Multiple Link Exchange for Distribution Network Optimization

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Abstract—This paper aims to develop two algorithms for the electric power distribution networks optimization, Single Link Exchange and Multiple Link Exchange, in order to minimize the real power losses. These algorithms were developed in MATLAB and tested on several networks. Towards testing the limits of the algorithms, the resistance of some branches was increased and random values of impedance were generated for the networks. Then, the algorithms were compared to an iterative method and to other distribution networks optimization algorithms. The obtained results showed that, unlike the Single Link Exchange, the Multiple Link Exchange is computationally very demanding, which leads to a huge runtime, but it rarely gets trapped in a local optimum solution.

Index Terms—Distribution networks reconfiguration, Minimal power losses, Single Link Exchange, Multiple Link Exchange

I. INTRODUCTION

Distribution networks are part of an electrical energy system, which comprises the production, transportation, distribution and consumption of electrical energy. The production of this energy is mostly held in thermal and hydro plants, which are located far from high consumption places, due to the unavailability of primary energy resources and infrastructure in these locations. Then, this energy enters the transport network, made up of high voltage lines. Afterwards, through transformers located in substations, the electric power reaches the distribution network, responsible for conducting it to the consumer.

Regarding the topological structure, these networks are mainly radial. A radial network is a network composed of lines that branch out from a feeding point, without ever meet. Such networks are characterized by lower reliability and lower cost than meshed ones.

The purpose of this paper is the development of two algorithms for the optimization of distribution networks: the Single Link Exchange (SLE) and the Multiple Link Exchange (MLE). This optimization targets the reduction of the real power losses in distribution networks. The algorithms will be compared and their limitations verified.

This paper is organized as follows. In Section II, the problem is formulated. In Section III, the developed algorithms are explained and tested in four distribution networks. In Section IV, the resistance of some branches in two networks are increased. In Section V, random values of impedance were generated for one network. In Section VI, an analogy of the Jacobi-Gauss-Seidel method to the developed algorithms is made. In Section VII, the conclusions of the paper are presented.

II. FORMULATION OF THE PROBLEM

In this section, the network reconfiguration problem for loss reduction is formulated.

A. Problem Statement

Network reconfiguration is defined by changes in the network structure, as a consequence of opening and closing branches. The main goals of this technique are to ensure the service restoration under contingencies, to reduce the power losses and to relieve the overloads in the network.

As the goal of this paper is the reduction of real power losses, the problem can be formulated as follows:

\[
\begin{align*}
\text{Minimize } P_L \\
\text{Subject to } \\
I_l &\leq I_{\text{max}} \quad l = 1, ..., \text{No. of branches} \\
V_{\text{min}} &\leq V_i \leq V_{\text{max}} \quad i = 1, ..., \text{No. of buses} \\
\end{align*}
\]

Where \( P_L \) is the real power loss of the system, \( I_l \) the current magnitude of branch \( l \), \( I_{\text{max}} \) the maximum current carrying capacity of branch \( l \), \( V_i \) the voltage magnitude at the bus \( i \) and \( V_{\text{min}} \) and \( V_{\text{max}} \) the minimum and maximum bus voltages at the bus \( i \) (0.9 p.u. and 1.1 p.u. respectively).

For a better understanding of the process of network reconfiguration and their limitations, we present Fig.1. By simplification, the loads along a feeder section are constant PQ loads, placed at the end of the lines.

![Fig. 1. Radial distribution system](image-url)
In Fig. 1, solid branches represent the lines that are in service. Some of these branches have switches, represented by rectangles, which are closed and named sectionalizing switches. About the dotted branches, they represent the lines with open switches called tie switches. The network can be reconfigured by closing a tie switch and opening a sectionalizing switch. This process is called branch exchange or link exchange.

Regarding the radial network structure restriction, it can be seen that if the branch 21 is closed, a loop formed by lines 1, 2, 3, 21, 7, 6 is created. So, it is necessary to open a branch with a sectionalizing switch (also known as a tie line), in order to restore the radial structure of the system. Any of the switches in the branches 1, 3 or 6 can be opened, to break the loop.

