Gauge Anomalies and Neutrino Seesaw Models

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Adding right-handed neutrino singlets and/or fermion triplets to the particle content of the Standard Model allows for the implementation of the seesaw mechanism to give mass to neutrinos and, simultaneously, for the construction of anomaly-free gauge group extensions of the theory. In this project, after briefly reviewing the SM and some theoretical aspects of anomalies, we discuss the anomaly cancellation and electric charge quantization in three popular (type I, II and III) seesaw mechanisms. We then study the seesaw realization of viable two-zero textures for the effective neutrino mass matrix, considering Abelian extensions based on an extra $U(1)_X$ gauge symmetry. By requiring gauge anomalies cancellation, we perform a detailed analysis in order to identify the charge assignments under the new gauge symmetry that lead to neutrino phenomenology compatible with current experiments. In particular, we study how the new symmetry can constrain the flavour structure of the Majorana neutrino mass matrix, leading to two-zero textures with a minimal extra fermion and scalar content. The possibility of distinguishing different gauge symmetries and seesaw realizations at colliders is also briefly discussed.

Keywords: Charge Quantization, Gauge Anomalies, Gauge Symmetries, Neutrino Physics, Seesaw Mechanism.

I. STANDARD MODEL

The Standard Model (SM) of particle physics is a theory concerning three of the fundamental forces/interactions (electromagnetic, weak and strong forces) in Universe, which predicts very accurately many of the experimentally verified phenomena. In July/2012, LHC experiments at CERN announced the discovery of a Higgs-like particle [1, 2], which is one of the most important discoveries in particle physics and a triumph for the SM predictive power.

From a theoretical point of view, SM is a quantum field theory (QFT) invariant under the gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \,, \tag{1}$$

which is broken spontaneously by the scalar Higgs field H when it acquires a non-zero vacuum expectation value (VEV). This Higgs field realizes the electroweak spontaneous symmetry breaking (SSB) and generates the mass of weak gauge bosons through the so-called Higgs mechanism [3–5]. The relevant parts of the SM Lagrangian for this mechanism are

$$\mathcal{L}_{Higgs} = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H)\,, \tag{2}$$

where the scalar potential $V(H)$ is

$$V(H) = \mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 . \tag{3}$$

The covariant derivative acting on H is

$$D_\mu H = \left(\partial_\mu - i g_W W_\mu^i T^i - i g_Y B_\mu Y_H\right) H\,, \tag{4}$$

where g_W and g_Y are, respectively, $SU(2)_L$ and $U(1)_Y$ coupling constants and Y_H is the Higgs field hypercharge. In the physical basis, where

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix} , \tag{5}$$

one obtain for the covariant derivative

$$\begin{aligned}\left(D_\mu H\right)^\dagger \left(D^\mu H\right) &= \frac{v^2}{8}\left[g_W^2 \left(W_\mu^1 W^{1\mu} + W_\mu^2 W^{2\mu}\right)\right] \\ &+ \frac{v^2}{8}\left[\left(g_W W_\mu^3 - g_Y B_\mu\right)\left(g_W W^{3\mu} - g_Y B^\mu\right)\right] + \cdots\,,\end{aligned} \tag{6}$$

and for the potential

$$V(H) = \cdots + \left(\frac{1}{2}\mu^2 + \frac{3}{2}\lambda v^2\right) h^2 = \cdots - \frac{1}{2}\left(2\mu^2\right) h^2 . \tag{7}$$

Rotating the weak bosons to the mass eigenstates (bosons W, Z and the photon A) we get for the complete mass spectrum

$$\begin{aligned} m_h &= \sqrt{-2\mu^2}\,, \quad m_W = \frac{v}{2} g_W\,, \\ m_Z &= \frac{v}{2}\sqrt{g_W^2 + g_Y^2}\,, \quad m_A = 0\,, \end{aligned} \tag{8}$$

and for the covariant derivative

$$\begin{aligned} D_\mu &= \partial_\mu - i g_W \left(W_\mu^+ T^+ + W_\mu^- T^-\right) - i Q e A_\mu \\ &- i \frac{g_W}{c_W}\left(T^3 - s_W^2 Q\right) Z_\mu\,, \end{aligned} \tag{9}$$

where $T^i = \sigma^i/2$ and

$$\begin{aligned} s_W &= \frac{g_Y}{\sqrt{g_W^2 + g_Y^2}}\,, c_W = \frac{g_W}{\sqrt{g_W^2 + g_Y^2}}\,, \\ e &= \frac{g_Y\, g_W}{\sqrt{g_W^2 + g_Y^2}}\,, \quad Q = T^3 + Y\,. \end{aligned} \tag{10}$$

The operator eQ is the electric charge operator since, when the electroweak gauge group is spontaneously broken to the QED one, eQ is coupled to the massless gauge boson A_μ.

It is clear that the Higgs mechanism is the connecting piece between the electroweak gauge group and massive bosons, allowing the unified theory of weak and electromagnetic forces to be consistent.

	Fields					
	H	$q_{L\alpha}$	$\ell_{L\alpha}$	$u_{R\alpha}$	$d_{R\alpha}$	$e_{R\alpha}$
$U(1)_Y$	1/2	1/6	−1/2	2/3	−1/3	−1
$SU(2)_L$	**2**	**2**	**2**	**1**	**1**	**1**
$SU(3)_C$	**1**	**3**	**1**	**3**	**3**	**1**

TABLE I. Representations under the SM gauge group and hypercharge assignments of the Higgs boson and the SM fermions.

In the SM, fermion masses are also generated through couplings with the Higgs fields since mass terms as $-m_f\overline{f}f = -m_f\left(\overline{f_R}f_L + \overline{f_L}f_R\right)$ are forbidden by the $SU(2)_L \otimes U(1)_Y$ symmetry. Therefore, one can include Yukawa interactions between fermions and the Higgs field and demand that the SSB mechanism gives the mass terms for fermions. The correct gauge invariant Yukawa Lagrangian is

$$\begin{aligned}\mathcal{L}_{Yukawa} = &- \mathbf{Y}_u^{\alpha\beta}\,\overline{q_{L\alpha}}\widetilde{H}u_{R\beta} - \mathbf{Y}_d^{\alpha\beta}\,\overline{q_{L\alpha}}Hd_{R\beta} \\ &- \mathbf{Y}_e^{\alpha\beta}\,\overline{\ell_{L\alpha}}He_{R\beta} + \text{H.c.}\,,\end{aligned} \tag{11}$$

where $\widetilde{H} = i\sigma^2 H^*$ and $\mathbf{Y}_{u,d,e}$ are respectively the up quark, down quark and charged-lepton Yukawa couplings matrices. To diagonalize these matrices we make the unitary transformations

$$f_L \to f_L' = \mathbf{L}_f f_L\,,\ f_R \to f_R' = \mathbf{R}_f f_R\,, \tag{12}$$

so that

$$\mathbf{L}_f^\dagger \mathbf{m}_f \mathbf{R}_f = \mathbf{d}_f = \text{diag}\left(m_{f_1}, m_{f_2}, m_{f_3}\right)\,. \tag{13}$$

Under these transformations the neutral currents remain unchanged, however the charged current (neglecting the H.c. terms) becomes

$$\begin{aligned}\mathcal{L}_{CC} &= \frac{g_W}{\sqrt{2}}\left(\overline{u_{L\alpha}'}\gamma^\mu d_{L\alpha}' + \overline{\nu_{L\alpha}'}\gamma^\mu e_{L\alpha}'\right)W_\mu^+ \Leftrightarrow \\ \mathcal{L}_{CC} &= \frac{g_W}{\sqrt{2}}\left(\overline{u_{L\alpha}}\,\mathbf{V}_{CKM}^{\alpha\beta}\gamma^\mu d_{L\beta} + \overline{\nu_{L\alpha}}\gamma^\mu e_{L\alpha}\right)W_\mu^+\,,\end{aligned} \tag{14}$$

where

$$\mathbf{V}_{CKM} = \mathbf{L}_u^\dagger \mathbf{L}_d = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{pmatrix} \tag{15}$$

is the Cabibbo-Kobayashi-Maskawa quark mixing matrix [6]. It follows that, at tree level, flavour can only be changed by charged currents. Therefore, there are no flavour changing neutral currents (FCNC) at tree level in the SM.

