Matrix Converter Based High Power High Frequency Modular Transformers for Traction Conversion Systems

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Thesis to obtain the Master of Science Degree in Electrical and Computer Engineering

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To my parents, my brother and friends.
Acknowledgements

This MSc thesis means the achievement of a stage in my life that would not be possible without the support and comfort of several people. Therefore, it is a pleasure for me to recognize them.

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Abstract

A new system based on a Power Electronic Transformer has been proposed in this thesis. It is installed in the traction substation and regulates the voltage, to the characteristics of the train crossing that section of the rail. It consists of a High Frequency Transformer with a Three Phase Matrix Converter in its input, to guarantee controllable output voltage and frequency, as well as bidirectional power flow. The Matrix Converter uses Space Vector Modulation, which has important advantages such as; a simplified algorithm control, maximum voltage transfer ratio without adding third harmonic components, and an innovative feature developed in this thesis, which also guarantees the non-saturation of the high frequency transformer. Finally, in the output of the transformer are three Single Phase Matrix Converters that restore the original waveform determined by the Space Vector Modulation. By combining the advantages of the Matrix Converters with a high frequency transformer, it is possible to produce controllable voltage, galvanic isolation and power quality improvements without any extra devices. Several features such as instantaneous current regulation and voltage sag compensation are combined with the Power Electronic Transformer. The proposed new Power Electronic Transformer configuration has been modelled using MATLAB/SIMULINK and the main advantages mentioned above have been verified by the simulation results.

Keywords

Resumo

Um novo sistema baseado na tipologia de um Transformador Electrónico de Potência é desenvolvido nesta tese. O sistema é instalado nas subestações de tracção e adapta o nível e frequência de tensão às características dos comboios que percorrem essa secção do carril. Este sistema consiste num transformador de alta frequência, alimentado através de um Conversor Matricial Trifásico, que permite controlar a amplitude e frequência das tensões de saída e garante trânsito de energia bidireccional. No Conversor Matricial é utilizada Modulação por Vectores Espaciais, que tem como vantagens um algoritmo de modulação simples, uma taxa máxima de transferência de tensão sem adicionar harmónicas de terceira ordem, e ainda um inovador atributo desenvolvido nesta tese, que assegura a não saturação do transformador. Na saída do transformador, existem três conversores matriciais que repõem a onda de tensão original obtida pela Modulação por Vectores Espaciais. Combinando as vantagens do Conversor Matricial com um transformador de alta frequência, é possível converter a saída para um nível de tensão e frequência desejado, garantir isolamento galvânico e melhorar a qualidade da energia eléctrica, sem a necessidade de dispositivos extras. O sistema desenvolvido permite ainda regular a tensão de saída e compensar algumas cavas. O sistema foi testado em ambiente MATLAB/SIMULINK, e as vantagens acima descritas confirmadas através dos resultados da simulação.

Palavras-chave

Conversor Matricial, Modulação com Vectores Espaciais, Regulação em Corrente, Tracção Eléctrica, Transformador de Alta Frequência.
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List of Acronyms

- AC
  - Alternate Current
- AC-AC
  - Electronic power conversion with an alternate current input and alternate current output.
- AC-DC
  - Electronic power conversion with an alternate current input and direct current output.
- DC
  - Direct Current
- DC-AC
  - Electronic power conversion with a direct current input and alternate current output.
- HFT
  - High Frequency Transformer
- MC
  - Matrix Converter
- PET
  - Power Electronic Transformer
- PETET
  - Power Electronic Transformer for Electric Traction
- PI
  - Proportional Integral Controller
- RMS
  - Root mean square
- SPMC
  - Single-Phase Matrix Converter
- SVM
  - Space Vector Modulation
- TGV TMST
  - Grande Vitesse TransMancheSuperTrain
- V
  - Volt, SI unit of voltage
- VSI
  - Voltage-Source Inverters
List of Symbols

- $\eta$  
  Efficiency of matrix converter

- $\xi$  
  Damping factor

- $\alpha_i$  
  Gain value of the current regulator

- $\alpha_v$  
  Gain value of the voltage regulator

- $\alpha_{\beta}$  
  System referenced to the $\alpha\beta$ coordinate plane

- $\omega_c$  
  Cut-off angular frequency of the input filter

- $\omega_i$  
  Angular frequency of the matrix converter input voltage

- $\omega_o$  
  Angular frequency of the matrix converter output voltage

- $\omega_s$  
  Matrix converter angular switching frequency

- $\phi_i$  
  Phase angle of the input load

- $\phi_o$  
  Phase angle of the output load

- $\phi_i$  
  Instantaneous phase of the input current reference vector

- $\phi_v$  
  Instantaneous phase of the output voltage reference vector

- $\theta_i$  
  Angle of the current reference vector related with the sector where it is located

