IMPACT OF DIFFERENT SOURCES OF UNCERTAINTY IN THE FREE SURFACE PROFILE IN A NATURAL RIVER IN FLOOD CONDITIONS. AN APPROACH BASED ON THE MONTE CARLO SIMULATION TECHNIQUE

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Abstract

The paper exemplifies how the uncertainty can be addressed when computing the free surface profile under flood conditions in a natural river reach.

Among the countless sources of uncertainty inherent to the adopted hydraulic model and to the implemented algorithm only the variables peak flood discharge, the Manning roughness coefficient and the river bed elevation were embedded of uncertainty. Based on these three variables four scenarios were analyzed corresponding, three of them to the consideration of uncertainty associated, independently, with each one of these variables and the fourth to the consideration of uncertainty associated to the three variables when analyzed simultaneously.

The uncertainty analysis performed relied on the application of Monte Carlo method as a way of, in each considered scenario, cover the largest possible number of cases likely to occur in reality. The using of this method allowed the calculation, in a simple and expeditious way, of a large number of flow heights, in order of thousands, for each of the sections that define the stretch of adopted channel for this study. This way it is possible to analyze what is the impact of the uncertainty in the three variables in study in the distribution of those heights.

From the developed methodology was possible to conclude that, of the three considered variables, the one that entails more uncertainty into the estimation of free surface configuration in flood conditions in natural channels is the peak flood discharge, although this conclusion is closely related to the characteristics of the sample of annual maximum instantaneous discharges sample that supported the analysis.

Keywords: uncertainty analysis, free surface profile, peak flood discharge, Manning roughness coefficient, ground elevation, Monte Carlo method.

1 INTRODUCTION

The research presented aimed to identify and characterize some of the uncertainties related to the estimation of free surface profile in natural rivers and to develop a methodology that allows analyzing the impact of those uncertainties in the flow heights. Such kind of analysis can be very useful for decision process related to flood analysis and flood risk assessment, among other applications.

The several sources of uncertainties that can be considered when computing the free surface profile in a natural river can be classified according to their origin, “location” in the analysis process and uncertainty degree.

In terms of the origin there are aleatory and epistemic uncertainties. The first one is a consequence of the natural and intrinsic variability of a given phenomenon (Naghettini, et al., 2011), being impossible to reduce either through sophisticated approaches or by collecting more data (Ganoulis, 1995, in Studart, et al., 2001). The epistemic uncertainty relates to imperfect and/or insufficient knowledge about the phenomenon and can be reduced either by collecting more data or by developing models that are more adequate to represent that phenomenon.

Regarding the location of the sources of uncertainty in the analysis process, Walker et al. (2003) and Van der Klis (2003) (in Noordam, 2005) refer that they can be classified as *i)* context uncertainty; *ii)* input uncertainty; *iii)* parameters uncertainty; and *iv)*model uncertainty.

According to Noordam, 2005, the first type of uncertainty results from the boundaries definition (e.g. consideration of Coriolis force to model large bodies of water). The input uncertainty is due to measurement errors, if the input is measured directly (e.g. the discharge or the flow height in a flow boundaries), or to estimation errors, if the input is estimated (e.g. the rainfall in a rainfall/runoff model), Willems, 2000 (in Timbe et al., 2004). The uncertainty in parameters is caused by the model parameters which can be classified into *i)* exact parameters which includes universal well-known constants (e or π numbers); *ii)* fixed parameters with values resulting from comprehensive research that allow considering them as values exact (gravity acceleration, g); *iii)* parameters that were previously defined meaning those parameters that are so difficult to calibrated that reasonable values are assumed to represent them; and *iv)* calibrated parameters which encompass the parameters with values resulting from a calibration process (Noordam, 2005).

Timbe et al., 2004, classified the parameters in three types: the measured parameters, the estimated parameters and the calibrated parameters with definitions similar to the previous ones.

Finally, the uncertainty of a model can be classified into structural and technical model uncertainty. The first one relates to the limitation of the modeler to perfectly describe the system (Noordam, 2005, and Timbe et al., 2004) and the second one to the uncertainty generated by software and hardware errors (Noordam, 2005).

Regarding the classification of the degree of uncertainty, five different levels of knowledge can be considered: *i)* complete deterministic understanding; *ii)* quantifiable uncertainty; *iii)* scenario analysis; *iv)* recognized ignorance; and *v)* total indeterminacy (Walker et al., 2003 in Noordam, 2005).

In what concerns the problem under analysis – the estimation of free surface profile in flood conditions along a natural channel – it involves a large number of uncertainties including the boundary condition in the downstream control section, the peak flood discharges even if the return period is fixed, the geometric and hydraulic characteristics of the channel and the hydraulic model itself. From the large number of variables that could be considered uncertain only three were chosen and further analyzed, namely the peak flood discharge, the Manning-Strickler roughness coefficient and the terrain elevations.