The particle group representations is another important aspect that allows us to construct gauge invariant terms in the SM Lagrangian. Since the Higgs boson does not couple to the photon, from Eq. (10), it must have $Y_H = \frac{1}{2}$. The hypercharge assignments of the SM fermions and their representations are summarized in Table I, which allows the Yukawa Lagrangian to be invariant.

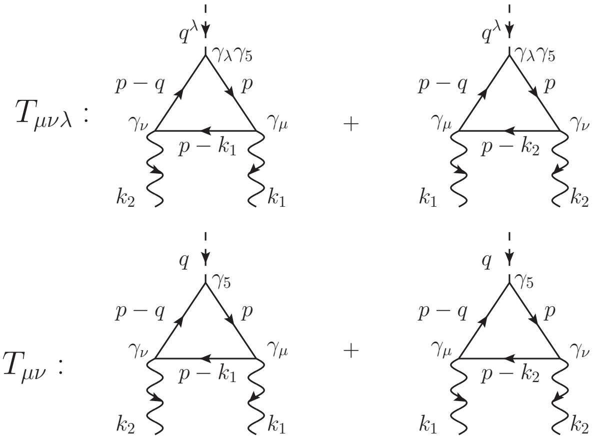

FIG. 1. Triangle diagrams with vertices vector-vector-axial and vector-vector-pseudoscalar.

II. GAUGE ANOMALIES

Another possible framework to discuss the hypercharge assignments is the consistency of the SM, i.e. being free of gauge anomalies. Anomalies appear when symmetries of the classical Lagrangian are not invariant of the functional integral or the path integral formulation of the theory. If this is a gauge or local symmetry, we have then a gauge anomaly. Since the SM fermions are chiral, we need to discuss the chiral gauge anomaly, and hence, the possibility of violation of the Ward Identities (WI)/non-renormalizability of the model [7].

The Abelian case is known as the Adler-Bell-Jackiw (ABJ) anomaly or the Abelian chiral anomaly [8, 9]. When we calculate the amplitude ($\mathcal{M}(k) = \epsilon_\mu(k)\mathcal{M}^\mu(k)$, $\epsilon_\mu(k)$ is the polarization vector of the gauge boson) for some physically possible scattering process of an Abelian theory, one expect that the WI ($k_\mu\mathcal{M}^\mu(k) = 0$) holds. However, computing the example $q^\lambda T_{\mu\nu\lambda}$ presented in Fig. 1 we obtain an anomalous term. The axial current conservation reads[1]

$$\partial_z^\lambda G_{\mu\nu\lambda} = 2miG_{\mu\nu} \Leftrightarrow q^\lambda T_{\mu\nu\lambda} = 2mT_{\mu\nu}\,, \tag{16}$$

where

$$\begin{aligned}G_{\mu\nu\lambda} &= \left\langle 0\left|Tj_\mu(x)j_\nu(y)\overline{\psi}\gamma_\mu\gamma_5\psi\right|0\right\rangle\,, \\ G_{\mu\nu} &= \left\langle 0\left|Tj_\mu(x)j_\nu(y)\overline{\psi}\gamma_5\psi\right|0\right\rangle\,,\end{aligned} \tag{17}$$

and $T_{\mu\nu\lambda}$, $T_{\mu\nu}$ are their respective Fourier transformations in momentum space. Using Feynman rules, we

[1] It is clear that the axial current is only conserved in the massless case, nevertheless, as we shall see, the anomalous term appears even if the particle is massless. In the same way, the quantum analogy of this relation, i.e. the axial WI (AWI), holds solely in the massless case. For our purpose, we shall consider that non-conservation exclusively occurs when an anomaly (anomalous term) is present.

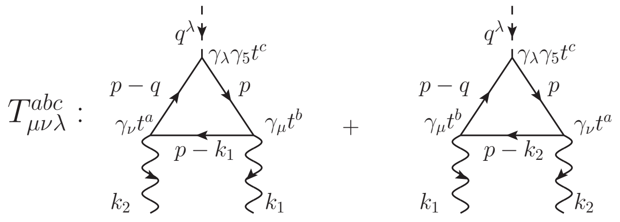

FIG. 2. Triangle diagrams with vertices vector-vector-axial for non-Abelian gauges.

can compute $T_{\mu\nu\lambda}$ as

$$T_{\mu\nu\lambda} = -\, i \int \frac{d^4p}{(2\pi)^4} \mathrm{tr} \left[\frac{i}{\not{p} - m} \gamma_\lambda \gamma_5 \frac{i}{\not{p} - \not{q} - m} \times \right. \\ \left. \gamma_\nu \frac{i}{\not{p} - \not{k}_1 - m} \gamma_\mu \right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \end{pmatrix} . \tag{18}$$

Thus, the axial WI (AWI) is written as

$$q^\lambda T_{\mu\nu\lambda} = 2m T_{\mu\nu} + \mathcal{A}_{\mu\nu} \,, \tag{19}$$

and conservation occurs only if $\mathcal{A}_{\mu\nu} = 0$. When calculating this anomalous term, one should use a regulator that preserves gauge invariance (e.g. Pauli-Villars regularization [10]) in order to obtain the physical/correct value (i.e. the physical value does not depend on the regularization scheme used and thus, we should compute it with a gauge invariant regulator). After a laborious algebra manipulation we obtain

$$\mathcal{A}_{\mu\nu} = \frac{1}{2\pi^2} \varepsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \,, \tag{20}$$

and hence, AWI does not hold. If we consider gauge bosons (massless case) as external legs coupled to the triangle diagram depicted in Fig. 1, the result would be the same: there is an anomaly/anomalous term.

