An Advanced Navigation System for Remotely Operated Vehicles

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Se eu não morresse, nunca! E eternamente buscases e conseguisse a perfeição das cousas!

Cesário Verde
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It is often said that a man is nothing more than the sum of his experiences. Similarly, everything one does is undeniably linked to his or her personality and character. Following this reasoning, the list of people who have had an impact on this work is far too vast to be included here. Special mentions will thus be given to the ones who had a more direct influence on this thesis, which is not to say that all others are disregarded or forgotten.

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Abstract

The usefulness and possible applications of Autonomous Underwater Vehicle (AUV) and Remotely Operated Vehicle (ROV) in underwater applications is often limited or defined by their navigation capabilities. While robots and autonomous systems of all kinds need some sort of navigation system, leading to it being a well known and developed field, the specificities of underwater robotics make it a particularly challenging environment.

This work will be concerned with the estimation of absolute position and linear velocity of underwater robots, taking into account the typical sensors available for them. Ultra-Short Baseline (USBL) and inertial sensors (measuring 3D orientation, angular velocities and linear accelerations) will be considered in particular, since they are common even amongst the smaller and cheaper vehicles.

Doppler Velocity Log (DVL) sensors, which are relatively expensive and heavy sensors, are also considered. In fact, assessing the performance limits with and without this sensor is one additional goal of this work in order to assist in the decision of installing it or not.

Finally, both DVL and USBL sensors require some calibration procedures in order to achieve their full potential. How to perform these in-situ is another focus of this thesis.

Keywords

Calibration, DVL, Navigation, Robotics, Underwater, USBL
Resumo

A utilidade e as áreas de aplicação de AUVs e ROVs são frequentemente limitadas ou definidas pelas suas capacidades de navegação. Ainda que todos os tipos de robots e sistemas autónomos necessitem de algum tipo de sistema de navegação, o que levou a que esta área da robótica esteja bem desenvolvida, as especificidades inerentes à robótica marítima levam a que este ambiente represente um desafio particularmente interessante.

Este trabalho é essencialmente dedicado a como estimar a posição absoluta e a velocidade linear de veículos subaquáticos, tendo em conta os sensores tipicamente disponíveis nesse ambiente. USBL e sensores inerciais (capazes de medir a pose tridimensional, velocidades angulares e acelerações lineares) vão ser alvo de particular atenção, já que são extremamente comuns mesmo entre os veículos mais pequenos e baratos.

DVLs, uma classe de sensores relativamente cara, também vão ser considerados. Perceber se a sua dispensa leva a uma inevitável e relevante redução de performance é um dos objectivos secundários deste trabalho.

Por último, para atingirem a sua máxima performance tanto os DVLs como os sensores da classe USBL requerem alguma calibração. Como realizar estes procedimentos no local (i.e: no mar) é outro foco deste trabalho.

Palavras Chave

Calibração, DVL, Navegação, Robóticas, Subaquática, USBL
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Abbreviations

AHRS  Attitude and Heading Reference System
ASC  Autonomous Surface Craft
AUV  Autonomous Underwater Vehicle
CRB  Cramér-Rao Bound
DOF  Degree of Freedom
DSOR  Dynamical Systems and Ocean Robotics
DVL  Doppler Velocity Log
EKF  Extended Kalman Filter
GIB  GPS Intelligent Buoys
GPS  Global Positioning System
IMU  Inertial Measurement Unit
INS  Inertial Navigation System
ISR  Institute for Systems and Robotics
IST  Instituto Superior Técnico
KF  Kalman Filter
LBL  Long Baseline
ML  Maximum Likelihood
MRU  Motion Reference Unit
NED  North-East-Down
NLO  Non-linear observer
RANSAC  Random Sample Consensus
RIB  Rigid Inflatable Boat
RMSE  Root Mean Square Error
ROS  Robotic Operating System
ROV  Remotely Operated Vehicle
RTK  Real Time Kinematic
SBL  Short Baseline
SVD  Singular Value Decomposition
UKF  Unscented Kalman Filter
USBL  Ultra-Short Baseline
UTM  Universal Transverse Mercator
VOS  Velocity of Sound
1 Introduction

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1.1 Problem Statement

In robotics, navigation typically refers to one of two distinct, although entwined topics: the estimation of the position (in a given space) or the generation of a desired trajectory, for a given goal. This thesis is concerned with the former.

The problem at hand is thus how to track the position over time of an underwater vehicle, be it an Autonomous Underwater Vehicle (AUV) or Remotely Operated Vehicle (ROV). Moreover, estimates of the linear velocities of the vehicle are also required. The 3D pose (and angular velocities) will be considered to be known for currently high-rate, high-accuracy drift-less sensors are available for this type of vehicles. Since at-sea experimental results will be given, all the navigation techniques considered need to take into account the specificities of underwater navigation, namely the type of sensors available.

While all sensors’ measurements are inevitably corrupted by some kind of noise, some consistently present error sources can be identified and corrected for with proper calibration. This work will also include a study of some of the most common and important error sources in typical underwater navigation sensor suites, along with in-situ (ie: at sea) calibration procedures that can enhance the navigation accuracy. These will be focused mostly on correcting assembly-time misalignments, both for Doppler Velocity Log (DVL)s and Ultra-Short Baseline (USBL)s transceivers.

1.2 Motivation

Although is it difficult to monitor the impact of maritime activities in Portugal’s economy, it is estimated that economic activities directly connected to the sea are responsible for generating about 75,000 jobs and represent around 2% of Portugal’s gross internal product. The National Strategy for the Sea includes in its objectives the increase of this percentage by 50% until 2020, which shows the growing relevance of maritime activities [9].

The roles played by AUVs and ROVs are diverse, including (but not limited to) mine counter-measure, deployment of surveillance sensors and/or cables, harbour monitoring, oceanographic missions, monitoring undersea infrastructure (communication cables, pipelines, etc...), anti-submarine warfare, submarine search and rescue and aquaculture patrol/monitoring.

It can be seen from the list above that most tasks are related to the oil industry, military/defence and scientific. There are certainly other tasks that can be performed by these types of vehicles, and their usefulness is often limited by their navigation capabilities [23].

The trend is to apply AUVs and ROVs more and more in the so called 3D situations: dull, dirty and dangerous. The advantages of their use are thus mainly non-exposure of humans to dangerous situations, accomplishment of tasks otherwise infeasible and cost reductions [24].

Although there is some difference in the type of tasks required of AUVs when compared to ROVs, the navigation challenges they face are similar in many aspects. The main difference probably relies in the fact that, for the latter, the tether may work as a fast communications link, allowing to get remote sensors’ information. For this reason throughout this work both cases will be referred to
mostly indiscriminately, with assumptions on data availability being given some focus.

Given the above, the importance of reliable navigation systems for underwater vehicles is clear. The need for *in-situ* calibration procedures for the navigation sensors arises from the fact that many of these vehicles can have variable payloads, and so it is not at all convenient (sometimes not even possible) to have some types of sensors assembled together so as to perform calibration at assembly time.

### 1.3 State of The Art

In what regards misalignments of USBL transceivers, they have been a concern for a long time, with calibration procedures proposed still before the beginning of the century [8]; a more recent one was described in [6]. However, both these methods fail to offer a closed form solution.

Recently an iterative method for the calibration of a transceiver misalignment was reported in [34], whereby the vehicle carrying the transceiver performs a line survey and the deformations observed on the trajectory given by the USBL (compared to the expected linear trajectory) are used to calibrate the array. While this method seems to require accurate manoeuvring, not always easy to perform at sea, it was experimentally evaluated with interesting results in [35].

*In-situ* calibration procedures for DVL misalignments are also present in the literature. The most intuitive methods integrate the velocities measured by the DVL and compare them with some external positioning sensors [19], [21]. A different approach is taken in [25], where no external sensors are used and the DVL velocity measurements are compared to acceleration measurements obtained through an internal Inertial Navigation System (INS) by means of integration of the latter or differentiation of the former. Those methods are extended in [26] to also incorporate accurate depth measurements coming from a depth cell. All these have been tested under laboratory conditions; the method proposed in [19] was further tested under actual at-sea experiments, also with clear improvements in performance.

An excellent survey of underwater vehicle navigation techniques was presented in [2], back in 2006. When reviewing the literature it is important to note which methods were successfully applied to at-sea trials and which were simply put to test in simulation environment.

One of the most simple and common navigation techniques are Doppler based odometry navigation, which are often coupled with other less accurate yet drift-less sensors so as to obtain bounded position error. In [15] position fixes provided by a Long Baseline (LBL) are used for comparison and to initialise the odometry procedures. A particle filter is used in [31] to fuse information from both a DVL and an USBL tested using logged data from a surface trial. In [30] a globally asymptotically stable position filter making use of an USBL and a DVL in water-lock mode is presented and tested in simulation. An Extended Kalman Filter (EKF) was chosen in [32] to fuse data from an Attitude and Heading Reference System (AHRS), an USBL and a DVL specifically addressing the issue of delayed position fixes.

Quite recently, an Unscented Kalman Filter (UKF) fusing data from a DVL, an USBL and an AHRS...
as well as a dynamical model for the vehicle, was presented and validated using simulations in [44]. A similar approach was taken in [14] with results extended to hardware-in-the-loop simulations.

A lot of attention has also been given to the case where a DVL is not present, mostly due to this sensor’s high cost. Most of the works in this field have focused on complementing inertial navigation with USBL or other acoustic positioning systems. A fusion technique based on an EKF, yet resolving the positioning and fusion problems separately, is presented in [38]. The same authors have proposed a more tightly-coupled fusion technique (also based on an EKF) exploiting directly the acoustic array spatial information in [39]. In both cases, results were limited to computer simulations.

There is also a growing trend, sometimes referred to as moving baseline navigation, whereas autonomous surface vehicles equipped with acoustic modems measure their respective range to the intended target vehicle. [12] is an example of such an attempt of multiple vehicle co-operative localisation, presenting experimental results, while a similar setup (GPS Intelligent Buoys (GIB)s) was examined with great detail in [29]. In [13], a diver positioning system through use of surface vehicles is described; this setup was the main goal of the Co3-AUV project, in which Institute for Systems and Robotics (ISR) was deeply involved, and so was the author to a lesser extent.

Other model-based observers have been proposed to estimate linear position and velocity. In [17], both a Kalman Filter (KF) based and a non-linear observer are proposed, assuming linear accelerations’ measurements and the presence of an acoustic positioning system, as well as known attitude, heading and angular velocities. A full state (position, velocity and 3D orientation) observer based on an INS, a dynamic model of the vehicle and a tightly coupled USBL was presented in [45]. An exact non-linear observer for the one Degree of Freedom (DOF) case, with both position and velocity measurements, was proposed in [36], and validated experimentally under laboratory conditions. A full six DOF state non-linear observer is considered in [7], with special care and a separate plant dedicated to the estimation of water currents. The results presented are yet again limited to computer simulation.

An interesting field of research, contraction analysis, can also be of importance to underwater navigation in the future. The theory in itself is presented in [47], while the authors propose some applications in [48]. More interesting are the relaxed criteria and application to an underwater vehicle observer presented in [49]. The author uses contraction analysis to prove exponential convergence of a non-linear one DOF full state observer, specifically addressing the fact that position fixes (measurements) are available at a much lower rate than the observer’s time step.

1.4 Thesis Outline

In chapter 2 a list of sensors relevant for underwater navigation, as well as a brief description of their working principles and error sources, will be provided. Next, in-situ calibration procedures aimed at diminishing or resolving some of those error sources will be presented in 3 and navigation techniques for underwater vehicles will be proposed in 4. The results obtained, both under simulation and real environments, will be presented in 5. Finally, chapter 6 holds some final remarks and suggestions for future work.
# Navigation Sensors

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This chapter concerns a number of sensors that can aid an AUV or ROV in navigation tasks. The list will be constrained to the sensors available at ISR used in sea trials and relevant for this work, so it is not extensive. Some other common sensors which do not fulfill those requirement will be briefly referred to. USBL and DVL sensors are of particular importance in this work, with in-situ calibration procedures for them being derived in chapter 3, and for that reason they will be the focus of further attention, in particular in what regards error sources that can corrupt their measurements.

2.1 GPS

GPS operates through use of a constellation of satellites maintained in earth orbit, emitting radio signals which can then be received by a passive radio receiver at or near the surface of the planet. Positioning of the receiver can then be performed recurring to trilateration, given that it receives signals from a large enough number of satellites (typically four or more).

The signal broadcast by each satellite consists of its position, as well as a time-stamp. By subtracting the time-stamp encoded in the message from the time of arrival of the message, the receiver is able to obtain the travelling time which, with knowledge of the radio wave speed, can be converted into a range measurement [18].

Although the performance of GPS is deliberately degraded for civilian use so that it is in the order of meters when it comes to absolute positioning, some techniques are available to achieve much better relative positioning precision. Real Time Kinematic (RTK) navigation uses the phase of the signal's carrier wave, in lieu of its content, as well as a fixed reference ground station proving real-time corrections to enhance the system's relative accuracy.

In underwater robotics the use of GPS is limited due to rapid attenuation of radio waves in water, and is thus basically constrained to the use on surface vehicles. Despite that, it is often used in the field, for instance for the positioning of a surface vehicle which in turn uses an acoustic positioning system (such as USBL) to estimate the relative position of a target.

Typical precisions for a GPS can vary from 1 cm (fixed RTK) to 10 m (autonomous receiver). Output rates are pretty fast, ranging around 1 – 10 Hz. Coverage in the Earth’s surface is almost total, while range in water is virtually zero [2].

2.2 Inertial Sensors and Navigation Systems

There are several inertial sensor suites which can be used in underwater robotics, such as MRU, IMU and AHRS. They typically consist of a set of three accelerometers, magnetometers and gyroscopes which allow the estimation of 3D linear accelerations, orientation and angular velocities. The difference between an Inertial Measurement Unit (IMU) and an AHRS is that the latter outputs attitude and heading solutions, while the former only feeds raw sensor measurements to an external device responsible for those calculations.

The precision and output rates obviously depend on the type (and consequently cost) of the equipment. Thus, precision in heading measurement can vary from 1° – 10° (magnetic compass) to as high
as $0.1^\circ - 0.01^\circ$ (ring-laser and fibre-optic gyroscopes). Roll and pitch measurements are typically more precise ($1^\circ - 0.01^\circ$) and output rates for all are typically in the range $1 - 10 Hz$, but can be as high as $1000Hz$. Angular velocity measurements can also achieve quite high precisions ($0.01^\circ s^{-1}$), and so do acceleration measurements ($0.001g$) [2] [26].

2.2.1 Navigation Equations

The orientation measured by the orientation sensor suite can be used to derive the rotation matrix from the body to the inertial (typically North-East-Down (NED) frame, $W_B R$. If a magnetic compass is used for heading computation, its solution will point to the magnetic north rather than true north or the Universal Transverse Mercator (UTM) north usually considered by GPS; this deviation is known as magnetic declination. Due to this fact, it is useful to consider an intermediate frame $\{M\}$ which considers the orientation sensor’s north, and a constant $W_M R = W_M R_Z$ performing the mapping to the desired inertial frame. Notice that this transformation is merely a rotation around the $Z$ axis (offset in yaw).

The measured angular velocities are body-fixed angular velocities (i.e.: they are the angular velocities of $\{B\}$ with respect to $\{M\}$, expressed in $\{B\}$), here $\nu_2$, and thus simple integration will not lead to accurate orientation ($\eta_2$) over time. In fact, the following equality holds [33]:

$$\dot{\eta}_2 = Q(\eta_2)\nu_2,
\quad (2.1)$$

where

$$Q(\eta_2) = \begin{bmatrix} 1 & s_{\phi t \theta} & c_{\phi t \theta} \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi & c_\phi \end{bmatrix}
\quad (2.2)$$

Notice that $2.2$ is singular for $\phi = 90^\circ$; in practice most vehicles do not approach that value of pitch in normal operation, which dissipates this issue. Nevertheless, if the vehicle is expected to go near those pitch values, euler angle representation should be dropped in favour of quaternions’. The time derivative of the body to inertial rotation matrix, $\dot W_B R$ can also be derived

$$W_B \dot R = W_B R S(\nu_2).
\quad (2.3)$$

The measured linear accelerations are also expressed in the body frame, and need to be rotated before integration; moreover, unless the inertial sensors suite does so internally, one also needs to sum the gravity vector since the accelerometers do not respond to gravitational acceleration, but rather measure deviations from freefall [18]. Thus,

$$W v(t) = \int_{t_0}^{t} W_B R(t) B a(t) + W v(t_0),
\quad (2.4)$$

with

$$B a(t) = \vec{a}(t) + \dot W_B R(t) W g,
\quad (2.5)$$

where $\vec{a}$ is the accelerometers’ output acceleration and $g$ is the gravity acceleration vector.
Concern is often given to a bias in the accelerometers’ readings [46]; if this is to be taken into consideration then the bias needs to be subtracted from 2.5 as well:

\[ B_a(t) = \bar{a}(t) + \int_{t_0}^{t} R(t)^{W} g - B_{\text{Bias}}. \]  

(2.6)

Notice that, while both the accelerometers’ bias and the gravity acceleration vector can be seen as constant, they are not constant in the same reference frame. While the former is constant in the body frame, the latter is constant in the inertial frame. Without this fact, the quantities would be indissociable.

It will also be necessary to derive an expression for the derivative of the velocity in the body frame. Recalling 2.4 and that \( B_v = B_W R_W v \):

\[ B \dot{v} = B_W R_W \dot{v} + B_W \dot{R}_W v \]

\[ = B_a - S(B_\omega)B_v. \]  

(2.7)

It could be of interest to compute the linear accelerations at another point of the rigid body (vehicle), such as its center of mass or another sensor’s position. For this purpose the position of the accelerometers, \( P_A \), can be expressed as a function of the position of the point of interest, \( P_C \), according to

\[ \hat{W} P_A = \hat{W} P_C + \hat{W} R_B \Delta_{A-C}. \]  

(2.8)

where \( \Delta_{A-C} = P_A - P_C \) is the displacement vector between the two points, which is constant in the body frame. Taking the time derivative of (2.8) yields

\[ \hat{W} \dot{P}_A = \hat{W} \dot{P}_C + \hat{W}_B R(B_\omega \times B \Delta_{A-C}). \]  

(2.9)

The differentiation of (2.9) then leads to

\[ \hat{W} \ddot{P}_A = \hat{W} \ddot{P}_C + \hat{W}_B R(B_\omega \times B \Delta_{A-C}) + B_\dot{\omega} \times B \Delta_{A-C} \]

\[(=) \]

\[ \hat{W} \ddot{P}_C = \hat{W}_B R(B_\dot{P}_A - S(B_\omega)(S(B_\omega)B \Delta_{A-C}) - S(B_\dot{\omega})B \Delta_{A-C}) \]  

(2.10)

The first term on the right-hand side of (2.10) can be obtained using the accelerometers’ readings through (2.6) and the second is computed using the angular velocities measured by the gyros and the solved attitude and heading; the third and final term, however, would require knowing \( B_\dot{\omega} \), which is not measured by any sensor. For this reason, and in order to avoid unnecessary approximations, when dealing with data from several sensors it is typically best to consider the accelerometers’ position as the reference point and adapt all other measurements accordingly.

### 2.3 Ultra-Short Baseline

An [USBL](#) system is but one of a family of acoustic positioning systems, where one can consider Short Baseline ([SBL](#)), [LBL](#), among others [3]. While [LBL] typically provide the best accuracy, their use
requires the deployment of seabed transponders, whose position needs to be carefully calibrated: a
time consuming and cost intensive procedure. Moreover, \textit{LBL} use leads to important restriction in the
area of operations, which is obviously constrained to the vicinity of the moored transponders. Inverted
\textit{LBL} which use transponders fixed to so-called intelligent buoys or Autonomous Surface Craft (ASC)
solve most of these issues, but are still not as easy to deploy as a \textit{SBL} or \textit{USBL} system. These require
just one surface vessel with an acoustic emitter and multiple receivers. The difference between a \textit{SBL}
and an \textit{USBL} lies on the way these are distributed. In the case of the former, the acoustic devices
are spread along the hull of the vessel. The latter, on the other hand, makes use of a transceiver,
which integrates an acoustic emitter and a very short (hence the name) array of receivers, which
interrogates and receives the acoustic replies from a transponder mounted on the target. The relative
position of the transponder from the transceiver is thus computed in terms of a range and a bearing,
being the first typically estimated by travelling time and the second through phase differences between
the signal received at each transducer.

\subsection*{2.3.1 Working Principles}

After a reply is received by a transducer, the slant range between it and the transponder can be
computed simply by
\begin{equation}
    d = \frac{c \cdot T}{2};
\end{equation}
where \( c \) is the sound speed in water and \( T \) is the two-way travelling time. Since there are multiple
transducers it is possible to improve the range estimation, for instance through a weighted average.

The phase difference obtained along each baseline (composed by a pair of transducers) is pro-
tional to the direction cosine along the baseline’s direction \[5\]
\begin{equation}
    \cos(\alpha) = \frac{\lambda \psi_\alpha}{2\pi d}.
\end{equation}
where \( \cos(\alpha) \) is the direction cosine along the baseline in consideration, \( \psi_\alpha \) is the measured phase
difference along the same baseline, \( \lambda \) is the acoustic signal’s wavelength and \( d \) is the length of the
baseline.

Notice that, considering three orthogonal baselines, the sum of the squares of the direction cosines
is equal to one. In practice this means that, in order to compute a three-dimensional relative position,
one only needs two baselines (with bearings \( \alpha \) and \( \beta \)) since the direction cosine along the third (with
bearing \( \gamma \)) will be given by
\begin{equation}
    \cos(\gamma) = \sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)}
\end{equation}

Typical accuracies for the range measurements are around \( \pm 0.2 \text{m} \), with \( \pm 3^\circ \) for the bearing mea-
surements. Output rates can vary from \( 2 \text{Hz} \) to as low as \( 0.05 \text{Hz} \).