To ascertain the consequences, in terms of flow height, of the uncertainty in each one of the previous variables, a distribution probability function was assigned to it and the Monte Carlo random sampling technique applied, via the generation of a large set of uniform random numbers. Considering that each one of those numbers represents a non-excedance probability and by inverting the cumulative distribution function assigned to the variable, a synthetic sample of values of the same is obtained. By introducing those values in the hydraulic model as many water surface profiles along the river reach as the number of generated values of the variable are attained. The uncertainty that outcomes from the large number of water profiles was expressed in terms of the empirical distribution of the flow height in each cross section of the river reach.

2 METHODOLOGY

2.1 MONTE CARLO METHOD

One of the ways to accomplish an uncertainty analysis utilized a large number of possible scenarios obtained based on the Monte Carlo technique. This technique provides a wide range of results to the decision makers, allowing predictions from the worst scenarios to the better ones including all the intermediate scenarios. The probability of occurrence of each scenario can be estimated using Monte Carlo method too (Palisade Corporation).

Prior to the application of Monte Carlo method in an uncertainty analysis it is necessary to define the parameters to consider in the analysis. After this definition the method proceeds in four phases. In the first one it is necessary to select/define the probability function that will represent each parameter. This selection can be based on the literature or result from statistical analyzes, depending on the knowledge about the processes involved and on the quantity of available data. When the probability distributions that represent the parameters are identified it becomes possible to generate random samples of values of the parameters according to those distributions and to estimate their probabilities of occurrence – the second phase of Monte Carlo technique. By introducing those parameters – either considered individually or together – in a mathematical model the approach proceeds with the third phase of simulation of the system. Each simulation of the model based on a certain value of a parameter or on a set of certain values of the several parameters is named iteration. By repeating the iteration hundreds or thousands times equal number of results for the output variable is obtained. These results are next treated also based on probability distribution functions, either empirical or theoretical (fourth phase).

The quality of obtained results depends of several factors (Brusamarello, 2011):

- mathematical models representativeness;
- quality of the input variables characterization;
- number of simulations.

In the study carried out, the implementation of the four phases of the Monte Carlo technique used repeatedly the random generation of equiprobable values in the interval [0;1] (random numbers uniformly distributed). By assigning the generated random numbers to non-exceedance probabilities and by inverting the probability distribution function that represents each parameter, a sample of values of the same is obtained.

To address the uncertainty associated to the free surface profile in a natural reach geometry defined by cross-sections the following steps were implemented:

1. Definition of the mathematical model that describes the free surface configuration in steady flow conditions.
2. Selection, between the different parameters and variables of the previous model, of those that will be considered embedded of uncertainty.
3. For each selected parameter/variable, definition of the more suitable probability function that represents its uncertainty.
4. Considering individually each parameter/variable, generation of a large number of its possible values.
5. For each generated value, application of hydraulic model to estimate the free surface configuration.
6. Treatment of the free surface elevations resulting for each cross section from the values of the parameter.
7. Repetition of steps iii. to vi. for another parameter.
8. Generation of a large number of possible values of the parameters considered together.
9. Repetition of steps v. to vi. for the values of set of parameters.

The previous approach required the development of a hydraulic model able to estimate the free surface configuration in a channel with variable geometry, as a natural channel. That model was developed and implemented in a MATLAB® environment.

2.2 HYDRAULIC MODEL

Hydraulic models can be classified as one-dimensional (1D) or two-dimensional (2D) being the first ones more common in flood propagation studies (Timbe et al, 2004). 1D models consider a uniform vertical velocity distribution (average velocity) of each cross-section, according to the predominant flow direction. 2D models require more time for the calculus and high computational capacity, being used, essentially, in localized flows.

Taking into account the relative complexity of 2D models and that in a natural channel the subcritical flow can be reasonably approximated by a one-dimensional model, it was decided to apply this last type of model in the analyses carried out.

More precisely, the hydraulic model applied to estimate the free surface configuration was based on the energy equation, also known as Bernoulli's equation, for a steady gradually varied flow of a perfect fluid:

$$\frac{d}{ds}\left(\frac{p}{\gamma}+y+\frac{u^2}{2g}\right)=-i \tag{1}$$

where i equals the energy dissipation along the trajectory (m m^{-1}), p is the pressure at any point of the path (Pa), y is the elevation head (m), γ is the unit weight of the liquid (N m^{-3}), u is the velocity (m s^{-1}) and g is the gravitational acceleration (m s^{-2}).

For the study of real flows in a natural channel considering the energy equation it is necessary to take into account the continuous and singular head losses and the vertical velocity distribution. Thus, considering a steady gradually varied flow in a low sloping channel, the energy equation can be written as (Cardoso, 1998):

$$h_m+y_{fm}+\alpha_m\frac{\mathrm{Q}^2}{\mathrm{S_m}^2 2g}=h_j+y_{fj}+\alpha_j\frac{\mathrm{Q}^2}{\mathrm{S_j}^2 2g}+J\Delta x+\Delta E \tag{2}$$

where m and j indentify upstream and downstream cross-sections respectively. The remaining variables are Q as discharge (m^3 s^{-1}), S as cross-section area of flow (m^2), y_f as elevation head (m), h as depth flow (m), J as head loss (m m^{-1}), α as Coriolis coefficient, which in a cross-section with uniform velocity, as this case, equals 1, Δx as distance between sections (m) and ΔE as the singular head losses (m).