### B. Power Flow Equations

Besides the initial power flow, it is necessary to perform a power flow on each reconfiguration of the network, for the purpose of calculating the real power losses.

The injected power is defined by the difference between the generated and consumed power, and it is given by:

\[ S_i = S_{Gi} - S_{Ci} \]  

By applying the Kirchhoff's current law to the bus \( i \), of the \( \pi \) equivalent diagram of the line connecting buses \( i \) and \( j \), Fig. 2, we obtain:

\[ S_i = \frac{V_i^*}{V_i} = \sum_{j=1}^{n} \left( \frac{1}{Z_L} \right) \frac{V_i}{V_j} + \sum_{j=1}^{n} \left( -\frac{1}{Z_L} \right) V_j \]  

Or

\[ \frac{S_i^*}{V_i^*} = y_{ii}V_i + \sum_{j \neq i}^{n} y_{ij}V_j \]  

Where

\[ y_{ii} = \sum_{j \neq i}^{n} \left( \frac{1}{Z_L} \right) \]  

\[ y_{ij} = y_{ji} = -\frac{1}{Z_L} \]  

Knowing that \( V_i = V_ie^{j\theta_i} \) and \( y = G + jB \), we obtain the real and reactive power:

\[ P_i = P_{Gi} - P_{Ci} = \sum_{j=1}^{n} V_i V_j G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j) \]  

\[ Q_i = Q_{Gi} - Q_{Ci} = \sum_{j=1}^{n} V_i V_j G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \]

Since the power flow equations are nonlinear, it is necessary to use an iterative method. The method chosen was the Newton-Raphson, which is the reference method in the power flow solution. Its steps are explained in [1].

### C. Calculation of the Objective Terms

After finding the solution of the power flow, it is time to compute the objective function, the real power losses, which is given by the sum of injected power in all buses:

\[ P_L = \sum_{i=1}^{n} P_{Gi} - P_{Ci} \]

### III. DEVELOPED ALGORITHMS

In this section, the two developed algorithms, SLE and MLE, are presented and tested in four networks.

#### A. Single Link Exchange

As the name implies, SLE is an algorithm that inserts and removes branches of the network, one by one.

Starting from an initial configuration, the algorithm will insert a branch in the network, find the loop created and, if there is loss reduction, remove a branch of the same loop. The process ends when the configuration with fewer losses, the optimal solution, is found. Note that, in each reconfiguration, it is necessary to perform a power flow, in order to find the real power losses for that configuration.

The steps of the SLE algorithm are as follows:

Step 1 – Perform a power flow, in order to estimate the initial power losses.

Step 2 – Insert a branch of the list of tie lines.

Step 3 – Find the loop created.

Step 4 – Remove the loop branches one by one and compute the real power losses after each branch removal, towards finding the branch which leads to the minimal power losses.

Step 5 – If the removal of the branch found in step 4 contributes to the reduction of the real power losses in the network, remove that branch, which goes to the list of tie lines.

Step 6 – Repeat steps 2 to 5, inserting the following tie lines one by one, until the optimal configuration is found.
B. Multiple Link Exchange

Unlike the SLE, the MLE inserts and removes multiple branches at a time. Due to the large increase of complexity as the number of branches to insert and remove at a time increases, the developed MLE is limited to two branch exchanges at a time.

Given an initial configuration, the algorithm inserts two branches in the network, finds the two loops created and, if there is loss reduction, removes one or two branches of the network (one of each loop).