\subsection*{2.3.2 Navigation Equations}

The first step to get a position estimate using a \textit{USBL} system is to combine the slant range with the
bearing measurements. Considering the simplest case, with bearings \( \alpha \) and \( \beta \) along two orthogonal
baselines and a slant range of $d$, a position estimate can be obtained through

$$H_{\text{relative}} = d \begin{bmatrix} \cos(\alpha) \\ \cos(\beta) \\ \sqrt{1 - \cos^2(\alpha) - \cos^2(\beta)} \end{bmatrix}.$$ \hspace{1cm} (2.14)

Notice that the result is in the hydrophone’s reference frame (H) and is thus of little use in this form. It can be converted to the desired inertial frame by use of the appropriate rotation:

$$W_{\text{relative}} = W_H R H_{\text{relative}},$$

where $W_H R$ is obtained through the following

$$W_H R = W_M R_M R_B H_{\text{relative}},$$ \hspace{1cm} (2.15)

In the above, $W_H R$ is the constant rotational mapping between the hydrophone and the body frame; both other matrices are according to the definition in section 2.2.

The position measurement is now in the inertial frame, but is still relative to the transceiver while an absolute position is usually desirable (except perhaps when the transceiver is static). This is achieved by simply adding the transceiver’s position

$$W_{\text{transponder}} = W_{\text{relative}} + W_{\text{transceiver}}.$$ \hspace{1cm} (2.16)

These equations are suitable for a simple two-baseline USBL system. In practice it is common to employ more than two baselines so as to improve accuracy, such as three in a tetrahedral geometry \[4\]. This is in fact the case of the unit owned by \[ISR\] and used at sea trials, the MicronNav by Tritech. With this setup one directly attains the three direction cosines $H_{cx}, H_{cy}$ and $H_{cz}$ and could thus generate the estimate

$$H_{\text{relative}} = d \begin{bmatrix} H_{cx} \\ H_{cy} \\ H_{cz} \end{bmatrix}.$$ \hspace{1cm} (2.17)

This, however, is definitely sub-optimal when considering noisy measurements. Firstly, it ignores the extra information that the squares of the direction cosines should add to one. Secondly and perhaps more importantly, the estimated distance to the transponder, given by $||P_{\text{relative}}||$, is not equal to the measured distance, $d$. This is both odd and undesirable, since in fact the direction cosines bear no information whatsoever on the distance to the target and thus the only reasonable estimate for that range is the measurement itself. This reasoning leads to the proposal of the intuitive estimator

$$H_{\hat{\text{relative}}} = \frac{d}{\sqrt{H_{cx}^2 + H_{cy}^2 + H_{cz}^2}} \begin{bmatrix} H_{cx} \\ H_{cy} \\ H_{cz} \end{bmatrix}.$$ \hspace{1cm} (2.18)

which positively addresses both issues reported. It is possible to arrive at this estimator through a theoretically more sound path, maximum likelihood estimation, for which a model for the measurements noise is needed. This derivation is included in appendix A.

2.3.3 Error Sources

USBL measurements are subject to several error sources, most of which tied to the limitations of sound propagation models. This section identifies some of them, with incidence on those that can be prevented or corrected.
2.3.3.A Limited sound speed in water

An unavoidable error source in USBL positioning systems is the limited propagation speed of sound waves in water. This leads to two different issues. First, both the transceiver and the transponder may move during the wave’s travelling time and secondly the measurements should have different time stamps, rather than simply the receiving time.

To further clarify these issues, consider three distinct and increasing time instants: the instant at which the interrogation is sent \( t_0 \), the instant upon which the signal reaches the transponder, here considered coincident with its reply \( t_r \), and the instant when the transceiver finally gets the response \( t_s \). This is still a simplified model, since it disregards any processing delays that may take place. Under these circumstances, one can notice that the range measurement will be given by

\[
\hat{d} = \| P_{t_r \text{transponder}}^{t_r} - P_{t_0 \text{transceiver}}^{t_0} \| + \| P_{t_r \text{transponder}}^{t_r} - P_{t_s \text{transceiver}}^{t_s} \|. \tag{2.19}
\]

The consequences of this are twofold. Firstly, simply halving the measured distance will not render the distance between the transceiver and the transponder at receiving time. In fact, the measurement itself cannot be uniquely assigned to any given time instant, as the different upper suffixes in 2.19 suggest. Secondly, the measurement only conveys information on the transponder’s position at the receiving time \( t_r \), in contrast to the instant where it becomes available, \( t_s \). The direction cosines measurements will be given by

\[
\hat{C} = \frac{P_{t_r \text{transponder}}^{t_r} - P_{t_s \text{transceiver}}^{t_s}}{\| P_{t_r \text{transponder}}^{t_r} - P_{t_s \text{transceiver}}^{t_s} \|}. \tag{2.20}
\]

Looking at 2.20 it is clear that the direction cosines measurements will suffer from the same issues as the ranges’.

2.3.3.B Sound speed error

Another very important source of error in USBL positioning is the assumed velocity of sound, \( \hat{c} \). Assuming a constant velocity of sound across the water column is an error source in itself, since it can vary according to a number of parameters. But even if one can neglect that effect, an error in the estimated velocity of sound, given by

\[
\hat{c} = s c, \tag{2.21}
\]

can lead to undesirable results. Going back to the simplified model for the range measurement, presented in 2.11, it is clear that the measurement will suffer from a scale factor

\[
\hat{d} = s d. \tag{2.22}
\]

Notice that, even for a range-independent noise model such as the one considered in A.1 a Velocity of Sound (VOS) mismatch will lead to the appearance of range-dependent noise. Not only that, the error in the VOS will also affect the direction cosine measurements. Recalling 2.12 and the fact that the wave length is estimated through

\[
\hat{\lambda} = \frac{\hat{c}}{f}. \tag{2.22}
\]
it becomes clear that the direction cosine measurements will also suffer from a scale factor error:

$$c_{\alpha} = s c_{\alpha}.$$  \hfill (2.23)

### 2.3.3.C Misalignment

Finally, a rotational misalignment during assembly of the transceiver is also a common and important source of error. In this case a new frame, the transceiver frame (T), must also be considered, changing \[2.15\] to

$$W_H R = W_M R_M^T R_T^T R_H R.$$  \hfill (2.24)

This is illustrated in figure 2.1.

It is important to stress that such a misalignment will only affect the direction cosine measurements, leaving the range measurements uncorrupted.

### 2.3.3.D Phase Shift Measurement Bias

Some concern has been shown in what regards the possibility of a bias in the measured phase shift by each pair of transducers [5]. Recalling \[2.12\] the phase shift is proportional to the direction cosine along that baseline which means that such a bias will induce a bias in the direction cosine measurements. It would not, however, influence the range measurements.

### 2.4 Doppler Velocity Log

A DVL is a sensor used to measure the speed of an underwater vehicle, relative to the fixed seabed (so called bottom-lock mode). It is also commonly employed to measure water currents, when fixed to a stationary base (water-lock mode). Although its accuracy renders them an important aid in navigation systems, DVLs’ typically large size and cost make their use somewhat undesirable for small low-cost AUV and ROV.
2.4.1 Operating Principles

A DVL measures velocities through the so-called Doppler frequency shift ($\Delta f$). The velocity (along each of the device’s beams) is thus estimated through

$$v = \frac{c \Delta f}{2 f_t}. \quad (2.25)$$

A derivation of 2.25 is present in appendix B.

A typical DVL has four such beams tilted from the vertical by an angle $\theta$ and separated by 90° each, such as illustrated in figure 2.2.

Throughout this work, unless otherwise specified, all references to DVL assume it is in bottom-lock mode.

DVL often offer precisions as high as $1 \text{mms}^{-1}$, with update rates of a few Hz.

2.4.2 Navigation Equations

DVLs are used to measure an AUV’s or ROV’s velocity, and thus can be directly used in odometry navigation. This consists in simple integration of the velocity, after it is expressed in an inertial frame:

$$W P(t) = W P(t_0) + \int_{t_0}^{t} W v(\tau) d\tau. \quad (2.26)$$

The question remains, though, on how to derive $W v$ from the values of the velocities along each beam. It can be noted that these radial velocities, computed through 2.25, are equivalent to

$$v_i = G^{e_i} G v \quad (2.27)$$

where $G$ is an arbitrary reference frame and $G^{e_i}$ is the direction cosine vector corresponding to beam $i$ expressed in that reference frame. Adopting the body frame as reference and stacking vertically all the terms on both hand sides of 2.27 one obtains the matrix form equality

$$V = E^B v, \quad (2.28)$$

where $V$ is the $N \times 1$ vector of radial velocities and $E$ is the $N \times 3$ matrix of stacked direction cosines. The vehicle velocity can thus be estimated through

$$B^v_{\text{DVL}} = E^+ V, \quad (2.29)$$

where $E^+$ is the pseudo-inverse of $E$ [10]. Equation 2.29 is the least square solution for $B^v_v$ given $N$ radial velocity measurements. Provided that the DVL comprises three or more (four is, as already...
stated, by far the most common) non co-linear acoustic beams, 2.29 has a single solution. Notice this condition is totally independent of the way the instrument is assembled, and thus it is easily satisfied. However, the beams must be pointed at the ground in order to allow (direct) reflection to occur.

One of the most common assembly configurations is the so-called Janus configuration, named after the Roman god, in which two beams point along the vehicle’s fore-aft and the others point towards starboard and port [16]. This is illustrated in figure 2.3. Another common solution is to rotate this configuration by 45° over the vertical axis; this sounds like a more optimised solution since all beams will measure velocities on the same order of magnitude, assuming that the vehicle moves mainly towards its front, not neglecting possible vertical movements.

Mind that 2.29 assumes that the instrument is mounted on the vehicle aligned with the body axes. When such is not the case, the velocity can easily be converted to the body axes by use of the assembly rotation matrix. Recalling 2.26 it is clear that a DVL needs auxiliary sensors (to measure orientation) in order to be used in odometry navigation. Angular velocities might also be of importance, if one wishes to correct the velocity measurements from the DVL position to the center of mass, or other point of reference. This is relevant since an AUV typically needs to accommodate multiple sensors, both for navigation and mission accomplishment purposes, which obviously cannot be physically superimposed. Thus considering a displacement

\[ \mathbf{w}_{\mathbf{P}_{CM}} = \mathbf{w}_{\mathbf{P}_{DVL}} + \mathbf{w}_{\mathbf{r}_{DVL}} \]

and taking its derivative

\[ \mathbf{w}_{\dot{\mathbf{P}}_{CM}} = \mathbf{w}_{\dot{\mathbf{P}}_{DVL}} + \mathbf{w}_{\mathbf{B}_{\mathbf{r}_{DVL}}} \times \mathbf{B}_{\mathbf{r}_{DVL}} \]

the correction term appears naturally.

2.4.3 Error Sources

Velocity measurements obtained through a DVL are subject to several error sources. These are most serious when they are consistent (e.g. they provoke a constant bias on the measurements),
due to error integration in odometry navigation. Nevertheless, even non consistent errors should be
singled out and corrected for when possible, so as to reduce short-term noise.

2.4.3.A Separation between DVL beam sources

The DVL is usually looked upon as a single sensor, ignoring the fact that there is a slight displace-
ment between each acoustic transducer, depicted in figure 2.4. Thus, not only does each transducer
measure the velocity along a different direction, it also measures it on a different point in space (be-
longing to the rigid body), the consequences of which were already displayed under equation 2.30.

![Figure 2.4: Physical displacement between beam sources](image)

Ignoring these displacements can lead to errors in the device’s estimated velocity, the derivation
of which can be seen under appendix C. From there it can be seen that the typical DVL will have a
measurement bias in $v_x$ and $v_y$ which is proportional to the pitch and roll rates, respectively, as well
as the constant $k$ given by C.8.

Since the distance $d$ is typically around $1dm$ and DVLs often advertise accuracies in the order of
$mms^{-1}$ it seems like relatively low rotation speed can induce significant error in the velocity mea-
urements. However, it is not so relevant when integrated over time, moreover since roll and pitch
usually exhibit oscillatory behaviour; simulation results corroborate that this error source is not a major
concern.

2.4.3.B Sound speed errors

Taking a quick glance at 2.25, 2.27 and 2.29 it is clear that any error in the estimated sound speed
will result in an error in the measured vehicle speed. In fact, similarly to what was shown in 2.3.3.B
if a scaling factor is considered to differentiate between the true and the estimated sound speed, the
same scale factor will corrupt the velocity measurements. Thus, the sound speed estimation error

$$\hat{c} = sc,$$

leads to the corruption of the radial velocities according to

$$\hat{v_i} = sv_i.$$  

The estimation of the body velocity will scale in the same manner

$$\hat{\mathbf{v}_B} = s\mathbf{v}_B.$$ (2.32)
2.4.3.C Misalignment

In 2.4.2 it was said that it matters not the alignment with which the DVL is mounted with respect to the body, given that it is known. In practice there is often a slight misalignment between the instrument and the body axes, which is indeed considered one of the most important error sources in Doppler Navigation [2]. When the DVL is intentionally mounted with a known rotation with respect to the body axes the issue still stands, in the form of a small error in that rotation matrix. Figure 2.5 illustrates a misalignment at assembly time and its consequences.

Mathematically, such a misalignment means that (2.29) needs to be changed to

\begin{equation}
B_v = B_D R E^+ V
\end{equation}

\begin{equation}
= B_D R^D v
\end{equation}

where \( \{D\} \) represents the reference frame of the DVL instrument.

2.4.3.D Bias in measured frequency-shift

If one considers a bias in the measured frequency shift then, disregarding any other error sources,

\begin{equation}
\Delta f = \Delta f + \Delta B,
\end{equation}

where \( \Delta B \) is the bias in the measured frequency shift. Recalling (2.25) and substituting \( \Delta f \) for the expression in (2.35) it can be seen that this bias will lead to a bias in the measured radial velocities:

\begin{equation}
v_i = v_i + K_{Bias},
\end{equation}

where \( K_{Bias} \) is given by

\begin{equation}
K_{Bias} = \Delta B \frac{c}{2f_i}.
\end{equation}

Going back to (2.27) and (2.29) and taking (2.36) into account one finally attains the effect on the final velocity measurement in the body axes

\begin{equation}
B_v = B_v + K_{Bias} E^{+1}_{4 \times 1}.
\end{equation}

A bias in the measured frequency-shift thus leads to another bias in the measured velocity in body axes that depends on the matrix \( E \). Interestingly enough if, after assembly, the beams are positioned pairwise-symmetrically with respect to the vertical axis (as is usually the case with 4-beam DVLs), the bias will have a single component in \( B_Z \).
3

Calibration Procedures

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This chapter is concerned with in-situ calibration procedures for some of the error sources identified in 2.3.3 and 2.4.3. These procedures are intended to be carried out in-situ and thus should only utilise data which is usually available during the sensors’ regular use, or that do not imply significantly more complicated logistics to be obtained. This is not to say, however, that these algorithms are designed to run with data collected during the vehicle’s regular mission; it seems reasonable to consider that some time window prior to the beginning of the vehicle’s mission will be given to calibration procedures, thus permitting the use of particular manoeuvres that provide adequate excitation for the calibration methods.

3.1 Calibration of an USBL

At least two of the error sources in USBL positioning identified in 2.3.3 can be calibrated out: the VOS error and the assembly misalignment error. This section is concerned with detailing the procedures to do so.

3.1.1 Sound speed error calibration

An error in estimated sound speed can be corrected with use of another sensor, a sound velocity probe, followed by appropriate scaling of the measurements. This subsection is concerned with how to make that correction when such sensor is not available, or when processing data from a trial where it was not employed.

Recalling 2.22, an error in the VOS will result in a scale factor corrupting the range measurements. Collecting a number of range measurements and comparing them to ground truth values it is possible to identify these scale factors.

Assuming the measurement model described in A.1, it is clear that the noise will be affected by the same scale factor and thus

\[ \bar{d} = s(d + \eta_d). \] (3.1)

Keeping 3.1 in mind, one could derive the likelihood function for the parameter \( s \). In practice it might be improper to consider that the noise variance, \( \sigma_d^2 \), is known, so it is best to consider it as another parameter to estimate, making \( \theta = [s \ \sigma_d]^T \). Defining \( \mathcal{D} \) as the set of range measurements and \( N \) as the number of measurements one attains

\[ L(\theta|\mathcal{D}) = -N \ln(s \sigma_d \sqrt{2\pi}) - \sum \frac{(\bar{d} - sd)^2}{2s^2 \sigma_d^2} - N \sigma_d. \] (3.2)

The partial derivatives with respect to both parameters can then be computed, yielding

\[ \frac{\partial L(\theta|\mathcal{D})}{\partial \theta} = \left[ \sum \frac{\bar{d}^2 - s\bar{d}d}{s^2 \sigma_d^2} - \frac{N}{s} \right] \left[ \sum \frac{(\bar{d} - sd)^2}{s^2 \sigma_d^2} - \frac{N}{\sigma_d} \right]. \] (3.3)

To try and derive the Maximum Likelihood (ML) estimators for the parameters the gradient 3.3 is set to zero, which leads to

\[ \hat{s} = \frac{\sum \bar{d}d}{\sum d^2} \] (3.4)
\[ \hat{\sigma}_d = \sqrt{\frac{\sum d^2 - \hat{s} \sum \bar{d} d}{N \hat{s}^2}}. \] \hspace{1cm} (3.5)

Next, one can compute the Hessian of the likelihood function

\[ \frac{\partial^2 L(\theta | \mathcal{D})}{\partial \theta^2} = \begin{bmatrix} \sum \frac{2s^2d^2}{\sigma_d^2} + \frac{N}{\sigma_d^2} \sum \frac{2(s^2d^2 - \bar{d}^2)}{s^2\sigma_d^2} - \frac{N}{\sigma_d^2} \sum \frac{3(\bar{d} - \bar{s})^2}{s^2\sigma_d^2} \end{bmatrix} \] \hspace{1cm} (3.6)

Similarly to the situation in 2.3.2 the Hessian matrix was not formally proven to be negative definite, nor is it clear whether its eigenvalues are negative at \( \theta = \hat{\theta} \). The proposed estimator could not be ascertained as the ML estimator for that reason.

Taking the expected value of 3.6 one attains the Fisher Information Matrix

\[ I(\theta) = \begin{bmatrix} \frac{2N}{\sigma_d^2} + \sum \frac{d^2}{s^2\sigma_d^2} & \frac{2N}{\sigma_d^2} \\ \frac{2N}{\sigma_d^2} & \frac{2N}{\sigma_d^2} \end{bmatrix} \] \hspace{1cm} (3.7)

and, inverting it, the Cramér-Rao Bound (CRB) for \( \hat{s}, \hat{\sigma}_d \)

\[ CRB(\theta) = \begin{bmatrix} \frac{s^2\sigma_d^2}{\sum d^2} & -\frac{s \sigma_d^2}{\sum d^2} \\ -\frac{s \sigma_d^2}{\sum d^2} & \frac{\sigma_d^3}{2(2N \sigma_d^2 + \sum d^2)} \end{bmatrix} \] \hspace{1cm} (3.8)

Expression 3.8 is simple enough to allow some qualitative comments. It can be seen that the bound gets lower with the increase in the number of measurements, which was obviously expected. Also quite intuitive is the fact that the bound depends (negatively) on \( \sigma_d \): noisier measurements leads to poorer results. It also improves with the true range the measurements were taken; this can be explained by two reasoning. On the one hand, larger ranges mean better signal to noise ratio with respect to the range-invariant noise \( \eta_d \). On the other hand, they also make the scaling factor \( s \) stand out more remarkably. Finally and probably most interesting, the bound indicates that results will be worse for larger \( s \). This can be blamed on the fact that the scaling factor will also lead to an increase in the noise levels.

As for assessing the performance of the estimated correction \( \hat{s} \), besides standard error metrics such as the Root Mean Square Error (RMSE), it would be interesting to test it in a dataset with a wide span of ranges. If the correction term is working, one should go from a range-dependent noise situation to a range-independent one.

For the sake of completeness the estimation of \( s \) with known \( \sigma_d \) is presented in D. The sharp-eye reader might have noticed that, according to 2.23 the direction cosine measurements also share a dependency on \( s \) and could thus aid in its estimation. While this is true, it must be kept in mind that these measurements are also subject to another extremely common source of error, already identified in 2.3.3 rotational misalignments. Under the influence of a misalignment 2.23 no longer holds and so the addition of the direction cosine measurements to the estimation of \( s \) can make it worse, instead of better. Nevertheless, a combined estimator for \( s, \sigma_d \) and \( \sigma_c \) is proposed in E along with the (CRB) for those parameters.
3.1.2 Misalignment Calibration

Before describing the calibration method, it is important to state the underlying assumptions. In this case, it is considered that the vehicle/unit equipped with the USBL transceiver has also 3D orientation sensing and an accurate positioning system (so that the position of the transceiver is known). The position of the transponder is also assumed to be known. Comparing to the conditions imposed in [6] (and most of the literature for that matter), the only difference lies in assuming a known (but possibly time-variant) transponder position instead of a moored transponder at an unknown position. However, the same reference describes a method used to estimate the (constant) position of the moored transponder at a pre-processing stage, and thus the misalignment calibration procedure thus in fact assume a known position. This pre-calibration procedure will also be described in appendix G, rendering the calibration method described herein as much as general.

The measurements of a USBL positioning system affected by a transceiver misalignment, but ignoring other sources of error, are given by

\[ d = ||P_{\text{Transponder}} - P_{\text{Transceiver}}|| \]

and

\[ H' C = H_T R_B R_W R_P Transponder - W_P Transceiver \]

Using one of the techniques discussed in 2.3.2 one could use 3.9 and 3.10 to derive a relative position measurement, which would be expressed in the transceiver’s frame due to the misalignment. Defining

\[ W_P USBL = W_P Transponder - W_P Transceiver \]

such measurement would correspond to

\[ \beta = \frac{B_T R_P USBL}{W_R W_P USBL} \]

and

\[ \alpha = \frac{W_R W_P USBL}{B_T R_P USBL} \]

where \( \beta \) is the misalignment matrix that needs to be estimated. Making \( \beta = \frac{B_P USBL}{W_R W_P USBL} \) it is possible to estimate \( \beta \) recurring to the procedures described in [27], [28], which use a Singular Value Decomposition (SVD) the derive the transformation parameters that minimise the square residuals. These are described in more detail in [F].