Having discarded the singular head losses, the only unknown variable in equation 4.2 in a reach with Δx length is J. This variable was considered equal to the arithmetic mean of the values of J in the cross-sections limiting that reach (Cardoso, 1998).

The head loss value in each cross-section for a given discharge, Q, was considered approximated equal to the head loss that would occur under uniform flow conditions. Accordingly the Manning-Strickler formula was applied to compute head loss at each cross-section:

$$J' = \frac{Q^2}{{K_s}^2 S^2 R^{4/3}} \tag{3}$$

where Q and S have the meaning and the units previously defined and J' is the uniform head loss in the cross-section (m m^{-1}), K_s is the Manning-Strickler coefficient ($m^{1/3}$ s^{-1}) and R is the hydraulic radius (m), which results from dividing the cross-section area of flow by wetted perimeter of flow, P (m).

The numerical method applied to the previous calculation was *standard-step method*. This is an iterative method that permits the estimation of free surface elevation at a cross-section if the free surface elevation in the downstream cross-section is known since the flow is subcritical and the geometry of both cross-sections is available.

For the beginning of calculation the values of the discharge and of the Manning-Strickler coefficient are provided to the program. To simplify the calculation Manning-Strickler coefficient was considered constant along the channel. The boundary condition at the most downstream cross-section (control cross-section for subcritical flow) was the free surface elevation given by the rating curve available at that section as it coincides with a stream gauging station (Almourol station). By assuming the same water-level in the cross-section immediately upstream it was checked if equation 2.2 was verified. If the difference between the two sides of equation is greater than 0.001 m, a new water-level is assumed for the upstream station (the average between the level initially arbitrated and the one obtained) and the calculation repeated. If the difference between the two sides of equation is smaller than 0.001 m the water level obtained for the upstream section is considered to be the correct one and the computations proceeds to the next upstream station by applying an equivalent procedure based on this level.

2.3 UNCERTAINTY MODELING

2.3.1 UNCERTAINTY IN PEAK FLOOD DISCHARGE

According to the design criteria more often applied in flood analysis, the free surface profile was calculated for the 100-year peak flood discharge.

For that purpose a statistic analysis was applied to the sample of annual maximum instantaneous discharges at Almourol stream gauging station, located at the most downstream section of the reach of Tejo River adopted as case study. That sample was obtained at SNIRH site (Sistema Nacional de Informação de Recursos Hídricos) and has 25 discharges.

The distributions postulated were the following ones: Normal or Gaussian, log-Normal or Galton, Gumbel, Goodrich, Pearson III and log-Pearson III. It should be stressed that the logarithmic laws are advantageous as they avoid negative values during the random generation procedure.

As a result of the visual evaluation of the adjustment the log-Normal distribution was selected. Before such law was adopted the normality of the logarithms of the discharges was confirmed by applying the Snedecor and Cochran test, 1989 (in Portela Ramos Silva, 1989). According to that test, the hypothesis that a sample comes from a normal population is rejected with a confidence level of η=1-α where α is the significance level if:

$$\left| \frac{C_{a_X}}{\left(\frac{6}{N}\right)^{0.5}} \right| > \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \tag{4}$$

where C_a is the skewness coefficient and N is the length of the sample and Φ^{-1} is the inverse of the standard Normal distribution. In the application carried out the variables were the logarithms of the annual maximum instantaneous discharges.

Once the log-Normal distribution was selected, to estimate the logarithm of the discharge with a given return period the probability factor method was applied according to the following equation:

$$\hat{X} = \bar{x} + Ks' \tag{5}$$

where $\hat{X}$ is the average and s' the standard deviation of the sample of the logarithms of the annual maximum instantaneous discharges and K is the probability factor given by the inverse of standard Normal distribution for the return period under consideration. The peak flood discharges are obtained by applying exponential function to the results from equation 5.

The probability factor, K, depends on the non-exceendace probability, F, which relates to the return period, T, according to:

$$F = 1 - \frac{1}{T} \tag{6}$$

To address the uncertainty related to the 100-year peak flood discharge a large number of synthetic samples of annual maximum discharges at Almourol stream gauging station were generated by applying the Monte Carlo method, each sample with a length equal to the length of the historical one, that is N=25 years. For that purpose, there were generated N aleatory and equiprobable numbers between zero and one being N the dimension of the collected sample. Each of those values corresponds to a non-exceendance probability for which was calculated the respective probability factor. By introducing those value in equation 5 considering the mean and standard deviation of the logarithmic values of the collected sample it was possible to estimate the peak flood discharge to that probability. Each series of N generated numbers corresponds to a synthetic sample of annual maximum instantaneous discharges. By applying the log-Normal distribution to each sample a 100-years return period discharge was calculated. This procedure was permitted the generation of 10000 synthetic samples and consequently 10000 peak flood discharges.

2.3.2 UNCERTAINTY IN MANNING COEFFICIENT

Johnson, 1996, refers several studies where the uncertainty in Manning coefficient was modeled. In his paper he presents the distribution and respective variation coefficient that each author, referred by him, suggests to model the uncertainty in Manning coefficient. Since any of those authors clarifies why they chose that specific distribution and variation coefficient, the uncertainty in Manning coefficient was modeled by a triangular distribution characterized by a variation coefficient of 0.08 (Yeh and Tung, 1993) to simplify the algorithm and because it seems to be the distribution that better represents the reality.