The steps of the MLE algorithm are as follows:

- **Step 1 -** Perform a power flow, in order to estimate the initial power losses.
- **Step 2 -** Insert a branch of the list of tie lines.
- **Step 3 -** Find the loop created.
- **Step 4 -** Repeat steps 2 and 3 to the next branch of the list of tie lines, removing the branch inserted in step 2, so that the loop found is different than the one found in step 3. Then, insert the branch again.
- **Step 5 -** Remove a branch of the first loop.
- **Step 6 -** If the removed branch is not common to the second loop, remove the branches of this loop one by one and compute the real power losses after each branch removal, towards finding the branch which leads to the minimal power losses. Otherwise, insert the removed branch.
- **Step 7 -** Repeat steps 5 and 6 for all the branches of the first loop.
- **Step 8 -** If the loops contain branches in common, go to step 9. Otherwise, go to step 12.
- **Step 9 -** Remove a branch of the second loop.
- **Step 10 -** If the removed branch is not common to the first loop, remove the branches of this loop one by one and compute the real power losses after each branch removal, towards finding the branch which leads to the minimal power losses. Otherwise, insert the removed branch.
- **Step 11 -** Repeat steps 9 and 10 for all the branches of the second loop.
- **Step 12 -** If the removal of the branches found in steps 6 and 10 contributes to the reduction of the real power losses in the network, remove these branches, which go to the list of tie lines.
- **Step 13 -** Repeat steps 2 to 12, inserting the following tie lines, until the optimal configuration is found.

C. Test Results

The SLE and MLE algorithms were tested in four distribution systems: 8-bus distribution system from [2], 16-bus distribution system from [5], 33-bus distribution system from [6], and 94-bus distribution system from [7].

The initial configuration, nominal real power loss and minimum node voltage of these test systems have been summarized in Table I and the results of the algorithms are given in Table II.

It can be observed, in Table II, that both algorithms reach the same configuration for the four test systems. About the computed power flows, the MLE algorithm performs a lot more than the SLE algorithm, which translates into a superior runtime.
IV. INCREASE OF RESISTANCES

To verify the behavior of the algorithms in adverse conditions, the resistances of some branches in the 8-bus and 16-bus networks were increased, while the other parameters of the networks remained unchanged.

In the 8-bus system, it was verified that when the resistance of one branch is increased, Fig. 3, the SLE reaches the same real power losses that the MLE. However, when the resistance of two branches is increased simultaneously, Figs. 4 and 5, there is a value of resistance from which the SLE cannot find the optimal network found by the MLE. This value of resistance is much smaller in branches connected to the substation, Fig. 5, than in other branches, Fig. 4.

Towards reaching conclusions, it was necessary to make the 16-bus distribution system a network where all the impedances and power loads were the same. The impedances were considered to be $0.04+j0.04$ p.u. and the power loads $2+j2$ MVA.
In the modified 16-bus system, the same was observed: Fig. 6 shows that when the resistance of only one branch is increased, the SLE reaches the same solution that the MLE and Figs. 7 and 8 show that there is a value of resistance from which the SLE cannot find the optimal network found by the MLE.

As these tests were made for several branches, it is safe to conclude that when the resistance of only one branch is increased, the SLE reaches the same final configuration found by the MLE. It can also be concluded that when the resistance of two branches connected to a substation is increased, as of a certain resistance, the SLE cannot find the network of minimal power losses found by the MLE. This also happens for branches that are not connected to a substation, although the resistance must be much higher and the branches have to belong to different loops.

V. RANDOM IMPEDANCES

In this section, the developed algorithms were tested when the 8-bus network is composed of random impedances.

In the first test the impedances were the ones in Tables III and IV, and in the second test the ones in Tables V and VI. The results are presented in Tables VII and VIII.

Since some values of resistance and reactance were negative, causing negative real power losses, one immediately realizes that this is not an electrical distribution problem, but an electrical circuits problem. However, it can be verified that the SLE obtains the same configuration as the MLE in some cases, but not in others, probably due to the high absolute value of the resistance in some branches.

VI. ANALOGY OF THE JACOBI-GAUSS-SEIDEL METHOD

The Jacobi-Gauss-Seidel method is used to compute the solution of a system of linear equations. This method is efficient, in terms of computation and storage, for systems with big dimensions, with a lot of zero inputs, since the small
dimension systems are solved by direct techniques, like the Gauss elimination.