This solution, from now on denoted \( P - Method \), might not be ideal. First of all, it depends on the method used to estimate \( T_T USBL \), since it is not a direct measurement from the USBL positioning system. But, most significantly, it will depend on the range measurements which are not at all affected by the misalignment, as seen in 2.3.3.C. This is, of course, undesirable since it will bring another error source into play which does not come along with any useful information. Moreover, it would implicitly give more weight to measurements at longer distances, since their residuals are bound to be larger.

This can be avoided by using the direction cosines directly. This is particularly interesting for a USBL whose transceiver uses a tetrahedral geometry, such as the one referred to in 2.3.2 since the
three direction cosines are then directly proportional to the measured phase shifts. Thus,

$$\frac{B^T R^T H^T C}{||P_{USBL}||} = \frac{R^W P_{USBL}}{||P_{USBL}||},$$

(3.12)

which now means that $\alpha = \frac{R^W P_{USBL}}{||P_{USBL}||}$ and $\beta = \frac{T^H R^H C}$. Notice that $\beta$ is simply the direction cosine measurements rotated by the constant $T^H R$, while $\alpha$ makes uses of the other sensors/assumptions mentioned above. This procedure, whereby named $dcos$–Method, has other uses: the scale factor returned by the transformation parameters estimation method should correspond to scaling error in sound speed. It can thus be used for sound speed calibration or, if that has already been done through one of the procedures described in 3.1.1, it can be used for cross-check. Another advantage of considering 3.12 instead of 3.11 for alignment calibration is that the estimated bias is also of use: it corresponds to a bias in the direction cosine measurements (rotated by $T^H R$) which, as stated in 2.3.3.D, might be due to a bias in the measured phase shifts. This is only valid, however, for a transceiver with three baselines that uses those phase shifts directly to compute the direction cosines.

### 3.1.3 Implementation Considerations

The calibration procedures referred above are based on two different types of setups: the first considers a (possibly) moving transponder with known position, while in the second the transponder is moored at an unknown position. In both cases a surface vessel with attitude sensors and GPS manoeuvres around the surface, with the transceiver.

The moored transponder setup is easy to visualise, since it simply requires the transponder to be attached to some anchor. It is depicted in figure 3.1. A possible variation would be to have the transponder attached to a wire pending from a surface buoy and an anchor; this is not optimal since some motion is bound to happen.

![Figure 3.1: USBL calibration setup using a moored transponder](image)

The other setup required the transponder to be carried by an AUV or ROV. Since its position must be known, the vehicle would need to be equipped with GPS and thus manoeuvre at or near the surface. This makes it difficult, in practice, to have the transponder deeply submerged. A draft of this setup can be seen in figure 3.2.
Considering the $P$ – Method and expanding equation [3.11] one can derive

\[
\begin{align*}
\mathbf{R}^{\mathbf{B}}_{\mathbf{T}} \mathbf{R}^{\mathbf{P}}_{\mathbf{USBL}} &= \mathbf{R}^{\mathbf{W}}_{\mathbf{T}} \mathbf{R}^{\mathbf{W}}_{\mathbf{Transponder}} - \mathbf{R}^{\mathbf{W}}_{\mathbf{Antenna}} - \mathbf{R}^{\mathbf{B}}_{\mathbf{T}} \mathbf{R}^{\mathbf{B}}_{\Delta \text{Transceiver} - \text{Antenna}} \\
&= \mathbf{R}^{\mathbf{W}}_{\mathbf{T}} \mathbf{R}^{\mathbf{W}}_{\mathbf{Transponder}} - \mathbf{R}^{\mathbf{W}}_{\mathbf{Antenna}} - \mathbf{R}^{\mathbf{B}}_{\Delta \text{Transceiver} - \text{Antenna}},
\end{align*}
\]

where $\mathbf{R}^{\mathbf{B}}_{\Delta \text{Transceiver} - \text{Antenna}}$ is constant. It is possible to interpret this displacement has a bias in the USBL measurement which is constant in the $\{\mathbf{B}\}$ frame, which is the one used for calibration. The calibration method which estimated this displacement along with the alignment will be referred to as $P_{\mathbf{w} / \Delta}$ Method.

Since USBL typically has the lowest output rate among other sensors, interpolation or holding of last value on these measurements would lead to great imprecisions. For this reason the calibration methods use the measurements unchanged, and estimate the transceiver’s (and transponder’s if not moored) position at those time instants by interpolating other sensors’ data.

3.1.4 Comments on USBL Calibration Procedures

Over this section in-situ procedures to calibrate an error in estimated sound speed, a transceiver misalignment (with respect to orientation sensor suite) and even a possible bias in the measured frequency shift (proportional to measured direction cosines) were given.

The proposed misalignment calibration procedure has the advantage of not using range information at all, since those measurements are not affected by the misalignment and thus would only bring extra noise into the process. Furthermore, it does not require the vehicle operating the transceiver to manoeuvre in any particular way, considering that basic excitation conditions are met.

It was stated that the calibration procedures requires the transponder either to be moored or at a known (not necessarily constant) position. In practice the latter can be achieved by having the transponder rigidly attached to a surface vessel, ROV or AUV equipped with GPS and attitude and orientation sensing. This may seems advantageous, since it alleviates the need to moor the transponder and estimate its position using range measurement only. However, in practice this will most surely lead to a situation where the transponder is at a quite low depth relative to the transceiver, with bearings close to the horizontal plane. This is undesirable since USBL systems are typically not designed to operate in such extreme environments, and its accuracy is bound to lower.

From the calibration procedures’ description in 3.1 it may seem that two trials need to be conducted when utilising a moored beacon: one to collect range data in order to compute the transponder’s position and another to collect the direction cosine measurements required for validation. However,
if ones notices that a circular trajectory, for instance, provides good excitation for both procedures it
seems clear that a single manoeuvre can be used for both purposes. Since the first step only uses
the range measurements and the second (using the $aCos – Method$) looks only to the direction cosine
measurements there is no fear of propagating and/or masking errors by doing so.

3.2 Calibration of a DVL

Although calibration methods not requiring the use of external sensors seem appealing, even more
since in [26] they are reported to compare favourably to the others, at the time of this work there were
no experimental data available with simultaneous use of an accurate INS and a DVL. It was thus
found best to focus on the other methods, which could be applied in practice and their results verified.

The method described in [19] is a batch mode method which uses the results of [27] and [28] that
describe the least-squares solution for the estimation of a rotation between two 3-D point patterns,
using SVD. A brief description of those results is included in F. This method will be from now on referred to as $LS – Method$. [21] proposes an adaptive identifier for the same rotation, with proof of
its convergence. This method will hereby be denoted as $Adaptive – Method$.

Both the $LS – Method$ and the $Adaptive – Method$ methods were preliminary implemented and
tested in simulation as well as with experimental data, and thus will be briefly described here. An
extension of the $LS – Method$, from now on $ELS – Method$, that allows simultaneous correction for
other sources of error, along with other characteristics, will then be presented.

3.2.1 LS-Method

As stated previously, this method makes use of the results from [27] and [28] which allow the
estimation of a rotation matrix, $R$, responsible for the mapping

$$\alpha = R\beta,$$

(3.14)

where both $\alpha$ and $\beta$ are sets of 3D points. The challenge is thus to find $\alpha$ and $\beta$ such that

$$\alpha = \frac{B}{D}R\beta.$$  

(3.15)

Recalling 2.33 it is trivial to note that

$$\frac{B}{W}R(t)^W v(t) = \frac{B}{D}R^D v(t).$$  

(3.16)

Although the problem might appear to be solved, it is not since there are no measurements available
for $W v(t)$ with the sensor suites available. Progress can be made by integrating both sides of 3.16
using the integration by parts on the left-hand side, to reach

$$\frac{B}{W}R(t)^WP - \int \frac{B}{W}R^W P(\tau)d\tau = \frac{B}{D}R\int D v(\tau)d\tau.$$  

(3.17)

$\frac{B}{W}R$, $WP$ and $Dv$ are obtained, respectively, through the measurements of the orientation sensors,
an external positioning system and the DVL. The more elusive $\frac{B}{W}R$ can be computed according to
and using both angular position and velocity measurements. The left-hand side of (3.17) thus corresponds to $\alpha(t)$ in (3.15) while the right-hand side, excluding the sought $\dot{R}$, is equal to $\beta(t)$.

Two remarks are in order with respect to (3.17). The first is that it omits the limits of integration; this was done on purpose since they are not referred in neither of the papers where it is presented. It can thus be implemented both by continuously integrating both sides or by doing by time steps. To be more precise and looking just at $\beta(t)$ for the sake of simplicity, one can consider

$$\beta(t) = \int_{t_0}^{t} D v(\tau) d\tau$$  \hspace{1cm} (3.18)

or

$$\beta(t) = \int_{t-T_s}^{t} D v(\tau) d\tau.$$  \hspace{1cm} (3.19)

Since the $ELS–Method$ will be based on (3.19) the $LS–Method$ here considered will use (3.18).

The second is that, since several different sensors are being employed to compute $\alpha$ and $\beta$, special care needs to be taken in what regards outliers. This was found to be problematic when dealing with experimental data.

### 3.2.2 Adaptive-Method

This method, contrary to $LS–Method$, uses an adaptive identifier and could thus be run in real time. Its derivation and proof of convergence will not be given here, instead referring the reader to [21]. The identifier is defined by the update law

$$\dot{\hat{R}}(t) = \hat{R}(t)S(\hat{R}^T(t)[\dot{\alpha}(t) \times \vec{\tau}(t)])$$  \hspace{1cm} (3.20)

where $\alpha(t)$ corresponds to the quantity derived in (3.2.1). This method, similarly to the $LS–Method$ guarantees that the solution is constrained to $SO_3$.

### 3.2.3 ELS-Method

This extension of the $LS–Method$ aims to attend to both issues raised in (3.2.1). In order to address the issue of outliers, an implementation using (3.19) was chosen in lieu of (3.18) since the latter will lead to a few outliers eventually corrupting all the subsequent valid measurements. The method was further extended using a Random Sample Consensus (RANSAC) approach, which is often used in image processing when estimating transformation parameters between sets of points [11] to get rid of any such possible corrupted measurements. Since the focus is on a rotational transformation, the error metric used was not the vector norm but rather the angular difference between $\alpha(t)$ and $\dot{\alpha}(t) = \dot{R}\beta(t)$, given by

$$\angle_{\dot{\alpha} - \alpha} = \arccos \left( \frac{\dot{\alpha} \cdot \alpha}{||\dot{\alpha}|| ||\alpha||} \right).$$  \hspace{1cm} (3.21)

Finally, although the original $LS–Method$ is concerned with rotational misalignments only, the transformation parameter estimation methods it is based upon also allows the estimation of a scale factor and a translation between the two sets of points. Utilising the chosen implementation these gain a
physical meaning in this context: the scale factor corresponds to the sound speed error considered in 2.4.3.B and the translation corresponds to a constant bias in the velocity measurements (in body frame), hypothetically due to the situation considered in 2.4.3.D. Moreover, the translation and scaling factor are computed after the rotation parameter, and thus there is no fear of them corrupting the misalignment estimative. Care should be given to over-fitting though.

### 3.2.4 Implementation considerations

The calibration methods described above require external position fixes to be available, besides 3D orientation and DVL measurements. In practice two distinct setups come to mind.

The first, which is the one of more practical importance, consists on having a ROV carrying the DVL while performing underwater manoeuvres. In this case the position fixes would most likely need to come from an acoustic positioning system such as USBL or LBL. Since the algorithms are not run on-line, delayed data is not of concern as long as it is properly timestamped. Care needs to be given to the displacement between the DVL and the acoustic unit (typically a transponder) mounted on the vehicle; the position needed for the computation of $\alpha$ is that of the DVL and so must be obtained from the other making use of orientation. This setup is exemplified in figure 3.3.

![DVL calibration setup using a ROV](image)

Figure 3.3: DVL calibration setup using a ROV

The second is interesting mostly for testing purposes, and has been adopted at sea trials by ISR due to its simplicity. It consists on having the DVL mounted off the side of a surface vessel, which can thus use GPS for positioning. The problem with this approach is that these vessels are typically heavily under-actuated, making it difficult to generate proper excitation for calibration. Again, the position of interest is not that of the GPS antenna; the appropriate correction must be performed. Figure 3.4 depicts this setup.

![DVL calibration setup using only a surface vessel](image)

Figure 3.4: DVL calibration setup using only a surface vessel

A hybrid approach can be considered where a ROV manoeuvres with the DVL at or quite near the surface, thus allowing it to also hold a GPS antenna. This has the advantage of exploiting both the
superior precision and output rates of GPS and the superior manoeuvrability of these vehicles.

Irrespective of the definition of $\alpha$ and $\beta$ (i.e. using whether 3.19 or 3.18) they must be sampled so as to generate the 3D point sets. This is most critical when using the 3.19 since the size of the time windows used for integration can have a major impact on the results. It should not be too large, so as to allow a large number of points to be available for calibration. The small it becomes however, the poorer the signal to noise ratio will become. Considering the setups above and after preliminary experiences a time window of $T_s = 5$ s was chosen.

The methods above make use of data provided by several different sensors, with different output rates, and are thus bound to arrive at different time instants. Since the algorithms are not intended to be run in real time, data interpolation is performed when integrating.

3.2.5 Comments on DVL calibration procedures

In this section several methods for estimating a rotational misalignment between the DVL and the attitude sensor suite were described, with an extension of a previously existing method being derived in 3.2.3. This method is also capable of detecting sound speed errors and biases in the velocity measurements, both previously identified as error sources in 2.4.3. Strictly speaking, the computation of the bias and the scale factor do not interfere with the rotational misalignment estmative, as these are performed afterwards. However, this is not completely true due to the use of the RANSAC tool. In truth, while they do not interfere with the rotation estmative obtained through a set of $\alpha - \beta$ pairs, they might interfere with the inlier selection and thus indirectly influence the final results. Due to this fact, before calibration it should be specified which parameters are being considered.

It is possible to further extend the calibration method with a weighting mechanism. This could be done, for example, to give more importance to the $\alpha - \beta$ pairs with more similar norms, since the difference of the norms can act as a noise indicator. This would, however, discard the possibility of bias estimation. After preliminary tests no improvements in the results was seen with the use of weighting mechanisms, and they were thus not used in this work. Nevertheless, they might have the potential to further extend the method’s accuracy.

Correction for another identified error source, the separation between the beams’ sources 2.4.3.A was not given since it trivially arrives from its description. In fact, one has but to subtract the bias term described in equation C.5

The Adaptive-Method seemingly has the advantage of running in real time; still, in practice there seems to be little interest in doing so, considering that it is a calibration procedure. Moreover, the batch method is also extremely fast and thus could also be theoretically run on site if needed. When extended with RANSAC its speed will obviously be linked to the number of iterations. Finally, there are some advantages that arise from running offline, such as being able to interpolate data with future values.

Recalling 2.4.3.D a constant bias in the measured Doppler shift will induce a bias in the measured vehicle velocity (in instrument axes) which is proportional to the sound speed in water. Therefore, should such a bias be identified, in order to use it under different sea conditions (e.g. different sound
speed in water) it should be scaled accordingly.

Similarly to what was already stated in 3.1.4, the sound speed error could probably be better
estimated through the use of a dedicated probe. Nevertheless, it remains a good thing that the
procedures described in this section are able to deal with error source.

Both the LS-Method and the ELS-Method make use of SVD and thus require that \( \text{rank}(\alpha\beta^T) \geq 2 \).
This obviously imposes some constraints on the motion required by the vehicle carrying the DVL. It
is not the vehicle’s trajectory that matters, however, but rather its trajectory in the velocities space.
Simply put, the vehicle’s velocity must have at least two non-zero components, and at least one of
them must change over time. This suggests that these methods should give better results when
applied to data collected from a fully (or close to) actuated vehicle, such as a typical ROV rather than
a single-propeller vehicle (which is the case of many AUV and surface vessels) which are typically
incapable of generating side-slip and/or vertical motion not induced by pitch. Vertical motion, while
providing valuable excitation for the calibration procedures, would mean that one could not use GPS
for external positioning, but rather have to rely on much less accurate underwater acoustic positioning
systems, which typically also have much lower output rates. It is thus expected that best results should
be achieved using a surface ROV capable of moving sideways, with perhaps limited vertical motion.

It might be interesting to note that the excitation requirements above refer to the velocity at the
point of assembly of the DVL and not the center of mass of the vehicle. It is theoretically possible
then for the vehicle to be stopped, with rotational motion only generating the excitation needed for
calibration. This, however, would require large values of rotational velocities and/or a large distance
between the DVL and the vehicle’s center of mass.
4

Navigation Algorithms

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This chapter is concerned with how to use the navigation aiding sensors reported in 2 in order to estimate the vehicle’s position over time, as well as its linear velocity. The 3D orientation of the vehicle, as well as its angular velocities, are assumed to be known through one of the sensor suites mentioned in 2.2. This can be considered a mild assumption, since reliable sensors with high output rates are commercially available. Two setups will be mainly considered in this chapter; the first concerns a vehicle equipped with a DVL, which is often used combined solely with orientation sensors for navigation, while the second makes use of an USBL positioning system as well as a set of accelerometers. Navigation algorithms using DVL are here presented simply to evaluate the performance of the calibration procedure described in 3.2. The second setup is of interest for the DEEPNET project, and thus several filters capable of fusing the different information sources will be presented.

4.1 Doppler Navigation

Navigating using a DVL only, besides orientation sensors, is commonly referred to as Doppler Navigation. It is basically a subtype of odometry navigation, consisting in simply rotating the velocities measured by the DVL to an inertial frame and then integrating. Given an initial position one thus obtains the trajectory over time according to 2.26. It is thus clear that Doppler Navigation is not a limited error procedure, and the position estimate will drift over time. This drift will be particularly severe in the presence of a misalignment between the DVL and the orientation sensors suite, since this will introduce errors which are consistent over time.

In order to reduce or eliminate this drift the DVL can be complemented with an external positioning system, such as GPS (requiring periodic resurfacing in order to re-initialise the position estimate) and USBL. When operating in water-lock mode the problem becomes increasingly challenging, since one also needs to estimate the unknown water current.

Since the Doppler navigation algorithms in this work will be used mostly for the evaluation of the calibration procedure, the simple but drift prone procedure resumed in 2.26 is sufficient.

4.2 Navigation through USBL/Inertial integration

It must again be stressed that, for the purpose of this work, the 3D orientation is considered to be given and thus does not need to be included in the state variables. In fact, if used in a loosely coupled manner, considering the orientation as given makes both the state and measurement equations linear, allowing the use of simple and well known techniques such as linear state feedback and the KF.

The challenge is thus to propose a state estimator for

\[ X = \begin{bmatrix} W P[k] \\ B v[k] \end{bmatrix} \]  

(4.1)

where \( W P[k] \) is the position in inertial coordinates of the target to track and \( B v[k] \) is its velocity expressed in the body frame.

As discussed previously the local gravity vector and a possible bias in the accelerometers are often included in the state for combined estimation, since an error in the adopted value for the former
and/or the existence of the latter might greatly compromise the filter’s accuracy. In that case, the extended state is given by

\[ X = \begin{bmatrix} W_P \\ B_v \\ W_g \\ B_{Bias} \end{bmatrix} \] (4.2)

Notice that both the bias and the gravity vector are included in the frame where they are constant.

For the sake of simplicity and compactness the leading superscripts of the entrances of \( X \) will often be omitted.

The use of an [USBL] means that, besides the target vehicle carrying the transponder, there is another rigid body (often but not necessarily a second vehicle) holding the transceiver. This leads to the existence of two difference body frames, and thus the notation \( \{B\} \) could lead to confusion. For that reason over this section \( \{BT\} \) will be used for the target's body, while \( \{BS\} \) will stand for the other body frame. The \( S \) derives from Support vessel, since that is the most usual way to carry the transceiver in the field.

The typical setup envisioned for the use of the techniques presented in this chapter is that of a [ROV] equipped with a transponder, connected to a support vessel carrying the transceiver by means of an umbilical (figure 4.1). This means that data can be readily sent and received between both and so accelerations and [USBL] measurements can be assumed to be available without delay. Typically the filter would be running on the support vessel, but it could be running on the [ROV] as well. If the filters are meant to be used on an [AUV] however, care should be given to the time needed to share information. This is particularly relevant since the accelerometers’ readings will be available at the vehicle, while the [USBL] measurements will be available at the support vessel (unless an inverted configuration is employed). In that situation it might be wise to use a delayed state approach.

The approximations referred \[2,3\] with respect to travelling can become unreasonable if the vehicle is operating very far (e.g: many hundreds or even thousands of meters) from the support vessel, particularly if it is moving fast. In that case it might be wise to use more elaborate techniques to take sound travelling time into account.

A common sensor fusion technique is the [KF] which has been widely used in linear state estimation with often good results. Its extensions to non-linear state estimation, namely the [EKF], are also widespread in that field. One of the attractions of these filters is that, besides an estimative of the state, they also provide an indicator of how good that estimative is believed to be. This can obviously be of interest for aiding an autonomous system make rational decisions or, likewise, to help a human
in the loop in making them. They thus represent a stochastic approach to the filtering problem. Both a KF and EKF for the problem at hand will be presented hereunder.

A novel Non-linear observer (NLO) which derives quite intuitively from the type of measurements available will then be presented, along with its inspiration.

Finally, irrespectively to the integration technique utilised, it is wise for it to accommodate some outliers’ removal technique, particularly since they are pretty common in underwater positioning systems due to multipath phenomena. These, along with other implementation considerations, will be presented before the conclusions on this section.

