\| - T_0 \Delta S \approx \|\Delta H\|$	The maximum work done by a fuel is the chemical work of its combustion. The term $T_0 \Delta S$ is the heat as a consequence of the entropy received.
Electricity	$B = E$	Electricity can be completely converted to work.
Heat	$B = E\left(1 - \frac{T_0}{T}\right)$	The maximum work done by a heat flow is the work that would be done by a Carnot cycle working between T and T_0.

Table A.1: Exergy content of different energy flows. T_0 is the temperature of the environment and T is the temperature of the reservoir from which heat is added.

The second-law efficiency is a measure of the performance of a device (or process) relative to the performance under reversible conditions for the same initial and final states. It expresses the distance of a process from its theoretical ideal. It differs from thermal efficiency, η_{th}, which measures the conversion between the useful output of a system and its energy input. Second-law efficiencies are given by:

$$\varepsilon = \frac{\eta_{th}}{\eta_{th,rev}} = \frac{W_u}{W_{rev}}; \tag{A.11}$$

Or, alternatively:

$$\varepsilon = \frac{desired\ exergy\ transfer}{relevant\ exergy\ input},\ with\ 0 \leq \varepsilon \leq 1; \tag{A.12}$$

Useful work values can only be obtained after estimation of ϵ. The second-law efficiency exhibits lower values as more exergy is destroyed in the process, and depends on the source and end-use of exergy.

Useful work then constitutes the measure of the exergy actually provided to end uses, after accounting for losses.

Appendix B

Mathematical appendix

B.1 Uzawa's Steady State theorem

Equivalence of sector-specific and aggregate regular growth paths

Assuming that all sector-specific key variables for the proposed two-sector model (Y^{NE}, C^E, C^{NE}, K^E, K^{NE}) follow regular growth paths as the one defined in chapter 2, with distinct α and β parameters for each variable[1], the first step is to show that, if the total aggregate variables (Y, C, K) for the model also follow similar regular growth paths then the characteristic parameters μ and ω for the aggregate variables will be the same as the μ and ω parameters of the respective disaggregate variables, e.g. $\mu_K = \mu_{K^{NE}} = \mu_{K^E}$ and $\omega_K = \omega_{K^{NE}} = \omega_{K^E}$. Consequently, the first-order growth rates of the total aggregate variables will be identical to the first-order growth rate of the respective disaggregate variables (e.g. $g_K = g_{K^{NE}} = g_{K^E}$).

Writing the aggregate variables as the sum of sector-specific variables, in terms of regular growth paths, gives:

$$\begin{cases} Y_0(1+\mu_Y\omega_Y t)^{1/\omega_Y} = Y_0^{NE}(1+\mu_{Y^{NE}}\omega_{Y^{NE}} t)^{1/\omega_{Y^{NE}}} + C_0^E(1+\mu_{C^E}\omega_{C^E} t)^{1/\omega_{C^E}} \\ \\ C_0(1+\mu_C\omega_C t)^{1/\omega_C} = C_0^{NE}(1+\mu_{C^{NE}}\omega_{C^{NE}} t)^{1/\omega_{C^{NE}}} + C_0^E(1+\mu_{C^E}\omega_{C^E} t)^{1/\omega_{C^E}} \\ \\ K_0(1+\mu_K\omega_K t)^{1/\omega_K} = K_0^{NE}(1+\mu_{K^{NE}}\omega_{K^{NE}} t)^{1/\omega_{K^{NE}}} + K_0^E(1+\mu_{K^E}\omega_{K^E} t)^{1/\omega_{K^E}} \end{cases} \tag{B.1}$$

The equalities in (B.1) must hold for all time t. If time is set to the initial instant $t = 0$, it results that:

[1] That is: α_i and β_i, with $i = Y^{NE}, C^{NE}, C^E, K^{NE}, K^E, L$.

$$\begin{cases} Y_0 = Y_0^{NE} + C_0^E; \\ C_0 = C_0^{NE} + C_0^E; \\ K_0 = K_0^{NE} + K_0^E; \end{cases} \tag{B.2}$$

Differentiating (B.1) once with respect to time gives:

$$\begin{cases} \dot{Y} = \mu_Y Y_0 (1 + \mu_Y \omega_Y t)^{\frac{1}{\omega_Y} - 1} = \mu_{Y^{NE}} Y_0^{NE} (1 + \mu_{Y^{NE}} \omega_{Y^{NE}} t)^{\frac{1}{\omega_{Y^{NE}}} - 1} + \mu_{C^E} C_0^E (1 + \mu_{C^E} \omega_{C^E} t)^{\frac{1}{\omega_{C^E}} - 1} \\ \dot{C} = \mu_C C_0 (1 + \mu_C \omega_C t)^{\frac{1}{\omega_C} - 1} = \mu_{C^{NE}} C_0^{NE} (1 + \mu_{C^{NE}} \omega_{C^{NE}} t)^{\frac{1}{\omega_{C^{NE}}} - 1} + \mu_{C^E} C_0^E (1 + \mu_{C^E} \omega_{C^E} t)^{\frac{1}{\omega_{C^E}} - 1} \\ \dot{K} = \mu_K K_0 (1 + \mu_K \omega_K t)^{\frac{1}{\omega_K} - 1} = \mu_{K^{NE}} K_0^{NE} (1 + \mu_{K^{NE}} \omega_{K^{NE}} t)^{\frac{1}{\omega_{K^{NE}}} - 1} + \mu_{K^E} K_0^E (1 + \mu_{K^E} \omega_{K^E} t)^{\frac{1}{\omega_{K^E}} - 1} \end{cases} \tag{B.3}$$