Given the system of \( n \) linear equations:

\[
Ax = b
\]  

(11)

And decomposing \( A \) in \( L, D \) an \( U \), where \( L \) is the lower triangular matrix, \( D \) the diagonal matrix and \( U \) the upper triangular matrix of the matrix \( A \), we obtain the iterative method:

\[
x^{(k+1)} = (D + L)^{-1}(b - Ux^{(k)})
\]  

(12)

Or

\[
x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j<i} a_{ij}x_j^{(k)} - \sum_{j>i} a_{ij}x_j^{(k)} \right),
\]

\[i = 1, 2, ..., n\]  

(13)

Starting from an initial vector \( x^{(0)} \), the algorithm computes the values of \( x^{(k+1)} \), until convergence. Note that the computation of \( x_i^{(k+1)} \) only uses the elements of \( x^{(k+1)} \) that had already been calculated. This method always converges, if the matrix \( A \) is strictly diagonally dominant, i.e.:

\[
|a_{ii}| > \sum_{j \neq i} |a_{ij}| \quad \text{or} \quad |a_{ii}| > \sum_{j \neq i} |a_{ij}|
\]  

(14)

The SLE algorithm can be compared to this algorithm, as it starts from an initial configuration of the network, with a set of opened branches (\( x^{(0)} \)), inserts and removes branches until the method converges to the optimal solution (\( x \)). At each step, the algorithm inserts a branch (\( x_i^{(k)} \)), from the tie lines, and removes the same branch, if its insertion does not benefit the network, or another branch. The removed branch is the solution \( x^{(k+1)} \). In the insertion of the next branch (\( x_i^{(k+1)} \)), the algorithm takes into account the changes made in the previous step. About the convergence criterion, there is no measurable criterion for the SLE, but it does not converge to the optimal solution in some situations, as observed in the previous sections.

In the MLE algorithm case, we are before systems like:

\[
Ax + Bx' = b
\]  

(15)

Where the solution is the vector \( x \), with dimension 2x1. As the system can be written as:

\[
Ax = b - Bx'
\]  

(16)

The Jacobi-Gauss-Seidel method can also be used to solve this system, as \( A \) is a square matrix and \( b - Bx' \) is the same size as \( x \). The iterative method, for each pair of solutions (as the MLE computes two solutions at a time), is given by:

\[
x^{(k+1)} = (D + L)^{-1}(b - Bx' - Ux^{(k)})
\]  

(17)

These two Jacobi-Gauss-Seidel methods (equation (12) is associated to the SLE and equation (17) to the MLE) were tested in several matrices. With \( b = \begin{bmatrix} 8 \\ -4 \\ 12 \end{bmatrix} \) constant, the results are presented in Table IX, for 100 iterations.

<table>
<thead>
<tr>
<th>Matrix A</th>
<th>Method</th>
<th>Solution ( x )</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\begin{bmatrix} 1 &amp; -1 \ 1 &amp; 2 \ 2 &amp; 1 \end{bmatrix}]</td>
<td>Jacobi-Gauss-Seidel-SLE</td>
<td>\begin{bmatrix} 1.4468 \ 1.9574 \ 1.1915 \end{bmatrix}</td>
<td>12</td>
</tr>
<tr>
<td>[\begin{bmatrix} 2 &amp; 1 &amp; -1 \ 1 &amp; -1 &amp; 2 \ 2 &amp; 1 &amp; 3 \end{bmatrix}]</td>
<td>Jacobi-Gauss-Seidel-MLE</td>
<td>\begin{bmatrix} 1.4468 \ 1.9574 \ 1.1915 \end{bmatrix}</td>
<td>7</td>
</tr>
<tr>
<td>[\begin{bmatrix} 1 &amp; 1 &amp; -1 \ 1 &amp; -1 &amp; 2 \ 1 &amp; 2 &amp; 2 \end{bmatrix}]</td>
<td>Jacobi-Gauss-Seidel-SLE</td>
<td>\begin{bmatrix} 1.1915 \ 1.4468 \ 1.9574 \end{bmatrix}</td>
<td>(10^5)</td>
</tr>
<tr>
<td>[\begin{bmatrix} 1 &amp; 1 &amp; -1 \ 1 &amp; -1 &amp; 2 \ 1 &amp; 2 &amp; 2 \end{bmatrix}]</td>
<td>Jacobi-Gauss-Seidel-MLE</td>
<td>\begin{bmatrix} 1.1915 \ 1.4468 \ 1.9574 \end{bmatrix}</td>
<td>(10^5)</td>
</tr>
</tbody>
</table>

