Neutrinos, Symmetries and the Origin of Matter

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In this thesis, we explore seesaw extensions of the Standard Model, where heavy states mediate neutrino mass generation and the smallness of these masses is naturally accounted for. Nonvanishing neutrino masses allow for leptonic mixing, whose structure strongly differs from that of quark mixing. The closeness of the lepton mixing matrix to the tribimaximal pattern points to the presence of discrete symmetries in the underlying high-energy theory – such as invariance under transformations of the A_4 group, considered in this work.

The Standard Model also fails to provide a satisfactory mechanism for the generation of the baryon asymmetry of the Universe. A remarkable feature of the seesaw extensions is the possibility that the out-of-equilibrium decays of the new heavy states are responsible for the dynamical generation of this asymmetry. This corresponds to the leptogenesis mechanism, whose efficiency is here determined by numerically solving a system of Boltzmann equations.

A particular model for spontaneous leptonic CP violation is analysed where neutrino masses and mixing are explained imposing an A_4 discrete symmetry. The implementation of the leptogenesis mechanism in this context is discussed in detail.

Keywords: Baryon asymmetry of the Universe; CP violation; Leptogenesis; Neutrino masses and mixing; Seesaw mechanism; Symmetries.

I. INTRODUCTION

Symmetries have played a fundamental role in the development of modern physics. They are to be understood as laws of invariance under specified operations, the description of which relies on the language of group theory, transcending a purely geometrical notion. In particular, gauge symmetries are at heart of our current understanding of the subatomic world, which relies on the Standard Model (SM) of particle physics.

Despite its remarkable successes and repeated experimental verification, the SM cannot provide a satisfatory answer to some questions which remain open. Among these is the fact that neutrino masses and mixing cannot be accounted for in the model, as required by neutrino oscillations. An additional puzzle arises as the observed leptonic mixing deviates strongly from the structure of quark mixing. Exploiting the normative aspect of symmetries, one may impose a family symmetry at high-energy (now broken) – operating in the leptonic sector – which produces the observed mixing pattern.

One might also wonder if the theory is sufficient to explain the observed imbalance between matter and antimatter, which allows for the richness of phenomena we observe in Nature (namely ourselves), as opposed to a dull, symmetric Universe filled by a sea of photons. As it turns out, the SM is insufficient to account for the observed baryon asymmetry of the Universe (BAU), even though the basic ingredients may seem to be there at a first glance. An alternate mechanism for the generation of this asymmetry (baryogenesis) is required.

A natural solution to both the neutrino mass and the BAU generation problems – to be addressed sequentially in what follows – is found within the seesaw framework, where the exchange of heavy particles induces low-energy neutrino masses. The out-of--equilibrium decays of these heavy states in the early Universe might then be responsible for producing the observed BAU with the aid of SM non-perturbative processes, in a mechanism known as leptogenesis.

II. NEUTRINOS IN THE SM

The Standard Model is a relativistic quantum Yang-Mills theory based on an underlying $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge symmetry group. The dynamics of the fields are encoded in the Lagrangian of the theory, which can be split into the following contributions:

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{Higgs}} + \mathcal{L}_{\mathrm{Gauge}} + \mathcal{L}_{\mathrm{Fermions}} + \mathcal{L}_{\mathrm{Yuk}}\,. \tag{1}$$

Following the requirement of gauge invariance under $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ (generators I^i and Y, respectively), a change to the usual Dirac and Klein-Gordon kinetic terms of the fermions and scalar Higgs doublet fields is in order, namely the substitution of the ordinary derivative by a covariant one [1]:

$$\partial_\mu \to D_\mu \equiv \partial_\mu - i\, g_2\, A^i_\mu\, I^i - i\, g_Y\, B_\mu\, Y, \tag{2}$$

where g_2 and g_Y are coupling constants associated with the gauge groups $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, respectively. This leads to the introduction of four real boson fields A^i_μ and B_μ in the theory, whose transformation properties are such that the kinetic terms are kept invariant under the action of the gauge group.

The term $\mathcal{L}_{\mathrm{Higgs}}$ in (1) contains the kinetic term for the Higgs doublet ϕ as well as the Higgs potential,

$$V(\phi) \equiv \mu^2 \phi^\dagger \phi + \lambda\big(\phi^\dagger\phi\big)^2. \tag{3}$$

Fermion and gauge boson kinetic terms are included in $\mathcal{L}_{\mathrm{Fermions}}$ and $\mathcal{L}_{\mathrm{Gauge}}$, respectively. Finally, $\mathcal{L}_{\mathrm{Yuk}}$ contains the Yukawa interactions between fermion chiral components and the Higgs doublet:

$$\begin{aligned}\mathcal{L}_{\mathrm{Yuk}} = &- \mathbf{Y}^l_{\alpha\beta}\, \overline{\ell_{\alpha L}}\, \phi\, l_{\beta R} - \mathbf{Y}^u_{\alpha\beta}\, \overline{q_{\alpha L}}\, \widetilde{\phi}\, u_{\beta R} \\ &- \mathbf{Y}^d_{\alpha\beta}\, \overline{q_{\alpha L}}\, \phi\, d_{\beta R} + \mathrm{H.c.}\,,\end{aligned} \tag{4}$$

which produce fermion mass terms of the Dirac-type after electroweak (spontaneous) symmetry breaking (EWSB). In the above, $\ell_{\alpha L}$ and $q_{\alpha L}$ denote lepton and quark doublets, while the fields u_α and d_α correspond to up- and down-type quarks, and ν_α and l_α to neutrinos and charged leptons. Greek indices run over three generations of fermions.

Since a right-handed component for the neutrino field is missing from the SM field content, one sees that the appearance of a Dirac mass term in $\mathcal{L}_{\rm SM}$ upon EWSB, as in the case of other fermion fields, is not possible. Hence, neutrinos are strictly massless in the SM. This implies that performing a rotation of the neutrino fields in flavour space is inconsequential as far as $\mathcal{L}_{\rm Yuk}$ is concerned, since there is no mass term to spoil: neutrinos have definite (zero) mass in all bases. Lepton mixing is thus absent from the SM.

Flavour neutrino fields $\nu_{e,\mu,\tau}$ are defined as the neutrino field combinations which are coupled by the charged current (CC) to each massive charged lepton e, μ, τ. Since interactions do not mix lepton flavours, lepton flavour numbers L_i – associated to the possibility of invariant rephasings of lepton fields – and, by extension, total lepton number $\mathrm{L} = \sum_i \mathrm{L}_i$, are conserved in the SM.

The SM depends on nineteen parameters, which are to be taken as experimental input. Fermion masses comprise a large number of these parameters. Since the theory offers no prediction for their values, their origin remains a mystery.