To address the same problem in non-Abelian gauge theories (e.g. in the SM), we can reproduce this calculation including the non-Abelian generators in the vertices (from Feynman rules, the group generator t^a modifies the vertex Γ^μ to $\Gamma^\mu t^a$). The respective diagram is presented in Fig. 2 and we get

$$T^{abc}_{\mu\nu\lambda} = -\, i \int \frac{d^4p}{(2\pi)^4} \mathrm{tr} \left[\frac{i}{\not{p} - m} \gamma_\lambda \gamma_5 t^c \frac{i}{\not{p} - \not{q} - m} \times \right. \\ \left. \gamma_\nu t^a \frac{i}{\not{p} - \not{k}_1 - m} \gamma_\mu t^b \right] + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu \leftrightarrow \nu \\ a \leftrightarrow b \end{pmatrix} . \tag{21}$$

Since the gamma matrices commute with group generators ($[\gamma^\mu, t^a] = [\gamma_5, t^a] = 0$), we can factorize $\mathrm{tr}_{\mathcal{R}} \left[t^c t^a t^b \right]$, which stands for the trace of the group generators in the representation $\mathcal{R}$ of the fields. This modification leads to a change in the anomalous term $A_{\mu\nu}$ given in Eq. (20),

$$\mathcal{A}^{abc}_{\mu\nu} = \frac{1}{4\pi^2} \varepsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \mathrm{tr}_{\mathcal{R}} \left[\left\{ t^a, t^b \right\} t^c \right] . \tag{22}$$

Due to the fact that fermions with opposite chirality contribute with opposite sign to the anomalous term [11], the anomaly-free condition is

$$\mathrm{tr}_{\mathcal{R}} \left[\left\{ t^a, t^b \right\} t^c \right] = 0 \,, \tag{23}$$

when summed over all fermions of the theory.

In the SM the hypercharges assignments and particles representations lead to a consistent theory since all gauge anomalies vanish.

Rather than checking if the SM is anomaly free, given a set of hypercharges, we could impose the anomaly-free conditions and verify whether or not these constraints lead to the uniqueness of hypercharges. If there is a single solution, then electric charge (hypercharge) is quantized [12, 13]. In this case, the relevant (generalized) anomaly-free conditions are

$$\begin{aligned} &[U(1)_Y]^3 : \sum_{i=1}^{n_G} \left(6Y_{q_i}^3 + 2Y_{\ell_i}^3 - 3Y_{u_i}^3 - 3Y_{d_i}^3 - Y_{e_i}^3 \right) = 0 \,, \\ &U(1)_Y \left[SU(2)_L \right]^2 : \sum_{i=1}^{n_G} \left(3Y_{q_i} + Y_{\ell_i} \right) = 0 \,, \\ &U(1)_Y \left[SU(3)_C \right]^2 : \sum_{i=1}^{n_G} \left(2Y_{q_i} - Y_{u_i} - Y_{d_i} \right) = 0 \,, \\ &U(1)_Y : \sum_{i=1}^{n_G} \left(6Y_{q_i} + 2Y_{\ell_i} - 3Y_{u_i} - 3Y_{d_i} - Y_{e_i} \right) = 0 \,, \end{aligned} \tag{24}$$

containing fifteen free parameters. Even with the constraints that arise from the gauge invariant Yukawa part of the SM Lagrangian, given in Eq. (11),

$$\begin{aligned} &-Y_{q_i} - Y_H + Y_{u_i} = 0 \,, \; -Y_{q_i} + Y_H + Y_{d_i} = 0 \,, \\ &-Y_{\ell_i} + Y_H + Y_{e_i} = 0 \,, \; i = 1, 2, 3 \,, \end{aligned} \tag{25}$$

there are only thirteen equations, which do not yield a unique solution for the sixteen hypercharges (one also needs to include Y_H). If we instead consider family universal charges, one obtains a unique solution and charge quantization. Thus in the SM, electric charge is quantized only if family universal charges are assumed.

III. NEUTRINOS AND SEESAW MECHANISMS

Neutrino oscillation experiments have firmly established the existence of neutrino masses and lepton mixing. Since neutrinos are strictly massless in the SM, this implies that new physics beyond the SM is required to account for these observations [14, 15].

The main concept of these oscillations is very similar to the transitions that change quark flavour. When neutrinos take part in weak interactions they are created with a specific lepton flavour (ν_e, ν_μ, ν_τ), although there is a non-zero probability of being in a different flavour state when they are measured. For three generations of neutrinos, the mismatch between interaction basis and mass basis is given by

$$|\nu'_\alpha\rangle = \sum_{i=1}^{3} \mathbf{U}_{\alpha i} \, |\nu_i\rangle \ , \tag{26}$$

where $\mathbf{U}$ is a 3×3 unitary matrix similar to the quark mixing matrix [16] and $|\nu_i\rangle$ is a mass eigenstate.

The fact that neutrino masses are tiny constitutes a puzzling aspect of nowadays particle physics. Since neutrinos are massive, the SM should be considered an effective theory and it is necessary to extend it with non-renormalizable terms that generate neutrino masses through new physics. The lowest order non-renormalizable operator, which generates Majorana neutrino masses after SSB, is the unique $d = 5$ Weinberg operator [17]

$$\mathcal{L}_{Weinberg} = -\frac{z^{\alpha\beta}}{\Lambda} \left(\overline{\ell_{L\alpha}}\widetilde{H}\right) C \left(\overline{\ell_{L\beta}}\widetilde{H}\right)^T + \text{H.c.} \,, \tag{27}$$

where $\mathbf{m}_\nu^{\alpha\beta} = v^2 z^{\alpha\beta}/\Lambda$ is the 3×3 effective neutrino mass matrix, $z^{\alpha\beta}$ are complex constants and Λ is the new high-energy physics cutoff scale, expected to be high enough to generate tiny masses. In this context, we can write the lepton mass terms in the interaction basis as

$$\mathcal{L}_{lep.mass} = -\mathbf{m}_e^{\alpha\beta}\, \overline{e'_{L\alpha}} e'_{R\beta} - \frac{1}{2}\mathbf{m}_\nu^{\alpha\beta}\, \overline{\nu'_{L\alpha}}\, \nu'^{c}_{L\beta} \ + \ \text{H.c.} \,. \tag{28}$$

In order to diagonalize these matrices we use the unitary transformations given in Eq. (12),

$$e'_{L\alpha} = \mathbf{L}_e^{\alpha\beta} e_{L\beta} \ , \ e'_{R\alpha} = \mathbf{R}_e^{\alpha\beta} e_{R\beta} \ , \ \nu'_{L\alpha} = \mathbf{L}_\nu^{\alpha i} \nu_{Li} \ , \tag{29}$$

which lead to

$$\begin{aligned} \mathbf{L}_e^\dagger \mathbf{m}_e \mathbf{R}_e &= \mathbf{d}_e = \text{diag}\left(m_e, m_\mu, m_\tau\right) \ , \\ \mathbf{L}_\nu^\dagger \mathbf{m}_\nu \mathbf{L}_\nu^* &= \mathbf{d}_n = \text{diag}\left(m_1, m_2, m_3\right) \ , \end{aligned} \tag{30}$$

where $\mathbf{d}_e$, $\mathbf{d}_n$ are the diagonal mass matrices and m_i $(i = 1, 2, 3)$ is the mass of the light neutrino ν_i. With these transformations the charged current (neglecting the H.c. terms) becomes

$$\frac{g_W}{\sqrt{2}} \left(\overline{u_{L\alpha}}\, \mathbf{V}_{CKM}^{\alpha\beta} \gamma^\mu d_{L\beta} + \overline{\nu_{Li}}\, \mathbf{U}_{PMNS}^{\dagger i\alpha} \gamma^\mu e_{L\alpha}\right) W_\mu^+ \ , \tag{31}$$

where

$$\mathbf{U}_{PMNS} = \mathbf{L}_e^\dagger \mathbf{L}_\nu = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} . \tag{32}$$

The unitary matrix $\mathbf{U}_{PMNS}$ is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix.