Like in (B.1), the equalities in (B.3) must hold for all time t. Hence, setting time $t = 0$ once more gives:

$$\begin{cases} \mu_Y Y_0 = \mu_{Y^{NE}} Y_0^{NE} + \mu_{C^E} C_0^E; \\ \mu_C C_0 = \mu_{C^{NE}} C_0^{NE} + \mu_{C^E} C_0^E; \\ \mu_K K_0 = \mu_{K^{NE}} K_0^{NE} + \mu_{K^E} K_0^E; \end{cases} \tag{B.4}$$

Where, substituting the initial values for the total variables Y_0, C_0 and K_0, as in (B.2), gives:

$$\begin{cases} (\mu_Y - \mu_{Y^{NE}}) Y_0^{NE} + (\mu_Y - \mu_{C^E}) C_0^E = 0; \\ (\mu_C - \mu_{C^{NE}}) C_0^{NE} + (\mu_C - \mu_{C^E}) C_0^E = 0; \\ (\mu_K - \mu_{K^{NE}}) K_0^{NE} + (\mu_K - \mu_{K^E}) K_0^E = 0; \end{cases} \tag{B.5}$$

In order for the equations in system (B.5) to be satisfied for any choice of initial conditions, the growth rates at time $t = 0$ for the sector-specific variables must be identical to the growth rate of the respective total aggregate variables in the same instant. That is:

$$\begin{cases} \mu_Y = \mu_{Y^{NE}} = \mu_{C^E}; \\ \\ \mu_C = \mu_{C^{NE}} = \mu_{C^E}; \\ \\ \mu_K = \mu_{K^{NE}} = \mu_{K^E}; \end{cases} \tag{B.6}$$

Moreover, we will also have, from the definition of total output under the two-sector model framework proposed, that $\mu_Y = \mu_{C^E} = \mu_C$. Differentiating now (B.3) once more with respect to time (or differentiating (B.1) twice with respect to time), results in:

$$\begin{cases} \ddot{Y} = (1-\omega_Y)g_Y^2 Y = (1-\omega_{Y^{NE}})g_{Y^{NE}}^2 Y^{NE} + (1-\omega_{C^E})g_{C^E}^2 C^E; \\ \\ \ddot{C} = (1-\omega_C)g_C^2 C = (1-\omega_{C^{NE}})g_{C^{NE}}^2 C^{NE} + (1-\omega_{C^E})g_{C^E}^2 C^E; \\ \\ \ddot{K} = (1-\omega_K)g_K^2 K = (1-\omega_{K^{NE}})g_{K^{NE}}^2 K^{NE} + (1-\omega_{K^E})g_{K^E}^2 K^E; \end{cases} \tag{B.7}$$

Again, (B.7) must be valid for all time t. Therefore, for the initial instant $t = 0$:

$$\begin{cases} (1-\omega_Y)\mu_Y^2 Y_0 = (1-\omega_{Y^{NE}})\mu_{Y^{NE}}^2 Y_0^{NE} + (1-\omega_{C^E})\mu_{C^E}^2 C_0^E; \\ \\ (1-\omega_C)\mu_C^2 C_0 = (1-\omega_{C^{NE}})\mu_{C^{NE}}^2 C_0^{NE} + (1-\omega_{C^E})\mu_{C^E}^2 C_0^E; \\ \\ (1-\omega_K)\mu_K^2 K_0 = (1-\omega_{K^{NE}})\mu_{K^{NE}}^2 K_0^{NE} + (1-\omega_{K^E})\mu_{K^E}^2 K_0^E; \end{cases} \tag{B.8}$$

Again, the initial values for the total aggregate variables were substituted as in (B.2) and the initial values for the growth rates (aggregate and disaggregate) were taken to be equal as in (B.6). This results in:

$$\begin{cases} (\omega_Y - \omega_{Y^{NE}})Y_0^{NE} + (\omega_Y - \omega_{C^E})C_0^E = 0; \\ \\ (\omega_C - \omega_{C^{NE}})C_0^{NE} + (\omega_Y - \omega_{C^E})C_0^E = 0; \\ \\ (\omega_K - \omega_{K^{NE}})K_0^{NE} + (\omega_K - \omega_{K^E})K_0^E = 0; \end{cases} \tag{B.9}$$

The equalities in system (B.9) are satisfied, for any choice of initital conditions, if and only if:

$$
\begin{cases}
\omega_Y = \omega_{Y^{NE}} = \omega_{C^E}; \\
\\
\omega_C = \omega_{C^{NE}} = \omega_{C^E}; \\
\\
\omega_K = \omega_{K^{NE}} = \omega_{K^E};
\end{cases}
\tag{B.10}
$$

The damping coefficient of the total aggregate variables of the model must be equal to the damping coefficients of the respective sector-specific variables. Moreover, like in (B.6), the total output definition for the proposed model implies that $\omega_Y = \omega_{C^E} = \omega_C$. Given the definition of regular growth (see chapter 2), the conditions (B.6) and (B.10) also imply that the first-order growth rates of total economic output Y and total consumption C will be identical, for all time t. That is:

$$
\left.
\begin{aligned}
\mu_Y &= \mu_C \\
\\
\omega_Y &= \omega_C
\end{aligned}
\right\} \Rightarrow g_Y = g_C;
\tag{B.11}
$$

This concludes the first part of the proof.

Equivalence of total output and total capital regular growth paths

It has been shown that assuming a regular growth path for the sector-specific and aggregate variables of the two-sector model implies that the first-order growth rates of the total variables are identical to the first-order growth rate of the respective disaggregate variables - Equations (B.6) and (B.10). It was also shown that under the framework of the model, total consumption and total output grow at the same rate - Equation (B.11).