To introduce the Manning coefficient uncertainty in free surface configuration calculation were generated 10000 equiprobable values in the range [0;1]. Each one of those values corresponds to a non-exceedance probability and by the inverse of cumulative triangular distribution it was possible to calculate the respective Manning coefficient. Thus, it was obtained 10000 values for Manning coefficient distributed around its average and according to a triangular distribution. For each Manning coefficient obtained was estimated the respective free surface configuration.

2.3.3 UNCERTAINTY IN TERRAIN ELEVATION

The coordinates of cross-sections can be measured by different methods and each measure is independent from the others. Therefore, to simplify this study, it was considered that the uncertainty in cross-section geometry is only associated to the elevation.

The uncertainty in cross-sections coordinates, namely the elevation, is defined by the uncertainty associated with measurement errors which differs with the adopted method to determine the cross-sections.

In this study the measurement of the cross-sections coordinates was made using based topographic maps. Pestana, 1999, states that in this case the uncertainty in measurement errors can be modeled by a Normal distribution with null average and a standard deviation given by:

$$\sigma_X = \frac{e}{3.29} \tag{5}$$

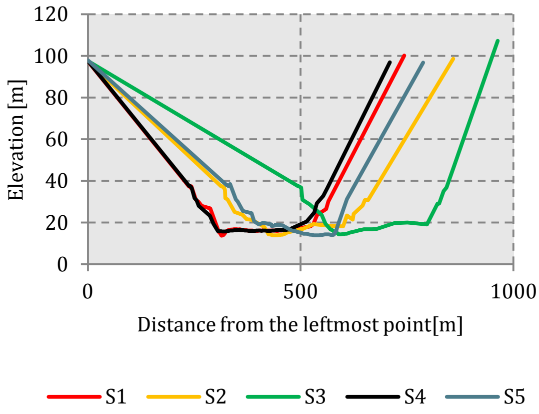
where e is the natural equidistant between contour lines.

The uncertainty associated with geometric characteristics of cross-sections was introduced in a way similar to that presented for Manning coefficient uncertainty. Thus it was generated a large number of equiprobable values in the range [0;1], namely 10000, each of them corresponding to a non-exceedance probability.

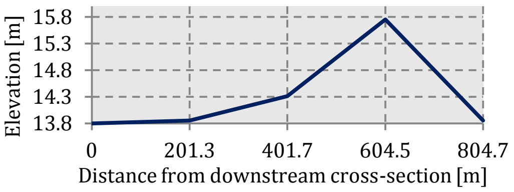
For each of these generated probabilities was calculated the respective measurement error using the inverse of cumulative Normal distribution which was added to the all elevation coordinates that define the river reach. For each considered error was estimated the free surface configuration.

3 CASE STUDY

The methodology developed for uncertainty analysis under flood conditions was applied to a reach of Tagus River with the length of 804.7 m. The geometry of that reach was characterized by five cross-sections (from downstream to upstream, S1, S2, S3, S4 e S5) the most downstream one being coincident with the stream gauging station of Almourol (control section for subcritical flow). The cross sections were prolonged – by extending the last points in each river bank – in under to allow the flow for all the generated peak flood discharges. The final geometry of the cross sections and the distances among the latter are presented in Graphics 1 and 2, respectively.



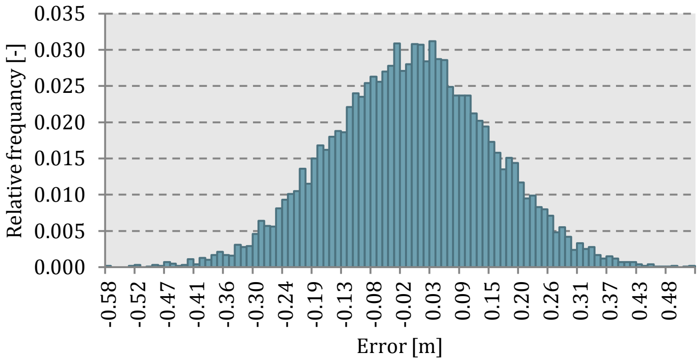
Graphic 1 Cross-sections of Tejo River.



Graphic 2 Distance between cross-sections and bottom elevation of each one.

The characterization of the studied river reach utilized topographic maps at the 1:500 scale with an equidistance of 0.5 m. According to the methodology implemented, the uncertainty in the elevations was modeled in terms of errors by a Normal distribution with null mean and a standard deviation of 0.15198 m, obtained by applying equation 5. By generating 10000 equiprobable values in the range [0;1], by assigning those values to non-exceedance probabilities and by inverting the cumulative Normal distribution, equal number of possible errors were obtained and added to elevations in every cross section.

The histogram of the relative frequency of the generated errors is presented in Graphic 3 which suggests a Normal distribution with parameters close to the ones assumed.