The question of why the experimental values are what they are seems to be metaphysical and unproductive. However, both the closeness of the $V_{\rm CKM}$ to the identity matrix as well as the presence of mass hierarchies between generations can be seen as suggestive of hidden relations between parameters.

Undeniable evidence for physics beyond the Standard Model arises when one considers the experimental evidence for neutrino oscillations [2]: neutrinos have small but nonzero masses. In the next section, simple extensions of the SM will be considered in order to generate neutrino masses which are naturally small.

III. NEUTRINOS BEYOND THE SM

A Dirac mass term for neutrinos can be defined consistently with the gauge symmetry in a straightforward extension to the SM, obtained by adding three right-handed sterile neutrino fields $\nu_{\alpha R}$ to the particle content. These extra fields are two-component spinors and weak isospin singlets with null hypercharge, and the extra Lagrangian terms read, before and after spontaneous symmetry breaking:

$$\begin{aligned}\mathcal{L}^{\nu}_{\rm Yuk} &= -\mathbf{Y}^{\nu}_{\alpha\beta}\,\overline{\ell_{\alpha L}}\,\widetilde{\phi}\,\nu_{\beta R} + \text{H.c.}\\ &\xrightarrow{\text{EWSB}} -\left(v + \frac{H}{\sqrt{2}}\right)\mathbf{Y}^{\nu}_{\alpha\beta}\,\overline{\nu_{\alpha L}}\,\nu_{\beta R} + \text{H.c.}\,.\end{aligned} \tag{5}$$

Diagonalization of the neutrino mass matrix $\mathbf{M}^\nu \propto \mathbf{Y}^\nu$ is achieved by rotating the neutrino fields into a basis of massive states $\nu_{1,2,3}$ and entails nonzero leptonic mixing.

In this framework, and due to the large abyss between the masses of neutrinos and those of other fermions, one sees that the entries of $\mathbf{Y}^\nu$ must be unnaturally smaller than those of $\mathbf{Y}^{l,u,d}$. One might expect that by virtue of having the same origin, namely electroweak symmetry breaking, the twelve elementary fermions would have comparable masses. The fact that neutrinos are a strong outlier seems to signal the presence of an alternative mass generation mechanism at work.

A. The Majorana Mass Term

As neutrinos are the only neutral fermions, it is possible to construct for them a Majorana mass term [3]. This is achieved by assuming that the chiral components of the neutrino field are not independent, identifying ν_L^C with ν_R (the superscript C denotes the charge conjugation transformation). The neutrino field then satisfies the Majorana condition, $\nu^C = \nu = \nu_L + \nu_L^C$. A Majorana-type mass term reads:

$$\mathcal{L}^{\nu\,\text{mass}}_{\rm Maj} = -\frac{1}{2}\,m\,\overline{\nu}\,\nu = -\frac{1}{2}\,m\left(\overline{\nu_L^C}\,\nu_L + \text{H.c.}\right). \tag{6}$$

One notices that the above mass term is allowed since ν_L^C not only behaves as a right-handed field ($P_L\,\nu_L^C = 0$, as expected), but also because it transforms as ν_L under a Lorentz transformation and, thus, $\mathcal{L}^{\nu}_{\rm Maj}$ preserves Lorentz invariance.

There is, nonetheless, a symmetry that is clearly broken by the Majorana Lagrangian, namely lepton number L – which automatically implies the breaking of B−L and B+L (B is baryon number) – as one has lost the liberty to rephase the neutrino field.

A mass term of the type (6) is forbidden in the SM since the ν_L are components of an $\mathrm{SU}(2)_L$ doublet and there is no field with the required electroweak assignments to aid in producing an invariant term of that type which is also renormalizable. If one forgoes this last requirement, then the lowest dimensional term which can be constructed using SM fields and induces Majorana neutrino masses is the 5-dimensional Weinberg operator [4], which can be written as [5]:

$$\mathcal{L}_{\rm Wein} = c_{\alpha\beta}\,\frac{1}{\Lambda}\left(\overline{\ell_{\alpha L}^{\,C}}\,\widetilde{\phi}^*\right)\left(\widetilde{\phi}^\dagger\,\ell_{\beta L}\right) + \text{H.c.} \tag{7}$$

where the $c_{\alpha\beta}$ are complex coefficients and Λ corresponds to an energy cutoff. Upon EWSB, one obtains a Majorana neutrino mass term from (7):

$$-\frac{1}{2}\,\mathbf{M}^{\nu}_{\alpha\beta}\,\overline{\nu_{\alpha L}^{\,C}}\,\nu_{\beta L} + \text{H.c.}\,, \tag{8}$$

where $\mathbf{M}^\nu$ is the neutrino (Majorana) mass matrix.

The above considerations are independent of the form of the high-energy theory which gives rise to the effective Weinberg operator $\mathcal{L}_{\rm Wein}$ at low-energy. In the next section, we consider seesaw extensions of the SM in which Majorana neutrino masses arise. Their values turn out to be very small compared to the rest of the fermions if a large enough scale Λ is considered, as suggested by (8).

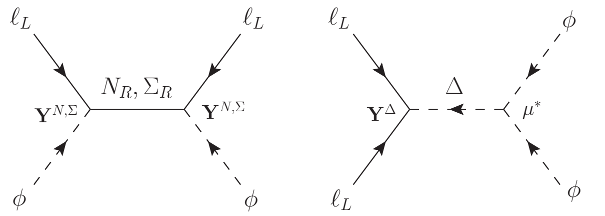

FIG. 1. Exchange interactions which give rise to the Weinberg operator of (7). Seesaw types I and III correspond to the exchange of fermion fields (left), while the type II seesaw mechanism is implemented through the exchange of scalar fields (right).

B. The Seesaw Mechanism

If one regards the SM as an effective description of particle physics, one forcibly has to add new degrees of freedom to the theory – taken to be heavy (with masses of order Λ) – which are typically available at high-energies but decouple from the theory at low-energy. In seesaw extensions of the SM, the 5--dimensional Weinberg operator arises after integrating out massive states from interactions in which they are exchanged. Such an interaction reduces, at lower energy, to a four-point interaction of the form $\phi\phi\ell\ell$, which produces neutrino mass terms following EWSB. Therefore, one must introduce fields ψ with interactions terms of the form $\psi\phi\ell$, corresponding to type I or type III seesaw mechanisms, or both $\psi\phi\phi$ and $\psi\ell\ell$, as is the case with the type II seesaw mechanism (see Fig. 1).