One of the most appealing theoretical frameworks to understand the smallness of neutrino masses is the so-called seesaw mechanism. In this context, the tree-level exchanges of new heavy states generate the effective neutrino mass matrix at low energies. Three simple possibilities consist of the addition of singlet right-handed neutrinos (type I seesaw), colour-singlet $SU(2)_L$-triplet scalars (type II)[2] or $SU(2)_L$-triplet fermions (type III).

To generate type I seesaw, n_R right-handed neutrino fields ν_{Ri} with the respective gauge group representations and hypercharge assignments $\sim (\mathbf{1}, \mathbf{1}, 0)$ are introduced [18–22]. The respective Lagrangian (neglecting the H.c. terms) is

$$\mathcal{L}_I = \mathcal{L}_{SM} + \frac{i}{2}\overline{\nu_{Ri}}\partial\!\!\!/\nu_{Ri} - \mathbf{Y}_\nu^{\alpha i}\overline{\ell_{L\alpha}}\widetilde{H}\nu_{Ri} - \frac{1}{2}\mathbf{M}_R^{ij}\, \overline{\nu_{Ri}^c}\nu_{Rj} \ , \tag{33}$$

where $\mathbf{Y}_\nu$ is a $3\times n_R$ complex Yukawa coupling matrix and $\mathbf{M}_R$ is a $n_R \times n_R$ symmetric matrix. Integrating out the heavy states since they decouple from the low energy theory, we can obtain

$$\frac{z^{\alpha\beta}}{\Lambda} \simeq -\left(\mathbf{Y}_\nu \mathbf{M}_R^{-1} \mathbf{Y}_\nu^T\right)^{\alpha\beta} \ . \tag{34}$$

Directly from Eq. (27), we then get the desired effective mass matrix of the light neutrinos, with $\mathbf{m}_D = v\mathbf{Y}_\nu$,

$$\mathbf{m}_\nu = -v^2 \mathbf{Y}_\nu \mathbf{M}_R^{-1} \mathbf{Y}_\nu^T = -\mathbf{m}_D \mathbf{M}_R^{-1} \mathbf{m}_D^T \,. \tag{35}$$

In order to generate the effective neutrino mass matrix within the context of type III seesaw, one includes n_Σ fermion triplets Σ_{Ri} to the SM particle content [23]. The respective gauge group representations and hypercharge assignments are $\sim (\mathbf{1}, \mathbf{3}, 0)$. Usually, one defines the field Σ_{Ri} as

$$\Sigma_{Ri} = \begin{pmatrix} \Sigma_{Ri}^0 & \sqrt{2}\Sigma_{Ri}^+ \\ \sqrt{2}\Sigma_{Ri}^- & -\Sigma_{Ri}^0 \end{pmatrix} \ , \tag{36}$$

and write the respective type III Lagrangian as

$$\begin{aligned} \mathcal{L}_{III} =& \mathcal{L}_{SM} + \frac{i}{2}\text{tr}\left(\overline{\Sigma_{Ri}}D\!\!\!\!/\,\Sigma_{Ri}\right) - \mathbf{Y}_T^{\alpha i}\overline{\ell_{L\alpha}}\Sigma_{Ri}\widetilde{H} \\ & - \frac{1}{2}\mathbf{M}_\Sigma^{ij}\text{tr}\left(\overline{\Sigma_{Ri}^c}\Sigma_{Rj}\right) + \text{H.c.} \,, \end{aligned} \tag{37}$$

where $\mathbf{Y}_T$ is a $3\times n_\Sigma$ complex Yukawa coupling matrix and $\mathbf{M}_\Sigma$ is a $n_\Sigma \times n_\Sigma$ symmetric matrix. This mass term includes Majorana masses for the fields $\left(\Sigma_{Ri}^0\right)^c + \Sigma_{Ri}^0$ and Dirac masses for the fields $\left(\Sigma_{Ri}^+\right)^c + \Sigma_{Ri}^-$. Looking closely to the type I Lagrangian given in Eq. (33), we identify similar terms by simply replacing $\{\nu_{Ri}, \mathbf{Y}_\nu, \mathbf{M}_R\}$ by $\{\Sigma_{Ri}, \mathbf{Y}_T, \mathbf{M}_\Sigma\}$. Therefore, it is straightforward to conclude that

$$\begin{aligned} \frac{z^{\alpha\beta}}{\Lambda} &\simeq -\left(\mathbf{Y}_T \mathbf{M}_\Sigma^{-1} \mathbf{Y}_T^T\right)^{\alpha\beta} \ , \\ \mathbf{m}_\nu &= -v^2 \mathbf{Y}_T \mathbf{M}_\Sigma^{-1} \mathbf{Y}_T^T = -\mathbf{m}_T \mathbf{M}_\Sigma^{-1} \mathbf{m}_T^T \,, \end{aligned} \tag{38}$$

with $\mathbf{m}_T = v\mathbf{Y}_T$.

To address the charge quantization problem in the context of seesaw models one needs to modify the anomaly-free equations to[3]

$$\begin{aligned}
[U(1)_Y]^3 &: \sum_{i=1}^{n_G} \left(6Y_{q_i}^3 + 2Y_{\ell_i}^3 - 3Y_{u_i}^3 - 3Y_{d_i}^3 - Y_{e_i}^3\right) - \sum_{i=1}^{n_\Sigma} 3Y_{\sigma_i}^3 - \sum_{i=1}^{n_R} Y_{\nu_i}^3 = 0\,,\\
U(1)_Y\,[SU(2)_L]^2 &: \sum_{i=1}^{n_G} (3Y_{q_i} + Y_{\ell_i}) - \sum_{i=1}^{n_\Sigma} 4Y_{\sigma_i} = 0\,,\\
U(1)_Y\,[SU(3)_C]^2 &: \sum_{i=1}^{n_G} (2Y_{q_i} - Y_{u_i} - Y_{d_i}) = 0\,,\\
U(1)_Y &: \sum_{i=1}^{n_G} (6Y_{q_i} + 2Y_{\ell_i} - 3Y_{u_i} - 3Y_{d_i} - Y_{e_i}) - \sum_{i=1}^{n_\Sigma} 3Y_{\sigma_i} - \sum_{i=1}^{n_R} Y_{\nu_i} = 0\,,
\end{aligned} \tag{39}$$

which contain $15 + n_R + n_\Sigma$ free parameters. Even with the gauge invariance constraints that arise from the type I Lagrangian (or type III Lagrangian or even in a mixed type I/type III scenario), charge is not quantized. However, if family universal charges are assumed, the electric charge quantization appears in the framework of type I, II or III seesaw models.

Despite the simplicity of the seesaw mechanism in explaining the smallness of the neutrino masses, the corresponding high-energy theory usually contains many more free parameters than those required at low energies. We recall that the effective neutrino mass matrix $\mathbf{m}_\nu$ can be written in terms of only nine physical parameters: 3 light neutrino masses and 3 mixing angles + 3 phases, that parametrize the PMNS mixing matrix. For instance, the type I seesaw Lagrangian given in Eq. (33) with n_R right-handed neutrino fields, contains altogether $7n_R - 3$ free parameters. The same parameter counting holds for the type III seesaw with the replacements $n_R \to n_\Sigma$, $\mathbf{Y}_\nu \to \mathbf{Y}_T$ and $\mathbf{M}_R \to \mathbf{M}_\Sigma$.