- $\theta_v$  
  Angle of the voltage reference vector related with the sector where it is located

- $\lambda_{ld}, \lambda_{ld}$  
  Current modulation indexes

- $C$  
  Concordia transformation matrix

- $C_{fin}$  
  Capacitance value of the input filter capacitor

- $C_{fout}$  
  Capacitance value of the output filter capacitor

- $d_y, d_\delta, d_o$  
  Duty cycles associated to the SVM

- $Dq$  
  System referenced to a coordinate plane $dq$

- $f_c$  
  Cut-off frequency of the input filter

- $f_i$  
  Matrix Converter input frequency

- $f_o$  
  Matrix Converter output frequency

- $f_s$  
  Matrix Converter switching frequency

- $H_{3q}(t)$  
  Function that commands the Three-Phase Matrix Converter semiconductors

- $H_{1q}(t)$  
  Function that commands the Single-Phase
Matrix Converter semiconductors

- $H_d, H_q$
- $I_y, I_0, I_o$
- $i_{α ref}, i_{β ref}$
- $i_a, i_b, i_c$
- $I_{ref α}, I_{ref β}, I_{ref c ref}$
- $i_{systemαβ}$
- $I_k$
- $I_A, I_B, I_C$
- $I_c$
- $I_{DC}$
- $I_i$
- $I_o$
- $i_{od}$
- $i_{oq}$
- $i_{oqref}$
- $i_{output filter}$
- $L_{in}$
- $L_{out}$
- $m_c$
- $m_v$
- $N_i$
- $N_v$
- $P_{DC}$
- $P_i$
- $P_{in}$
- $P_o$
- $P_{out}$
- $r_i$

- Voltage commands of the output currents
- Vectors component, of the reference current vector
- Instantaneous value, in $αβ$ coordinates, of the Matrix Converter output reference currents
- Matrix converter input currents
- Input reference currents of matrix converter
- Input reference current vector in $αβ$ coordinates
- PETET current, in $αβ$ coordinates
- Matrix Converter input current vector $k = \{1,2,3,4,5,6,7,8,9\}$
- Matrix Converter output currents
- Current output capacitor
- Direct current of the intermediate stage
- RMS value of the matrix converter input current
- RMS value of the matrix converter output current
- $d$ component of the load current
- $q$ component of the load current
- Reference value of $i_{oq}$ and $i_{od}$ current
- RMS value of the rated current in the filter
- Self-inductance value of the input filter coil
- Self-inductance value of the output filter coil
- Input current modulation index
- Output voltage modulation index
- Number of the sector where the input current reference vector is located
- Number of the sector where the output voltage reference vector is located
- Instantaneous power in the intermediate stage of the Matrix Converter
- Input Power Factor
- Input Power of the Matrix Converter
- Load losses of the transformer
- Output Power of the Matrix Converter
- Negative incremental resistance of the input filter
• Rs
• \( r_{\text{out}} \)
• \( r_p \)
• S
• \( S_c \)
• \( S_T \)
• \( S_{kj} \)
• \( T_c \)
• \( T_d \)
• \( T_s \)
• \( V_y, V_\delta, V_0 \)
• \( V_a, V_b, V_c \)
• \( V_{AB}, V_{BC}, V_{CA} \)
• \( V_{ABref}, V_{BCref}, V_{CAref} \)
• \( V_{ca\beta} \)
• \( V_{DC} \)
• \( V_i \)
• \( V_{lc} \)
• \( V_{\text{Load}} \)
• \( V_o \)
• \( V_{\text{aref}}, V_{\beta\text{ref}} \)
• \( V_{\text{aref}_{\alpha\beta}} \)
• \( V_{oc} \)

- Load resistance for the purpose of scaling the input filter
- Resistance value equivalent of the output filter
- Total resistance of the losses in the primary and secondary windings of the transformer
- Matrix of 3x3 elements that represents the state of the matrix converter bidirectional switches
- Matrix that relates line-to-line output voltages with line-to-neutral input voltages
- Transpose of matrix S
- Bidirectional switch that connects the output phase \( k = \{1, 2, 3\} \) to input phase \( j = \{1, 2, 3\} \) of a three-phase converter
- Switching period of the Matrix Converter
- Average delay of the converter
- Variable time period of the vector with \( \delta\beta \) components
- Vectors component, of the reference voltage vector
- Matrix converter line-to-neutral input voltages
- Matrix converter line-to-neutral output voltages
- Matrix converter line-to-line output voltages
- Matrix converter line-to-line output voltages reference
- Output capacitor voltage, \( \), in \( \alpha\beta \) coordinates
- Voltage of the Matrix Converter intermediate stage
- RMS value of the matrix converter input line-to-neutral voltage
- RMS value of the matrix converter input line-to-line voltage
- Output load voltage
- RMS value of the matrix converter line-to-neutral output voltage
- Instantaneous value, in \( \alpha\beta \) coordinates, of the Matrix Converter line-to-line output reference voltage
- Reference vector of the line-to-line MC output voltage, in \( \alpha\beta \) coordinates
- RMS value of the matrix converter line-to-line
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{of}$</td>
<td>Magnitude of the output filter impedance</td>
</tr>
<tr>
<td>$Z_{i}$</td>
<td>Impedance value of the input filter</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