1. Type I Seesaw

In the type I seesaw paradigm [6–10], n' sterile neutrino fields $N_{iR} \sim (1,0)$ are added to the SM field content. Aside from a Yukawa term of the form (5), one is free to add a Majorana mass term as that of (6) for the N_{iR}. The relevant part of the extended Lagrangian reads [5, 11]:

$$-\big(\mathbf{Y}^{N\dagger}\big)_{\alpha i}\,\overline{\ell_{\alpha L}}\,\widetilde{\phi}\,N_{iR} - \frac{1}{2}\,\mathbf{M}^N_{ij}\,\overline{N^{\,C}_{iR}}\,N_{jR} + \text{H.c.}, \quad (9)$$

where $\mathbf{Y}^N$ is a general $n' \times 3$ complex matrix and $\mathbf{M}^N$ is $n' \times n'$ and symmetric. Thanks to the sterility of the N_{iR}, the covariant derivative in the kinetic term reduces to a regular one and the presence of an L-violating Majorana mass term is not forbidden by gauge symmetry.

Integrating out the heavy fields induces the Weinberg operator and the neutrino mass matrix reads:

$$\mathbf{M}^\nu = -\mathbf{Y}^{N\,T}\,\frac{v^2}{\mathbf{M}^N}\,\mathbf{Y}^N\,. \quad (10)$$

A type III seesaw implementation, where the added fermions are arranged into $\mathrm{SU}(2)_L$ triplets, results in a similar structure for the neutrino mass matrix.

2. Type II Seesaw

An alternative to the above scenario is found in type II seesaw framework [12–16], which requires the addition of scalar triplets $\vec{\Delta}_i = (\Delta_i^{++}, \Delta_i^{+}, \Delta_i^{0})$ with hypercharge $Y = 1$ to the SM content.

Defining the Δ_i matrix as:

$$\Delta_i \equiv \begin{pmatrix} \Delta_i^0 & -\Delta_i^+/\sqrt{2} \\ -\Delta_i^+/\sqrt{2} & \Delta_i^{++} \end{pmatrix}\,, \quad (11)$$

the relevant extended Lagrangian terms read:

$$\begin{aligned} &\big(D^\mu\,\vec{\Delta}_i\big)^\dagger\big(D_\mu\,\vec{\Delta}_i\big) - \big(\mathbf{M}^\Delta_{ij}\big)^2\,\mathrm{Tr}\big[\Delta_i^\dagger\Delta_j\big] \\ &-\Big(\mathbf{Y}^{\Delta_i}_{\alpha\beta}\,\overline{\ell^{\,C}_{\alpha L}}\,\Delta_i\,\ell_{\beta L} + \mu_i\,\widetilde{\phi}^T\,\Delta_i\,\widetilde{\phi} + \text{H.c.}\Big), \end{aligned} \quad (12)$$

where the $\mathbf{Y}^{\Delta_i}$ are $n \times n$ symmetric matrices and $\mathbf{M}^\Delta$ is an $n' \times n'$ Hermitian matrix. The low-energy neutrino mass matrix is in this context given by:

$$\mathbf{M}^\nu = -\sum_i \frac{2\,\lambda_i^*\,v^2}{M_i}\,\mathbf{Y}^{\Delta_i}\,, \quad (13)$$

where $\lambda_i \equiv \mu_i/M_i$ (M_i are triplet mass eigenvalues). One notices here an intersting feature, namely that the flavour structure at high-energies is muddled up, as in seesaw types I and III (see Eq. (10)), but preserved when the low energy limit is taken.

An alternative way of obtaining the above contribution for the neutrino mass matrix is by considering that, thanks to EWSB, both ϕ^0 (the lower field component of the Higgs double) and Δ_i^0 acquire nonvanishing VEVs. In the approximation $M_i \gg v$, the Higgs VEV v remains unchanged, while the triplet induced VEV is given by [17]:

$$u_i \equiv \langle \Delta_i^0 \rangle = -\frac{\lambda_i^*\,v^2}{M_i}\,, \quad (14)$$

and one has $|u_i| \ll v$ for all i.

Having bestowed neutrinos with nonzero masses (and before addressing the production of the BAU), one turns to lepton mixing and the role of family symmetries in the shaping of the mixing pattern.

IV. LEPTON MIXING AND FAMILY SYMMETRIES

As in the case of quarks, lepton mixing arises from the mismatch of lepton states which are massive and those which participate in CC interactions. Assuming henceforth that neutrinos possess Majorana masses, one sees that the rotation of the fields to the mass basis leads to

$$\mathcal{L}^\ell_{\text{CC}} \to \frac{g_2}{\sqrt{2}}\,\overline{\nu_{iL}}\,\gamma^\mu\,(U^\dagger_{\text{PMNS}})_{i\alpha}\,l_{\alpha L}\,W_\mu + \text{H.c.}, \quad (15)$$

where one has defined the lepton mixing matrix U_{PMNS}, known as the Pontecorvo-Maki-Nakagawa--Sakata matrix [18–20]. The relation between states of definite flavour (greek indices) and definite mass (roman indices) is given by:

$$\nu_{\alpha L} = \left(U_{\rm PMNS}\right)_{\alpha i}\, \nu_{iL}\,. \tag{16}$$

The number of physical parameters contained in $U_{\rm PMNS}$ can be determined as in the quark case with one important caveat: due to the Majorana nature of neutrinos one has lost the freedom to invariantly rephase the three corresponding fields. Therefore, for n generations, one can only remove n phases through the rotation of charged leptons instead of $2n-1$ from the unitary mixing matrix (the global $\mathrm{U(1)_L}$ is also broken). One ends up, for $n = 3$, with $n(n-1)/2 = 3$ mixing angles and $n(n+1)/2 - n = n(n-1)/2 = 3$ physical phases. A parametrization of the $U_{\rm PMNS}$ is given by [21]:

$$U_{\rm PMNS} = \underbrace{\begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix}}_{\equiv V_{\rm PMNS}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}}_{\equiv K_{\rm PMNS}}, \tag{17}$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ refer to mixing angles $\theta_{12}, \theta_{13}, \theta_{23} \in [0, \pi/2]$. The $\delta \in [0, 2\pi[$ phase is a Dirac-type phase, while $\alpha_1, \alpha_2 \in [0, 2\pi[$ arise due to the Majorana character of neutrinos. As mentioned in Section III A, lepton mixing breaks lepton flavour numbers (and the total L in the Majorana case). This leads to the possibility of observing charged lepton flavour violating processes such as $\mu \to e\,\gamma$, $\mu - e$ conversion in nuclei, and $\mu \to 3\,e$ [22]. So far, searches have yielded negative results, setting upper limits of order 10^{-12} on branching ratios and conversion rates [23–25].

As a consequence of lepton mixing, neutrinos produced with a definite flavour are allowed to oscillate between flavours. Neutrino oscillations are only sensitive to the mass-squared differences $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$, providing no information on the absolute neutrino mass scale. Since the sign of Δm^2_{31} is indeterminate, whereas Δm^2_{21} is positive, there is room for two possible orderings of the mass spectrum: normal ordering, with $m_1 < m_2 < m_3$, and inverted ordering, for which $m_3 < m_1 < m_2$.