The second part of the proof shows how the growth rates of total output and total capital are also identical under the developed two-sector framework, assuming regular growth paths for the key variables. Invoking the laws of motion for both Ne-Sector and E-Sector capital stocks, as defined in the two-sector model:

$$
\begin{cases}
\dot{K}^{NE} = I^{NE} - \delta_{NE} K^{NE} \\
\\
\dot{K}^{E} = I^{E} - \delta_{E} K^{E}
\end{cases}
\tag{B.12}
$$

Investment can be written, for a closed economy, as $I = Y - C$. Total investment is allocated to the NE-Sector and the E-Sector according to the fractions σ and $(1-\sigma)$, respectively. Substituting the key variables in (B.12) for their regular growth definitions, and setting $t = 0$ gives:

$$
\begin{cases}
\mu_K K_0^{NE}(1+\mu_K\omega_K t)^{\frac{1}{\omega_K}-1} = \sigma \cdot \left[Y_0(1+\mu_Y\omega_Y t)^{\frac{1}{\omega_Y}} - C_0(1+\mu_C\omega_C t)^{\frac{1}{\omega_C}}\right] - \delta_{NE} K_0^{NE}(1+\mu_K\omega_K t)^{\frac{1}{\omega_K}}; \\
\\
\mu_K K_0^{E}(1+\mu_K\omega_K t)^{\frac{1}{\omega_K}-1} = (1-\sigma) \cdot \left[Y_0(1+\mu_Y\omega_Y t)^{\frac{1}{\omega_Y}} - C_0(1+\mu_C\omega_C t)^{\frac{1}{\omega_C}}\right] - \delta_{NE} K_0^{E}(1+\mu_K\omega_K t)^{\frac{1}{\omega_K}};
\end{cases}
\tag{B.13}
$$

The values for $(\mu_{K^{NE}}, \mu_{K^E})$ and $(\omega_{K^{NE}}, \omega_{K^E})$ were substituted, respectively, by μ_K and ω_K, as in (B.6) and (B.10). Equation (B.13) for initial time $t = 0$ becomes:

$$
\begin{cases}
\sigma \cdot [Y_0 - C_0] = (\mu_K + \delta_{NE}) K_0^{NE}; \\
\\
(1-\sigma) \cdot [Y_0 - C_0] = (\mu_K + \delta_E) K_0^{E};
\end{cases}
\tag{B.14}
$$

Differentiating (B.13) once with respect to time results in:

$$
\begin{cases}
(1-\omega_K) g_K^2 K^{NE} = \sigma \cdot \left[\mu_Y Y_0 (1+\mu_Y \omega_Y t)^{\frac{1}{\omega_Y}-1} - \mu_C C_0 (1+\mu_C \omega_C t)^{\frac{1}{\omega_C}-1}\right] - \mu_K \delta_{NE} K_0^{NE} (1+\mu_K \omega_K t)^{\frac{1}{\omega_K}-1}; \\
\\
(1-\omega_K) g_K^2 K^{E} = (1-\sigma) \cdot \left[\mu_Y Y_0 (1+\mu_Y \omega_Y t)^{\frac{1}{\omega_Y}-1} - \mu_C C_0 (1+\mu_C \omega_C t)^{\frac{1}{\omega_C}-1}\right] - \mu_K \delta_{E} K_0^{E} (1+\mu_K \omega_K t)^{\frac{1}{\omega_K}-1};
\end{cases}
\tag{B.15}
$$

For $t = 0$ this becomes:

$$
\begin{cases}
(1-\omega_K) \mu_K^2 K_0^{NE} = \sigma \cdot (\mu_Y Y_0 - \mu_C C_0) - \mu_K \delta^{NE} K_0^{NE}; \\
\\
(1-\omega_K) \mu_K^2 K_0^{E} = (1-\sigma) \cdot (\mu_Y Y_0 - \mu_C C_0) - \mu_K \delta^{E} K_0^{E};
\end{cases}
\tag{B.16}
$$

Since $\mu_Y = \mu_C$ (from (B.11)), $\sigma \cdot (Y_0 - C_0)$ and $(1-\sigma) \cdot (Y_0 - C_0)$ can be substituted from (B.14) as $(\mu_K + \delta^{NE}) K_0^{NE}$ and $(\mu_K + \delta^{E}) K_0^{E}$, respectively. Then, dividing both sides of the first equation in (B.16) by the initial endowment of non-energy capital K_0^{NE}, and both sides of the second equation in (B.16) by the initial endowment of energy capital, K_0^{E}, results in:

$$
\begin{cases}
(1-\omega_K) \mu_K^2 = \mu_Y (\mu_K + \delta^{NE}) - \mu_K \delta^{NE}; \\
\\
(1-\omega_K) \mu_K^2 = \mu_Y (\mu_K + \delta^{E}) - \mu_K \delta^{E};
\end{cases}
\tag{B.17}
$$

The l.h.s. of both equations in (B.17) is the same. Therefore, it is possible to set the r.h.s. of both equations equal to each other, as:

$$
\mu_Y (\mu_K + \delta^{NE}) - \mu_K \delta^{NE} = \mu_Y (\mu_K + \delta^{E}) - \mu_K \delta^{E};
\tag{B.18}
$$

The previous equation can be written as:

$$
(\delta^{NE} - \delta^{E}) \mu_Y = (\delta^{NE} - \delta^{E}) \mu_K;
\tag{B.19}
$$