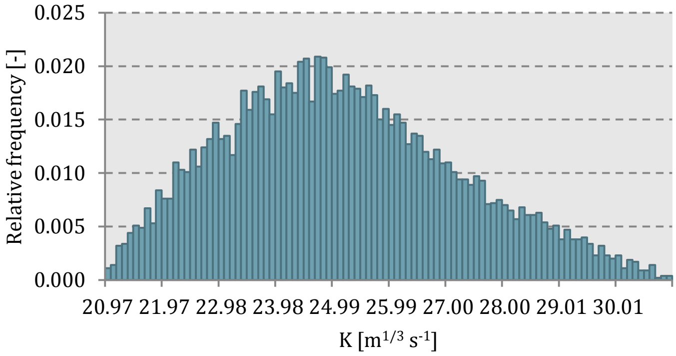


Graphic 3 Histogram of the relative frequency of the 10000 errors generated according to a Normal distribution.

As shown in the previous graphic the generated errors can be either positive or negative. When positive they represent an increase in the elevations and when negative, a decrease. As previously mentioned, each error was considered to affect the elevation of all cross-sections.

With respect to Manning coefficient it was necessary to establish the average value of triangular distribution adopted to represent the uncertainty derived from that parameter. Assuming that the values of Manning coefficient can vary likewise around its average symmetric values were considered for the upper and lower limits of the triangular distribution. For the average the value of 0.04 s $m^{-1/3}$ mentioned in Portela, 2008, for discharges in the study river reach higher than 13850 m^3 s^{-1} was adopted. Based on that value and on the variation coefficient previously defined (*cv* = 0.08) the symmetric triangular distribution is completely characterized.

To model the uncertainty that comes from the Manning coefficient, 10000 equiprobable values in the range [0;1] were generated each one representing a non-excedaance probability. By the inverting the cumulative triangular distribution for each non-exceedance probability a Manning coefficient value is obtained. Graphic 4 presents the results thus achieved expressed in terms of the inverse of the Manning coefficient, the Manning-Strickler coefficient. As expected the shape of the histogram is approximately triangular.



Graphic 4 Histogram of the relative frequency of 10000 Manning-Strickler coefficients generated according to a triangular distribution.

The sample of annual maximum instantaneous discharges at Almourol station, adopted to characterize the peak flood discharge is presented in Table 1, either in terms of discharges or of their logarithms. To represent such the sample an empirical non-exceedance probability was assigned to each annual maximum instantaneous discharges according to the Weibull formula:

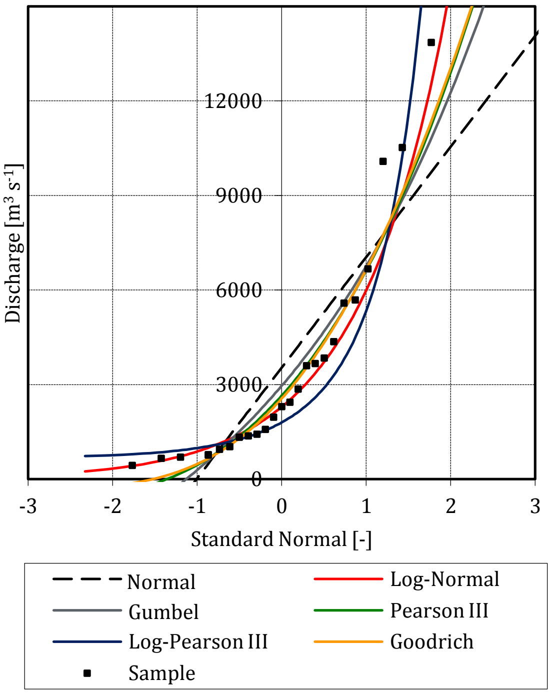
$$F = \frac{i}{N+1} \tag{6}$$

where *i* is the rank of the discharge in the sample ordered by ascending values, and *N* is the length of sample.

From the analysis of Graphic 5 it is possible to conclude that the distribution that best fit the sample of annual maximum instantaneous discharges is the log-Normal distribution, with the advantages that come with this logarithmic distribution, namely, the guarantee that all the discharge generated values will be positives.

Hydrologic year	Annual maximum instantaneous discharge (Q_i) [$m^3\ s^{-1}$]	ln (Q_i)
1	1331.00	7.19
2	764.00	6.64
3	5586.00	8.63
4	10521.00	9.26
5	13853.07	9.54
6	1426.96	7.26
7	700.39	6.55
8	3843.54	8.25
9	948.59	6.85
10	3600.71	8.19
11	3669.41	8.21
12	1580.36	7.37
13	1967.65	7.58
14	2441.48	7.80
15	1374.29	7.23
16	10082.00	9.22
17	2302.87	7.74
18	780.02	6.66
19	662.82	6.50
20	6672.00	8.81
21	2859.00	7.96
22	4363.00	8.38
23	438.70	6.08
24	1032.00	6.94
25	5686.16	8.65
Mean	3539.5 $m^3\ s^{-1}$	7.7
Standard deviation	3506.7 $m^3\ s^{-1}$	1.0
Skewness coefficient	1.6488	0.1762

Table 1 Annual maximum instantaneous discharges sample and the respective logarithms. Samples and statistical parameters.