Results from the latest global fit to neutrino oscillation data [26] are summarized in Table I, and can be given a visual interpretation through the diagram of Fig. 2. From inspection of Fig. 2 it is apparent that the experimental mixing data bears a close resemblance to the tribimaximal (TBM) pattern. The TBM hypothesis, put forward by Harrison, Perkins, and Scott [27], corresponds to taking $|V_{e3}|^2 = 0$, $|V_{\mu 3}|^2 = 1/2$, and $|V_{e2}|^2 = 1/3$, where V here denotes $V_{\rm PMNS}$. By redefining Majorana phases and rephasing charged leptons, one may write:

$$U_{\rm TBM} \equiv e^{i\varphi} \underbrace{\begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{\equiv V_{\rm TBM}} K_{\rm PMNS}, \tag{18}$$

where $K_{\rm PMNS} = \mathrm{diag}(1, e^{i\alpha_1}, e^{i\alpha_2})$ is given in terms of the new phases.

The possibility of a TBM mixing pattern motivated the study of discrete family symmetries which might govern the leptonic sector. However, exact TBM is excluded by the aforementioned results on a nonzero reactor angle, given by the Daya-Bay, RENO and Double Chooz collaborations [28–30] after hints from the T2K and MINOS collaborations [31, 32]. Nevertheless, one can still pursue the family symmetry approach, without having to resort to mass anarchy [33]. Taking symmetry as a guiding principle, one may accommodate the above results by regarding specific mixing patterns – like the TBM – as a leading-order approximation, to be perturbed when higher-order corrections are taken into account.

Parameter	**Result of Global Fit within 1σ {3σ}**	
	Normal Order	Inverted Order
Δm^2_{21}	$7.62 \pm 0.19 \left\{^{+0.58}_{-0.50}\right\}$	
Δm^2_{31}	$2.53\,^{+0.08}_{-0.10} \left\{^{+0.24}_{-0.27}\right\}$	$-\left(2.40\,^{+0.10}_{-0.07} \left\{^{+0.28}_{-0.25}\right\}\right)$
$\sin^2\theta_{12}$	$0.320\,^{+0.015}_{-0.017} \left\{\pm 0.050\right\}$	
$\sin^2\theta_{23}$	$0.49\,^{+0.08}_{-0.05} \left\{^{+0.15}_{-0.10}\right\}$	$0.53\,^{+0.05}_{-0.07} \left\{^{+0.11}_{-0.14}\right\}$
$\sin^2\theta_{13}$	$0.026\,^{+0.003}_{-0.004} \left\{^{+0.010}_{-0.011}\right\}$	$0.027\,^{+0.003}_{-0.004} \left\{^{+0.010}_{-0.011}\right\}$
δ	$\left(0.83\,^{+0.54}_{-0.64}\right)\pi$ {any}	$\left(0.07\,^{+1.93}_{-0.07}\right)\pi$ {any}

TABLE I. Global fit results taken from [26] for the three--neutrino oscillation parameters and for both ordering possibilities. Δm^2_{21} and Δm^2_{31} are presented in units of 10^{-5} eV^2 and 10^{-3} eV^2, respectively.

V. AN A_4 MODEL WITH SPONTANEOUS CP VIOLATION

In the present section, we turn to the analysis of the model of Ref. [34], which implements an A_4 family symmetry (popular in the literature, [35–38]) in a type II seesaw framework with spontaneous CP violation. Aside from two real flavon fields, $\mathbf{\Phi}$ and $\mathbf{\Psi}$, which are singlets of $\mathrm{SU(2)}_L$ with null hypercharge,

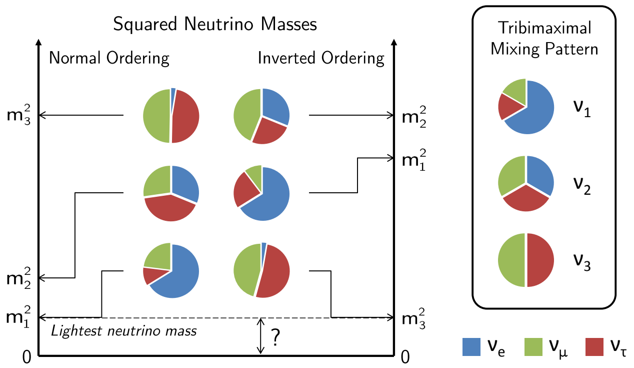

FIG. 2. Depiction of lepton mixing for both a normally ordered and an inverted neutrino mass spectrum, where the global fit data of Table I has been considered (left), to be compared with the tribimaximal *ansatz* (right).

one adds two $\mathrm{SU}(2)_L$ triplets $\Delta_{1,2}$ with $Y = 1$ and one complex scalar field S, which is a singlet of the gauge group, to the SM particle content. A $\mathbb{Z}_4$ shaping symmetry is also considered. For details concerning the scalar potential and symmetry assignments, the reader is referred to Section 3.3 of the present thesis.

An interesting property of the present model is the absence of explicit CP violation at the Lagrangian level. The unique source all CP violation corresponds to the complex phase of the VEV $\langle S\rangle = v_S\, e^{i\alpha}$. As for the Yukawa Lagrangian, the usual SM charged lepton term is forbidden by the $\mathbb{Z}_4$ symmetry, as well as the standard type II seesaw coupling of triplets $\Delta_{1,2}$ to lepton doublets (a coupling of the Higgs type, $\Delta\phi\phi$, is only allowed for Δ_2). Instead, terms of these forms will arise as effective operators. Up to $\mathcal{O}(1/\Lambda, 1/\Lambda')$, where Λ and Λ' are taken to correspond to an assumed unique flavon scale and to the scale of S decoupling, respectively, one obtains the following Yukawa Lagrangian, compatible with the postulated symmetries:

$$
\begin{aligned}
\mathcal{L}^l &= -\frac{y_e^l}{\Lambda}\left(\overline{\ell_L}\,\mathbf{\Phi}\right)_{\mathbf{1}}\phi\, e_R - \frac{y_\mu^l}{\Lambda}\left(\overline{\ell_L}\,\mathbf{\Phi}\right)_{\mathbf{1}''}\phi\,\mu_R - \frac{y_\tau^l}{\Lambda}\left(\overline{\ell_L}\,\mathbf{\Phi}\right)_{\mathbf{1}'}\phi\,\tau_R + \text{H.c.},\\
\mathcal{L}^\nu &= \frac{1}{\Lambda'}\,\Delta_1\left(\ell_L^T\,C^\dagger\,\ell_L\right)_{\mathbf{1}}\left(y_1\,S + y_1'\,S^*\right) + \frac{y_2}{\Lambda}\,\Delta_2\left(\ell_L^T\,C^\dagger\,\ell_L\,\mathbf{\Psi}\right)_{\mathbf{1}} + \text{H.c.},
\end{aligned}
\tag{19}
$$

where the bold superscripts indicate which field combination is chosen from the decomposition of the triplet tensor product.