Assuming that the depreciation rates for non-energy (δ^{NE}) and energy capital (δ^{E}) are different from one another, Equation (B.19) gives:

$$
\mu_Y = \mu_K;
\tag{B.20}
$$

Equation (B.20) shows that the initial value for the growth rates of total output and total capital is identical. An analogous conclusion can be obtained for the damping coefficients for these two variables. Differentiating (B.15) once more with respect to time results in:

$$\begin{cases} (1-\omega_K)(1-2\omega_K)g_K^3 K^{NE} = \sigma\left[(1-\omega_Y)g_Y^2 Y - (1-\omega_C)g_C^2 C\right] - (1-\omega_K)\delta^{NE} g_K^2 K^{NE}; \\ \\ (1-\omega_K)(1-2\omega_K)g_K^3 K^{E} = (1-\sigma)\left[(1-\omega_Y)g_Y^2 Y - (1-\omega_C)g_C^2 C\right] - (1-\omega_K)\delta^{E} g_K^2 K^{E}; \end{cases} \tag{B.21}$$

Assuming $t = 0$ and that $\mu_Y = \mu_C$ and $\omega_Y = \omega_C$, gives:

$$\begin{cases} (1-\omega_K)(1-2\omega_K)\mu_K^3 K_0^{NE} = \sigma(1-\omega_Y)\mu_Y^2\left[Y_0 - C_0\right] - (1-\omega_K)\delta_{NE}\mu_K^2 K_0^{NE}; \\ \\ (1-\omega_K)(1-2\omega_K)\mu_K^3 K_0^{E} = (1-\sigma)(1-\omega_Y)\mu_Y^2\left[Y_0 - C_0\right] - (1-\omega_K)\delta_{E}\mu_K^2 K_0^{E}; \end{cases} \tag{B.22}$$

Substituting again $\sigma(Y_0 - C_0)$ and $(1-\sigma)(Y_0 - C_0)$ from (B.14), and dividing both sides of the first equation in (B.22) by K_0^{NE} and both sides of the second equation by K_0^E, like before, results in:

$$\begin{cases} (1-\omega_K)(1-2\omega_K)\mu_K^3 = (1-\omega_Y)\mu_Y^2(\mu_K + \delta_{NE}) - (1-\omega_K)\delta_{NE}\mu_K^2; \\ \\ (1-\omega_K)(1-2\omega_K)\mu_K^3 = (1-\omega_Y)\mu_Y^2(\mu_K + \delta_{E}) - (1-\omega_K)\delta_{E}\mu_K^2; \end{cases} \tag{B.23}$$

Once again, the r.h.s. of both equations in (B.23) can be set as equal to one another:

$$(1-\omega_Y)\mu_Y^2(\mu_K - \delta^{NE}) - (1-\omega_K)\delta^{NE}\mu_K^2 = (1-\omega_Y)\mu_Y^2(\mu_K - \delta^{E}) - (1-\omega_K)\delta^{E}\mu_K^2; \tag{B.24}$$

Now, using the result obtained in (B.20) ($\mu_Y = \mu_K$) the previous equation can be simflified as:

$$(\delta^{NE} - \delta^E)\omega_Y = (\delta^{NE} - \delta^E)\omega_K; \tag{B.25}$$

Assuming, like before, that the depreciation rates δ^{NE} and δ^E are distinct, the solution to equation (B.26) implies that:

$$\omega_Y = \omega_K; \tag{B.26}$$

That is, the damping coefficient for total output, ω_Y, is equal to the damping coefficient for total capital, ω_K. The initial time growth rates for total output and total capita are also identical, from (B.20). Therefore, these aggregated variables grow at the same first-order rate:

$$\left.\begin{aligned} \mu_Y &= \mu_K \\ \\ \omega_Y &= \omega_K \end{aligned}\right\} \Rightarrow g_Y = g_K; \tag{B.27}$$

This shows that, assuming a regular growth path for the aggregate and disaggregate variables of the two-sector model in chapter 2 (except labor L) implies that all these variables will grow at the same rate, i.e. $g_C = g_{C^{NE}} = g_{C^E} = g_{Y^{NE}} = g_Y = g_K = g_{K^{NE}} = g_{K^E}$. This concludes the second part of the proof.

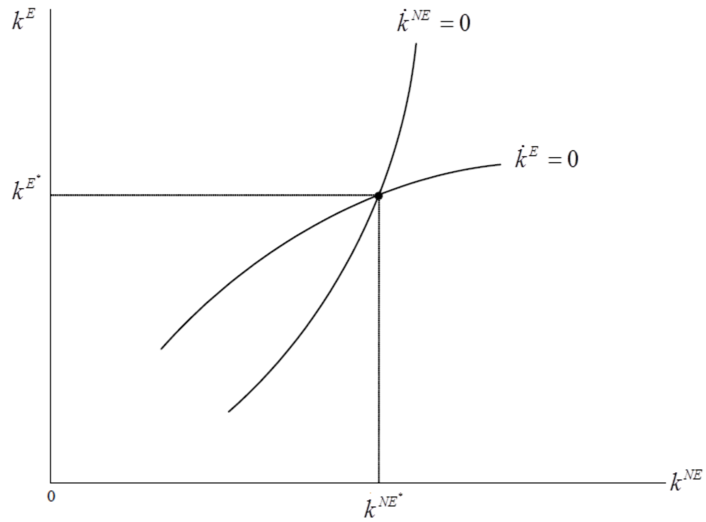

Figure B.1: Steady-state equilibrium in the two-sector model, with energy and non-energy capital.

B.2 Existence and uniqueness of a steady state for the model.