Both methods converge to the right solution, when the matrix \( A \) is diagonally dominant. Reducing the values of the diagonal leads to the non-convergence of the first method, although the second method converges to the right solution. Finally, when the matrix is far from diagonally dominant, both methods can’t find the solution of the system.

These results were very similar to the ones obtained throughout the paper, as the algorithms reach the same real power losses for the distribution networks tested but, as the disorder of the systems increases, like when the resistances of some branches are increased, first the SLE and then both algorithms cannot find a configuration better than the initial one.

VII. COMPARISON WITH OTHER ALGORITHMS

In this section, the SLE and MLE algorithms are compared to other algorithms of reconfiguration of distribution networks: Ant Colony Search Algorithm, Genetic Algorithm and Tabu Search Algorithm.
The first algorithm mimics the behavior of ants: when an ant finds a good path from the colony to a food source, the other ants are more likely to follow that path, due to the pheromone trails laid down by the ants.

In the second algorithm, a population of individuals evolves towards better solutions. The fittest individuals are chosen to form new generations and the algorithm ends when a maximum number of generations has been produced or the population value of the objective function in the optimization problem has been reached.

The third algorithm is an algorithm which manages local searches, using memory structures in order to avoid getting trapped in local optimum solutions.

The results are presented in Table X. Due to the fact that the Genetic and Tabu Search algorithms were tested in a 135-bus system, from [8], the developed algorithms were also tested. The initial real power losses in this network are 320.268 kW.

Table X: Comparison Results for Test Distribution Systems

<table>
<thead>
<tr>
<th>Test System</th>
<th>Method</th>
<th>Opened branches</th>
<th>Real Power Losses (kW)</th>
<th>Runtime (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33-bus</td>
<td>SLE</td>
<td>8-7, 14-15, 10-9, 32-33, 25-29</td>
<td>139.551</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>MLE</td>
<td>8-7, 14-15, 10-9, 32-33, 25-29</td>
<td>139.551</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>Genetic</td>
<td>8-7, 14-15, 10-9, 32-33, 25-29</td>
<td>139.6</td>
<td>5</td>
</tr>
</tbody>
</table>

In Table X, it can be observed that the two developed algorithms reach approximately the same configuration as the other algorithms. Furthermore, for small networks, the SLE runtime is identical or less than the compared algorithms. About the MLE, its runtime is much superior to the other algorithms in all the tested networks.

VIII. CONCLUSIONS

In this paper, two algorithms of distribution network optimization (SLE and MLE) were developed and compared.

The SLE algorithm, for uniformed or real networks, reaches the same final configuration that the MLE and other algorithms reach. However, when the conditions of the network are adverse, like when two branches connected to a substation have high values of resistance, or in an electrical circuits problem, the SLE gets trapped in a local optimum solution.

These algorithms can be compared to a simple iteration method as the Jacobi-Gauss-Seidel method, indicating the inferior complexity of these algorithms over other distribution network optimization algorithms.

In the end, we conclude that the MLE, unlike the SLE, is computationally very demanding, leading to a huge runtime, but it is an algorithm which presents greater confidence in obtaining the minimal power losses configuration, since the it rarely gets trapped in a local optimum solution.

REFERENCES


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