It then becomes clear that for a high energy seesaw theory to be predictive the number of free parameters should be somehow reduced. A well-motivated framework is provided by the so-called zero textures of the Yukawa coupling matrices. In some cases, such zeros also propagate to $\mathbf{m}_\nu$, implying relations among the neutrino observables. These textures can be obtained, for instance, in the presence of flavour symmetries or additional local gauge symmetries.

The neutrino mass matrix $\mathbf{m}_\nu$ is a symmetric matrix with six independent entries. There are $6!/[n!(6-n)!]$ different textures, each containing n independent texture zeros. Since each matrix entry is a complex number, there are $2n$ constraints. It can be shown that any pattern of $\mathbf{m}_\nu$ with more than two independent zeros ($n > 2$) is not compatible with current neutrino oscillation data. Clearly, one-zero textures in $\mathbf{m}_\nu$ have much less predictability than the two-zero textures. Their phenomenological implications have been studied in Refs. [24–28] and we shall not discuss them any further here.

To conclude this section we present the possible two-zero textures of $\mathbf{m}_\nu$. There are fifteen textures which can be classified into six categories $(\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, \mathbf{F})$:

$$\mathbf{A}_1 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \ \mathbf{A}_2 : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix};$$

$$\mathbf{B}_1 : \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}, \ \mathbf{B}_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix},$$

$$\mathbf{B}_3 : \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \ \mathbf{B}_4 : \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix};$$

$$\mathbf{C} : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}; \ \mathbf{D}_1 : \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \ \mathbf{D}_2 : \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix};$$

$$\mathbf{E}_1 : \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}, \ \mathbf{E}_2 : \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, \ \mathbf{E}_3 : \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix};$$

$$\mathbf{F}_1 : \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \ \mathbf{F}_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}, \ \mathbf{F}_3 : \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix};$$

the symbol "$*$" denotes a nonzero matrix element. In the flavour basis, where the charged-lepton mass matrix $\mathbf{m}_e$ is diagonal, i.e. $\mathbf{m}_e = \mathbf{d}_e$, only seven patterns, to wit $\mathbf{A}_{1,2}$, $\mathbf{B}_{1,2,3,4}$ and $\mathbf{C}$ [29], are compatible with the present neutrino oscillation data [30]. Since any ordering of the charged leptons in the flavour basis is allowed, any permutation transformation acting on the above patterns is permitted, provided that it

[2] Since for our purposes, namely the study of anomaly-free gauge extensions of the SM and their connection with the flavour structure of $\mathbf{m}_\nu$, the type II seesaw mechanism does not lead to relevant constraints (due to its bosonic character), we shall not consider it in this work.

[3] The complete discussion of quantized charges in type I, II and III seesaw extensions to the SM is presented on the thesis.

leaves $\mathbf{m}_e$ diagonal. In particular, the following permutation sets can be constructed:

$$\begin{aligned}
\mathcal{P}_1 &\equiv (\mathbf{A}_1, \mathbf{A}_2, \mathbf{B}_3, \mathbf{B}_4, \mathbf{D}_1, \mathbf{D}_2), \\
\mathcal{P}_2 &\equiv (\mathbf{B}_1, \mathbf{B}_2, \mathbf{E}_3), \\
\mathcal{P}_3 &\equiv (\mathbf{C}, \mathbf{E}_1, \mathbf{E}_2), \\
\mathcal{P}_4 &\equiv (\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3).
\end{aligned} \tag{40}$$

Starting from any pattern belonging to a particular set, one can obtain any other pattern in the same set by permutations.

IV. ANOMALY-FREE GAUGE SYMMETRIES IN NEUTRINO SEESAW FLAVOUR MODELS

Abelian symmetries naturally arise in a wide variety of grand unified and string theories. One of the interesting features of such theories is their richer phenomenology, when compared with the SM. In particular, the spontaneous breaking of additional gauge symmetries leads to new massive neutral gauge bosons which, if kinematically accessible, could be detectable at the Large Hadron Collider (LHC). Clearly, the experimental signatures of these theories crucially depend on whether or not the SM particles have non-trivial charges under the new gauge symmetry. Assuming that the SM fermions are charged under the new gauge group, and that the new gauge boson has a mass around the TeV scale, one expects some effects on the LHC phenomenology.

In this section (discussed in detail in Ref. [31]), we consider Abelian extensions of the SM based on an extra $U(1)_X$ gauge symmetry, with $X \equiv a\,B - \sum_\alpha b_\alpha L_\alpha$ being an arbitrary linear combination of the baryon number B and the individual lepton numbers L_α. Then, we consider all possible gauge symmetries to perform a systematic study, thus complementing previous works on several aspects.

By requiring cancellation of gauge anomalies, we study the allowed charge assignments under the new gauge symmetry, when two or three right-handed neutrino singlets or fermion triplets are added to the SM particle content. We then discuss the phenomenological constraints on these theories, requiring consistency with current neutrino oscillation data. In particular, by extending the SM with a minimal extra fermion and scalar content, we study how the new gauge symmetry can lead to predictive two zero textures in the effective neutrino mass matrix. We also briefly address the possibility of distinguishing different charge assignments (gauge symmetries) and neutrino textures at collider experiments.

A. Anomaly Constraints on the Extended Gauge Group

We consider a renormalizable theory containing the SM particles plus a minimal extra fermionic and scalar content, so that light neutrinos acquire seesaw masses. We include n_R singlet right-handed neutrinos ν_R and n_Σ color-singlet $SU(2)$-triplet fermions Σ to implement type-I and type-III seesaw mechanisms, respectively. Besides the SM Higgs doublet H that gives masses to quarks and leptons, a complex scalar singlet field S is introduced in order to give Majorana masses to ν_R and Σ.

We assume that each fermion field f have a charge x_f under the new $U(1)_X$ gauge symmetry. For quarks, a family universal charge assignment is assumed, while leptons are allowed to have non-universal X charges.

As we have seen in previous sections, in the presence of extra fermion degrees of freedom, the anomaly conditions may change. Furthermore, when we extend the gauge group, for instance by including a $U(1)_X$ Abelian symmetry, extra conditions should be satisfied to render the theory free of the $U(1)_X$ anomalies. Following the same line of reasoning of Section II, we obtain the system of constraints

$$\begin{aligned}
U(1)_X\,[SU(3)_C]^2 &: n_G\,(2\,x_q - x_u - x_d) = 0\,, \\
U(1)_X\,[SU(2)_L]^2 &: \frac{3n_G}{2} x_q + \frac{1}{2}\sum_{i=1}^{n_G} x_{\ell i} - 2\sum_{i=1}^{n_\Sigma} x_{\sigma i} = 0\,, \\
U(1)_X\,[U(1)_Y]^2 &: n_G\left(\frac{x_q}{6} - \frac{4x_u}{3} - \frac{x_d}{3}\right) + \sum_{i=1}^{n_G}\left(\frac{x_{\ell i}}{2} - x_{ei}\right) = 0\,, \\
[U(1)_X]^2\,U(1)_Y &: n_G\left(x_q^2 - 2x_u^2 + x_d^2\right) + \sum_{i=1}^{n_G}\left(-x_{\ell i}^2 + x_{ei}^2\right) = 0\,, \\
[U(1)_X]^3 &: n_G\left(6\,x_q^3 - 3\,x_u^3 - 3\,x_d^3\right) + \sum_{i=1}^{n_G}\left(2\,x_{\ell i}^3 - x_{ei}^3\right) - \sum_{i=1}^{n_R} x_{\nu i}^3 - 3\sum_{i=1}^{n_\Sigma} x_{\sigma i}^3 = 0\,, \\
U(1)_X &: n_G\,(6x_q - 3x_u - 3x_d) + \sum_{i=1}^{n_G}(2\,x_{\ell i} - x_{ei}) - \sum_{i=1}^{n_R} x_{\nu i} - 3\sum_{i=1}^{n_\Sigma} x_{\sigma i} = 0\,.
\end{aligned} \tag{41}$$