The steady state condition that ($\dot{k}^{NE} = 0, \dot{k}^E = 0$) implies that the laws of motion for capital per effective-labor become:

$$\begin{cases} s \cdot \sigma f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right) = (\delta^{NE} + n + \lambda)k^{NE^*} \\ s \cdot (1-\sigma) f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right) = (\delta^{E} + n + \lambda)k^{E^*} \end{cases} \tag{B.28}$$

The production function f^{NE} verifies a trivial steady-state $f^{NE}(0,0) = 0$ for ($k^E = 0, k^{NE} = 0$). This steady-state is ignored and focus is given on a steady-state equilibria with $k^{E^*} > 0$ and $k^{NE^*} > 0$. The uniqueness of this non-trivial steady-state can be shown, heuristically, in Figure (B.1).

The curves drawn in (k^{NE}, k^E) space, as shown in Figure (B.1), represent the equations in (B.28). Both lines are upward sloping. For example, in the first equation of (B.28) a higher level of k^{E^*} implies greater $f^{NE}(k^{NE^*}, \gamma k^{E^*})$, from the properties of the neoclassical production function. Therefore, the level of k^{NE^*} that will satisfy the equation will be higher. The same reasoning applies to the second equation in (B.28). The proof presented below shows that $s \cdot (1-\sigma) f^{NE}\left(k^{NE^*}, \gamma k^{E^*}\right) = (\delta^E + n + \lambda)k^{E^*}$ is always shallower in the (k^{NE}, k^E) space, and therefore the two curves can only intersect once.

First, considering the slope of the curve corresponding to $\dot{k}^{NE} = 0$, the implicit function theorem is used to determine:

$$\left.\frac{dk^E}{dk^{NE}}\right|_{\dot{k}^{NE}=0} = \frac{(\delta_{NE} + n + \lambda) - \sigma s f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right]}{\sigma s f^{NE}_{k^E}\left[k^{NE^*}, \gamma k^{E^*}\right]} \tag{B.29}$$

Here, $f^{NE}_i \equiv \partial f^{NE}/\partial i$. Dividing the first equation in (B.28) by the steady state level k^{NE^*} gives

$$\frac{\sigma s f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{k^{NE^*}} - \left(\delta^{NE} + n + \lambda\right) = 0 \tag{B.30}$$

The output per effective labor function f^{NE} is strictly concave, from the neoclassical property of positive and diminishing marginal products, and $f^{NE}\left[0, \gamma k^{E^*}\right] \geq 0$, from the essentiality property, so:

$$f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right] > f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right] + f^{NE}\left[0, \gamma k^{E^*}\right] > f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right] \tag{B.31}$$

Therefore, $(\delta_{NE} + n + \lambda) - \sigma s f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right] > 0$, and (B.29) is strictly positive. Applying the implicit function theorem to the $\dot{k}^E = 0$ locus gives an analogous result:

$$\left.\frac{dk^E}{dk^{NE}}\right|_{\dot{k}^E=0} = \frac{(1-\sigma) s f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right]}{(\delta_{NE} + n + \lambda) - (1-\sigma) s f^{NE}_{k^E}\left[k^{NE^*}, \gamma k^{E^*}\right]} \tag{B.32}$$

Applying the same line of reasoning used for (B.29), equation (B.32) is also strictly positive. The following step is to prove that (B.29) is steeper than (B.32) whenever the steady state conditions (B.28) hold, so that there can be at most only one intersection between the two. First, one verifies that:

$$\left.\frac{dk^E}{dk^{NE}}\right|_{\dot{k}^E=0} < \left.\frac{dk^E}{dk^{NE}}\right|_{\dot{k}^{NE}=0} \Leftrightarrow$$

$$\Leftrightarrow \frac{(1-\sigma) s f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right]}{(\delta_{NE} + n + \lambda) - (1-\sigma) s f^{NE}_{k^E}\left[k^{NE^*}, \gamma k^{E^*}\right]} < \frac{(\delta_{NE} + n + \lambda) - \sigma s f^{NE}_{k^{NE}}\left[k^{NE^*}, \gamma k^{E^*}\right]}{\sigma s f^{NE}_{k^E}\left[k^{NE^*}, \gamma k^{E^*}\right]} \Leftrightarrow$$

$$\Leftrightarrow (1-\sigma)\sigma s^2 f^{NE}_{k^{NE}} f^{NE}_{k^E} < (1-\sigma)\sigma s^2 f^{NE}_{k^{NE}} f^{NE}_{k^E} + (\delta_E + n + \lambda)(\delta_{NE} + n + \lambda) -$$

$$(\delta_E + n + \lambda)\,\sigma s f^{NE}_{k^{NE}} - (\delta_{NE} + n + \lambda)(1-\sigma) s f^{NE}_{k^E} \tag{B.33}$$

Now, using (B.28) and substituting for $(\delta^{NE} + n + \lambda) = \sigma \cdot s f^{NE}(k^{NE^*}, \gamma k^{E^*})/k^{NE^*}$ and $(\delta^E + n + \lambda) = (1-\sigma) \cdot s f^{NE}(k^{NE^*}, \gamma k^{E^*})/k^{E^*}$, equation (B.33) is equivalent to:

$$f^{NE}(k^{NE^*}, \gamma k^{E^*}) > f^{NE}_{k^{NE}}(k^{NE^*}, \gamma k^{E^*}) k^{NE^*} + f^{NE}_{k^{E^*}}(k^{NE^*}, \gamma k^{E^*}) k^{E^*} \tag{B.34}$$