Graphic 5 Sample of annual maximum instantaneous discharges and adjustment of the Normal, log-Normal, Gumbel, Pearson III, log-Pearson III and Goodrich distributions.

The result from the Snedecor e Cochran test, 1989 (in Portela Ramos Silva, 1989) – equation 4 – applied to the logarithm values of the annual maximum instantaneous discharge is:

$$\left|\frac{0.1762}{\left(\frac{6}{25}\right)^{0.5}}\right| > \Phi^{-1}(0.975) \Leftrightarrow 0.36 \ngtr 1.96$$

And so the hypothesis that the sample comes from a population normally distributed is not rejected.

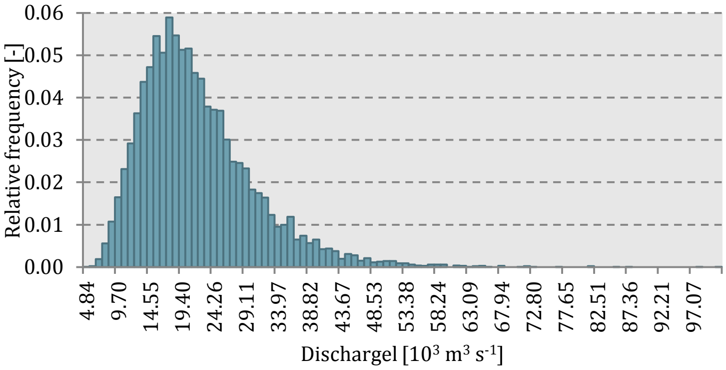
Following the methodology adopted to introduce the uncertainty in the peak flood discharge, 10000 synthetic samples composed by 25 discharges each were generated according to a log-Normal distribution. For each sample the 100-year peak flood discharge was computed based on the non-exceedance probability of:

$$F = 1 - \frac{1}{100} = 0.99$$

and on the probability factor of:

$$K = \phi^{-1}(0.99) = 2.326$$

The 1000 discharges thus obtained are presented in Graphic 6.



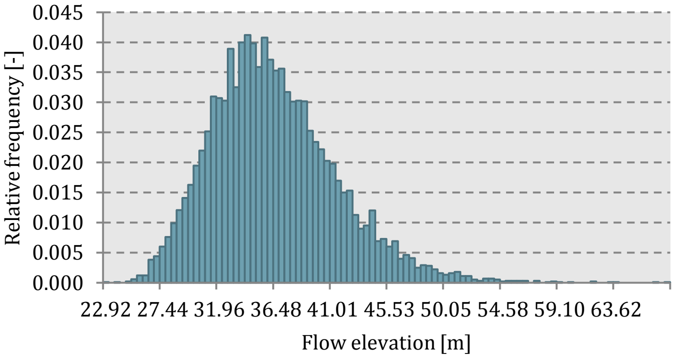
Graphic 6 Histogram of the relative frequency of 10000 flood discharges with a 100-return period generated according to a log-Normal distribution.

For each generated peak flood discharge the flow elevation at the downstream control cross-section was obtained by applying the rating curve of Almourol stream gauging station (zero of the scale at the elevation of 12.25 m). The equation of the rating curve is given by:

$$\begin{cases} Q = 23.33045(h + 0.75)^{2.408} & para\ h \leq 3.70\ m \\ Q = 47.178(h + 0.75)^{1.902} & para\ h > 3.70\ m \end{cases} \tag{7}$$

wherein Q is the discharge (m^3 s^{-1}) and h the hydrometric height (m).

The histograms of the flow elevations at cross section S1 are presented in Graphic 7.



Graphic 7 Free water surface elevations in the most downstream cross-section (control section) for the generated discharges.

4 RESULTS

The uncertainty analysis was applied in two phases: the first one encompassed the uncertainty that affects each one of the variables elevation, roughness coefficient and peak flood discharge when considered separately and the second phase resulted from considering that those three variables were simultaneously affected by uncertainty.

Thus, to analyze the uncertainty in free surface configuration estimation that comes from uncertainty in peak flood discharge it were realized so many simulations as the number of generated discharges namely 10000 simulations. In each simulation was considered a different value for the peak flood discharge while the Manning-Strickler coefficient was fixed at 25 m$^{1/3}$ s^{-1} and the terrain elevation coordinates were not associated with any error.

To analyze what is the impact of the Manning-Strickler coefficient uncertainty in the estimation of free surface configuration, that coefficient was made vary accordance with the foregoing *Case Study*. As in the discharge uncertainty analysis there were made 10000 simulations wherein each simulation considers a different value for Manning-Strickler coefficient and the peak flood discharge and the error in elevation coordinates were fixed at 21390.4 m^3 s^{-1} and 0 m, respectively. That peak flood discharge respects the discharge for a 100-year return period estimated for the collected sample of annual maximum instantaneous discharges and to it is associated a flow height of 36.427 m at downstream cross-section.