A. Nonzero Reactor Neutrino Mixing Angle

In order to attain an acceptable agreement with experimental data within the presented framework, one must account for the deviation $\theta_{13} \neq 0$ from the TBM pattern. This is achieved by considering perturbations to the flavon alignment which directly leads to TBM mixing and which might arise due to non-renormalizable corrections to the flavon potential. Consider the perturbed alignment:

$$\langle\mathbf{\Phi}\rangle_{\text{TBM}} = (r,0,0),\ \langle\mathbf{\Psi}\rangle = s(1, 1+\varepsilon_1, 1+\varepsilon_2)\,. \tag{20}$$

The (leptonic) Yukawa Lagrangian then becomes:

$$
\begin{aligned}
\mathcal{L}^l &= -\,\mathbf{Y}^l_{\alpha\beta}\,\overline{\ell_{\alpha L}}\,\phi\, l_{\beta R} + \text{H.c.},\\
\mathcal{L}^\nu &= -\,\mathbf{Y}^{\Delta_1}_{\alpha\beta}\,\overline{\ell^{\,C}_{\alpha L}}\,\Delta_1\,\ell_{\beta L} - \mathbf{Y}^{\Delta_2}_{\alpha\beta}\,\overline{\ell^{\,C}_{\alpha L}}\,\Delta_2\,\ell_{\beta L} + \text{H.c.},
\end{aligned}
\tag{21}
$$

where one recognizes the familiar charged lepton Yukawa and type II seesaw terms, with:

$$\mathbf{Y}^l = \frac{1}{\Lambda}\begin{pmatrix} y_e & 0 & 0\\ 0 & y_\mu & 0\\ 0 & 0 & y_\tau\end{pmatrix},\ \mathbf{Y}^{\Delta_1} = y_{\Delta_1}\begin{pmatrix} 1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \tag{22}$$

$$\mathbf{Y}^{\Delta_2} = \frac{y_{\Delta_2}}{3}\begin{pmatrix} 2 & -1-\varepsilon_2 & -1-\varepsilon_1\\ -1-\varepsilon_2 & 2(1+\varepsilon_1) & -1\\ -1-\varepsilon_1 & -1 & 2(1+\varepsilon_2)\end{pmatrix}. \tag{23}$$

In the above, $y_{e,\mu,\tau} \equiv r\, y^l_{e,\mu,\tau}/\Lambda$, $y_{\Delta_1} \equiv \frac{v_S}{\Lambda'}\left(y_1\, e^{i\alpha} + y_1'\, e^{-i\alpha}\right)$, and $y_{\Delta_2} \equiv s\, y_2/\Lambda$. The useful definitions $2\, u_1\, y_{\Delta_1} \equiv z_1 e^{i\beta}$ and $2\, u_1\, y_{\Delta_1} \equiv z_2$ (with z_1 and z_2 real) are considered (recall Eq. (14)).

The neutrino mass eigenvalues are given, as a function of perturbations, by $m_2^2 \simeq z_1^2$ and:

$$
\begin{aligned}
m_{1,3}^2 \simeq z_1^2 + \frac{1}{3}\Big(& z_2^2\,(3 + 2\varepsilon_1 + 2\varepsilon_2)\\
&\pm 2\, z_1\, z_2\,(3 + \varepsilon_1 + \varepsilon_2)\,\cos\beta\Big)\,.
\end{aligned}
\tag{24}
$$

For small enough perturbations, $|\varepsilon_{1,2}| \ll 1$, only normal ordering is allowed. As for the mixing pattern, one obtains the following deviations to TBM:

$$
\begin{aligned}
\sin^2\theta_{12} &\simeq \frac{1}{3}\left[1 + \frac{2}{3}\,(\varepsilon_1 + \varepsilon_2)\right],\\
\sin^2\theta_{23} &\simeq \frac{1}{2}\left[1 + \frac{1}{3}\,(\varepsilon_1 - \varepsilon_2)\right],\\
\sin^2\theta_{13} &\simeq \frac{1}{2}\left(\frac{\varepsilon_1 - \varepsilon_2}{6\cos\beta}\right)^2.
\end{aligned}
\tag{25}
$$

As far as mixing phases are concerned, one can measure Dirac-type CP violation through the following invariant [11, 39] (U is short for U_{PMNS}):

$$J_{\text{CP}} = \frac{1}{8}\Big(\textstyle\prod_{\theta_{ij}}\,\sin(2\,\theta_{ij})\Big)\,\cos\theta_{13}\,\sin\delta\,. \tag{26}$$

For $\Delta m_{31}^2 \gg \Delta m_{21}^2$, one obtains the result $J_{\text{CP}} \simeq \tan\beta\,(\varepsilon_2 - \varepsilon_1)/36 \Rightarrow \sin\delta \simeq \pm\sin\beta$.

One is now in a position to consider experimental constraints on the perturbed version of model. This is done in Fig. 3: the left part illustrates the allowed regions of the $(\varepsilon_1, \varepsilon_2)$ plane, while the right part shows the corresponding possible values of $J_{\rm CP}$. One obtains a limit $0.02 \lesssim |J_{\rm CP}| \lesssim 0.05$ at 3σ. This perturbed

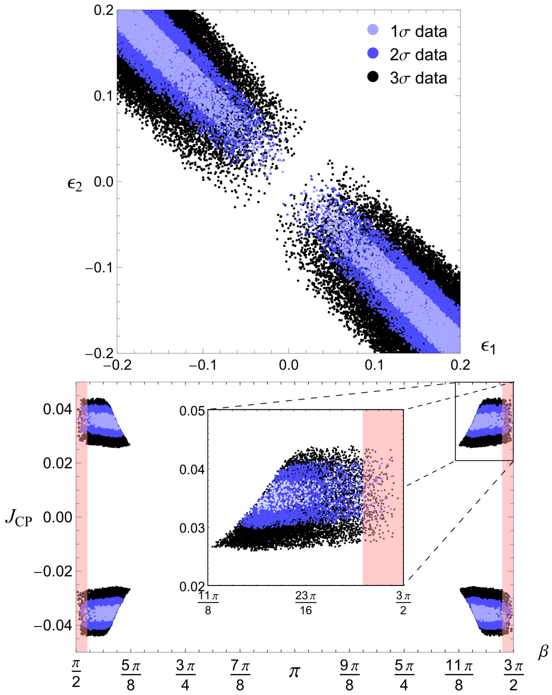

FIG. 3. Scatter plot of the experimentally allowed regions in the $(\varepsilon_1, \varepsilon_2)$ plane (up), where exact TBM is seen to be excluded, and corresponding regions of the $(J_{\rm CP}, \beta)$ plane (down).

version of the model will be reconsidered later on, as we analyse the viability of producing the BAU within its context.