4.2.1 KF approach

The KF, named after the Hungarian mathematician and engineer Rudolf Kálmán, is the optimal filter under the conditions that both the state and measurement equations are linear and corrupted by zero-mean white Gaussian noise [42]. The filter’s equations and assumptions are referred here, while its complete derivation is omitted for the sake of simplicity. The interested reader is kindly referred to [42].

Thus, the system is modelled through

\[ X[k+1] = A[k+1]X[k] + B[k+1]u[k+1] + \eta_X[k+1] \] (4.3)

and the measurements by

\[ z[k] = C[k]X[k] + \eta_z[k]. \] (4.4)

In the above, \( X_{N \times 1}[k] \) is the state vector, \( A_{N \times N}[k] \) is the state matrix, \( B_{N \times l}[k] \) is the input matrix, \( u_{l \times 1}[k] \) is the system’s input, \( z_{m \times 1}[k] \) is the measurement vector, \( C_{m \times N}[k] \) is the measurement matrix and \( \eta_X[k] \sim \mathcal{N}(0, Q[k]) \) and \( \eta_z[k] \sim \mathcal{N}(0, R[k]) \) are, respectively, the process and measurement noise.

The KF algorithm is divided into two steps: the predict step and the update step. The former predicts the state vector and covariance taking 4.3 into account, while the latter updates them upon receiving new measurements, now considering 4.4. The prediction step is performed through

\[ \hat{X}[k+1] = A[k+1]\hat{X}[k] + B[k+1]u[k+1], \] (4.5)

\[ P[k+1] = A[k+1]P[k]A^T[k+1] + Q[k+1], \] (4.6)

where the left-hand side of the first equation is commonly referred to as predicted a priori (state) estimate. Likewise, the left-hand side of the second equation corresponds to predicted a priori estimate covariance. The names derive from the fact that the calculations do not make use of any measurements, thus they are a prediction (through use of a model) prior to the receipt of information. For the same reason, the estimates following the update step are typically preceded by “updated a posteriori”.

A word is in order with respect to notation. In the literature around the KF it is common to represent the state estimative and its covariance prior to the update step with an over-line. This notation was kept here in order to remain consistent with most other sources; this should not be confused with the over-line used above measurements.
The update step is more complex, and is sometimes divided into five steps instead of just two

\[
\hat{y}[k+1] = z[k+1] - C[k+1]\hat{X}[k+1], \tag{4.7}
\]

\[
S[k+1] = C[k+1]P[k+1]C^T[k+1] + R[k+1], \tag{4.8}
\]

\[
K[k+1] = P[k+1]C^T[k+1]S^{-1}[k+1], \tag{4.9}
\]

\[
\hat{X}[k+1] = \hat{X}[k+1] + K[k+1]\hat{y}[k+1], \tag{4.10}
\]

\[
P[k+1] = (I_N \times N - K[k+1]C[k+1])P[k+1]. \tag{4.11}
\]

The left-hand side of 4.7, 4.8, 4.9, 4.10 and 4.11 are known as, respectively, the innovation (also known as measurement residual), the innovation covariance, the Kalman gain, the updated a posteriori state estimate and the updated a posteriori estimate covariance.

Given an initial estimative of the state and its covariance the filter will update them recursively. Notice that the Kalman gain is computed through 4.9 and thus the tuning parameters of the filter are simply the process and measurement noises’ covariance (which will in turn obviously affect the gain) and the initial conditions.

In order to make use of the KF, it is thus necessary to map the problem stated above into this mathematical framework. Recalling 4.1 and 2.7, the state matrix can be defined as

\[
A[k] = \begin{bmatrix}
I_{3 \times 3} & h * \frac{W}{B^T R}
\end{bmatrix}
\]

(4.12)

where \(h\) is the time step used in the filter.

The system’s input will be the accelerations in body axes, which are obtained from the accelerometers readings through 2.5. The input matrix is thus constant and given by

\[
B[k] = \begin{bmatrix}
0_{3 \times 3}
\end{bmatrix}
\]

(4.13)

The measurements are absolute position fixes in the inertial frame, computed using the USBL measurements recurring to 2.16 and so the corresponding measurement matrix is

\[
C[k] = \begin{bmatrix}
I_{3 \times 3}
\end{bmatrix}
\]

(4.14)

While in practice both the measurement and process noises’ covariance are usually adjusted experimentally, their general form can be decided based on theoretical principles so as to facilitate that process.

The process noise is considered to be of the form

\[
\eta_X = \begin{bmatrix}
0_{3 \times 1}
\end{bmatrix}
\]

where \(\eta_v\) is meant to model accelerometers’ noise and error arising from discretisation and sensor data sampling. The process noise covariance matrix \(Q\) will thus be of the form

\[
Q = \begin{bmatrix}
0_{3 \times 3} & 0_{3 \times 3}
\end{bmatrix}
\]

(4.15)

In what regards the measurement noise covariance, \(R\), good arguments can be given against the use of a constant matrix. Any error in the estimated direction cosines will scale proportionally with
the distance between transceiver and transponder, which could be taken into account. A diagonal form for the matrix is often desirable since it reduces the number of parameters to adjust and is simpler to interpret. Still, ignoring important correlations between the variables can reveal itself a more important error source. If using a tetrahedral USBL transceiver, it seems reasonable to assume the direction cosines’ measurements (in the $\{H\}$ frame) to be independent of one another, as discussed in 2.3.2. Generating the position fix through 2.18 there will unavoidably exist some correlation between the error in each direction ($X, Y, Z$) arising both from the error in the range measurements and the normalisation factor; this is, however, independent of the range itself and thus should be crippling to ignore it. When considering the position measurement in the inertial frame though, which is the one of interest, additional correlations will appear due to the mapping $W_H$. Based on all the above the suggested structure for $R$ is

$$R = W_H R_0 W_H R^T,$$  

(4.16)

where $R_0$ is range (measurement) dependent and given by

$$R_0 = (\sigma^2_d + \bar{d}^2 * \sigma^2_c) * I_{3 \times 3}.$$  

Mind that (4.16) depends on both the range (measurement) and the orientation of the transceiver (enclosed in $W_H$) and thus needs to be computed each time a new measurement arrives.

It is important to recall that the output rate of an USBL is seldom higher than 1 Hz, while the filter is intended to run at much higher rates, moreover since inertial sensors typically achieve those rates. Therefore, in practice most of the state estimatives $X[k + 1]$ will be obtained using the predict step only; the update step is only run when new measurements arrive.

Another important remark is that the above equations assume that the accelerometers are spatially coincident with the transponder. When the displacement between both is large enough to be of concern, a point of the rigid body should be chosen for tracking and the appropriate corrections made. The simplest choice would be to pick the accelerometers’ position as the tracking point and thus simply correcting the position measurement by addition of the adequate (orientation dependent) offset.

If the extended state proposed in 4.2 is to be adopted, it is necessary to change $A[k], u[k], B[k], C$ and $Q$. It is no longer possible to correct the accelerometers’ readings through 2.6 without using information from the state, since both the gravity vector and the accelerometers’ bias are needed. Thus, the input $u[k]$ is now simply the accelerometers readings and the correction is performed through the matrix $A$, which takes the form

$$A[k] = \begin{bmatrix} I_{3 \times 3} & h * W_H R & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} - h * S(B \omega) & h * B_H R & -h * I_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}.$$  

(4.17)

Finally, input matrix $B$ and the covariance matrix $Q$ are extended with zeros, since both the gravity vector and the accelerometers’ bias are not measured directly and are assumed to be constant. Small values could also be used if they are considered to be slowly varying; this could be interesting, for
instance, if the vehicle is expected to operate over a large area during a long mission, particularly if near a gravity anomaly.

The system’s input will be the accelerations in body axes, which are obtained from the accelerometers readings through 2.5.

### 4.2.2 EKF approach

The filtering procedure described in 4.2.1 assumes that position measurements are available, thus requiring an additional intermediate step in order to generate the position measurement from the raw USBL readings. This has both advantages and disadvantages; on the one hand it allows easier fusion of the measurements’ information, but on the other hand makes the measurement noise harder to characterise. Sometimes better performance can be achieved through a more tight coupling technique, which can be performed at different levels. Adopting the range measurement and the direction cosines as our measurements is sound in the sense that they derive directly from travelling time and phase shift measurements and thus the noise corrupting them is more likely to be adequately modelled by a normal distribution and to be independent among each others.

These measurements, however, are not a linear function of the state and thus do not fit in the mathematical framework of the KF. The EKF is applicable to non-linear system models and operates by linearisation around the current state estimative and covariance. Similarly to the previous subsection, a complete derivation of the filter will be omitted, referring the reader to [42].

In this framework the system is modelled through

\[ X[k + 1] = f(X[k], u[k + 1]) + \eta_X[k + 1] \]  

and the measurements by

\[ z[k] = h(X[k]) + \eta_z[k], \]  

where \( f(X[k], u[k + 1]) \) and \( h(X[k]) \) are differentiable but not necessarily linear functions.

The predicted state \textit{a priori} estimative is computed in much the same way as in 4.2.1 the previous estimative and the system’s input are simply put through the mapping \( f \):

\[ \hat{X}[k + 1] = f(\hat{X}[k], u[k + 1]). \]  

The linearisation referred is needed to compute the new predicted covariance since it cannot simply be put through the non-linear mapping. The predicted covariance is thus given by

\[ P[k + 1] = F[k + 1]P[k]F^T[k + 1] + Q[k + 1], \]  

where \( F[k + 1] \) is the matrix derivative of \( f \) with respect to \( X \) (see [1] for more on the concept of matrix derivative) evaluated at \( \hat{X}[k] \).

Similarly to the computation of the predicted state, the innovation is also obtained by passing the state estimative through the mapping \( h \):

\[ \tilde{y}[k + 1] = z[k + 1] - h(\hat{X}[k + 1]). \]
The innovation covariance is obtained through

\[ S[k+1] = H[k+1]P[k+1]H^T[k+1] + R[k+1], \tag{4.23} \]

where \( H[k+1] \) is the matrix derivative of \( h \) with respect to \( X \) evaluated at \( \hat{X}[k+1] \). The Kalman gain is then computed according to

\[ K[k+1] = P[k+1]H^T[k+1]S^{-1}[k+1], \tag{4.24} \]

finally allowing the update of both the state estimative and covariance

\[ \hat{X}[k+1] = \hat{X}[k+1] + K\hat{y}[k+1], \tag{4.25} \]

\[ P[k+1] = (I_{N \times N} - K[k+1]H[k+1])P[k+1]. \tag{4.26} \]

It can be noticed that, should \( f \) be a linear mapping, 4.20 and 4.21 will degenerate into 4.5 and 4.6 respectively, making this step equal to that of KF. Likewise, if \( h \) is linear, then 4.22, 4.23, 4.24, 4.25 and 4.26 will degenerate into their KF counterparts as well (respectively 4.7, 4.8, 4.9, 4.10 and 4.11). Finally, if both \( f \) and \( h \) are linear then the whole EKF degenerates into the KF.

Due to the fact that it uses a linearisation, moreover around the estimative of the state instead of the true state, the EKF is typically not guaranteed to converge. Nevertheless, it has been experimentally validated in numerous fields with good results.

For the problem of observing an underwater vehicle’s position and velocity given a kinematic model, the state transition function is linear (as seen above). Therefore, the predict step for the EKF is the same as for the KF. However, the measurement vector is now given by

\[ z = \begin{bmatrix} d \\ C \end{bmatrix} \tag{4.27} \]

where \( d \) and \( C \) are the range and direction cosine measurements as defined in 2.3.1, and so the mapping \( h \) is clearly not linear. One thus needs to compute its Jacobian, and for that it is convenient to define

\[
\theta = W P_{\text{Transponder}} - W P_{\text{Transceiver}} \\
= P - W P_{\text{Transceiver}} \\
= \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}.
\]

\[ d = ||\theta||, \] such that

\[ z = \begin{bmatrix} d + \eta_d \\ \eta_d/	heta_x + \eta_C \end{bmatrix}. \tag{4.29} \]

The Jacobian \( H \) can then be expressed as

\[ H = \begin{bmatrix} \frac{\partial}{\partial x} R(\frac{1}{2} * I_{3 \times 3} - \frac{1}{2} \theta \theta^T) & 0_{1 \times 3} \\ 0_{1 \times 3} & 0_{3 \times 3} \end{bmatrix}. \tag{4.30} \]

Similarly to 4.2.1 some modifications are in order if the displacement between the accelerometers and the transponder are to be taken into account. In this case, however, it is not possible to adjust
the measurements to the accelerometers position, since that would require entwining the range and
direction cosines measurements. For that reason, throughout this work the EKF is used to track
the transponder’s position, and thus the accelerometers’ readings are corrected with recourse to \( \Delta \) and knowledge of \( \Delta_{\text{Transponder–Accelerometers}} \). This also holds for the case of the NLO proposed hereunder.

### 4.2.3 Proposed NLO

Considering the measurements as presented in (range measurement and direction cosine
measurements separately) a simple observation can be made: the range measurements only give
information on the range between transponder and transceiver, while the direction cosines only give
information on the bearings. While a seemingly innocuous observation, it suggests that the range
measurements should only affect the estimated range between the instruments, which in turn should
be left unchanged by the direction cosines measurements’ (and vice-versa); this condition is not
satisfied by the EKF proposed here above (and obviously not by the KF which uses both types of
measurements combined).

To achieve this goal was the main motivation behind the derivation of the new NLO. Notice that
this only concerns the way in which the measurements are incorporated; for this reason the proposed
observer is also divided into two steps, predict and update, with the former being equal to the ones
used in the KF and EKF.

For the derivation of the observer, consider that new measurements \( \vec{d} \) and \( \vec{C} \) have arrived. The
position included in the state to estimate can be decomposed into

\[
W \hat{P} = W_{P_{\text{Transceiver}}} + \hat{d} W \hat{C},
\]

(4.31)

where \( W_{P_{\text{Transceiver}}} \) is, as stated previously assumed to be known. Looking at (4.31) the conditions
necessary to achieve the goal proposed appear naturally: the range measurement \( \vec{d} \) should only
affect \( \hat{d} \), while \( \vec{C} \) should have an impact on \( \hat{C} \) (which must be kept normalised). The first step for
the measurements’ integration is thus to compute \( \hat{d} \) and \( \hat{C} \), which is done quite simply by subtracting
\( W_{P_{\text{Transceiver}}} \) from \( W \hat{P} \): the norm of the derived vector is the estimated range while the normalisation
of the vector itself will generate the expected direction cosines. Then, the distance between sensors
is updated by a simple proportional law:

\[
\hat{d}' = \hat{d} + K_{d}(\vec{d} - \hat{d}).
\]

(4.32)

where \( K_{d} \) is one of the observer's gains (tuning parameter).

Updating the estimated direction cosines is trickier and the proposed solution was based on the
adaptive identifier for a DVL misalignment reported in [21] and already mentioned in 3.2. The rea-
soning is as follows: since one wants the estivative of the direction cosines to remain a normalised
vector, then the update must be of the form

\[
\hat{C}' = R_{\text{update}} \hat{C},
\]
where $R_{\text{update}}$ is the rotation matrix responsible for the update. It is then possible to link the two problems by considering that the current rotation matrix is the identity matrix ($\hat{C} = I_{3 \times 3}$); the update would then be given, as seen in 3.2.2 by

$$\dot{R}_{\text{update}} = R_{\text{update}} S( R_{\text{update}}^T (W \hat{C} \times W \hat{C}) ).$$

(4.33)

Notice that $W \hat{C}$ is expressed in the inertial frame, and thus the measurements need first be rotated using $W R$. When $R_{\text{update}} = I_{3 \times 3}$ then 4.33 takes the simplified form

$$\dot{R}_{\text{update}} = S( W \hat{C} \times W \hat{C} )$$

$$= I_{3 \times 3} S( W \hat{C} \times W \hat{C} ).$$

(4.34)

The second line in 4.34, although seemingly less simplified than the first, was included to illustrate that the equation is in the general form for the derivative of a rotation matrix and that thus

$$\omega = W \hat{C} \times W \hat{C}$$

$$= [\omega_x \omega_y \omega_z]^T$$

(4.35)

can be seen the body-fixed angular velocity. While 4.35 can only be computed at each measurement’s time of arrival one can consider a small angular step through integration of 4.35 over a (virtual) small time step ($T_s$). Defining $\alpha, \beta, \gamma$ such that

$$\hat{C}' = \text{Rot}(\alpha, \beta, \gamma) \hat{C}$$

(4.36)

and recalling 2.1 one attains

$$\dot{\alpha} = \omega_x + s \alpha t \beta \omega_y + c \alpha t \beta \omega_y$$

$$\dot{\beta} = c \alpha \omega_y - s \alpha \omega_z$$

$$\dot{\gamma} = s \alpha c \beta \omega_y + c \alpha c \beta \omega_z$$

Over a small time interval $T_s$ and with zero as initial conditions the angles resulting from integration of the above can be approximated through

$$\alpha \sim T_s \ast \omega_x,$$

(4.37)

$$\beta \sim T_s \ast \omega_y,$$

(4.38)

$$\gamma \sim T_s \ast \omega_z.$$  

(4.39)

$T_s$ can thus be interpreted as a gain, and for that reason will from now on be referred to as $K_c$. Combining 4.36 with 4.37, 4.38 and 4.39 the final expression for $C'$ can be derived. The update step for the position estimative is then wrapped-up with

$$\hat{P}' = W P_{\text{Transceiver}} + \hat{d} \ast \hat{C}'$$

(4.40)

It is important to notice that, while the range measurements can be incorporated on their own (and thus can be discarded as invalid by themselves as well), the three direction cosines are needed in
order to compute $\alpha$, $\beta$ and $\gamma$. This means that none of the three measurements can be discarded, if the information of the other two is to be used. Still, those measurements’ fusion remains completely independent of the range measurement’s, which would seem more important.

To update the velocities estimatives no similar intuition is available. For that reason it was decided to use a loose coupling for the update, which is then given by

$$
\dot{\hat{v}}' = \dot{\hat{v}} + K_v \ast W R \ast (W \hat{P} - W \hat{P}), 
$$

(4.41)

where $W P$ is computed according to 2.18 and mapped to the inertial frame with $\hat{W} B R$. This step can thus only be performed when all four measurements are considered to be valid.

There are yet other important remarks to be made. The computation of $\hat{C}$ requires the division by $\hat{d}$, which may seem troublesome when it approaches zero. In fact, near $\hat{d} = 0$ the angles $\alpha$, $\beta$ and $\gamma$ will tend to increase, for the same noise levels in $C$. This, however, is not that different from what would happen with other filtering techniques, such as the KF and EKF proposed above. When the target is near the transceiver, a small position fix will result in a large angular displacement, which is simply concealed by the fact that the update is performed in Cartesian coordinates.

A perhaps more important issue is the fact that, looking at 4.35, it seems like no angular correction will be performed if the direction cosines are rotated by $\pi$ radians. In fact, the adaptive identified upon which the update procedure was inspired is only locally stable, and does not converge if the initial estimative $\hat{R}(t_0)$ is not within $\pi$ radians of the actual parameter $R$ [21]. In this case, however, the velocity updates will eventually steer the estimative in the right direction until the angular estimative is close enough to allow convergence; the observer will still take quite longer than usual to converge though. While this may seem an alarming issue, truth is in practice the estimated direction cosines are not likely to be so extremely off-target, unless the distance between and transponder and transceiver is very low (in this case convergence will be fast). The only situation where this can be expected to happen is in the transient when starting the filter, if the initial conditions are grossly wrong. This can be easily presented by proper initialisation of the filter. Still, safety procedures to detect this issue (e.g: if several consecutive direction cosine measurements are way off) can be considered so as to re-initialise the filter in this situation.

Finally, although no formal proof of convergence was achieved for this observer, preliminary tests in simulation environment suggest that it works well, apart from the situations identified above.

### 4.2.4 Implementation Considerations

All the techniques described in this section were implemented using a discrete implementation; this is consistent with the discrete nature of the measurements. The time step was set to $h = 0.01s$, since the output rate of the fastest sensors (measuring accelerations and attitude) are typically $100Hz$.

All observers’ equations are divided into a predict and an update set; this makes it easy to take into account that, during most time step, no new measurements will be available.

The observers were included with outlier rejection blocks since these are common in acoustic positioning, due to, among others, multipath phenomena. The Kalman family observers discard mea-
surements if

\[ \tilde{y}^T S^{-1} \tilde{y} \geq \gamma, \]  

(4.42)

where \( S = H \hat{P} H^T + R \) and \( \gamma \) is a scalar threshold [40]. This makes use of the covariance propagated by the filter, thus fully exploiting the filters’ stochastic nature and aids in avoiding deadlocks. The [NLO] being deterministic, does not allow the same procedure to be performed. Thus, the range measurements are simply discarded if

\[ |\tilde{d} - \hat{d}| \geq \gamma_d \]  

(4.43)

and

\[ \frac{180}{\pi} ||logm(I_{3x3}^{T} \ast Rot(\alpha, \beta, \gamma))||_2 \geq \gamma_{gd}, \]  

(4.44)

where \( \gamma_d \) is a threshold on the geodesic distance between \( Rot(\alpha, \beta, \gamma) \) and the identity matrix, expressed in degrees. In practice [4.43] and [4.44] do not allow measurements which are heavily inconsistent with the observer’s current estimated state to be incorporated. This can, of course, lead to deadlocks if the estimates drift too far away from the actual state. While this behaviour was not seen in simulation or trials with real data, a re-initialisation mechanism after several discards could be considered so as to minimise its effects.
5 Results

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5.3 Results regarding the use of position and linear velocity observers .... 66
In this chapter results obtained with the procedures described in the previous chapters are presented. These will consist on simulation results, as well as results obtained with data logged in real trials, whenever available. In the first case the simulation condition will be described, while in the second one a short description of the trial conditions and objectives will be given.