n_R	n_Σ	Anomaly constraints	Symmetry generator X
2	0	$b_i + b_j = 3a,\, b_k = 0$	$B - 3L_j - b'_i(L_i - L_j)$
		$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
0	2	$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
2	1	$b_i + b_j = 3a,\, b_k = 0$	$B - 3L_j - b'_i(L_i - L_j)$
		$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
1	2	$b_i + b_j = 0,\, b_k = 3a$	$B - 3L_k - b'_i(L_i - L_j)$
		$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
3	0	$b_i + b_j + b_k = 3a$	$(B - L) + (1 - b'_i)(L_i - L_j) + (1 - b'_k)(L_k - L_j)$
		$b_i + b_j + b_k = 0$	$-b'_i(L_i - L_k) - b'_j(L_j - L_k)$
0	3	$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
3	1	$b_i + b_j = 3a,\, b_k = 0$	$B - 3L_j - b'_i(L_i - L_j)$
		$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
1	3	$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$
2	2	$b_i + b_j = 0,\, b_k = 0$	$L_i - L_j$

TABLE II. Anomaly-free solutions for minimal type I and/or type III seesaw realizations and their symmetry generators. In all cases, $i \neq j \neq k$ and $b'_i \equiv b_i/a$. Cases with $a = 0$ correspond to a purely leptonic symmetry.

Since the anomaly equations are nonlinear and contain many free parameters, some assumptions are usually made to obtain simple analytic solutions. We shall consider models where $X \equiv a\,B - \sum_{i=1}^{n_G} b_i L_i$ is an arbitrary linear combination of the baryon number B and individual lepton numbers L_i, simultaneously allowing for the existence of right-handed neutrinos and fermion triplets that participate in the seesaw mechanism to generate Majorana neutrino masses. Under the gauge group $U(1)_X$, the charge for the quarks q_L, u_R, d_R, is universal, $x_q = x_u = x_d = a/3$ while the charged leptons ℓ_{Li}, e_{Ri} have the family non-universal charge assignment $x_{\ell i} = x_{ei} = -b_i$, with all b_i different. The latter condition guarantees that the charged lepton mass matrix is always diagonal (i.e. it is defined in the charged lepton flavour basis), assuming that the SM Higgs is neutral under the new gauge symmetry. The right-handed neutrinos ν_R and/or the triplets Σ are allowed to have any charge assignment $-b_k$, where $k = 1 \ldots n_G$.

Substituting the $U(1)_X$ charge values into the anomaly equations (41), we obtain the constraints

$$\begin{aligned}
&\sum_{k\leq n_\Sigma} b_k = 0,\\
&\sum_{i=1}^{n_G} b_i = \sum_{j\leq n_R} b_j = n_G\, a,\\
&\sum_{i=1}^{n_G} b_i^3 - \sum_{j\leq n_R} b_j^3 - 3\sum_{k\leq n_\Sigma} b_k^3 = 0\,.
\end{aligned} \tag{42}$$

The solutions of this system of equations and the corresponding symmetry generators X are presented in Table II, for minimal type I and type III seesaw realizations with $n_R + n_\Sigma \leq 4$. We note that in the absence of right-handed neutrinos only purely leptonic ($a = 0$) gauge symmetry extensions are allowed. This is a direct consequence of the second constraint in Eq. (42). Given the charge assignments, one can identify the maximal gauge group corresponding to each solution. For instance, when $n_R = 3$ and $n_\Sigma = 0$, the maximal anomaly-free Abelian gauge group extension is $U(1)_{B-L} \times U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau}$ [32].

B. Phenomenological Constraints

For our study, besides the usual SM Yukawa interactions, the relevant Lagrangian terms in the context of (minimal) type I and type III seesaw models are

$$\begin{aligned}
&\mathbf{Y}_u\, \overline{q_L} u_R \widetilde{H} + \mathbf{Y}_d\, \overline{q_L} d_R H + \mathbf{Y}_e\, \overline{\ell_L} e_R H + \mathbf{Y}_\nu\, \overline{\ell_L} \nu_R \widetilde{H}\\
&+ \frac{1}{2}\mathbf{m}_R \nu_R^T C \nu_R + \mathbf{Y}_1\, \nu_R^T C \nu_R S + \mathbf{Y}_2\, \nu_R^T C \nu_R S^*\\
&+ \frac{1}{2}\mathbf{m}_\Sigma \mathrm{Tr}\left(\Sigma^T C\, \Sigma\right) + \mathbf{Y}_T\, \overline{\ell_L} i\tau_2 \Sigma\, H\\
&+ \mathbf{Y}_3\, \mathrm{Tr}\left(\Sigma^T C\, \Sigma\right) S + \mathbf{Y}_4\, \mathrm{Tr}\left(\Sigma^T C\, \Sigma\right) S^* + \mathrm{H.c.}\,,
\end{aligned} \tag{43}$$

We assume that the SM Higgs doublet is neutral under the $U(1)_X$ gauge symmetry, and that the complex singlet scalar field S has a $U(1)_X$ charge equal to x_s. Here $\mathbf{Y}_{1,2}$ are $n_R \times n_R$ symmetric matrices, while $\mathbf{Y}_{3,4}$ are $n_\Sigma \times n_\Sigma$ symmetric matrices.

Notice that, in general, the $U(1)_X$ symmetry does not forbid bare Majorana mass terms for the right-handed neutrinos and fermion triplets. For matrix entries with $X = 0$, such terms are allowed. In turn, entries with $X \neq 0$ are permitted in the presence of the singlet scalar S, charged under $U(1)_X$. The latter gives an additional contribution to the Majorana mass terms once S acquires a VEV.