VI. THE BARYON ASYMMETRY OF THE UNIVERSE

We live in an apparently biased Universe in which there is, as far as one can see [40], a clear preference for the presence of matter over antimatter. How could this imbalance come to be? One assumes that it is either a consequence of the initial conditions of the Universe or it is dynamically generated as the Universe cools down. The reason why the accidental asymmetry case is usually rejected is twofold. On the one hand, generating the observed asymmetry would require one extra quark for every $\sim 10^7$ antiquarks in the early Universe [41] (an unnatural scenario). On the other hand, a constraint arises if the effects of inflation – which is predicted to exponentially dilute any initial asymmetry – are considered.

In the dynamical case, the symmetry violating processes should be weak enough in order to produce the observed values of the BAU, which can be quantified in the present epoch through the parameter η, defined as the ratio between the number density of baryonic number, $n_{\rm B}$, and that of photons, n_γ:

$$\eta \equiv \frac{n_{\rm B}}{n_\gamma} \equiv \frac{n_b - n_{\bar b}}{n_\gamma} \simeq \frac{n_b}{n_\gamma}, \tag{27}$$

where n_b and $n_{\bar b}$ represent the densities of baryons and antibaryons. The value of η also signals the imbalance between matter and radiation in the present-day Universe. The abundances of light elements and the anisotropies of the cosmic microwave background (CMB) constrain it independently. One verifies that both conditions give compatible values for the BAU parameter, which when combined give [21]:

$$\eta = (6.19 \pm 0.15) \times 10^{-10}. \tag{28}$$

An equivalent description is given by $Y_{\rm B} \simeq \eta/7.04$, corresponding to the ratio between $n_{\rm B}$ and the current entropy density of the Universe s $(Y_i \equiv n_i/s)$.

Equilibrium thermodynamics considerations lead to the conclusion that the Universe must have possessed a baryon asymmetry already at early times. To put matters into perspective, Fig. 4 depicts important stages in the thermal history of the Universe.

The evolution of contents of the Universe has been driven by departures from thermal equilibrium. As a rule of thumb, one may consider an interaction to be out of equilibrium if the corresponding rate Γ is not fast enough to accompany the expansion of the Universe, $\Gamma \ll H(T)$ (H represents the Hubble parameter, and T the temperature of the Universe). On the contrary, if interactions are fast, $\Gamma \gg H(T)$, one takes the corresponding particle species to be in equilibrium. The non-trivial case is located in between, with $\Gamma \sim H(T)$, and demands a more careful and quantitative treatment, which relies on the Boltzmann transport equation.

VII. THERMAL LEPTOGENESIS

Taking an initial state of zero baryon number, a baryon asymmetry can be generated if the following sufficient conditions, presented by Andrei Sakharov in 1967 [42], hold:

- **B symmetry** is violated.
- **C and CP symmetries** are violated.
- Departures from **thermal equilibrium** occur.

Although the Sakharov conditions are not necessary conditions for the BAU generation [43], trying to get around any one of the three is generally not easy and might require troublesome assumptions.

The SM appears to be insufficient in explaining the presently observed BAU in light of the Sakharov conditions: it offers neither a first-order electroweak phase transition (EWPT) nor enough CP violation. One interesting alternative scenario is that of leptogenesis [44], in which the decays of heavy particles – such as the seesaw mediators of Section III B – violate lepton number L. Lepton-number asymmetries would then be converted into baryon-number asymmetries by electroweak sphaleron processes, whereas the required CP violation would arise from complex couplings in the heavy particle interaction Lagrangian.

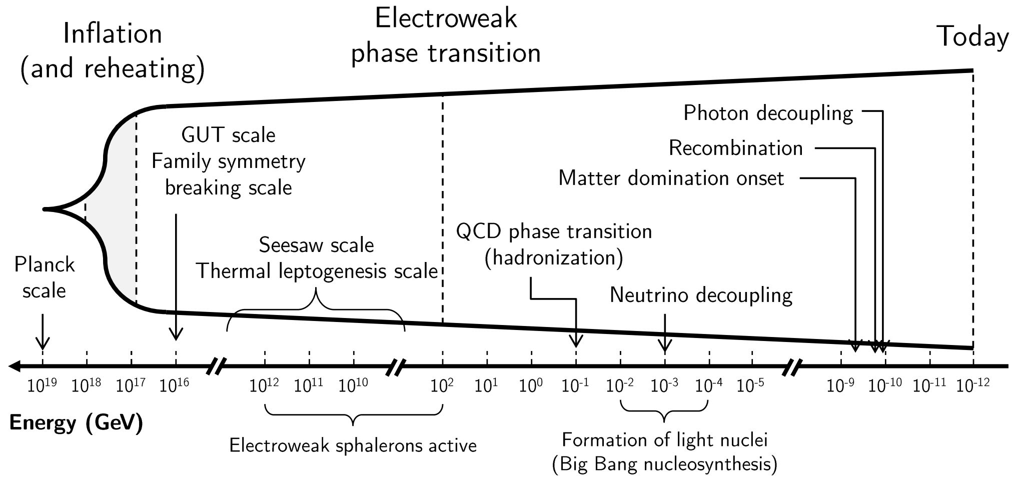

FIG. 4. Brief thermal history of the Universe. The family symmetry breaking scale is often taken to be of the order of the GUT scale.

We will henceforth focus on the case of *thermal* leptogenesis, meaning that the heavy particles (here taken to correspond to seesaw mediators) are produced thermally, following reheating, by scattering processes in the plasma.

Having accounted for possible deviations from thermal equilibrium due to the extreme mass of the new states and the expansion of the Universe, one turns to the remaining Sakharov conditions.

If the decays of the new species violate L alone, then B−L is also broken. This is a crucial aspect of baryogenesis models operating at high scales $T \gg T_{EW}$, as electroweak sphalerons (which tend to wash out all asymmetries) cannot erase B−L. For a second-order EWPT, the following relation between lepton and baryon numbers at the electroweak scale is imposed by fast sphaleron interactions [45, 46]:

$$Y_{\mathrm{B}} = \frac{12}{37}\, Y_{\mathrm{B-L}} \Rightarrow Y_{\mathrm{B}} = -\frac{12}{25}\, Y_{\mathrm{L}} = -0.48\, Y_{\mathrm{L}}\,. \tag{29}$$

The remaining Sakharov condition, CP violation, is satisfied through CP asymmetries in the quantum amplitudes for decay processes. These generally arise, at leading order, from the interference between a tree-level and a one-loop diagram.