5.1 Results regarding the use of **USBL**

This section is devoted to the evaluation of the calibration procedures and navigation equations relative to the **USBL** sensor, presented in 2.3.2 and 3.1 and will be divided into three main subsections. The first will be concerned with the performance of the ML based estimator for the relative position between transponder and transducer (given range and direction cosine measurements). The second will scrutinise the ML based estimators for the measurement noise parameters and sound speed scale factor. Finally, the third will be focused on the misalignment calibration procedure described and will be complemented with experimental results.

5.1.1 Relative Position Estimators

To analyse the performance of the relative position ML based estimator proposed (2.18) Matlab simulations were performed. A random position \( P \) was first picked, defining the true range and direction cosines \( (d,C) \). Then a large number of measurements \( (N) \) was simulated by adding zero mean Gaussian noise to both types of measurements, with covariance equal to \( \sigma^2_d \) and \( \Sigma_C \), respectively. For evaluation purposes, the mean value \( \hat{\mu} \) of the estimates was computed, along with its covariance \( \hat{\Sigma} \) and RMSE. The CRB is also computed by inverting A.10 and substituting all the algebraic variables for their numerical value, and is compared to the estimator’s covariance both by direct inspection and through their eigenvalues.

So as to allow comparison, the same metrics are also computed for the simpler estimator which consists of just multiplying the range by the direction cosines (2.14). These will be presented with the trailing subscript *simple*.

A large number of random positions was tested so as to reduce the likelihood of chance results due to any particular geometry/configuration. The conditions and results for one of the runs are summarised in tables 5.1 and 5.2, respectively. The covariance matrices and the CRB are shown in 5.2. The tables show the following:

<table>
<thead>
<tr>
<th>( P[m] )</th>
<th>( \sigma_d[m] )</th>
<th>( \Sigma_C[m^2] )</th>
<th>( N )</th>
<th>( \text{eig} (\text{CRB}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[7.269 – 3.535 24.191]</td>
<td>0.7</td>
<td>( \delta(0.025^\circ) )</td>
<td>100000</td>
<td>[0.407 0.407 0.490]</td>
</tr>
</tbody>
</table>

Table 5.1: Simulation variables for the assessment of relative position estimators

<table>
<thead>
<tr>
<th>Estimator</th>
<th>( \mu[m] )</th>
<th>RMSE ( [m] )</th>
<th>( \text{eig} (\Sigma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML Based</td>
<td>[7.265 – 3.531 24.179]</td>
<td>1.140</td>
<td>[0.407 0.409 0.484]</td>
</tr>
<tr>
<td>Simple Estimator</td>
<td>[7.270 – 3.533 24.194]</td>
<td>1.3068</td>
<td>[0.407 0.409 0.891]</td>
</tr>
</tbody>
</table>

Table 5.2: Simulation results of relative position estimators
\[
\hat{\Sigma}_{\text{simple}} = \begin{bmatrix}
0.447 & -0.019 & 0.131 \\
-0.019 & 0.419 & -0.065 \\
0.131 & -0.065 & 0.842
\end{bmatrix}
\] (5.1)

\[
\hat{\Sigma}_{\text{MLBased}} = \begin{bmatrix}
0.413 & -0.003 & 0.021 \\
-0.003 & 0.411 & -0.011 \\
0.021 & -0.011 & 0.476
\end{bmatrix}
\] (5.2)

\[
CRB_{\text{simple}} = \begin{bmatrix}
0.413 & -0.003 & 0.023 \\
-0.003 & 0.408 & -0.011 \\
0.023 & -0.011 & 0.482
\end{bmatrix}
\] (5.3)

The mean values of the estimatives of both estimators are quite near the real value of the position, \(P\), (simulations indicate that they become even closer for larger \(N\)), which suggests that they are both unbiased. Looking at the covariance obtained with the simple estimator, it does not seem to be efficient. The ML based estimator, however, does seem to reach the CRB and so appears to be efficient. Other simulation runs always rendered comparable results.

The simple estimator can be argued to have the advantage of keeping the noise along each direction as uncorrelated as possible; in fact they are only correlated (at least in the assumed model) due to noise in the range measurement. The ML based estimator does not mimic that behaviour. While looking at (5.1) and (5.2) that may not seem obvious, the difference becomes more visible when the distance to the target is larger, since then the direction cosines’ measurement noise starts to dominate the overall positioning error. (5.4) and (5.5) are the covariances obtained in a simulation run with the same conditions, except that the relative position was randomly chosen but constrained to \(|P| = 1000m\), that is, the transponder is 1000m away from the transceiver (much farther than with the conditions above, where the same distance was around 50m).

\[
\hat{\Sigma}_{\text{simple}} = \begin{bmatrix}
626.740 & -0.060 & -1.916 \\
-0.060 & 620.207 & 0.891 \\
-1.916 & 0.891 & 626.110
\end{bmatrix}
\] (5.4)

\[
\hat{\Sigma}_{\text{MLBased}} = \begin{bmatrix}
606.990 & 86.886 & 61.885 \\
86.886 & 228.991 & -287.791 \\
61.885 & -287.791 & 415.328
\end{bmatrix}
\] (5.5)

Notice that the off-diagonal entries of (5.4), unlike those of (5.5), are very close to zero. However, this increase in simplicity comes at the cost of a higher RMSE. Moreover, this independence only holds in the \{H\} frame, where the measurements are made, and is lost in the frames of interest (\{W\} and \{B\}). For these reasons this seemingly advantage is not thought to be of great importance. Also notice how the uncertainty is now much larger in all directions (for both estimators); this is a consequence of the direction cosines’ noise scaling with the true range.

There is another argument which can be given in favour of the efficient estimator. If there is an error in the estimated sound speed, it was seen in 2.3.3.B that it would affect both the range measurement and the direction cosines by the scale factor. It can be expected that the normalisation performed by
the ML based estimator over the norm of direction cosines’ vector somehow diminishes this effect. Table 5.3 displays the results obtained for a simulation run with the same conditions, but with an estimated sound speed wrong by a factor of $\hat{s} = 1.1$.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>$\mu [m]$</th>
<th>RMSE [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML Based</td>
<td>$[7.990 - 3.890 26.594]$</td>
<td>2.828</td>
</tr>
<tr>
<td>Simple Estimator</td>
<td>$[8.794 - 4.281 29.270]$</td>
<td>5.584</td>
</tr>
</tbody>
</table>

Table 5.3: Simulation results of relative position estimators with sound speed error

Both estimators are now obvious and predictably biased, and have larger RMSE. Still, the simpler estimator has suffered more noticeably from the sound speed error. In fact, it can be seen that the mean of the estimatives appear to converge to $\frac{P_s}{2}$, whereas the ML based estimator’s converged to around $\frac{P_s}{2}$. While sound speed can and should be calibrated through other means it is nevertheless a good thing for the estimator to be more robust to small changes in it.

5.1.2 Sound Speed and Noise Parameters Estimators

To assess the performance of the sound speed and noise parameters estimators proposed in 3.1.1, simulations similar to the ones referred above were performed. The performance metrics will remain the same, but the parameters to estimate will vary between estimators. Other difference is that, for each simulation run, several measurements are considered since these estimators are not expected to perform with a single measurement (which was the case in 5.1.1). For that reason there are now two new simulation parameters, the number of iterations $N_{iter}$ and the number of measurements per iteration $N_{measurements}$. Mind that the latter can actually influence the CRB for each set of parameters, while the former (as in the previous section) only affects our assessment of the estimators.

Much alike 5.1.1 many simulations were performed to evaluate these estimators. The one presented in here is just a single illustrative example. All other simulations performed lead to comparable results.

The estimators will be named after the number of parameters they aspire to estimate; thus the estimator for $s$ using range measurements only and with known $\sigma_d$ (resumed to D.3) will be denoted as $\hat{s}_1 = \hat{s}_1$; the combined estimator for $s$ and $\sigma_d$ (equations 3.4 and 3.5) will be named $\hat{\theta}_2 = [\hat{s}_2 \hat{\sigma}_d_2]$ and, finally, the estimator $s$, $\sigma_d$, and $\sigma_{c_d}$ using both range and direction cosines measurements (resumed in E.7, E.8 and E.9 will be represented by $\hat{\theta}_3 = [\hat{s}_3 \hat{\sigma}_d_3 \hat{\sigma}_c_3]$.

The transponder is considered to be stopped (e.g: moored) at coordinates $P_{transponder} = [0 0 30] m$, and the transceiver to be moving in a straight line with $Y = Z = 0$, and $X$ ranging from $-40 m$ to $159 m$; measurements are taken at each integer value of $X$. These basically correspond to a vehicle moving in a straight line at $1 m s^{-1}$, with USBL measurements every second, for a total of two hundred seconds/measurements. For the sake of simplicity orientation and attitude are not considered here, and the position coordinates referred are assumed to be already in the $\{H\}$ frame (where the direction cosine measurements are expressed); while seemingly odd this is of no interest/concern
for this purpose.

The rest of the simulation conditions are summarised under table 5.4 whereas the eigenvalues of CRB for each parameter set are in table 5.5. The results obtained for all three estimators are comprised in table 5.6. The full covariances matrices obtained for \( \theta_2 \) and \( \theta_3 \) are shown 5.6 and 5.8 with the respective CRBs being depicted in 5.7 and 5.9.

<table>
<thead>
<tr>
<th>( s )</th>
<th>( \sigma_d[m] )</th>
<th>( \sum C[m^2] )</th>
<th>( N_{\text{est}} )</th>
<th>( N_{\text{measurements}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>0.7</td>
<td>( \delta(0.025^2) )</td>
<td>10000</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 5.4: Simulation variables for the assessment of sound speed and noise parameters estimators

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>eig(CRB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\theta}_1 )</td>
<td>( 3.027e-7 )</td>
</tr>
<tr>
<td>( \hat{\theta}_2 )</td>
<td>( [3.027e-7, 1.225e-3] )</td>
</tr>
<tr>
<td>( \hat{\theta}_3 )</td>
<td>( [2.747e-7, 5.212e-7, 1.223e-3] )</td>
</tr>
</tbody>
</table>

Table 5.5: Eigenvalues of the CRB for each parameter set for a simulation run

<table>
<thead>
<tr>
<th>Estimator</th>
<th>( \mu )</th>
<th>( \text{RMSE (per parameter)} )</th>
<th>( \text{RMSE (eig C)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\theta}_1 )</td>
<td>0.980</td>
<td>5.553e-4</td>
<td>3.084e-7</td>
</tr>
<tr>
<td>( \hat{\theta}_2 )</td>
<td>[0.980 0.697]</td>
<td>[5.554e-4 3.564e-2]</td>
<td>[3.083e-7 1.264e-3]</td>
</tr>
<tr>
<td>( \hat{\theta}_3 )</td>
<td>[0.980 0.698 0.025]</td>
<td>[5.294e-4 6.750e-1 7.058e-4]</td>
<td>[2.799e-7 4.980e-7 1.265e-3]</td>
</tr>
</tbody>
</table>

Table 5.6: Simulation results for each sound speed and noise parameters estimators

\[
\hat{\Sigma}_{\theta_2} = \begin{bmatrix}
3.085e-7 & -4.772e-7 \\
-4.771e-7 & 1.264e-3
\end{bmatrix}
\]  \hspace{1cm} (5.6)

\[
CRB_{\theta_2} = \begin{bmatrix}
3.027e-7 & -2.162e-7 \\
-2.162e-7 & 1.225e-3
\end{bmatrix}
\]  \hspace{1cm} (5.7)

\[
\hat{\Sigma}_{\theta_2} = \begin{bmatrix}
2.802e-7 & -4.707e-7 & -5.030e-9 \\
-4.707e-7 & 1.265e-3 & -3.330e-7 \\
-5.030e-9 & -3.330e-7 & 4.980e-7
\end{bmatrix}
\]  \hspace{1cm} (5.8)

\[
CRB_{\theta_2} = \begin{bmatrix}
2.750e-7 & -1.964e-7 & -5.030e-9 \\
-1.964e-7 & 1.223e-3 & -5.010e-9 \\
-7.014e-9 & 5.010e-9 & 5.210e-7
\end{bmatrix}
\]  \hspace{1cm} (5.9)

As can be seen, all estimators appear to be unbiased. Moreover, the simulations seem to indicate that they reach the respective CRBs, and thus are efficient.

Comparing the estimators between themselves, the results for \( \hat{s} \) appear to be extremely similar and thus in practice \( \hat{\theta}_2 \) would probably be the best best: \( \hat{\theta}_1 \) requires prior knowledge of \( \sigma_d \) which, unless experimentally estimated in a previous trial, is not available and \( \hat{\theta}_3 \) makes use of the direction cosines’ measurement and thus could worsen the estimates if even a slight misalignment is present. \( \hat{\theta}_3 \) can still be of interest if one desires to estimate \( \sigma_c \); in that case a careful misalignment calibration procedure should be previously performed.
5.1.3 Misalignment Calibration

The evaluation of the misalignment (along with sound speed error and direction cosine's bias) calibration procedure reported in 3.1.2 was done with both simulations and experimental data. The simulations performed tried to reproduce the conditions of an actual sea trial, obviously featuring some kinds of noise and keeping in mind the vehicles’ physical limitations in terms of motion.

5.1.3.A Simulations

The simulations here included are but a few of the many performed, and were chosen for being considered to represent the general performance of the algorithms. The setup is the one considered in 3.1.3 with a moored transponder at \( W_{\text{Transponder}} = [0 \ 0 \ 15] \text{m} \). The transponder’s position is assumed to be unknown. The support vessel carrying the transceiver will perform two circular manoeuvres with different radius (30m and 50m), since a single circle could lead to a wrong vertical coordinate due to an error in the estimated sound speed. For a better simulation of a realistic scenario the circles are de-centred and deformed, with some vertical motion as well. The vehicle’s trajectory is depicted in figure 5.1. Nominal velocity is \( 0.5 \text{ms}^{-1} \) with constant yaw rate, which would allow a complete circle to be made in \( \sim 380 \text{s} \). All angular velocities are summed with sinusoidal terms to simulate waving. Normal distributed noise with zero mean was added to all sensors measurements; its standard deviations can be seen in table 5.7 along with the sensors’ output rates. Moreover, the USBL range measurements were corrupted with 10% outliers, since these are pretty common in practice, mostly due to multipath phenomena.

![Figure 5.1: Simulated support vessel trajectory](image)

<table>
<thead>
<tr>
<th>Sensor Simulated</th>
<th>Measured Variable</th>
<th>( \Sigma ) [s.u]</th>
<th>( f ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[GPS]</td>
<td>( WP_{\text{Antenna}} )</td>
<td>( \delta([0.1^2 \ 0.1^2 \ 0.1^2]) )</td>
<td>10</td>
</tr>
<tr>
<td>[MRU]</td>
<td>( \eta_2 )</td>
<td>( \delta([0.017^2 \ 0.002^2 \ 0.002^2]) )</td>
<td>100</td>
</tr>
<tr>
<td>[USBL]</td>
<td>( H_C )</td>
<td>( \delta([0.025^2 \ 0.025^2 \ 0.025^2]) )</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.7: Noise parameters for the simulation run used in transceiver misalignment calibration

Although the transponder is moored, sensor noise will lead to jumps in the measured position. Notice that this includes but is not limited to noise in the USBL measurements; recalling 2.3.2 it is clear that the measured transponder position will be affected by noise in the USBL, GPS and attitude and orientation sensors. On top of that, a transceiver misalignment and an error in the estimated
sound speed used for the USBL measurements are also considered in the simulation. These are parametrised, respectively, by the trio of euler angles $\phi, \theta, \psi$ and the scale factor $s$. The values adopted for the simulation can be seen under table 5.8. The displacement between the GPS antenna and the transceiver and the $T_H R$ mapping are also presented in the same table.

<table>
<thead>
<tr>
<th>$\phi$ [°]</th>
<th>$\theta$ [°]</th>
<th>$\psi$ [°]</th>
<th>$s$</th>
<th>$B \Delta_{\text{Transceiver-Antenna}}$ [m]</th>
<th>$T_H R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>-0.8</td>
<td>3.1</td>
<td>0.98</td>
<td>$[-0.3 \ 0.55 \ 3]$</td>
<td>$\text{Rot}(-45^\circ, \text{atan} (\sin (45^\circ))$</td>
</tr>
</tbody>
</table>

Table 5.8: Angular misalignment and sound speed error used in simulation run

The 2D absolute position of the transponder, as obtained through the sensors’ measurements, is shown in figure 5.2. The doughnut shape is quite typical of a misaligned transceiver, with a moored transponder. Notice that the figure is zoomed, and so the outliers are not visible.

![Transponder position (XY) before sensors’ calibration](image)

Figure 5.2: Transponder position (XY) before sensors’ calibration

First, the transponder’s position was estimated using the algorithm described in G. The initial estimative for the 2D position was given by the median of the vessel’s positions (recall it was trying to perform circles centred on the target); the vertical coordinate was initialised at an estimated value (as discussed in G it is not unreasonable to assume such an estimative is available); in this case it was 4m off ($\hat{Z} = 11m$). The values for the thresholds were chosen as $T_{\text{large}} = 20$ and $T_{\text{small}} = 2$; these should be adjusted to the actual expected ranges of course. The results are summarised in table 5.9.

<table>
<thead>
<tr>
<th>$W X_{\text{Transponder}}$ [m]</th>
<th>$W Y_{\text{Transponder}}$ [m]</th>
<th>$W Z_{\text{Transponder}}$ [m]</th>
<th>$\hat{s}$</th>
<th>% of data used</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.023</td>
<td>0.096</td>
<td>14.924</td>
<td>0.981</td>
<td>90.0</td>
</tr>
</tbody>
</table>

Table 5.9: Results for transponder positioning algorithm

The estimated position for the transponder is about 10cm away from the actual position, while the sound speed correction factor converged to the real value. Moreover, the algorithm used the expected number of inliers, which is another good performance indicator.

Next, the alignment calibration algorithms reported in 3.1.2 were run; the results are comprised in table 5.10. The entry $d_g$ stands for the geodesic distance between the estimated and the real alignment (rotation) matrix, as defined in (25).
Looking at table 5.10, all methods produced alignment angles close to the real ones up to the tenth of degree; the alignment obtained with the \textit{dCos – Method}, despite the expectations, seems to have performed slightly worse with a higher \( gd \). The \textit{Pw/Delta – Method}, besides estimating the alignment correctly, also estimated \( \Delta_{\text{Transceiver–Antenna}} \) although the accuracy of the later seems to be poor. This, however, does not seem to have hindered the calibration procedure. Other simulations corroborate this trend.

While the \textit{dCos – Method} might have the advantage of being able to correct an eventual bias in the direction cosines’ measurements, simulation results suggest that, under the conditions above, with estimates such biases within 0.01 on each coordinate. It is likely for any existing bias to on that order of magnitude or even small, which in practice renders the method useless. If a bias is suspected, longer calibration trials, a more precise position fix on the transponder and/or better sensors would thus be necessary.

Figure 5.3 shows the 2D absolute position of the transponder, as obtained through the sensors’ measurements, using \( \hat{s} \) and \( \hat{R} \) obtained with the \( \Delta_{\text{Transceiver–Antenna}} \) method. Comparing with 5.2 the improvements are clear: the doughnut shape is gone and the point cloud is now narrower and centred on the actual transponder position.

However, using for validation the same data used by calibration could be considered a bad practice. For this reason a line trajectory was also simulated for validation, under the same conditions; the results before and after calibration can be seen in figures 5.4 and 5.5. Again the improvements are clear: before calibration the moored transponder appears to mimic the transceiver’s trajectory, while after calibration the narrower point cloud centred on the true position is obtained.
Perhaps also important to notice is that, the two position estimators seem to perform equally well (under visual inspection) after calibration, the ML based estimator seems to perform slightly better before calibration. This is more clearly seen in figure 5.2. The reason behind this is the increased robustness to errors in sound speed estimation, already discussed in 5.1.1.

5.1.3.B Experimental Results

During the 9th of August 2013, experiments were conducted at the Tagus River with USBL misalignment calibration in mind. The setup consisted in the ISeaTec, a research vessel owned by ISR equipped with a GPS antenna (functioning in fixed RTK mode), a MRU and the Micronav (by Tritech) USBL transceiver. The transponder was moored at a fuzzily known location. The ISeaTec, before loading of the equipment, can be see in figure 5.6.

The Micronav USBL transceiver, transponder and and its interface hub can all be seen in figure 5.7. The technical specifications are summarised under table 5.11, while the equipments’ physical dimensions and depth ratings can be consulted in table 5.12. Although the Micronav transceiver also provides internal attitude measurements, these were not used. The offset between the transceiver and the GPS antenna is given by $\Delta T_A = [0.04, -0.03, 3.286] m$. The rotation matrix from the $\{H\}$ to the $\{BS\}$ frame is the same as the one used in 5.1.3.A.
Table 5.11: MicronNav USBL specifications

<table>
<thead>
<tr>
<th>Range Accuracy</th>
<th>Bearing Accuracy</th>
<th>Beam-width</th>
<th>Tracking Range (XY-Z)</th>
<th>Output Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>±0.2 m</td>
<td>±3°</td>
<td>180°</td>
<td>500 m - 150 m</td>
<td>0.1 - 2 Hz</td>
</tr>
</tbody>
</table>

Table 5.12: MicronNav USBL equipments’ physical properties

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Height [cm]</th>
<th>Diameter [cm]</th>
<th>Weight in Air [Kg]</th>
<th>Depth Rating [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transceiver</td>
<td>27.0</td>
<td>7.5</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>Transponder</td>
<td>7.6</td>
<td>5.6</td>
<td>0.225</td>
<td>750</td>
</tr>
</tbody>
</table>

In figure 5.8, the mechanism used to moor the transponder at the sea bottom can be seen (depth around 17.5m).