Since a universal $U(1)_X$ charge is assigned to quarks, the new gauge symmetry does not impose any constraint on the quark mass matrices. However, our choice of a non-universal charge assignment for charged leptons, with all b_i different, forces the charged lepton mass matrix to be diagonal. Thus, leptonic mixing depends exclusively on the way that neutrinos mix. As discussed in Section III, the effective neutrino mass matrix $\mathbf{m}_\nu$ is obtained after the

Symmetry generator X	$\lvert x_s \rvert$	$\mathbf{M}_R$	$\mathbf{m}_\nu$
$B + L_e - L_\mu - 3L_\tau$	2	$\mathbf{D}_2$	$\mathbf{A}_1$
$B + 3L_e - L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{23} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_e - 6L_\tau$	3		
$B + 9L_e - 3L_\mu - 9L_\tau$	6		
$B + L_e - 3L_\mu - L_\tau$	2	$\mathbf{D}_1$	$\mathbf{A}_2$
$B + 3L_e - 5L_\mu - L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{23} = 0$	
$B + 3L_e - 6L_\mu$	3		
$B + 9L_e - 9L_\mu - 3L_\tau$	6		
$B - L_e + L_\mu - 3L_\tau$	2	$\mathbf{B}_4$	$\mathbf{B}_3$
$B - L_e + 3L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{13} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_\mu - 6L_\tau$	3		
$B - 3L_e + 9L_\mu - 9L_\tau$	6		
$B - L_e - 3L_\mu + L_\tau$	2	$\mathbf{B}_3$	$\mathbf{B}_4$
$B - L_e - 5L_\mu + 3L_\tau$	2	$(\mathbf{M}_R)_{12} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B - 6L_\mu + 3L_\tau$	3		
$B - 3L_e - 9L_\mu + 9L_\tau$	6		

TABLE III. Anomaly-free $U(1)$ gauge symmetries that lead to phenomenologically viable two-zero textures of the neutrino mass matrix $\mathbf{m}_\nu$ in a type I seesaw framework with 3 right-handed neutrinos. In all cases, the Dirac-neutrino mass matrix $\mathbf{m}_D$ is diagonal and the charge assignment $x_{\nu i} = x_{\ell i} = x_{ei} = -b_i$ is verified. The solutions belong to the permutation set $\mathcal{P}_1$. For a mixed type I/III seesaw scenario with $n_R = 3$ and $n_\Sigma = 1$ only the solutions with $|x_s| = 3$ remain viable.

decoupling of the heavy right-handed neutrinos and fermion triplets. In the presence of both (type I and type III) seesaw mechanisms it reads as

$$\mathbf{m}_\nu \simeq -\mathbf{m}_D\ \mathbf{M}_R^{-1}\ \mathbf{m}_D^T - \mathbf{m}_T\ \mathbf{M}_\Sigma^{-1}\ \mathbf{m}_T^T\,, \tag{44}$$

where, according to Eq. (43),

$$\begin{aligned} \mathbf{m}_D &= \mathbf{Y}_\nu \langle H\rangle, \quad \mathbf{M}_R = \mathbf{m}_R + 2\mathbf{Y}_1\langle S\rangle + 2\mathbf{Y}_2\langle S^*\rangle\,, \\ \mathbf{m}_T &= \mathbf{Y}_T \langle H\rangle, \quad \mathbf{M}_\Sigma = \mathbf{m}_\Sigma + 2\mathbf{Y}_3\langle S\rangle + 2\mathbf{Y}_4\langle S^*\rangle\,. \end{aligned} \tag{45}$$

In what follows we restrict our analysis to minimal seesaw scenarios with $n_R + n_\Sigma \leq 4$. The requirement that charged leptons are diagonal ($b_1 \neq b_2 \neq b_3$) imposes strong constraints on the matrix textures of $\mathbf{m}_D$ and $\mathbf{m}_T$. Indeed, considering either a type I or a type III seesaw framework, only those matrices with a single nonzero element per column are allowed. Furthermore, matrices with a null row or column are excluded since they lead to a neutrino mass matrix with determinant equal to zero, not belonging to any pattern of those given in Section III, however mixed type I/III seesaw mechanisms can relax this constraint.

We look for anomaly-free $U(1)_X$ gauge symmetries that lead to phenomenologically viable two-zero textures of the neutrino mass matrix $\mathbf{m}_\nu$, namely to patterns $\mathbf{A}_{1,2}$, $\mathbf{B}_{1,2,3,4}$ and $\mathbf{C}$ given in Section III. Solutions were found only within a type I seesaw framework with three right-handed neutrinos, or in a mixed type I/III seesaw scenario with three right-handed neutrinos and one fermion triplet. In Table III we show the allowed solutions, for the cases when the Dirac-neutrino mass matrix $\mathbf{m}_D$ is diagonal, which implies the charge assignment $x_{\nu i} = -b_i$. All the solutions belong to the permutation set $\mathcal{P}_1$ [see Eq. (40)]. We remark that, for each pattern of $\mathbf{m}_\nu$, there are another 20 solutions corresponding to matrices $\mathbf{m}_D$ with 6 zeros (i.e. permutations of the diagonal matrix) and their respective charge assignments. Thus, all together there exist 96 viable solutions. No other anomaly-free solutions are obtained in our minimal setup. Solutions leading to $\mathbf{M}_R = \mathbf{D}_1, \mathbf{D}_2, \mathbf{B}_3, \mathbf{B}_4$ have been recently considered in Ref. [32]. The remaining solutions, to our knowledge, are new in this context. For a mixed type I/III seesaw with $n_R = 3$ and $n_\Sigma = 1$, only the set of solutions with $|x_s| = 3$ in Table III are allowed, since the anomaly equations imply that the b_k coefficient associated to the fermion triplet charge is always zero.

Notice also that, starting from any pattern given in Table III, other patterns in the table can be obtained by permutations of the charged leptons. For instance, starting from the symmetry generators that lead to the $\mathbf{A}_1$ pattern, those corresponding to $\mathbf{A}_2$ and $\mathbf{B}_3$ are obtained by $\mu \leftrightarrow \tau$ and $e \leftrightarrow \mu$ exchange, respectively. Similarly, the $\mathbf{B}_4$ texture can be obtained from $\mathbf{A}_2$ through the $e \leftrightarrow \tau$ exchange.

C. Gauge Sector and Flavour Model Discrimination

For the effects due to the new gauge symmetry to be observable, the seesaw scale should be low enough. One expects a phenomenology similar to the case with a minimal $B - L$ scalar sector [33]. Nevertheless, by studying the Z' resonance and its decay products, one could in principle distinguish the generalized $U(1)_X$ models from the minimal $B - L$ model.

Due to their low background and neat identification, leptonic final states give the cleanest channels for the discovery of a new neutral gauge boson. In the limit that the fermion masses are small compared with the Z' mass, the Z' decay width into fermions is

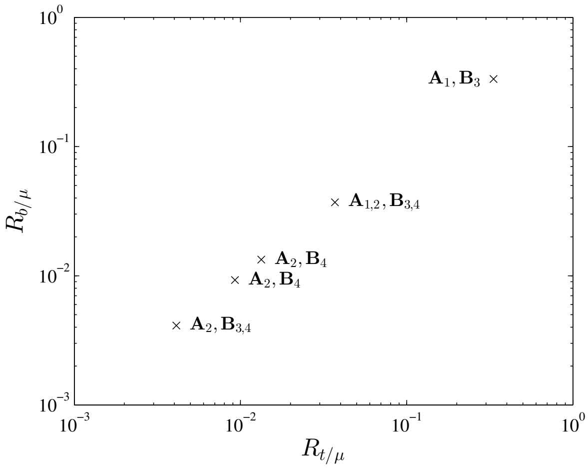

FIG. 3. $R_{t/\mu} - R_{b/\mu}$ branching ratio plane for the anomaly-free solutions of Table III, leading to neutrino mass matrix patterns of type $\mathbf{A}_{1,2}$ and $\mathbf{B}_{3,4}$.