Equation (B.34) is satisfied by the fact that $f^{NE}(k^{NE^*}, \gamma k^{E^*})$ is a strictly concave function. This proves uniqueness of the steady-state. To establish existence of the steady-state, one must recall the Inada conditions and rewrite them for effective-labor output f^{NE}:

$$\begin{gathered} \lim_{k^{NE}\to\infty}\left(\frac{\partial f^{NE}}{\partial k^{NE}}\right) = 0,\ \lim_{k^{NE}\to 0}\left(\frac{\partial f^{NE}}{\partial k^{NE}}\right) = \infty; \\ \lim_{k^{E}\to\infty}\left(\frac{\partial f^{NE}}{\partial k^{E}}\right) = 0,\ \lim_{k^{E}\to 0}\left(\frac{\partial f^{NE}}{\partial k^{E}}\right) = \infty; \end{gathered} \tag{B.35}$$

Since primary exergy inputs and energy capital are proportional to each other, having $k^E \to 0$ is equivalent to having $b^U \to 0$ (with b^U being the ratio of useful work per effective-labor). By l'Hôpital's rule, the limits as k^E or k^{NE} approach zero for the average products f^{NE}/k^E and f^{NE}/k^{NE} are the same as those of the respective marginal products, $f^{NE}_{k^E}$ and $f^{NE}_{k^{NE}}$. Therefore (B.35) is equivalent to:

$$\begin{aligned} &\lim_{k^{NE}\to\infty}\left(\frac{f^{NE}}{k^{NE}}\right) = 0,\ \lim_{k^{NE}\to 0}\left(\frac{f^{NE}}{k^{NE}}\right) = \infty; \\ &\lim_{k^{E}\to\infty}\left(\frac{f^{NE}}{k^{E}}\right) = 0,\ \lim_{k^{E}\to 0}\left(\frac{f^{NE}}{k^{E}}\right) = \infty; \end{aligned} \tag{B.36}$$

In Figure (B.1), the curve corresponding to $\dot{k}^{NE} = 0$ is below the curve corresponding to $\dot{k}^E = 0$ as $k^{NE} \to 0$ and $k^E \to \infty$. However, for $k^{NE} \to \infty$ and $k^E \to 0$, the curve $\dot{k}^{NE} = 0$ lies above $\dot{k}^E = 0$. This implies that the two curves must intersect at least once a steady-state exists for (B.28).

B.3 Comparative Statics

By performing simple comparative statics on the two-model introduced in chapter 2, it is possible to gain an understanding of what features of this framework lead to higher levels of income per effective labor and more rapid economic growth.

The steady state conditions $\dot{k}^{NE} = 0$ and $\dot{k}^E = 0$ are written as:

$$\begin{cases} \dfrac{f^{NE}(k^{NE^*}, \gamma k^{E^*})}{k^{NE^*}} - \dfrac{(\delta^{NE} + n + \lambda)}{\sigma \cdot s} = 0 \\[2ex] \dfrac{f^{NE}(k^{NE^*}, \gamma k^{E^*})}{k^{E^*}} - \dfrac{(\delta^{E} + n + \lambda)}{(1-\sigma) \cdot s} = 0 \end{cases} \tag{B.37}$$

Recalling the constant returns to scale and positive and diminishing marginal products properties of the neoclassical production function, the marginal product of labor can be written as:

$$\frac{\partial F^{NE}}{\partial L} = \frac{\partial\left[A^L L f^{NE}(k^{NE}, \gamma k^E)\right]}{\partial L} = A^L\left[f^{NE} - f^{NE}_{k^{NE}} \cdot k^{NE} - f^{NE}_{k^E} \cdot k^E\right] > 0 \tag{B.38}$$

Total labor-augmenting technological progress, A^L, is assumed to be positive in value at all times. It results that:

$$f^{NE} > f^{NE}_{k^{NE}} \cdot k^{NE} + f^{NE}_{k^E} \cdot k^E \tag{B.39}$$

With this established, the comparative statics are obtained through the application of the implicit function theorem to (B.37) for any given parameter. For example, the saving rate s:

$$\frac{\partial k^{NE^*}}{\partial s}=\frac{-\dfrac{\partial}{\partial s}\left(\dfrac{f^{NE}\left[k^{NE^*},\gamma k^{E^*}\right]}{k^{NE^*}}-\dfrac{\left(\delta^{NE}+n+\lambda\right)}{\sigma s}\right)}{\dfrac{\partial}{\partial k^{NE^*}}\left(\dfrac{f^{NE}\left[k^{NE^*},\gamma k^{E^*}\right]}{k^{NE^*}}-\dfrac{\left(\delta^{NE}+n+\lambda\right)}{\sigma s}\right)}=\frac{\left(\delta^{NE}+n+\lambda\right)\left(k^{NE^*}\right)^2}{\sigma\left(s\right)^2\left[f^{NE}-f^{NE}_{k^{NE^*}}\cdot k^{NE^*}\right]} \tag{B.40}$$

The r.h.s. of (B.40) is always positive, since all the parameters and variables assume positive values, and $f^{NE}-f^{NE}_{k^{NE}}\cdot k^{NE}$ is always positive from (B.39). Therefore a higher value for the saving rate s will lead to a higher steady state value for non-energy capital:

$$\frac{\partial k^{NE^*}}{\partial s}>0 \tag{B.41}$$

Similarly, one can obtain the effects on non-energy capital from changing the technological growth rate λ, the population growth rate n, and the non-energy capital depreciation rate δ^{NE}:

$$\frac{\partial k^{NE^*}}{\partial \lambda}<0;\quad \frac{\partial k^{NE^*}}{\partial n}<0;\quad \frac{\partial k^{NE^*}}{\partial \delta^{NE}}<0 \tag{B.42}$$