After mooring the transponder, a calibration manoeuvre consisting of several circles with different radius and orientations were performed. The circles were roughly centered at the transponder’s projection at the surface, with radius starting at 20m and then enlarged to 40m. Turns were made both in clockwise and anti-clockwise orientation. A validation trial was then conducted, consisting of a back-and-forth line trajectory, followed by a drifting period and finally another straight line with induced roll. As proposed previously in 3.1.4, the range data collected during these manoeuvres was used to estimate the transponder’s position, along with the sound propagation speed in water. The estimated position, along with the USBL measurements along the calibration trials, can be seen in figure 5.9, while the plots for the validation trial can seen in figure 5.4.
Looking at the $Z$ coordinate it is clear that the measurements are biased depending on the current transceiver position (not displayed), which could be a sign of the presence of a misalignment. The corresponding error metrics are under table 5.13.

<table>
<thead>
<tr>
<th>Trial</th>
<th>$\sigma_x$ [m]</th>
<th>$\sigma_y$ [m]</th>
<th>$\sigma_z$ [m]</th>
<th>RMSE [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>1.443</td>
<td>1.162</td>
<td>1.724</td>
<td>2.359</td>
</tr>
<tr>
<td>Validation</td>
<td>1.439</td>
<td>1.311</td>
<td>1.602</td>
<td>2.679</td>
</tr>
</tbody>
</table>

Table 5.13: Error metrics for both trials, before calibration

As with the simulation runs, all three calibration algorithms were run, leading to the results displayed in table 5.14. Obviously now there is no ground truth to compare to, so the alignment estimatives have to be evaluated based on re-navigation.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\hat{\phi}$ [$^\circ$]</th>
<th>$\hat{\theta}$ [$^\circ$]</th>
<th>$\hat{\psi}$ [$^\circ$]</th>
<th>% of data used</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\cos$ Method</td>
<td>-3.522</td>
<td>1.895</td>
<td>-0.757</td>
<td>64.774</td>
</tr>
<tr>
<td>$P$ Method</td>
<td>-1.256</td>
<td>-0.120</td>
<td>-0.725</td>
<td>64.890</td>
</tr>
<tr>
<td>$P_{\omega}/\Delta$ Method</td>
<td>1.612</td>
<td>-1.278</td>
<td>-1.026</td>
<td>43.337</td>
</tr>
</tbody>
</table>

Table 5.14: Alignment estimatives for sea trials in Lisbon

As can be seen, the estimates do not seem consistent among themselves, except perhaps for the value of $\hat{\psi}$, which is obviously not a good sign. The re-navigated trajectories obtained with the $d\cos$ Method alignment are shown in figures 5.11 and 5.12 while the error metrics are kept under table 5.15. This particular method was chosen both because it rendered the best RMSE and due to it separating the range measurements from the direction cosines’ entirely, as mentioned previously.
The plots do not seem indicate any clear improvement. The error metrics also seem to indicate that the calibration procedure did not seem to provide the expected and desired improvements.

While this failure could be blamed on a number of different causes, arguments can be made over a particular one. If the estimated position for the moored transponder is grossly wrong, that would of course have an impact over the misalignment calibration. Figure 5.13 shows the range measurements as well as the estimated range as estimated error for the validation trial. It can seen that the error is clearly biased in some parts of the trial, corresponding mostly to the back-and-forth line trajectory where the ISeaTec sailed away far from the transponder in two different direction. This behaviour can be explained by an incorrect estimated transponder position, which may have compromised the calibration procedures.
5.1.4 Comments on the results regarding the use of USBL

The estimators proposed for the sound speed and noise parameters seem, based on simulation results, to be unbiased and to reach the CRB.

As for the relative position estimation, using range and direction cosines information, the ML based estimator also appears to be efficient which, added to its increased robustness to errors in the estimated sound speed, may be an important argument towards its use in lieu of other techniques.

In what regards in-situ misalignment calibration, simulation results indicate that the proposed methods can indeed be useful and lead to a general improvement in the USBL measurements. However, they did not seem to work well under actual sea trials. The problem however, appears to precede them and rely on the estimation of the moored transponder’s position. While it cannot be entirely ascertained why the method used apparently failed to render accurate results, it is possible that the acoustic noise coming from the boat’s motors interfered with the USBL equipment. Another possibility is that the transceiver was still too close to the sea surface.

It should be said that, although not featured in this thesis, other trials regarding the use of USBL took place on the 24th of April 2013. Those trials used the moving transponder approach, depicted in figure 3.2 with the transponder attached to a MEDUSA vehicle. While the data collected in that day would in theory be suitable for calibration testing, unresolved synchronisation issues rendered it less useful. It was thus chosen not to include data from those trials in this work.

5.2 Results regarding the use of DVL

Results concerning the calibration procedures relative to the DVL sensor will now be provided. Since it is not common to have the sorts of a ground truth for the 3D linear velocity, the evaluation of the calibration procedures referred in 3.2 will be performed through Doppler Navigation, as described in 2.4.2. The results will be divided into two subsections: the first will present simulation results whereas the second will display results with data obtained during actual sea trials.
5.2.1 Simulation Results

The simulations here presented intend to reproduce, within its possibilities, the experimental setups described in 3.2.4. A first simulation will feature a single heavily under-actuated surface vessel carrying the DVL while the second mimics a ROV with actuation over more DOFs and a supporting surface vessel with an acoustic position system (eg USBL).

5.2.1A Single surface vessel

The vessel carrying the DVL is assumed to be only capable of inducing forward motion. For this reason, the vessel's behaviour is limited to accelerating/decelerating along the $BX$ axis. The calibration trial lasts twenty minutes and a constant surface water current ($1ms^{-1}$) is considered to be present. So as to exploit as much as possible this source of excitation, the vessel will align itself with the current for the first half of the trial, and then turn 90° so as to generate maximum side-slip. Meanwhile it will accelerate and decelerate sinusoidally between 0.5 and 1.5ms$^{-1}$. To avoid generating excitation difficult to replicate in practice, roll, pitch and the other two components of velocity in body axes are considered to be always zero (except when induced by the water current). This approximates the conditions obtained with very calm sea states. As in 5.1.3A Gaussian noise with zero mean was added to all sensors’ measurements; its standard deviations and the sensors’ output rates can be seen in table 5.16.

<table>
<thead>
<tr>
<th>Sensor Simulated</th>
<th>Measured Variable</th>
<th>Σ [s.u]</th>
<th>f [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>$\delta WP_{\text{Antenna}}$</td>
<td>$\delta(0.1^2 0.1^2 0.1^2)$</td>
<td>10</td>
</tr>
<tr>
<td>MRU</td>
<td>$\eta_2$</td>
<td>$\delta(0.017^2 0.002^2 0.002^2)$</td>
<td>100</td>
</tr>
<tr>
<td>MRU</td>
<td>$\nu_2$</td>
<td>$\delta(0.002^2 0.002^2 0.002^2)$</td>
<td>10</td>
</tr>
<tr>
<td>DVL</td>
<td>$D_v$</td>
<td>$\delta(0.005^2 0.005^2 0.005^2)$</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 5.16: Noise parameters for the simulation run used in DVL misalignment calibration

An error in the estimated sound speed used for the DVL measurements and a misalignment (parametrised, as before, by three euler angles) were also considered to be present. The values for both, along with the displacement between the GPS antenna and the DVL can be seen under 5.17.

<table>
<thead>
<tr>
<th>φ [°]</th>
<th>θ [°]</th>
<th>ψ [°]</th>
<th>s</th>
<th>$B \Delta_{DVL-Antenna}$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>-0.9</td>
<td>2.8</td>
<td>0.97</td>
<td>[0.8 − 0.5 2.3]</td>
</tr>
</tbody>
</table>

Table 5.17: Angular misalignment and sound speed error used in simulation run

The 2D absolute position of the DVL as obtained through Doppler Navigation, is shown in figure 5.14 as well as the vertical component in the adjacent plot. Both are presented along with the true trajectory for comparison.
Looking at both figures, the existing drift is evident, particularly in the vertical component; this is the signature mark of a misalignment in this type of sensor. From the three calibration methods presented in 3.2, the one that managed to produce reasonable estimatives for the alignment matrix was the $\text{LS – Method}$. The results are presented in table 5.18.

<table>
<thead>
<tr>
<th>$\hat{\phi}$ [$\circ$]</th>
<th>$\hat{\theta}$ [$\circ$]</th>
<th>$\hat{\psi}$ [$\circ$]</th>
<th>$d_g$ [$\circ$]</th>
<th>$\hat{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.272</td>
<td>-1.067</td>
<td>2.780</td>
<td>0.597</td>
<td>0.969</td>
</tr>
</tbody>
</table>

Table 5.18: Results for misalignment calibration algorithm

While the $\text{ELS – Method}$ and the $\text{Adaptive – Method}$ failed to generate valid results, the $\text{LS – Method}$ lead to an alignment estimative quite close to the true value, as well as a good estimative for $s$. The trajectories obtained after alignment and sound speed correction can be seen in figure 5.15. The Doppler navigation procedure is not yet drift-less; still clear improvements can be seen. Looking at the vertical component in figure 5.15, it is also clear why the popular performance metric consisting on the final displacement between true and Doppler navigation trajectories can be misleading: it is quite close to zero just by chance, if the trial ended a bit later or earlier the result would be worse.

Again, a validation trial is required to further test the alignment obtained; a simple ten minutes circular manoeuvre was chosen (constant yaw rate). The results of Doppler navigation before calibration can be seen in figure 5.16, whereas 5.17 holds the results after calibration. Again, the drift is still present, but somewhat mitigated.
It is important to stress that the actuation restrictions of the surface vessel greatly limit the calibration procedures. In fact, for the same conditions and noise levels, reducing the water current speed to $0.5 m/s^{-1}$ greatly harms the results, since the procedure is relying on it for excitation. While in practice one can except further velocity components to appear due to waving, these are bound to be residual.

Another important fact is that the $LS - Method$, the only one that was able to render good acceptable results for this scenario, is not capable of dealing with a bias in the DVL velocity measurements. Extending the trial's duration did not lead to great improvements in the results.

## 5.2.1.B ROV and support vessel

The ROV considered in this simulation is expected to be able to accelerate in all three directions. Sensor noise is still as described in 5.2.1.A apart from the GPS entry since this sensor is not used any more. In turn, a generic acoustic position system (e.g: USBL or LBL) is considered, with a standard deviation of $1m$ along each direction. The calibration trial consists in the vehicle moving in a seemingly random pattern for $20m$, with sinusoidal velocities in the three body-axes, frequencies of which are low (under $0.05Hz$) so as to minimise errors arising from the interpolation of the position fixes, which could be in issue due to their low output rates. $BV_Z$ was made to oscillate between $\pm 1m/s^3$, while $BV_Y$ is limited to half that amount. $BV_X$ oscillated around $0.8m/s^{-1} \pm 0.2m/s^{-1}$, so that the vehicle’s velocity (in absolute value) never approaches zero. The trajectory of the ROV along with the one obtained through Doppler Navigation before calibration, can be seen in 5.18. While the 3D trajectory is quite complex, the misalignment induced drift is hard to miss and the final displacement reached 56.345m.
Table 5.19 holds the results for the calibration using the three methods considered in this work. Again, the *LS* – *Method* seems to exhibit the best performance, while the *ELS* – *Method* still gave a reasonable result. Since it appears to be the most reliable, results with the *LS* – *Method*’s alignment and $\hat{s}$ will be presented.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\hat{\phi}$ [$^\circ$]</th>
<th>$\hat{\theta}$ [$^\circ$]</th>
<th>$\hat{\psi}$ [$^\circ$]</th>
<th>$d_{\Delta}$ [$^\circ$]</th>
<th>$\hat{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adaptive – <em>Method</em></td>
<td>-0.295</td>
<td>-0.191</td>
<td>0.503</td>
<td>2.482</td>
<td>-</td>
</tr>
<tr>
<td><em>ELS</em> – <em>Method</em></td>
<td>-0.381</td>
<td>-0.338</td>
<td>2.789</td>
<td>0.883</td>
<td>0.947</td>
</tr>
<tr>
<td><em>LS</em> – <em>Method</em></td>
<td>-0.120</td>
<td>-1.144</td>
<td>2.505</td>
<td>0.572</td>
<td>0.970</td>
</tr>
</tbody>
</table>

The re-navigated trajectory obtained with after calibration is presented in figure 5.19; the trajectories are clearly more aligned, which contributed to the improvement in the final displacement, which went down to 6.676m.

A validation trial consisting of a circular-like survey, with the ROV manoeuvring at the surface with constant yaw rate and surge oscillating between 0.8ms$^{-1}$ and 1.2ms$^{-1}$ was also considered. The trajectories obtained before and after calibration can be seen in figures 5.20 and 5.21. Improvements are again clear, while a drift is still present. Final displacement decreased from 7.166m to 2.234m.
Since the ROV setup provides superior excitation comparing to the one using a simple vessel one can expect to be able to estimate an eventual bias in the DVL measurements (such as the referred in \[2.4.3.D\]) along with the alignment using the ELS – Method. However, simulations have shown that, with the noise levels considered above, the algorithm is incapable of estimating a bias with accuracy up to $\text{ms}^{-1}$. The algorithm did succeed in estimating such bias when the noise levels of the attitude and orientation sensor suite were reduced by an order of magnitude ($0.01^\circ$), and the positioning system’s precision and output rates was set to those typical of a GPS. In practice this could be obtained through use of a high precision MRU and a ROV equipped with GPS manoeuvring at the surface.

### 5.2.2 Experimental Results

The DVL misalignment calibration procedures were tested with experimental data collected during two sets of sea trials. The first took place in the Azores, during the week starting at July 22nd 2012. The second took place at August 9th 2013 in Lisbon, along the river Tagus.

#### 5.2.2.A Trials in the Azores

In these trials two different DVLs were used: the NavQuest 600 Micro (from LinkQuest, California, USA) and the Workhorse Navigator 600 kHz (from RDI, California, USA). The former will hereby be denoted as $NQ$ and the latter as $RDI$. The instruments can be seen in figure 5.22.
Both the \textit{NQ} and the \textit{RDI} employ a 4 Beam Convex configuration (as shown previously in figure 2.2). However, they were installed in different ways: the \textit{NQ} was assembled in a Janus configuration (as seen in 2.3) while the \textit{RDI} departed from that configuration by $45^\circ$ rotation along the vertical. The beam disposition for both equipments can be seen in 5.23. As their names suggest, both equipments work at 600kHz; other specifications can be seen in table 5.20 while 5.21 holds their physical dimensions.

During the week starting at July 22nd 2012 both sensors were used in two different environments: one with favourable and the other with adverse conditions. The former were thus conducted inside a harbour, being the terrain mostly flat, the currents very small and the waves small (the harbour entrance had some exposure to waves). The latter took place in the open sea, not far from the coastline, in one of the areas envisioned for the future MORPH trials, over rugged terrain with steep slopes and 3D complex bottom morphology. Figure 5.24 shows the areas in Faial used for the trials;
the green trajectory was the one followed in trial 2 (flat terrain) while the black one represents the vehicle's position along trial 1 (rugged terrain).

Figure 5.24: Trial location, rugged (black) and flat (green) terrain areas

In all the trials a Rigid Inflatable Boat (RIB) was used and driven manually along predefined tracks. The vehicle was equipped with a GPS operating in fixed RTK mode providing 3D position, a MRU measuring the vehicle's 3D orientation and angular velocities, the DVLs mentioned above (one at a time) measuring the vessel's linear velocity, and an acquisition system developed by Instituto Superior Técnico (IST) based on Robotic Operating System (ROS) for proper data synchronization and time stamping. The boat, equipped with the MRU GPS antenna the NQ DVL can be seen in figure 5.25.

Figure 5.25: Setup used for the trials in the Azores

Similarly to 5.2.1, the evaluation of the calibration procedures will be performed by visual comparison of the trajectories before and after calibration, as well as by the final displacement \(d\). The results for the four trials, prior to calibration, can be seen under table 5.22. A decay in performance can be seen for the trials where environment conditions are more adverse.

<table>
<thead>
<tr>
<th>Trial</th>
<th>DVL</th>
<th>Environment Conditions</th>
<th>Time Elapsed [s]</th>
<th>Distance Travelled [m]</th>
<th>(d) [m/Km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RDI</td>
<td>Adverse</td>
<td>3600</td>
<td>3391</td>
<td>13.670</td>
</tr>
<tr>
<td>2</td>
<td>RDI</td>
<td>Favourable</td>
<td>2650</td>
<td>2673</td>
<td>1.703</td>
</tr>
<tr>
<td>8</td>
<td>NQ</td>
<td>Adverse</td>
<td>3300</td>
<td>3321</td>
<td>20.518</td>
</tr>
<tr>
<td>9</td>
<td>NQ</td>
<td>Favourable</td>
<td>1900</td>
<td>2241</td>
<td>8.083</td>
</tr>
</tbody>
</table>

Table 5.22: Results for each trial before calibration
Trials 2 and 9, which featured more favourable conditions, were chosen for calibration, and the others for validation. The re-navigated trajectories for all trials can be seen in figures 5.26, 5.27, 5.28 and 5.29. As in 5.2.1, the vertical component is presented separately so as to allow better inspection.

![Figure 5.26: Re-navigated position for trial 2, before calibration](image)

![Figure 5.27: Re-navigated position for trial 1, before calibration](image)

![Figure 5.28: Re-navigated position for trial 9, before calibration](image)

![Figure 5.29: Re-navigated position for trial 8, before calibration](image)

In the trials where the \( NQ \) was used the signs of a misalignment are clear: there is a severe drift in the vertical component. Moreover, the drift seems consistent along the whole trial, which is expected.
since the RIB’s motion should be almost limited to surge. In the calibration trial using the RDI (trial 2, shown in figure 5.26) this is not so evident; in trial 1, however, a misalignment seems apparent again.

The calibration results for trials 2 and 9 can be seen under tables 5.23 and 5.24 respectively. In both cases the LS – Method and ELS – Method have similar estimates, while the Adaptive – Method gives a different result. The oddly large \( \hat{\psi} \) given by the latter suggests that the result is not reliable. In fact, re-navigating through the calibration trials using the alignment given by the Adaptive – Method lead to terrible results, while the other two calibrations rendered similar results. For trial 9 the LS – Method lead to a slightly better \( d \) (2.479m/Km) and was thus adopted as the alignment estimative for the NQ trials. In the case of trial 2, even though the alignment estimatives are close the ELS – Method lead to much better results than the LS – Method \( (d = 1.146m/km) \) and was thus adopted for the RDI trials.

<table>
<thead>
<tr>
<th>Method</th>
<th>ˆφ [°]</th>
<th>ˆθ [°]</th>
<th>ˆψ [°]</th>
<th>s</th>
<th>% of data used</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS – Method</td>
<td>1.014</td>
<td>0.066</td>
<td>-1.784</td>
<td>1.006</td>
<td>71.63</td>
</tr>
<tr>
<td>LS – Method</td>
<td>1.475</td>
<td>0.129</td>
<td>-1.581</td>
<td>1.002</td>
<td>-</td>
</tr>
<tr>
<td>Adaptive – Method</td>
<td>1.347</td>
<td>-0.626</td>
<td>-11.637</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.23: Estimated misalignments for trial 2 (RDI DVL)

<table>
<thead>
<tr>
<th>Method</th>
<th>ˆφ [°]</th>
<th>ˆθ [°]</th>
<th>ˆψ [°]</th>
<th>s</th>
<th>% of data used</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS – Method</td>
<td>-0.786</td>
<td>-0.289</td>
<td>-0.784</td>
<td>0.999</td>
<td>83.71</td>
</tr>
<tr>
<td>LS – Method</td>
<td>-1.816</td>
<td>-0.358</td>
<td>-0.424</td>
<td>1.007</td>
<td>-</td>
</tr>
<tr>
<td>Adaptive – Method</td>
<td>0.573</td>
<td>-0.901</td>
<td>-11.114</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.24: Estimated misalignments for trial 9 (NQ DVL)

Both for the NQ and the RDI trials the values of \( s \) are very close to one; the sound speed used for navigation thus seem to be about correct.

The re-navigated trajectories obtained for the calibration trials using the estimated alignments are shown in figures 5.30 and 5.31. The improvement in trial 9 is easily spotted, while it is not clear if there are any improvements at all for trial 2.
Finally, the plots for the validation trials after calibration can be in figures 5.32 and 5.33. There is a clear improvement in trial 8, confirming that the alignment estimated in trial 9 lead to increase navigational capabilities. The normalised drift for this trial equalled $d = 18.971 \text{m/km}$. In trial 1, however, the results do not seem to be any better than before calibration, with $d = 12.100 \text{m/Km}$.

5.2.2.B Trials in Lisbon

During the 9th of August 2013 trials aimed at DVL misalignment calibration and evaluation were conducted, besides the ones already referred to in 5.1.3.B using an USBL. In this case the ISeaTec was equipped with a GPS antenna (functioning in fixed RTK mode), a MRU and the NQ DVL.

Two trials were performed using the DVL. The first, lasting around thirty minutes, was intended for calibration, while the second lasted around twenty five minutes and consisted on a star manoeuvre, for validation. In the calibration trial there was concern with trying to generate proper excitation to the calibration algorithms; for that reason different surge velocities were induced, and the vehicle was
also oriented perpendicular to the water current so as to generate sway. Unfortunately, the water current was quite diminished at that time of the day.

Plots of the re-navigated trajectories, before calibration, can be seen in figures 5.34 and 5.35. The normalised drifts were, respectively, of 5.355m/km and 3.534m/km. The effects of a misalignment are clearly visible, resembling the patterns seen in the simulations presented in 5.2.1.A, namely the consistent drift in the vertical coordinate.