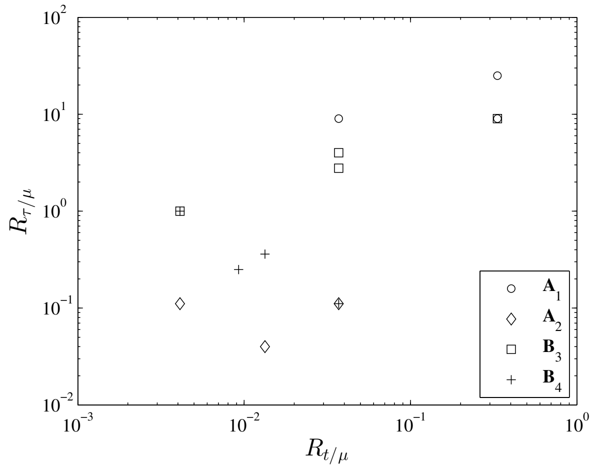

FIG. 4. $R_{t/\mu} - R_{\tau/\mu}$ branching ratio plane for the anomaly-free solutions of Table III, leading to neutrino mass matrix patterns of type $\mathbf{A}_{1,2}$ and $\mathbf{B}_{3,4}$.

approximately given by

$$\Gamma(Z' \to f\bar{f}) \simeq \frac{g'^2}{24\pi} m_{Z'} \left(x_{fL}^2 + x_{fR}^2\right) , \tag{46}$$

where x_{fL} and x_{fR} are the $U(1)_X$ charges for the left and right chiral fermions, respectively. Moreover, the decays of Z' into third-generation quarks, $pp \to Z' \to b\,\bar{b}$ and $pp \to Z' \to t\,\bar{t}$ can be used to discriminate between different models, having the advantage of reducing the theoretical uncertainties [34, 35]. In particular, the branching ratios $R_{b/\mu}$ and $R_{t/\mu}$ of quarks to $\mu^+\mu^-$ production,

$$\begin{aligned} R_{b/\mu} &= \frac{\sigma(pp \to Z' \to b\,\bar{b})}{\sigma(pp \to Z' \to \mu^+\mu^-)} \simeq 3K_b \frac{x_q^2 + x_d^2}{x_{\ell 2}^2 + x_{e2}^2} , \\ R_{t/\mu} &= \frac{\sigma(pp \to Z' \to t\,\bar{t})}{\sigma(pp \to Z' \to \mu^+\mu^-)} \simeq 3K_t \frac{x_q^2 + x_u^2}{x_{\ell 2}^2 + x_{e2}^2} , \end{aligned} \tag{47}$$

could serve as discriminators. The $K_{b,t} \sim \mathcal{O}(1)$ factors incorporate the QCD and QED next-to-leading-order correction factors. Substituting the quark and charged-lepton $U(1)_X$ charges, we obtain

$$R_{b/\mu} \simeq \frac{K_b}{3} \frac{a^2}{b_2^2} , \quad R_{t/\mu} \simeq \frac{K_t}{3} \frac{a^2}{b_2^2} , \tag{48}$$

yielding $R_{b/\mu} \simeq R_{t/\mu}$. Fig. 3 shows the $R_{t/\mu} - R_{b/\mu}$ branching ratio plane for the anomaly-free solutions given in Table III, which lead to the viable neutrino mass matrix patterns $\mathbf{A}_{1,2}$ and $\mathbf{B}_{3,4}$, with two independent zeros. As can be seen from the figure, the solutions split into five different points in the plane, which correspond to the allowed values of the b_2 coefficient, $|b_2| = 1, 3, 5, 6, 9$, assuming $a = 1$. The allowed $\mathbf{m}_\nu$ patterns are shown at each point.

The ratio $R_{\tau/\mu}$ of the branching fraction of $\tau^+\tau^-$ to $\mu^+\mu^-$ has also proven to be useful for understanding models with preferential couplings to Z' [35]. It is approximately given in our case by

$$\begin{aligned} R_{\tau/\mu} &= \frac{\sigma(pp \to Z' \to \tau^+\tau^-)}{\sigma(pp \to Z' \to \mu^+\mu^-)} \\ &\simeq K_\tau \frac{x_{\ell 3}^2 + x_{e3}^2}{x_{\ell 2}^2 + x_{e2}^2} \simeq K_\tau \frac{b_3^2}{b_2^2} . \end{aligned} \tag{49}$$

Clearly, this ratio can be used to distinguish models with generation universality ($R_{\tau/\mu} \simeq 1$) from models with non-universal couplings, as those given in Table III. The $R_{t/\mu} - R_{\tau/\mu}$ branching ratio plane is depicted in Fig. 4. In this case, the neutrino mass matrix patterns exhibit a clear discrimination in the plane, having overlap of two solutions in just three points.

In conclusion, by studying the decays of the Z' boson into leptons and third-generation quarks at collider experiments, it is possible to discriminate different gauge symmetries and the corresponding flavour structure of the neutrino mass matrix.

V. CONCLUSIONS

The recent discovery of a Higgs-like particle at the LHC reinforces the great success of the SM as the effective low energy theory for the electroweak interactions. In spite of this, there remain a few aspects that cannot be explained within the SM. In particular, neutrino oscillation experiments have confirmed that neutrinos have non-vanishing masses and mix. The well-known seesaw mechanism is an appealing and economical theoretical framework to explain the tiny neutrino masses. In this context, the addition of new heavy particles (fermions or bosons) to the theory allows for the generation of an effective neutrino mass matrix at low energies. As is well known, theories that contain fermions with chiral couplings to the gauge fields suffer from anomalies and, to make them consistent, the chiral sector of the new theory should be arranged so that the gauge anomalies cancel. One attractive possibility is to realize the anomaly cancellation through the modification of the gauge symmetry.

We have considered extensions of the SM based on Abelian gauge symmetries that are linear combinations of the baryon number B and the individual lepton numbers $L_{e,\mu,\tau}$. In the presence of a type I and/or type III seesaw mechanisms for neutrino masses, we have then looked for all viable charge assignments and gauge symmetries that lead to cancellation of gauge anomalies and, simultaneously, to a predictive flavour structure of the effective Majorana neutrino mass matrix, consistent with present neutrino oscillation data. Our analysis was performed in the physical basis where the charged leptons are diagonal. This implies that the neutrino mass matrix patterns with two independent zeros, obtained via the seesaw mechanism, are directly linked to low-energy parameters. We recall that, besides three charged lepton masses, there are nine low-energy leptonic parameters (three neutrino masses, three mixing angles, and three CP violating phases). Two-zero patterns in the neutrino mass matrix imply four constraints on these parameters. Would we consider charge assignments that lead to nondiagonal charged leptons, then the predictability of our approach would be lost, since rotating the charged leptons to the diagonal basis would destroy, in most cases, the zero textures in the neutrino mass matrix.

Working in the charged lepton flavour basis, we have found that only a limited set of solutions are viable, namely those presented in Table III, leading to two-zero textures of the neutrino mass matrix with a minimal extra fermion and scalar content. All allowed patterns were obtained in the framework of the type I seesaw mechanism with three right-handed neutrinos (or in a mixed type I/III seesaw framework with three right-handed neutrinos and one fermion triplet), extending the SM scalar sector with a complex scalar singlet field.

Finally, we briefly addressed the possibility of discriminating the different charge assignments (gauge symmetries) and seesaw realizations at the LHC. We have shown that the measurements of the ratios of third generation final states (τ, b, t) to μ decays of the new gauge boson Z' could be useful in distinguishing between different gauge symmetry realizations, as can be seen from Figs. 3 and 4. This analysis provides a complementary way of testing flavour symmetries and their implications for low-energy neutrino physics.

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