VIII. TYPE II LEPTOGENESIS

In the context of leptogenesis, Boltzmann equations (BEs) can be used to quantify the evolution of an asymmetry in lepton number. They are here applied to the case where the decaying particles generating the lepton asymmetries are the heavy scalar triplets Δ_i of the type II seesaw mechanism. The flavoured parameters which quantify CP asymmetries in decays and inverse decays of the Δ_i are given by:

$$\varepsilon_i^{\alpha\beta} = 2\,\frac{\Gamma(\Delta_i^* \to \ell_\alpha\, \ell_\beta) - \Gamma(\Delta_i \to \overline{\ell}_\alpha\, \overline{\ell}_\beta)}{\Gamma_{\Delta_i} + \Gamma_{\Delta_i^*}} \tag{30}$$

where Γ_{Δ_i} is the total triplet decay rate. These arise from the interference bewteen the diagrams presented in Fig. 5. For $M_j \gg M_i$, one obtains the CP asymmetries of Eq. (31).

$$\varepsilon_i^{\alpha\beta} = -\,\frac{2}{1+\delta_{\alpha\beta}}\,\frac{1}{2\pi}\sum_{j\neq i}\frac{M_i}{M_j}\,\frac{\mathrm{Im}\Big(\lambda_i^*\lambda_j\,\mathbf{Y}^{\Delta_i}_{\alpha\beta}\mathbf{Y}^{\Delta_j *}_{\alpha\beta} + (M_i/M_j)\,\mathrm{Tr}\big[\mathbf{Y}^{\Delta_i\,\dagger}\,\mathbf{Y}^{\Delta_j}\big]\mathbf{Y}^{\Delta_i}_{\alpha\beta}\mathbf{Y}^{\Delta_j *}_{\alpha\beta}\Big)}{\mathrm{Tr}\big[\mathbf{Y}^{\Delta_i\,\dagger}\,\mathbf{Y}^{\Delta_i}\big] + |\lambda_i|^2}\,. \tag{31}$$

The Boltzmann equation formalism has been developed in Chapter 4 of the present thesis. The full network of Boltzmann equations, needed to compute number densities of asymmeties is explicitly given, for the type II seesaw leptogenesis framework, in Section 5.2 of the thesis.

Consider, henceforth, the particular context of the perturbed A_4 model developed in Section V. Some working assumptions (see Section 5.4), including that of hierarchical triplets, $M_2 \gg M_1$, have been made. The maximum values which the CP asymmetries may attain are, as a function of the parameters of the model, given by:

$$\varepsilon^0_{1,\mathrm{max}} \simeq \frac{\sqrt{\Delta m^2_{31}}}{12\sqrt{6}\pi v^2}\left[1-\frac{1}{3}(\varepsilon_1+\varepsilon_2)\right] M_1\,\sin\beta\,, \tag{32}$$

$$\varepsilon^0_{2,\mathrm{max}} \simeq -\frac{\sqrt{\Delta m^2_{31}}}{48\pi v^2}\left[1-\frac{1}{9}(\varepsilon_1+\varepsilon_2)\right] M_2\,\tan\beta\,. \tag{33}$$

The magnitudes of these maximum values are shown as a contour plot in the (β, M_i)-plane in Fig. 6.

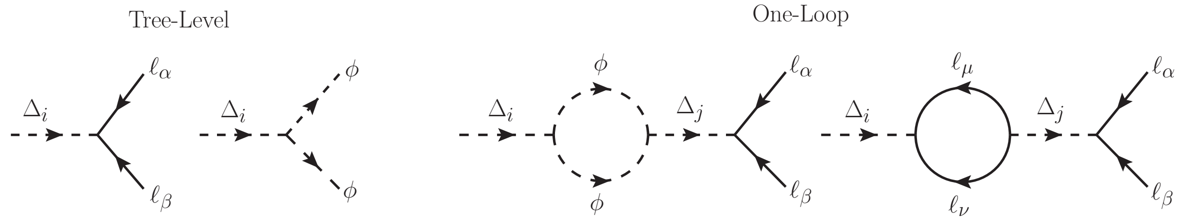

FIG. 5. Tree-level diagrams for the decays of type II seesaw scalar triplets and one-loop diagrams contributing to the decay process $\Delta_i \to \ell_\alpha \ell_\beta$.

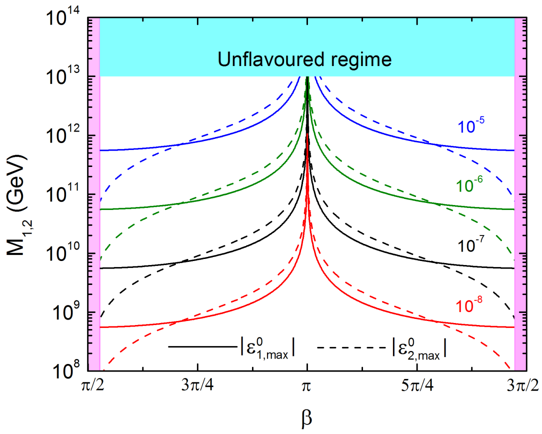

FIG. 6. Contours of the (magnitude of the) maximum CP asymmetries in the decays of hierarchical scalar triplets $\Delta_{1,2}$.

The plot shows that, for certain points in the (β,M_i) plane, relatively large CP asymmetries are obtained. To finally solve the system of BEs one needs to compute the reaction densites for the relevant processes, given in Fig. 7. These densities are obtained from the integration of reduced cross sections, whose explicit form is presented (for the model at hand) in Section 5.3 of this thesis. The BE integration was performed numerically through a Fortran-based programme. Fig. 8 shows the results for the baryon asymmetry density Y_B for two choices of M_1, after compatibility with neutrino oscillation data has been assured.

We now restrict ourselves to a particular point, chosen to lie in the region above the green band (details are given in Section 5.4). Reaction densities and the evolution of asymmetries for this particular perturbed version of the model are given in Fig. 9. A baryon asymmetry $Y_{\rm B} \simeq 5.93 \times 10^{-10}$ is obtained for this point, following the decoupling of the lightest triplet, by summing over all three $Y_{{\rm BL}\alpha}$ and converting the resulting $Y_{\rm B-L}$ asymmetry into a purely baryonic one through (29).

We have seen that the model is sufficient to account for the observed baryon asymmetry of the Universe. One has assumed that triplet masses are hierarchical, taken to imply that asymmetries produced prior to the decay of the lightest triplet have been washed out. The fact that CP asymmetries in the out-of-equilibrium decays of Δ_2 can overshadow those of Δ_1 by as much as an order of magnitude (see Fig. 6) challenges this assumption. Flavour considerations may nevertheless come to our rescue, as a sufficiently massive Δ_2 may decay during the unflavoured regime, for which all different lepton flavours are out-of-equilibrium and indistinguishable. The structure of the model prevents, in such a case, the generation of asymmetries.