![Figure 5.34: Re-navigated position for calibration trial, before calibration](image1)
![Figure 5.35: Re-navigated position for validation trial, before calibration](image2)

The estimated alignments, using the calibration trial, can be seen in 5.25. Only the ELS – Method provided reasonable results, still the misalignment in roll estimated by the other two method are far too large not to be detected by visual inspection. Re-navigation using the alignments confirms that the other two alignments lead to much worse results.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\phi$ [°]</th>
<th>$\theta$ [°]</th>
<th>$\psi$ [°]</th>
<th>$\dot{s}$</th>
<th>% of data used</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELS – Method</td>
<td>2.378</td>
<td>-0.324</td>
<td>1.019</td>
<td>1.015</td>
<td>46.10</td>
</tr>
<tr>
<td>LS – Method</td>
<td>19.395</td>
<td>-4.290</td>
<td>1.814</td>
<td>0.970</td>
<td>-</td>
</tr>
<tr>
<td>Adaptive – Method</td>
<td>47.910</td>
<td>-9.292</td>
<td>-7.326</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.25: Estimated misalignments for the calibration trial

The trajectories obtained for both trials using the alignments estimated by the ELS – Method can be seen in figures 5.36 and 5.37. The normalised drifts are of 10.1772m/Km and 4.7843m/Km for the calibration and validation trials, respectively. There was thus no improvement in terms of $d$ for both trials. Still, looking at 5.36 particularly the right-hand plot, one can see that most of the drift occurred in the first 400s of the trial. This can perhaps be attributed to another fact. Figure 5.38 shows the absolute value of the velocity measured by the DVL it is easy to see that part of the trial corresponds
to the stronger accelerations/decelerations. While they are not so large to make the output rate of
5Hz be of concern, they are linked to motor activity and thus it is possible that its acoustic noise
impaired the instrument’s accuracy.

Figure 5.36: Re-navigated position for calibration trial, after calibration

Figure 5.37: Re-navigated position for validation trial, after calibration

Figure 5.38: Absolute value of the velocity (calibration trial), as measured by the DVL

5.2.3 Comments on DVL calibration results

Simulations have shown that, when using a GPS for positioning, the results improve when the
offset $\delta_{DVL-Antenna}$ is smaller. This can be explained by the fact that this offset will make any
orientation and attitude errors spread into a DVL position error.

It was previously stated that the error source deriving from the separation between beam sources,
explain in 2.4.3.A did not have a relevant impact on the performance of the DVL. While this appears
to hold true for the process of Doppler Navigation, the calibration procedures are much more sen-
sitive and lead to different results whether correction for that error source is performed or not. All
experimental results throughout this document were obtained using the correction, since in general it seemed to lead to better results.

The in-situ misalignment calibration methods have proved themselves useful when applied to real data, allowing increased performance for the trials using the NQ even when the calibration was performed in different trial during a different day. The Adaptive − Method failed to present good estimates however, which may indicate an increased sensitivity to noise.

Nevertheless, the calibration procedures did not lead to any improvements in the trials with the RDI. There may be a number of reasons for this. First of all, since the RDI had to be re-attached and to the vessel (the trials took place in different days) the possibility that the alignments for both trials were not exactly the same should not be disregarded. The fact that trial 2 without calibration had such good results, while 8 exhibited a much larger and more consistent drift seems to point in this direction. Another possible explanation is that in trial 2 the RDI was outputting the velocities directly in the instrument frame and, according to the manual, may be performance internal corrections for beam misplacement/misalignment. Notice that, as discussed in 2.4.3.B, this is not the same as a misalignment between the DVL and the attitude unit, and thus is not captured by the calibration procedure. In trial 1, the instrument was outputting the radial velocities along each beam which in that case would mean that error source would remain uncorrected. Finally, as discussed in 3.2.4, the use of a surface vessel only is not ideal since it has heavy restraints on its motion capabilities. The fact that the calibration trial took place at an harbour, with residual currents only, and was not planned specifically for calibration may have further decreased the data’s richness in terms of excitation for the calibration procedures.

5.3 Results regarding the use of position and linear velocity observers

This section concerns the evaluation of the observers presented in 4.2, both using simulated and experimental data. While in the first case ground truth comparison can be done both for the position and linear velocity quantities, that is not true for the latter, where the velocities were not being measured directly.

5.3.1 Simulation Results

In the simulation herein the vehicle will first dive up to a given depth, both by vertical motion and use of pitch. It will then level itself and perform a circular 2D trajectory. In the end the vehicle returns to the surface using vertical motion only. The offset between the transponder and the accelerometers, in the \{BT\} frame, is given by $\Delta_{T-A} = [0.3 - 0.05 0.02]$. The whole trial has the duration of twenty minutes. The orientation and velocity of the ROV (or AUV) can be seen in figures 5.39 and 5.40 respectively. For the sake of simplicity, the USBL transceiver is considered to be stationary at $W_{P_{\text{Transceiver}}} = [30 30 0]$, while the transponder is carried by the AUV. The parameters of the (Gaussian zero-mean) noise can be seen under table 5.26. Keep in mind that, while the orientation
and position of the support vessel are kept constant throughout the trial, their readings are still subject to noise.

![Figure 5.39: Orientation of the simulated vehicle along trial](image)

![Figure 5.40: Velocities of the simulated vehicle along trial](image)

<table>
<thead>
<tr>
<th>Sensor Simulated</th>
<th>Measured Variable</th>
<th>$\Sigma$ [s.u]</th>
<th>$f$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRU</td>
<td>$\eta_2$</td>
<td>$\delta(0.017^2, 0.002^2, 0.002^2)$</td>
<td>100</td>
</tr>
<tr>
<td>MRU</td>
<td>$\eta_P$</td>
<td>$\delta(0.002^2, 0.002^2, 0.002^2)$</td>
<td>100</td>
</tr>
<tr>
<td>USBL</td>
<td>$d$</td>
<td>$0.7^2$</td>
<td>1</td>
</tr>
<tr>
<td>USBL</td>
<td>$H_C$</td>
<td>$\delta(0.025^2, 0.025^2, 0.025^2)$</td>
<td>1</td>
</tr>
<tr>
<td>GPS</td>
<td>$H_P$</td>
<td>$\delta(0.1^2, 0.1^2, 0.1^2)$</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5.26: Noise parameters for the simulation run used in observers’ evaluation

In all cases the filters were initialised a few meters off the actual position and with zero velocities. The time step $h$ used was of $0.01\, s$, since that was the sampling time of the fastest sensors. Performance will be evaluated both by visual inspection of the estimated trajectories and velocities but also by checking the standard deviation in the positioning error, as well as the maximum displacement during the whole trial (not counting the first five minutes due to initialisation).

The results obtained using the KF can be seen under figures 5.41 and 5.41 as well as table 5.27. The parameters used were $\sigma_d = 0.7\, m$, $\sigma_c = 0.025$ and $\eta_v = 0.0113\times3\, ms^{-1}$. 
As can be seen the filter managed to provide driftless estimates both for position and velocity. Moreover, the position estimates seem to be consistently closer to the ground truth values than the USBL measurements, further assessing the value of this type of navigation. It should be noticed that, as discussed previously, the filter is tracking the position of the accelerometers. The other two filters mentioned herein will track the position of the transponder.

The results obtained with the EKF are displayed in figures 5.43 and 5.44 while the error metrics are kept under table 5.28. Measurement and process noise were set to the same values as in the previous case.
Figure 5.44: Velocities as estimated by EKF in simulation

<table>
<thead>
<tr>
<th>$\sigma_x [m]$</th>
<th>$\sigma_y [m]$</th>
<th>$\sigma_z [m]$</th>
<th>$d_{\text{max}} [m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.72</td>
<td>2.16</td>
<td>1.54</td>
<td>9.27</td>
</tr>
</tbody>
</table>

Table 5.28: Error metrics for the EKF observer during simulation trial

It can be seen in the plots that the EKF displayed stronger oscillations than its linear counter-part, which lead to worse error metrics as well. Still, the filter clearly remains unbiased and does not drift over time.

Finally, results using the NLO can be seen in figures 5.45 and 5.46 with the error metrics under table 5.29. The gains used were $K_d = 0.1$, $K_c = 0.1$ and $K_v = 0.01$.

Figure 5.45: Position as estimated by NLO in simulation

Figure 5.46: Velocities as estimated by NLO in simulation
The NLO seems to have a rougher time during initialisation, with a clearly visible overshoot in the vertical component and in the velocities. After the initial transient, however, it displays a similar behaviour to the other two filters and the error metrics in fact compare favourably with those of the EKF.

### 5.3.1.A Effects of accelerometers bias and errors in the estimated gravity vector

Consider using the KF for the same simulation as in the previous section, but with two new error sources. The accelerometers’ measurements are now corrupted by a bias, equal to \( \mathbf{\pi}_{\text{Bias}} = [0.01 \ 0.01 \ 0.02] \text{ms}^{-2} \). Moreover, the simulation will now use the gravity acceleration vector \( \mathbf{g} = [0.171 \ 0.9809] \text{ms}^{-2} \), while the filter will still be using \( \tilde{\mathbf{g}} = [0 \ 0 \ 9.810] \text{ms}^{-2} \). The value of \( \tilde{\mathbf{g}} \) can be obtained from \( \mathbf{g} \) simply by rotating \( 1^\circ \) along the Y axis. Plots of the estimated velocities and vertical component can be seen in figures 5.47 and 5.48.

**Table 5.29: Error metrics for the NLO observer during simulation trial**

<table>
<thead>
<tr>
<th>( \sigma_x \ [m] )</th>
<th>( \sigma_y \ [m] )</th>
<th>( \sigma_z \ [m] )</th>
<th>( d_{\text{max}} \ [m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
<td>1.67</td>
<td>1.28</td>
<td>6.43</td>
</tr>
</tbody>
</table>

**Figure 5.47:** Velocities as estimated by KF with extra error sources

**Figure 5.48:** Vertical component as estimated by KF with extra error sources

Besides having an overall worse performance, the filter drifts after a while due to the outliers rejection block not accepting any measurements, since they differ too much from the estimated state.
This could be perhaps improved by changing the threshold $\gamma$, the process and the measurements covariances but would inevitably lead to a decrease in performance. Simply discarding the outliers rejection block would render the filter quite sensitive to outliers, while re-initialising it when drifts are detected would lead to periodic error booms. It is thus clear that the presence of accelerometers’ bias and an error in $\hat{g}$ are quite harmful.

Usage of the KF with extended state, as previously described in 4.2, leads to the results displayed in figures 5.49 and 5.50 and to the error metrics under table 5.30.

![Figure 5.49: Position as estimated by KF with extended state in simulation](image1)

![Figure 5.50: Velocities as estimated by KF with extended state in simulation](image2)

<table>
<thead>
<tr>
<th>$\sigma_x [m]$</th>
<th>$\sigma_y [m]$</th>
<th>$\sigma_z [m]$</th>
<th>$d_{max} [m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>2.00</td>
<td>1.47</td>
<td>16.70</td>
</tr>
</tbody>
</table>

Table 5.30: Error metrics for the KF with extended state in simulation

Notice that the relevant process and measurement noise parameters were kept the same, as well as the threshold $\gamma$, so as to allow comparison. It is likely that fiddling with the latter could lead to an improve in the results, particularly in the (huge) initial drift. At any rate it can clearly be seen that, as the filter learns the correct values for the bias and gravity, the estimates improve drastically. Final estimates for those parameters were $\hat{a}_{Bias} = [0.010 \ 0.010 \ 0.025] ms^{-2}$ and $\hat{g} = [0.171 \ 0.9815] ms^{-2}$, which are pretty close to the actual values used in the simulation. Also notice that the trajectory was not planned in order to make this estimation easier; further roll and pitch motion in the beginning of the trial would certainly lead to faster convergence and better results.
5.3.2 Experimental Results

In order to assess the observers’ performance with actual data, trials were conducted at the Tagus River on the 24th of April 2013. Again, the ISeaTec vessel was employed, equipped with GPS and attitude units, as well as the USBL transceiver. A Medusa vehicle equipped with its internal IMU and GPS antenna was used as a surface ROV. Pictures of both can be seen in figures 5.51 and 5.52. A rope knotted around the Medusa’s lower cylinder then held the suspended transponder, as well as an AHRS by SBG.

![Figure 5.51: Medusa vehicle inside the ISeaTec, before take off](image)

![Figure 5.52: Medusa vehicle operating as surface ROV](image)

The trajectory of the transponder is known with some accuracy due to the Medusa’s GPS, combined with the attitude information and known offsets. The velocities, however, were not being measured by any sensor and thus no ground truth is available for comparison.

The results obtained with the KF can be seen in figures 5.53 and 5.54. Since the position plots’ viewing window is clearly dominated by some error peaks, a zoom of the most central part is also shown in 5.55. The parameters were tuned to $\sigma_d = 0.1m$, $\sigma_c = 0.0025$, $\eta_v = 0.1I_{3\times3}$. The outliers’ rejection threshold was set to $\gamma = 10$. 

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The results are clearly much worse than those obtained in the simulations. In fact, the filter’s position estimates do not seem to lie inside the USBL measurements, leading to no improvement in accuracy. They do, instead, seem to diverge rapidly in-between position fixes. This also becomes obvious when noticing the large amplitudes of the oscillations in all velocity components.

As for the EKF its parameters were set to $\sigma_d = 1.5m$, $\sigma_c = 0.1$, $\eta_v = 0.05I_{3x3}$. The outliers’ rejection thresholds used were $\gamma_d = 25$ and $\gamma_c = 0.05$. The results can be seen in figures 5.56 and 5.57.
Results with the EKF are even worse, with time periods where the positioning error grows largely to a few dozens of meters. The velocities’ estimates exhibit the same erratic behaviour, which obviously does not correspond to the vehicle’s actual velocities, although no ground truth is available for comparison.

Finally, the NLO was run with $K_d = 0.75$, $K_C = 0.35$ and $K_v = 0.1$; the thresholds related to outliers were set to $\gamma_d = 50$ m and $\gamma_{\theta_d} = 10^\circ$. The results are displayed in figures 5.58 and 5.59.

Although perhaps slight better than the EKF’s, the NLO’s behaviour is still quite disastrous. Positioning error again reaches several dozens of meters, with highly oscillatory velocity estimates.
5.3.3 Comments on the results regarding proposed observers

In simulation, all three proposed observers proved to be able to estimate position and linear velocities without biases or drifts. The velocity estimates, however, displayed a relevant oscillatory component, even when the vehicle was undergoing pretty smooth motion. To improve this, higher grade accelerometers and attitude units (eg: lower noise in those sensor simulated measurements) would likely need to be employed.

For the stochastic filters a process noise model with a sparse covariance matrix was adopted, where the non-zero entries corresponded to errors in the derivative of the velocities arising from accelerometers and attitude noise, as well as discretisation errors. While, for a given attitude and velocity, the model used for the derivative of the position should be exact (hence the zeros in the process covariance), it is still subject to discretisation errors. Moreover, errors in attitude would obviously also have a direct impact on it; these are particularly serious for lower-end and cheaper attitude units with reduced accuracy. It is thus possible that the use of a full covariance matrix better models the system and would lead to better results.

When running with data collected from an actual sea trial, all filters lead to unacceptably large errors. There are a number of reasons which might help explain this fact. First of all, the USBL measurements appear to exhibit some (non constant) bias, most likely due to an uncorrected transceiver misalignment. None of the proposed filters are prepared to deal with such error source, and it is likely to be the cause of some havoc. Still, from the very beginning, the velocity estimatives seem to be diverging quite quickly. This can be due to both an error in the estimated local gravity vector or, similarly, a misalignment between the accelerometers and the attitude unit. A bias in the accelerometers’ readings could also partly help to explain the terrible results. In fact, as it is, removing the accelerometers readings altogether leads to an improvement in results.

Even though an extended version of the KF and the EKF, capable of dealing with errors in the estimated gravity acceleration vector and accelerometers’ bias, was presented, the misalignment issue mentioned above and the poor excitation of the vehicle’s motion render it quite useless in this situation.

Finally, unresolved synchronisation issues were detected. These are particularly relevant for the case of the transceiver’s attitude, since it changes fairly fast due to wave induced motion.

New trials under more controlled conditions are thus certainly in need to further evaluate the filters’ usefulness.

In simulation, the simple KF seemed to exhibit the best behaviour. Still, the NLO appears to render comparable performance when at close range. Enlarging the distance between the transceiver and the transponder lead to a decrease in performance for both the EKF and the NLO, with the KF standing out more clearly. In the case of the NLO improvements can probably be made by the use of non-constant gains, particularly $K_c$ and $K_v$. The first, concerning rotational corrections, should be reduced for larger distances since it will lead to large position changes. The latter might follow the same reasoning, since when the distance between transceiver and transponder is larger so is the expected range measurement error (eg: deviation), and thus the gain should be more modest.
In simulation the effects of an increase in the range between transponder and transceiver were also studied briefly. While this leads to a decrease in performance with all observers, due to the USBL measurements being less reliable, the KF seem to suffer the least from this. In the case of the NLO its response could most likely be improved by the gain adjustment referred above.
Conclusions and Future Work
The first and perhaps major conclusion of this work, easily perceived by any reader of it and/or its references, is that the navigation problem is far from solved in the context of autonomous or remotely operated underwater vehicles. The heavy constraints imposed by the diminished penetrability of electromagnetic waves, and limited speed of sound waves, in water make it particularly challenging for deep operation vehicles.

The results regarding the use of DVLs, presented in 5.2, corroborate the general claim found in the literature that the most important error source in Doppler Navigation is an angular misalignment. While the methods here presented and tested did seem capable of providing fairly accurate misalignment estimatives and thus some improvements in the overall navigation capabilities, there is still undoubtedly room for improvement. These could come, for instance, under the form of estimating the optimal behaviour the vehicle should exhibit in order to provide the best excitation possible for calibration.

Results regarding the USBL calibration methods proposed were not entirely consistent. While in simulation the calibration procedures using a moored transponder lead to precise misalignment estimatives and huge improvements in positioning error, these could not be replicated in practice. Unfortunately it is not clear whether this failure was due to unresolved synchronisation issues, gross errors in the models used, acoustic noise generated by the boat's motors or other yet unconsidered error source. Further trials are therefore in need; these should be carefully and specifically aimed at discarding each of the expected bottlenecks.

Even though the $d\cos$ Method proposed is theoretically capable of estimating biases in the USBL's direction cosines measurements, the simulations performed seem to indicate that in practice that is only possible with high-accuracy attitude and heading solutions and manoeuvring capabilities hard to replicate in practice.

The zero-mean white Gaussian noise assumed for the USBL measurement model, used for the relative position, sound speed and noise parameters estimators, should be given some attention in the future. In fact, some work was already performed towards that end, and the range measurements' noise did seem to be modelled acceptably well through this means. However, the range of the ranges available was not that large in order to allow confident remarks to be made in that respect; for that reason this study was omitted from this thesis. A similar preliminary analysis was also conducted with the direction cosines measurements; there, the Gaussian model did not seem to fit the experimental data well. However, since misalignment were expected to be presented (and were not successfully estimated and corrected for) such a fit would not seem likely.

The observers proposed, while positively validated in simulation, did not seem to provide reasonably accurate estimates when dealing with data from actual sea trials. While the conditions of these were not ideal, namely the depth of acoustic transducers and the quality of the ground truth available, it is of the utmost importance to ascertain whether logistical issues were the sole reason for their poor performance. For that, further trials under more controlled conditions should be planned.

Analysing simulation results, the simpler KF seems to outperform the other more complex fusion techniques. However, this may be just due to the fact that these have different and perhaps more
difficult to tune parameters. In the end, the response to experimental data should be the decisive factor when choosing the navigation filter.

The use of stochastic filters, such as the ones from the Kalman family, has the advantage of providing the user not only with a state estmative but also with a measure of its accuracy. Whether this is accurate on itself is, of course, a completely different question, one that can be addressed by checking the filter’s consistency when in presence of a ground truth. Consistency checks, instead of simple actual and estimated state comparison, can be an interesting way to tune the filter’s parameters.

As referred in 4.2, formal proofs of convergence for the proposed NLO were not obtained. Should it prove useful, more effort should be put towards that end. An analysis which takes into account the sparse nature of USBL measurements, perhaps recurring to contraction theory, could be particularly important.

An interesting and promising area for future work is the study of the impact of different types of trajectories in the observers’ performance. This goes for both the target vehicle, carrying the transponder, and the support vehicle on the surface.

Although building a realistic simulator was not one of this thesis’ goals, simulations were often needed and used to validate and/or test algorithms and procedures. There is clearly the need for a more realistic simulator, which better encapsulates the effects of currents and waves.

The use of a dynamical model of the vehicle, instead of the simpler kinematic model, certainly has the potential to aid in reaching extended navigation accuracy. This could be used, for instance, as a substitute for the somewhat unreliable accelerometers, or as a complement to them.