Similarly to the previous procedures the impact of uncertainty in terrain elevation coordinates was analyzed by the estimation of free surface configuration for 10000 errors to associate with these coordinates. For each of those errors was realized a simulation in each of which the peak flood discharge and the Manning-Strickler coefficient did not vary and were adopted equal to 21390.4 m^3 s^{-1} and 25 m$^{1/3}$ s^{-1} respectively. As referred above, the flow height that is associated to the adopted peak flood discharge equals 36.427 m.

The previous three analyses respect the first phase and for each one were obtained 10000 flow heights in each cross-section. The mean, standard deviation and asymmetry coefficient for the flow heights obtained are presented in Table 2.

Table 2 Mean, standard deviation and asymmetry coefficient for flow heights distributions when the uncertainty is individually associated with each analyzed variable.

	Peak flood discharge				Manning-Strickler coefficient				Terrain elevation			
	S2	**S3**	**S4**	**S5**	**S2**	**S3**	**S4**	**S5**	**S2**	**S3**	**S4**	**S5**
Mean [m]	22.841	22.498	21.136	22.990	22.721	22.363	20.994	22.832	22.719	22.361	20.991	22.829
Standard deviation [m]	4.99	4.96	4.90	4.86	0.02	0.04	0.06	0.09	0.15	0.16	0.16	0.16
Asymmetry coefficient [-]	0.75	0.77	0.77	0.79	0.15	0.13	0.13	0.12	-0.03	-0.03	-0.02	-0.02

In the second phase were realized 10000 simulations, too. However in this analysis for each simulation were adopted a different value generated according to exposed at *Case Study* for peak flood discharge, for Manning-Strickler coefficient and for the error in terrain elevation coordinates.

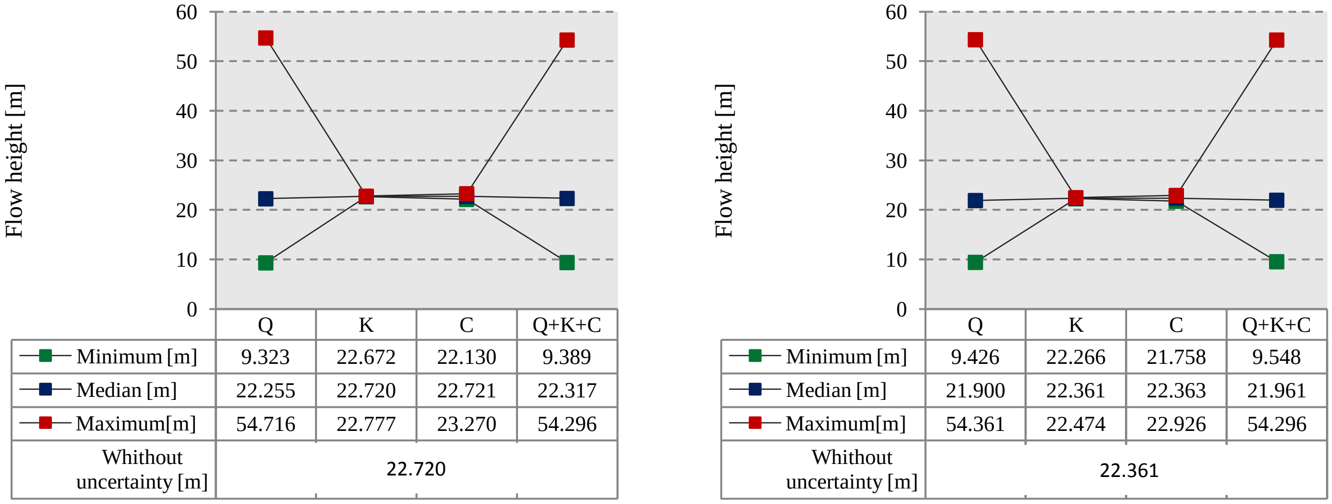
The mean, standard deviation and coefficient asymmetry that characterize the flow heights obtained distribution are presented in Table 3.

Table 3 Mean, standard deviation and skewness coefficient for flow heights distributions when the uncertainty is simultaneously associated with the three analyzed variable.

	S2	**S3**	**S4**	**S5**
Mean [m]	22.864	22.521	21.160	18.014
Standard deviation [m]	5.00	4.96	4.90	4.86
Asymmetry coefficient [-]	0.75	0.77	0.77	0.79

Beyond the characterization of the obtained flow heights presented above it is important to evaluate what were the median of those flow heights and the maximum and the minimum of them for the realized uncertainty analyses. Those values are presented in Graphic 7 which covers the results for each of the five cross-sections for the four uncertainty analyses realized. To have a better notion of those results it was opted to represent in the label the flow height of the free surface for a free uncertainty scenario. That scenario considers the peak flood discharge for a return period of 100 years calculated for the collected sample of annual maximum instantaneous discharges, 21390.4 m^3 s^{-1}, a Manning-Strickler coefficient of 25 $m^{1/3}$ s^{-1} and a no error associated with elevation coordinates.

In the X-axis of Graphic 6 ate identified the different scenarios that were analyzed namely the one that the uncertainty is associated with peak flood discharge indentified by *Q*, the scenario where Manning-Strickler coefficient is embedded in uncertainty identified by *K*, the scenario that considers the uncertainty associated with terrain elevation identified by *C* and, at last, the scenario that considers the uncertainty in these three variables identified by *Q+K+C*.