The procedure for obtaining comparative statics for energy capital is in all analogous to the one for non-energy capital exemplified above. First, the implicit function theorem gives:

$$\frac{\partial k^{E^*}}{\partial s}=\frac{-\dfrac{\partial}{\partial s}\left(\dfrac{f^{NE}\left[k^{NE^*},\gamma k^{E^*}\right]}{k^{E^*}}-\dfrac{\left(\delta^{E}+n+\lambda\right)}{(1-\sigma)s}\right)}{\dfrac{\partial}{\partial k^{E^*}}\left(\dfrac{f^{NE}\left[k^{NE^*},\gamma k^{E^*}\right]}{k^{E^*}}-\dfrac{\left(\delta^{E}+n+\lambda\right)}{(1-\sigma)s}\right)}=\frac{\left(\delta^{E}+n+\lambda\right)\left(k^{E^*}\right)^2}{(1-\sigma)\left(s\right)^2\left[f^{NE}-f^{NE}_{k^{E^*}}\cdot k^{E^*}\right]} \tag{B.43}$$

As before, the r.h.s. of (B.43) is always positive, for the same reasons as (B.40). And so, like non-energy capital, a change in the saving rate increases the steady state level of energy capital:

$$\frac{\partial k^{E^*}}{\partial s}>0 \tag{B.44}$$

The comparative statics for λ, n, and δ^{E} are also analogous to the respective statics for non-energy capital:

$$\frac{\partial k^{E^*}}{\partial \lambda}<0;\quad \frac{\partial k^{E^*}}{\partial n}<0;\quad \frac{\partial k^{E^*}}{\partial \delta_{E}}<0 \tag{B.45}$$

Now, calculating the comparative statics for energy and non-energy capital depending on the variation of the fraction of total investment alocated to the Non-Energy Sector, σ, results in, for non-energy capital:

$$\frac{\partial k^{NE^*}}{\partial \sigma} = \frac{-\frac{\partial}{\partial \sigma}\left(\frac{f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{k^{NE^*}} - \frac{(\delta_{NE}+n+\lambda)}{\sigma s}\right)}{\frac{\partial}{\partial k^{NE^*}}\left(\frac{f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{k^{NE^*}} - \frac{(\delta_{NE}+n+\lambda)}{\sigma s}\right)} = \frac{(\delta_{NE}+n+\lambda)\left(k^{NE^*}\right)^2}{(\sigma)^2 s\left[f^{NE} - f^{NE}_{k^{NE^*}} \cdot k^{NE^*}\right]} \tag{B.46}$$

And, for energy capital:

$$\frac{\partial k^{E^*}}{\partial \sigma} = \frac{-\frac{\partial}{\partial \sigma}\left(\frac{f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{k^{E^*}} - \frac{(\delta_{E}+n+\lambda)}{(1-\sigma) s}\right)}{\frac{\partial}{\partial k^{E^*}}\left(\frac{f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{k^{E^*}} - \frac{(\delta_{E}+n+\lambda)}{(1-\sigma) s}\right)} = -\frac{(\delta_{E}+n+\lambda)\left(k^{E^*}\right)^2}{(1-\sigma)^2 s\left[f^{NE} - f^{NE}_{k^{E^*}} \cdot k^{E^*}\right]} \tag{B.47}$$

In this case, the r.h.s. of equation (B.46) is positive, but the r.h.s. of equation (B.47) is negative. This means that the steady state level of non-energy capital is higher if σ is higher, while the steady state level of energy capital is lower as σ rises. That is:

$$\frac{\partial k^{NE^*}}{\partial \sigma} > 0; \quad \frac{\partial k^{E^*}}{\partial \sigma} < 0; \tag{B.48}$$

Comparative statics for the NE-Sector output, f^{NE}, are simpler and do not require the implicit function theorem. The variation of $f^{NE}(k^{NE^*}, \gamma k^{E^*})$ depending on the saving rate s is

$$\frac{\partial f^{NE}\left[k^{NE^*}, \gamma k^{E^*}\right]}{\partial s} = f^{NE}_{k^{NE^*}} \frac{\partial k^{NE^*}}{\partial s} + f^{NE}_{k^{E^*}} \frac{\partial k^{E^*}}{\partial s} > 0; \tag{B.49}$$

Similarly, for δ^{NE}, δ^{E}, λ and n:

$$\frac{\partial f^{NE^*}}{\partial \delta_E} < 0; \quad \frac{\partial f^{NE^*}}{\partial \delta_{NE}} < 0; \quad \frac{\partial f^{NE^*}}{\partial \lambda} < 0; \quad \frac{\partial f^{NE^*}}{\partial n} < 0; \tag{B.50}$$

The steady state level of output for the NE-Sector is higher if the saving rate rises, and is lower as the depreciation of both types of capital, population growth and technological advances rise. As for the case of σ, the results are not as clear:

$$\frac{\partial f^{NE^*}}{\partial \sigma} = f^{NE}_{k^{NE^*}} \frac{\partial k^{NE^*}}{\partial \sigma} + f^{NE}_{k^{E^*}} \frac{\partial k^{E^*}}{\partial \sigma} \tag{B.51}$$