One of the goals of this work was to evaluate the severity of the losses in performance when not using a DVL. This was motivated by the DVL being a relatively large and expensive sensor, making it hard to include in small and low cost ROVs or AUVs. While the position and linear observers proposed in 4.2 did not include such a sensor in their suite, it was seen in 5.2 that the DVL solely aided by an attitude unit, delivers wonderful accuracy in the short term. The results obtained with the observers, presented in section 5.3 do not seem to be able to replicate that accuracy, even in simulation. Moreover, it does not share the issue common to USBL sensors of having the measurement error grow larger with depth, since it does not exchange any information with the surface. The results here presented thus seem to indicate that not using a DVL will most likely lead to important drops in navigation capabilities. While it is true that the use of higher grade AHRS can lead to a drastic improvement in the position and velocity estimatives, it would also mitigate the cost advantage of not using a Doppler sensor.
Bibliography


Estimation of USBL relative position
It is hereby considered that the measurements \( \overline{d}, \overline{cx}, \overline{cy} \) and \( \overline{cz} \) are corrupted by white zero-mean Gaussian noise with variances \( \sigma_d^2 \) (for the range measurements) and \( \sigma_c^2 \) for the direction cosines. Note that this is equivalent to assuming that the travelling time and the phase differences are corrupted by a similarly white zero-mean Gaussian noise with scaled variances, which seems more reasonable since both of which are direct measurements. The measurements are also assumed to be independent; this may be the weakest assumption since the phase-differences usually are computed with respect to a common receiver. This issue should thus be given further attention in the future. The measurement models are depicted in equations A.1 and A.2

\[
\overline{d} = d + \eta_d; \quad \eta_d \sim N(0, \sigma_d^2). \tag{A.1}
\]

\[
\overline{C} = C + \eta_C; \quad \eta_C \sim N(0_{3 \times 1}, \Sigma_C), \tag{A.2}
\]

where

\[
C = \begin{bmatrix} \overline{cx} \\ \overline{cy} \\ \overline{cz} \end{bmatrix},
\]

\[
\overline{C} = \begin{bmatrix} \overline{cx} \\ \overline{cy} \\ \overline{cz} \end{bmatrix},
\]

and

\[
\Sigma_C = \sigma_c^2 \cdot I_{3 \times 3}.
\]

Using this model and defining

\[
\theta = H P_{\text{relative}}
\]

\[
= \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\]

as the vector of parameters to estimate,

\[
X = \begin{bmatrix} d \\ \overline{cx} \\ \overline{cy} \\ \overline{cz} \end{bmatrix}
\]

as the vector of measurements,

\[
\mu = E\{X\}
\]

\[
= \begin{bmatrix} d \\ \overline{cx} \\ \overline{cy} \\ \overline{cz} \end{bmatrix}
\]

\[
= \begin{bmatrix} \|\theta\| \\ \theta \end{bmatrix}
\]
as the expected value of the measurements and

\[ \Sigma = \begin{bmatrix} \sigma^2_d & 0_{1 \times 3} \\ 0_{3 \times 1} & \Sigma_C \end{bmatrix} \]

as the measurements' covariance, the log-likelihood function is given by

\[ L(\theta | X) = -\frac{1}{2} (X - \mu)^T \Sigma^{-1} (X - \mu) - \ln(4\pi^2 \sqrt{|\Sigma|}). \]

Notice that a zero distance leads to a singularity in the direction cosines. In practice this situation would mean that the transponder and the transceiver were superimposed, which is quite far from the situations of interest. All calculations and remarks will thus assume that \( d > 0 \). The gradient is equal to

\[ \frac{\partial L}{\partial \theta} = (X - \mu)^T \Sigma^{-1} \frac{\partial \mu}{\partial \theta}, \quad (A.3) \]

where

\[ \frac{\partial \mu}{\partial \theta} = \begin{bmatrix} \frac{\theta^T}{\| \theta \|^2} \\ \frac{\partial \theta}{\| \theta \|^2} \end{bmatrix} \]

Making \( \frac{\partial L(\theta | X)}{\partial \theta} = 0_{1 \times 3} \) the single critical point of \( L(\theta | X) \) is found:

\[ \hat{\theta} = \frac{\partial}{\partial \theta} \Sigma, \quad (A.4) \]

which corresponds to the estimator proposed first suggested in [2.18]. Taking the second derivative with respect to \( \theta \),

\[ \frac{\partial^2 L(\theta | X)}{\partial \theta^2} = -\frac{\partial \mu}{\partial \theta}^T \Sigma^{-1} \frac{\partial \mu}{\partial \theta} + (I_{3 \times 3} \otimes (X - \mu)^T \Sigma^{-1} \frac{\partial^2 \mu}{\partial \theta^2}. \quad (A.5) \]

The full expression for equation \( \frac{\partial^2 \mu}{\partial \theta^2} \) can be seen in [A.8], while the Hessian of the likelihood function is fully depicted in [A.9]. It is not clear whether \( \frac{\partial^2 L(\theta | X)}{\partial \theta^2} \prec 0 \), or even if \( \frac{\partial^2 L(\theta | X)}{\partial \theta^2} |_{\theta = \hat{\theta}} \prec 0 \) for that matter, so \( \hat{\theta} \) cannot be positively assessed as the ML estimator for \( \theta \). Still, this result is useful to derive the CRB:

\[ CRB(\theta) = I(\theta)^{-1}, \quad (A.6) \]

with the Fisher information matrix being given by

\[ I(\theta) = -E \left\{ \frac{\partial^2 L(\theta | X)}{\partial \theta^2} \right\}. \quad (A.7) \]

The complete expressions for equation \( A.7 \) can be found in [A.10]. The explicit formulation of the CRB was not included due to its large size.

\[ \frac{\partial^2 \mu}{\partial \theta^2} = d^{-5} \begin{bmatrix} d^2(y^2 + z^2) & -d^2 xy & -d^2 xz \\ 3x(x^2 - d^2) & y(3x^2 - d^2) & z(3x^2 - d^2) \\ y(3x^2 - d^2) & x(3y^2 - d^2) & 3xyz \\ z(3x^2 - d^2) & 3xyz & x(3z^2 - d^2) \\ -d^2 xy & d^2(x^2 + z^2) & -d^2 yz \\ y(3x^2 - d^2) & x(3y^2 - d^2) & 3xyz \\ x(3y^2 - d^2) & 3y(3y^2 - d^2) & z(3y^2 - d^2) \\ 3xyz & z(3y^2 - d^2) & y(3z^2 - d^2) \\ -d^2 xz & -d^2 yz & d^2(x^2 + y^2) \\ z(3x^2 - d^2) & 3xyz & x(3z^2 - d^2) \\ 3xyz & z(3y^2 - d^2) & y(3z^2 - d^2) \\ x(3z^2 - d^2) & y(3z^2 - d^2) & 3z(z^2 - d^2) \end{bmatrix} \quad (A.8) \]
\[
\frac{\partial^2 L(\theta | X)}{\partial \theta^2} = 
\begin{bmatrix}
H_{11} & H_{12} & H_{13} \\
H_{12} & H_{22} & H_{23} \\
H_{13} & H_{23} & H_{33}
\end{bmatrix}
\]  \tag{A.9}

with

\[
H_{11} = \frac{\partial^2 (3x^2y - yd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
H_{22} = \frac{\partial^2 (3y^2x - xd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
H_{33} = \frac{\partial^2 (3z^2y - yd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
H_{12} = \frac{\partial^2 (3x^2y - yd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
H_{13} = \frac{\partial^2 (3x^2z - zd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
H_{23} = \frac{\partial^2 (3y^2z - zd^2)}{d^5 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2} + \frac{\partial}{\partial d^3 \sigma_c^2},
\]
\[
I(\theta) = \frac{x^2}{d^2 \sigma_c^2} + \frac{y^2}{d^2 \sigma_c^2} + \frac{z^2}{d^2 \sigma_c^2} + \frac{xy}{d^2 \sigma_c^2} + \frac{yz}{d^2 \sigma_c^2} + \frac{xz}{d^2 \sigma_c^2} - \frac{x^2}{d^2 \sigma_c^2} - \frac{y^2}{d^2 \sigma_c^2} - \frac{z^2}{d^2 \sigma_c^2} + \frac{d^2}{d^2 \sigma_c^2} + \frac{d^2}{d^2 \sigma_c^2} + \frac{d^2}{d^2 \sigma_c^2} \tag{A.10}
\]
Velocity estimation through Doppler effect
Doppler frequency shift is the name given to the frequency shift measured by a receiver when there is relative motion between it and the transmitter. More specifically, if the signal is emitted by a moving object, a stationary receiver will measure the frequency

\[ f_r = \frac{f_t}{1 \pm \frac{v}{c}}; \]  

(B.1)

where \( f_t \) and \( f_r \) are, respectively, the transmitted and the received frequency, \( c \) is the wave propagation speed and \( v \) is the component of the emitter’s velocity which is parallel to the direction of propagation. Similarly, if the emitter is static but the receiver is moving, the following holds

\[ f_r = f_t \left( 1 \pm \frac{v}{c} \right). \]  

(B.2)

In both cases the upper sign is valid for approaching objects, while the lower one applies for separations.

When used in bottom-lock mode the vehicle is both a moving transmitter and receiver, while the reflection at the sea floor can be analysed as a stationary receiver immediately followed by a stationary emitter [10]. Combining B.1 and B.2 one gets the expression for the received frequency

\[ f_r = f_t \frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}} = f_t \frac{(1 \pm \frac{v}{c})^2}{(1 \mp \frac{v}{c})(1 \pm \frac{v}{c})} \]  

(B.3)

It is assumed that the vehicle’s speed does not suffer relevant changes during the wave’s travelling time. Expanding B.3 and ignoring the squared terms that appear (the fraction \( \frac{v}{c} \) is assumed to be very small) one reaches

\[ f_r = f_t \left( 1 \pm 2 \frac{v}{c} \right) \]

and, finally, the expression that allows the computation of the vehicle’s velocity along the beam’s direction:

\[ v = \frac{c \Delta f}{2 f_t}. \]
Approximations due to the separation between beam sources in a DVL
Naming $B \Delta_0$ as the (constant) displacement between the vehicle’s center of mass and the DVL’s center, and $B \Delta_i$ as the (also constant) displacement between the latter and the position of beam $i$ source, it can be seen that $v'_i$, the velocity along beam $i$ taking the displacements into account, is given by

$$v'_i = B e_i \cdot (B v_{CM} + B \omega \times (B \Delta_0 + B \Delta_i)). \quad (C.1)$$

Notice that this is different from the attained by ignoring the beams displacement (eg: $\Delta_i = 0$), which would render

$$v_i = B e_i \cdot (B v_{CM} + B \omega \times B \Delta_0). \quad (C.2)$$

In fact, looking at C.1 and C.2 the following equality can be derived

$$v'_i = v_i + B \omega \cdot (B \Delta_i \times B e_i) = B v_i + B \omega \cdot B \theta_i, \quad (C.3)$$

where

$$B \theta_i = B \Delta_i \times B e_i.$$  

Stacking vertically all the $v'_i$ terms to produce the $N \times 1$ matrix $V'$ one thus attains

$$V' = V + \Theta B \omega, \quad (C.4)$$

where $\Theta$ is the $N \times 3$ matrix that arises from vertically stacking all $\theta_i$ elements.

This error would not be of concern if it did not propagate into $B v'_{DVL}$, since one is not interested in the velocity at each beam’s center along its pointing direction, but rather in one single three dimensional velocity measurement corresponding to a unique point (here the center of the DVL for the sake of simplicity). It is thus important to ascertain whether the inequality $v'_i \simeq v_i$ will lead into the more concerning $B v'_{DVL} \simeq B v_{DVL}$.

Recalling 2.29 and C.4

$$B v'_{DVL} = E^+ V' = E^+ (V + \Theta B \omega) = B v_{DVL} + E^+ \Theta B \omega = B v_{DVL} + M_{Bias} B \omega, \quad (C.5)$$

where $M_{Bias}$ is the (constant) $3 \times 3$ matrix given by

$$M_{Bias} = E^+ \Theta. \quad (C.6)$$
For a typical DVL with four beams pointing at the bottom with same angle with the horizontal and separated by 90° from each other, the bias matrix has the structure

$$M_{Bias} = \begin{bmatrix} 0 & -kd & 0 \\ kd & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$  \hspace{1cm} (C.7)

which means that the resulting bias, \( B_{Bias} \), only depends on the roll and pitch rates \( (p \text{ and } q) \) and has a null vertical component. The constant \( k \) is determined by the DVL geometry and is given by

$$k = \frac{\sin(\theta)}{2\cos(\theta)},$$  \hspace{1cm} (C.8)

where \( \theta \) is the angle by which the beams are depressed from the horizontal, typically around 60°.
Estimation of sound speed error with known measurement standard deviation
The likelihood function, in this simplified case, is given by

\[ L(s | \mathcal{D}) = -N \ln(s \sigma_d \sqrt{2\pi}) - \frac{\sum (\bar{d} - sd)^2}{s^2 \sigma_d^2} \]  

(D.1)

and its derivative

\[ \frac{\partial L(s | \mathcal{D})}{\partial s} = \sum d^2 - s \bar{d}d - \frac{N}{s}. \]  

(D.2)

Taking the critical points of the likelihood function, the candidate estimators appear

\[ \hat{s}_{1,2} = \frac{\sum \bar{d}d \pm \sqrt{(\sum \bar{d}d)^2 + 4N\sigma_d^2 \sum \bar{d}}}{-2N\sigma_d^2}. \]  

Since a negative scale factor is obviously of no interest, the proposed estimator is

\[ \hat{s} = \frac{\sum \bar{d}d - \sqrt{(\sum \bar{d}d)^2 + 4N\sigma_d^2 \sum \bar{d}}}{-2N\sigma_d^2}. \]  

(D.3)

The second derivative of the likelihood function is given by

\[ \frac{\partial^2 L(s | \mathcal{D})}{\partial s^2} = \frac{N}{s^2} + 2s \sum \bar{d}d - 3 \frac{\sum \bar{d}^2}{s^4 \sigma_d^2}, \]  

(D.4)

and the negation of its expected value by

\[ I(s) = \frac{2N\sigma_d^2 + \sum d^2}{s^2 \sigma_d^2}. \]  

(D.5)

The CRB is thus easily obtained:

\[ \text{CRB}(s) = \frac{s^2 \sigma_d^2}{2N\sigma_d^2 + \sum d^2}. \]  

(D.6)

Once again it does not seem trivial to prove analytically that \( \hat{s} \) is the ML estimator for \( s \), or any other properties such as being unbiased and attaining the CRB.
Estimation of sound speed scale factor using all USBL measurements
The likelihood function for $\theta = [s, \sigma_d, \sigma_c]^T$ is given by

$$L(\theta|D, C) = - \frac{\sum (d - sd)^2}{2s^2\sigma_d^2} - 2Nd\ln(\sigma_d\sqrt{2\pi}) - \frac{\sum (\bar{c}_x - scx)^2 + \sum (\bar{c}_y - scy)^2 + \sum (\bar{c}_z - scz)^2}{2s^2\sigma_c^2}$$

$$- (N_x + N_y + N_z)\ln(\sigma_c\sqrt{2\pi}),$$

(E.1)

(E.2)

where $N_d, N_x, N_y$ and $N_z$ correspond, respectively, to the number of available range measurements and the number of available measurements for each of the cosines and $C$ corresponds to the set of all direction cosine measurements. Although in theory they would be same, since each USBL measurement comes with the four quantities, in practice splitting them allows for separate outlier removal in a pre-processing stage. $N$, without any subscript, will from now on be equivalent to $N_d + N_x + N_y + N_z$, and $N_{xyz}$ to $N_x + N_y + N_z$.

The three partial derivatives that form the gradient are then computable:

$$\frac{\partial L(\theta|D, C)}{\partial s} = \frac{\sum (d - sd)^2}{s^3\sigma_d^2} + \frac{d\sum (d - sd)}{s^2\sigma_d^2} + \frac{\sum (\bar{c}_x - scx)^2 + \sum (\bar{c}_y - scy)^2 + \sum (\bar{c}_z - scz)^2}{s^3\sigma_c^2}$$

$$+ \frac{\sum cx(\bar{c}_x - scx) + cy(\bar{c}_y - scy) + cz(\bar{c}_z - scz)}{s^2\sigma_c^2} - \frac{N_x}{s},$$

(E.3)

$$\frac{\partial L(\theta|D, C)}{\partial \sigma_d} = \frac{\sum (d - sd)^2}{s^2\sigma_d^3} - \frac{Nd}{\sigma_d},$$

(E.4)

$$\frac{\partial L(\theta|D, C)}{\partial \sigma_c} = \frac{\sum (\bar{c}_x - scx)^2 + \sum (\bar{c}_y - scy)^2 + \sum (\bar{c}_z - scz)^2}{s^2\sigma_c^2} - \frac{N_x + N_y + N_z}{\sigma_c}. $$

(E.5)

Taking (E.6), (E.3) and (E.5) and equalling them to zero one obtains, amongst other irrelevant ones, the critical point

$$\hat{\sigma}_d = \sqrt{\frac{\sum (d - \bar{sd})^2}{Nd\bar{s}^2}},$$

(E.7)

$$\hat{\sigma}_c = \sqrt{\frac{\sum (\bar{c}_x - \bar{scx})^2 + \sum (\bar{c}_y - \bar{scy})^2 + \sum (\bar{c}_z - \bar{scz})^2}{(N_{xyz})\bar{s}^2}},$$

(E.8)

and

$$\hat{s} = -\frac{b + B + \frac{3a}{B}}{3a},$$

(E.9)

with

$$a = -N(\sum cx^2 + \sum cy^2 + \sum cz^2) \sum d^2,$$

$$b = Nd(\sum cx + \sum cy + \sum cz) \sum d^2 + N(\sum cx^2 + \sum cy^2 + \sum cz^2) \sum \bar{d}d$$

$$+ N(\sum (\bar{c}_x + \sum cy + \sum cz) \sum d^2 + N_{xyz}(\sum cx^2 + \sum cy^2 + \sum cz^2) \sum \bar{d}d,$$
\[ c = -N_d \left( \sum cx^2 + \sum cy^2 + \sum cz^2 \right) \sum d^2 - 2N \left( \sum cx \sum cy + \sum cy \sum cz + \sum cz \sum cx \right) \sum d^2 \\
- N_{xyz} \left( \sum cx^2 + \sum cy^2 + \sum cz^2 \right) \sum d^2, \]

\[ d = N_d \left( \sum cx^2 + \sum cy^2 + \sum cz^2 \right) \sum d^2 + N_{xyz} \left( \sum cx \sum cy + \sum cy \sum cz + \sum cz \sum cx \right) \sum d^2, \]

\[ \Delta_0 = b^2 - 3ac, \]

\[ \Delta_1 = 2b^3 - 9abc + 27a^2d, \]

\[ B = \sqrt[3]{\frac{\Delta_1 + \sqrt{\Delta_1^2 - 4\Delta_0^3}}{2}}. \]

The reader may have recognised \[ E.9 \] as the expression for one of the roots of a cubic function. Although not proved analytically in this document, simulation results indicate that this is the single real root of the polynomial of interest (the other two being complex conjugates), thus being adopted as the proposed estimator for \[ \sigma \]. Likewise, both the estimators for \[ \sigma_d \] and \[ \sigma_c \] had one other solution: its negation. Obviously that would not be a reasonable estimator for the parameters, so \[ E.7 \] and \[ E.8 \] were adopted.
Estimation of 3D transformation parameters between two set points
This appendix merely describes how to obtain the transformation parameters (rotation, scaling and translation) between two 3D set points using SVD; for a complete derivation of this method the user is referred to [27] and [28].

Given two $3 \times N$ (noisy) sets of points, X and Y, which follow

$$y_i = sRx_i + t,$$

the parameters which satisfy the least-squares criterion are given by

$$\hat{R} = USV^T,$$

$$\hat{s} = \frac{1}{\sigma^2} tr(DS)$$

and

$$\hat{t} = \mu_y - \hat{s}\hat{R}\mu_x,$$

where $\mu_x$ and $\mu_y$ are the centroids of the two sets of points, $UDV^T$ is a singular value decomposition of $YX^T$ and

$$S = \begin{cases} I_{3\times3}, & \text{if} \ det(YX^T) \leq 0 \\ \delta(1,1,-1), & \text{if} \ det(YX^T) < 0 \end{cases}$$

The parameters have a unique solution as long as $\text{rank}(YX^T) \geq 2$. h
Estimation of a Moored Transponder’s Position
It would seem that this issue could be addressed simply by averaging some USBL position measurements. However, one must keep in mind that the purpose of this is estimation is to render a ground truth transponder position to be used in a misalignment calibration. Therefore, averaging the position estimatives from a misaligned transceiver will not lead to a viable solution.

Recalling 2.3.3.C, such a misalignment does not affect the range measurements. It is thus preferable to estimate the transponder’s position using range measurements alone, given that the position of the transceiver is known at the time of each measurement. This problem is analogous to that of estimating the position of a transponder given the range to a set of landmarks, studied in detail in [29]. Notice that, while in this situations the measurements take place at different time instants, such is not a concern since the transponder is known to be moored (stopped).

Still, the closed form solutions proposed in [29] require at least four measurements taken with non-coplanar transceiver positions to render accurate 3D positions; in practice the transceiver is often attached to a surface vessel and is thus incapable of vertical motion, apart from residual motion due to waves and other perturbations. Still, they can be used to initialise iterative and more powerful algorithms. Another way to derive a good initialisation point would be to use the XY coordinates of the drop-out point, and an estimative of the relative depth between transponder and transceiver for the vertical coordinate.

I might also be important to take into account a possible mismatch in the estimated sound speed, if it was not previously calibrated. For this purpose one can try and minimise the sum of square differences $\epsilon_i$:

$$\epsilon_i = d_i - \|P_{\text{Transponder}} - P_{\text{Transceiver}}(t_i)\|. \quad (G.1)$$

Another possibility is to have the surface vehicle with the transceiver perform a circular trajectory centred around the projection on the surface of the moored transponder’s position. In this case, a sound speed error will not affect 2D positioning [34], merely reflecting itself in the vertical component. This could be estimated in another way, for instance through use of a depth in proximity with the transponder. However, this approach might not be reliable since the trajectories will surely be offsetted and deformed.

Whatever procedure is used to estimate the moored transponder’s position, it may be useful for it to include outlier rejection techniques. For this purpose a RANSAC approach could thus be used [11]. Still, the approach described in [41] seems particularly fit for this purpose, since it attempts to estimate which measurements are consistent with each other (i.e: the spherical surfaces generated by them intersect to some tolerance) prior to any estimative of the transponder position. For a more detailed description of this method the reader is referred to the original description.

A simple algorithm to estimate the moored transponder’s position, including outliers rejection step, is presented step by step below. Some of the steps will not be represented mathematically, since they can be solved in many ways (e.g: the methods above).

- Compute the expected distance, $\hat{d}$, at the time instant of each measurement using an initial estimative for the transponder’s position, $P_0$.
• Check the absolute value of the difference between the expected and measured distance (the latter hereby $\tilde{d}$), and remove any measurements that exceed a (large) threshold, $T_{h_{\text{large}}}$;

• Minimise $G.1$ using an iterative method;

• Compute the expected distance for the new estimated transponder position and scaling factor;

• Check the absolute value of the difference between $\hat{d}$ and $\tilde{d}$, and remove any measurements that exceed a smaller threshold, $T_{h_{\text{small}}}$;

• Minimise $G.1$ again with the inliers only.

The last three steps can obviously be repeated a few times if improvements in results are detected.