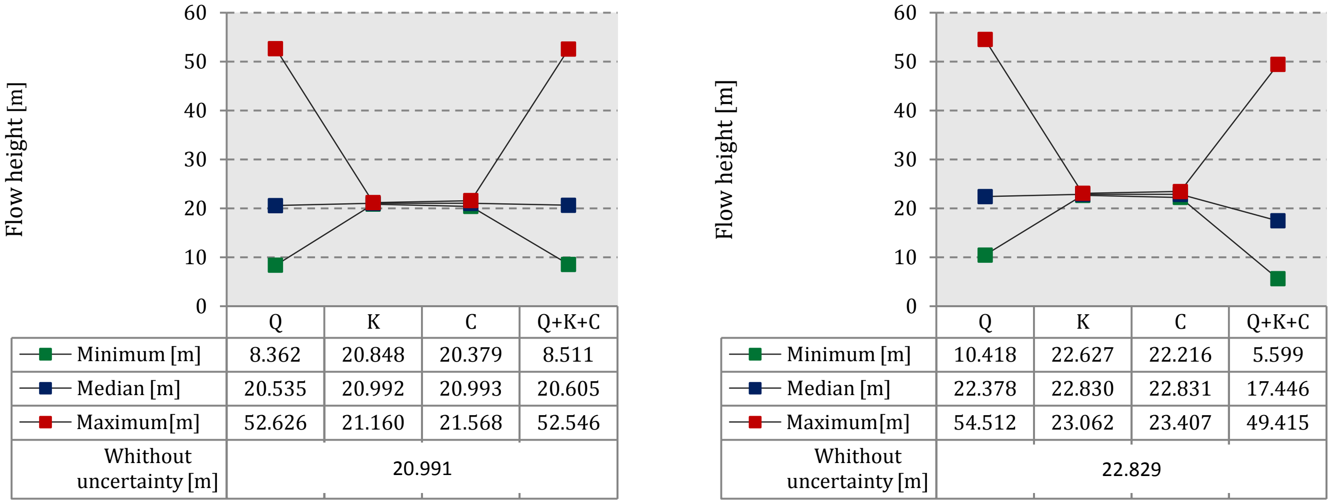


	Q	K	C	Q+K+C
🟩 Minimum [m]	9.323	22.672	22.130	9.389
🟦 Median [m]	22.255	22.720	22.721	22.317
🟥 Maximum[m]	54.716	22.777	23.270	54.296
Whithout uncertainty [m]	22.720			

a) Section S2.

	Q	K	C	Q+K+C
🟩 Minimum [m]	9.426	22.266	21.758	9.548
🟦 Median [m]	21.900	22.361	22.363	21.961
🟥 Maximum[m]	54.361	22.474	22.926	54.296
Whithout uncertainty [m]	22.361			

b) Section S3.



	Q	K	C	Q+K+C
🟩 Minimum [m]	8.362	20.848	20.379	8.511
🟦 Median [m]	20.535	20.992	20.993	20.605
🟥 Maximum[m]	52.626	21.160	21.568	52.546
Whithout uncertainty [m]	20.991			

c) Section S4.

	Q	K	C	Q+K+C
🟩 Minimum [m]	10.418	22.627	22.216	5.599
🟦 Median [m]	22.378	22.830	22.831	17.446
🟥 Maximum[m]	54.512	23.062	23.407	49.415
Whithout uncertainty [m]	22.829			

d) Section S5.

Graphic 7 Maximum, minimum and median obtained in each cross-section for considered scenarios. Flow heights with no uncertainty associate.

5 CONCLUSIONS

Based on the obtained and presented results it is evident that the uncertainty in peak flood discharge is the one that leads to a wider range of flow heights. However it can be stated that for the uncertainty in peak flood discharge and in all the three analyzed variables are achieved unrealistic flow heights around 50 m while the median value of all considered scenarios and the flow heights arising from the free uncertainty scenario are around 20 m.

Excessive variability of flow heights may have its origin in the sample variability of annual maximum instantaneous discharge on its own combined with the small size of that sample and the weak fit of the selected statistical distribution to the collected discharge values. Those aspects themselves are uncertainty sources which contribute to a wide range of discharge values in a generated process.

When the uncertainty is considered simultaneously associated with the three analyzed variables the variation in flow heights that stems from the peak flood discharge is so superior to the uncertainty induced by the other two variables that the results from this scenario translate the effects of the uncertainty associated with peak flood discharge. Obviously, the range of flow heights are similar in the scenario that only considers the uncertainty in peak flood discharge and the scenario that considers the uncertainty associated with all the analyzed variables.

The uncertainty in Manning-Strickler coefficient and in terrain elevation coordinates has a low impact in the estimation of free surface configuration; however between these two variables the second is the one that induced higher flow heights.

It is important to note that the similar results obtained for the flow heights in the free uncertainty scenario and the mean of flow heights in the four scenarios where it was considered the presence of uncertainty is expected and desirable once that this similarity permits to validate the generation methodology.

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