IX. CONCLUDING REMARKS

The most popular SM extensions accommodating massive neutrinos in a natural way are those in which neutrino masses are generated through the tree-level exchange of new heavy particles. We have analysed the phenomenological viability of a specific type II seesaw model based on an A_4 non-Abelian discrete symmetry. In this scenario, CP-violating effects stem from a single complex phase associated to the VEV of a scalar singlet. The model is compatible with all data provided by neutrino oscillation experiments and predicts large CP-violating effects, detectable in future experiments.

The connection between neutrino physics and cosmology may be established within the leptogenesis scenario, where the excess of baryons in the Universe relies on a lepton asymmetry. For the specific case of type II seesaw leptogenesis, the starting point to generate that lepton asymmetry is the out-of-equilibrium decay of the heavy-triplet scalars. The viability of the leptogenesis scenario in the framework of the aforementioned A_4 model has also been explored. We have concluded that the model not only allows to reproduce the current neutrino mass and mixing pattern, but also generates a sufficiently large baryon asymmetry of the Universe, in accordance with the experimental result.

Our conclusions justify the purpose of this thesis: to relate neutrino masses and mixing, symmetries and the origin of matter. What we have presented is just an example (among many others) of how complementary studies may shed some light on answering open questions in fundamental physics, the future of which hinges on the combined exploration of physical phenomena at the energy, intensity and cosmic frontiers.

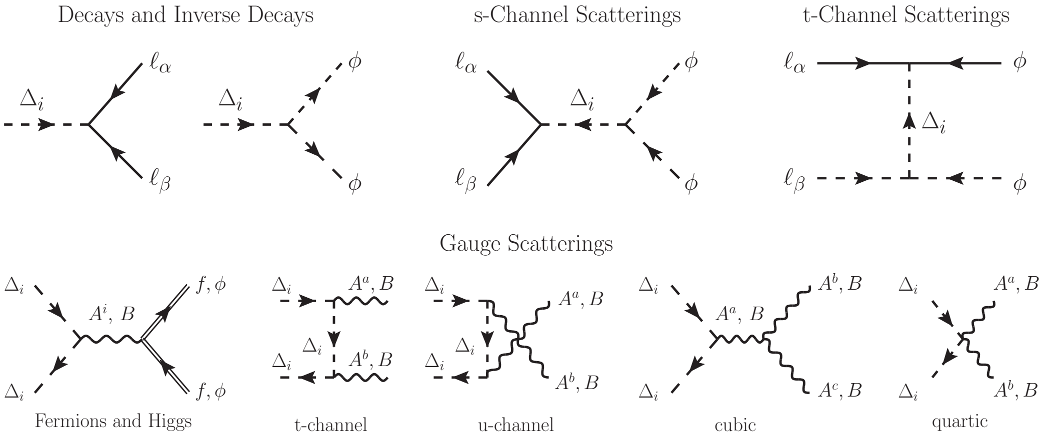

FIG. 7. Scalar triplet interactions relevant to the BE out-of-equilibrium analysis, where one considers the diagrams presented for decays, inverse decays and s- and t-channel scatterings and their charge conjugates, as well as gauge scattering reactions.

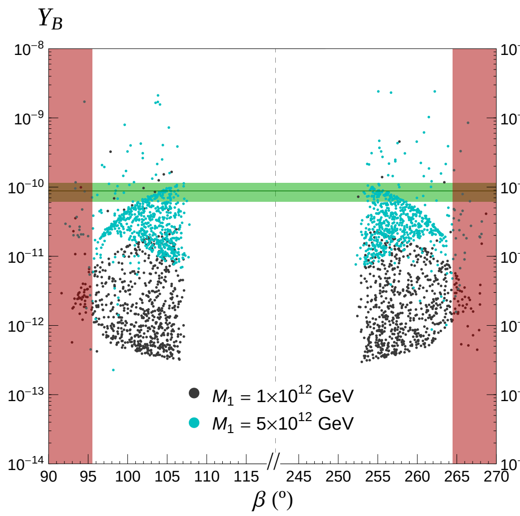

FIG. 8. Scatter plot of the baryon asymmetry generated in randomly-chosen perturbed versions of the model, for $M_1 = 10^{12}$ GeV (black) and $M_1 = 5 \times 10^{12}$ GeV (cyan).

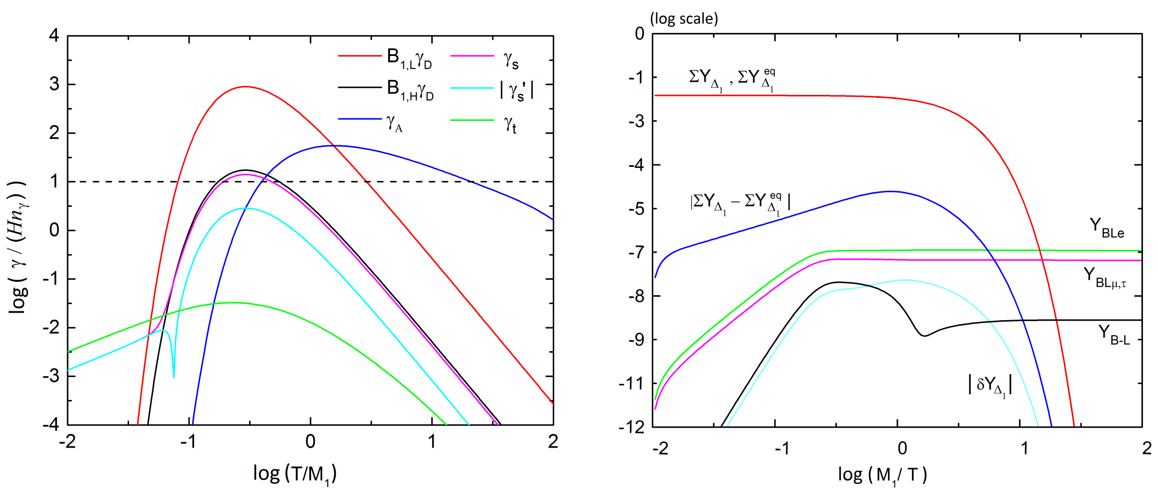

FIG. 9. Reaction densities normalized to the product $H(T)\, n_\gamma(T)$ (left) and evolution of the various densities considered in the BE network (right). Gauge scatterings keep triplets close to equilibrium. The relevant densities here are the flavoured $Y_{\mathrm{BL}\alpha} \equiv Y_{\mathrm{B}}/3 - Y_{\mathrm{L}_\alpha}$ (for a description of the remainder, see Chapter 5).

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