As was shown in (B.46) and (B.47), the first term in the r.h.s. of (B.51) is always positive, while the second term on the r.h.s. is always negative. Depending on the total value of the two terms, there will be three possible cases:

$$\begin{cases} \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| > \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right| \Leftrightarrow \dfrac{\partial f^{NE^*}}{\partial \sigma} > 0; \\ \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| = \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right| \Leftrightarrow \dfrac{\partial f^{NE^*}}{\partial \sigma} = 0; \\ \left| f^{NE}_{k^{NE^*}} \dfrac{\partial k^{NE^*}}{\partial \sigma} \right| < \left| f^{NE}_{k^{E^*}} \dfrac{\partial k^{E^*}}{\partial \sigma} \right| \Leftrightarrow \dfrac{\partial f^{NE^*}}{\partial \sigma} < 0; \end{cases} \tag{B.52}$$

For the comparative statics on the variation of the fraction of useful work produced by the Energy sector used in production by the Non-energy sector, γ, the results are, for energy and non-energy capital:

$$\frac{\partial k^{NE^*}}{\partial \gamma} = \frac{(k^{NE^*}) f^{NE}_{k^{E^*}} \cdot k^{E^*}}{\gamma \left[f^{NE} - f^{NE}_{k^{NE}} \cdot k^{NE} \right]} > 0; \quad \frac{\partial k^{E^*}}{\partial \gamma} = \frac{(k^{E^*}) f^{NE}_{k^{E^*}} k^{E^*}}{\gamma \left[f^{NE} - f^{NE}_{k^{E}} \cdot k^{NE} \right]} > 0; \tag{B.53}$$

And so, for steady-state output:

$$\frac{\partial f^{NE^*}}{\partial \gamma} = \frac{k^E}{\gamma} f^{NE}_{k^E} > 0 \tag{B.54}$$

This concludes the comparative statics analyzed under the assumptions of the two-sector model introduced in chapter 2.

Appendix C

Glossary

Capital stock (K): One of the factors of production. Durable goods (equipment and structures) already produced by the economy and used in the production of goods and services. Typically measured in terms of the value of the equipment.

Energy capital (K^E): Capital used in E-Sector conversion processes. Includes every equipment and device that performs the conversion of primary exergy (B^P) into actual useful work (B^U);

Non-energy capital (K^{NE}): Capital used in NE-Sector production of goods and services;

Consumption (C): Amount of total output (Y) of goods and services purchased by households, government or non-profitable institutions serving households (NPISH).

Energy sector consumption (C^E): Fraction of E-Sector output (i.e. useful work B^U) directly consumed by households, government and NPISH;

Non-energy sector consumption (C^{NE}): Fraction of NE-Sector output (Y^{NE}) directly consumed by households, government and NPISH;

Depreciation rate (δ): Rate at which the gradual decrease in economic value of K occurs.

Energy capital depreciation rate (δ_E): Depreciation rate for K^E;

Non-energy capital depreciation rate (δ_{NE}): Depreciation rate for K^{NE};

Factors of production: Inputs to the production processes;

Gross Domestic Product (GDP): A measure of economic activity in a country. It can be determined by summing the output of every enterprise and subtracting intermediates (production approach), by summing the incomes paid to households for factors of production (income approach), or by summing the total expenditure in the economy (expenditure approach);

Interest rate (r): Payment received by households on funds lent to other households. In absence of uncertainty, capital and loans are perfect substitutes as stores of value and the interest rate is equivalent to the rate of return to a household per unit of K: $r = R - \delta$;

Investment (I): Amount of total output (Y) used to increase the capital stock K. According to economic theory, this amount is equivalent to total output saved (see saving rate s);

Energy sector investment (I^E): Fraction of I used to increase the stock of energy capital K^E;

Non-energy sector investment (I^{NE}): Fraction of I used to increase the stock of non-energy capital K^{NE};

Labor (L): One of the factors of production. A measure of the work done by human beings, measured in terms of employment or number of employees. The price of labor is equivalent to the wages (w);

Marginal product: Measure of extra output that can be produced by using one more unit of a given input (see factors of production);

Price of primary exergy (p_{B^P}): Price paid to households for the rental of one unit of primary exergy (B^P);

Price of useful work (p_{B^U}): Monetary value attributed to useful work (B^U);

Primary exergy (B^P): Energy extracted from nature that is available to perform useful work (B^U) and that has not been subjected to any conversion or transformation process;

Production function (F): A mathematical way to describe the relationship between the quantity of inputs (e.g. capital K or labor L) used by a firm and the quantity of output (Y) it produces with them;

Rental price of capital (R): Price paid to households for the rental of one unit of capital stock (K);

Returns to scale: A term that refers to changes in output resulting from a proportional change in all inputs (where all inputs increase by a constant factor). Returns to scale may be constant (see appendix C), decreasing, or increasing;

Saving rate (s): The fraction of total output (Y) devoted to investment (I), i.e. not spent in consumption (C);

Technical change (A): Level of knowledge or technology. A term used to describe a change in the set of feasible production possibilities (see total factor productivity);

Non-energy sector technical change (A^{NE}): Level of technology associated with the production processes in the Non-energy Sector (labor-augmenting);

Energy sector technical change (A^E): Level of technology associated with the conversion of primary exergy (B^P) into useful work (B^U) (energy capital augmenting);

Total Factor Productivity (TFP): A variable which accounts for effects in total output (Y) not caused by traditionally measured inputs of labor (L) and capital (K). If all inputs are accounted for, TFP can be taken as a measure of long-term technological change (A).

Total output (Y): What is produced (goods and services) with the factors of production;

Energy sector output (B^U): Useful work converted from primary Exergy (B^P) in the E-Sector, using energy capital (K^E) (a.k.a. useful work - see appendix A);

Non-energy sector output (Y^{NE}): Non-energy related goods and services produced by the NE-Sector, using inputs K^{NE}, L and B^U;

Wage rate (w): The price of labor. Remuneration paid by an employer to an employee;