This chapter gives a brief overview of the thesis. Before establishing work targets and original contributions, the scope and motivations are brought up. At the end of the chapter, the work structure is provided.
1.1 Overview

The railway transportation began in the early years of the IX century, in England. The first transportation of goods and passengers on regular schedule started in 1825. Back then the locomotive pulled 21 loaded coal cares and 450 passengers with an average speed of 15 km/h. Rail travel was the cheapest way of transportation despite the investment of constructing rail lines, and also a lot faster than other types of transportation. At that time, steam engines powered all locomotives, steam boats and factories, and therefore acted as the foundation of the Industrial Revolution.

More than half a century later in 1879, the first electric train was designed by the German scientist Werner von Siemens, reaching a speed of 13 km/h and supplied by 150 V, Direct Current (DC). The pollution caused by the steam powered trains, led to an increased use of electric trains, especially around cities. For that reason, the development of electric locomotives adjusted to the necessities of transportation inside the cities, and subsequently the construction of the underground in London, in 1890.

Later on, DC motors were developed and improved, and the 1.5 kV and 3 kV DC systems were adopted for several countries and are still in use today. The line of Cascais in Portugal is one of those examples, providing a 1.5 kV DC system to power the locomotives, [Gued92].

Not only the discoveries and improvements in electrical technologies, but also the power transmission network expansion set the course of the electric railway transportation. In the beginning of the XX century, the adoption of alternate current (AC) in power transmission lines and the low reliable mercury rectifiers used in DC systems that limited a higher voltage and power of the locomotive motors, led to new successful experiments in locomotives supplied by AC systems, [Holt13]. The Hungarian engineer Kálmán Kandó, who dedicated his life to the development of electric traction, in 1931 and after remarkable achievements in Switzerland and Italy, developed a 16 kV AC, 50 Hz system to supply the locomotives running between Budapest and Komárom. Initially, his research did not attract the attention of railway operators outside Hungary, still, his solution showed a way for the future.

In 1909 another AC system was adopted in Germany and Switzerland, and afterwards in Austria, Norway and Sweden. The 15 kV, 16.7 Hz system, exactly one third of the electrical grid frequency of 50Hz, at that time provided advantages such as the need of a smaller and less costly railway power generator by reducing the number of poles, preserving the same shaft speed. This system is still being used in those countries, [Lang10].

In 1951, the most operated electric railway system today was implemented in southern France, initially at 20 kV but converted to 25 kV two years later. The use of 25 kV, 50 Hz system was then adopted as a standard in France and in several countries such as Portugal.
In recent decades, given the significant technological advances of power semiconductors, power electronic converters have experienced great development and are well-known today due to their high reliability and robustness in a wide range of applications, [SPPB03].

Matrix Converters (MC), which received significant improvements in recent years [FrKo12], are power electronic converters with high switching frequency and are able to generate three phase output voltages with variable frequency and, at the same time, with controllable input power factor. When compared to the conventional Voltage-Source Inverter (VSI), MCs do not need a bench of electrolytic capacitors since there is no intermediate DC-link, which contributes to limit the usual VSIs lifetime [PaPo10], increasing electric losses, volume and costs.

Furthermore, MCs allow bidirectional power flow and do not contribute significantly to the harmonic degradation of the input waveform voltage. This is extremely important as it led to an endorsement of this converter, as an innovative and clean solution from the harmonic point of view, [WRCE02].

The MC has also been implemented recently in multi-level locomotives due to their light weight and small size, in order to replace the conventional back-to-back converters on board of railway vehicles, [DPPC11] as well as compact power sources for electromechanical variable speed drives, in this case AC motors, [NgTL12].

1.2 Motivation

The choice for electric traction over other systems such as diesel locomotives is an economical question in which the return on investment must be analyzed. There are important factors to take into consideration for this analysis and electric traction has significant advantages that are important to mention:

- High energy efficiency of locomotives and multiple units
- High power-to-weight ratio that results in fewer locomotives and higher speeds
- Environment-friendly operation, low noise
- Possibility of energy recovery when breaking
- Low maintenance cost
- Usability of hydroelectric power and others renewable sources
- Low dependence on crude oil as fuel

Despite all those advantages the electric traction has to be supported by a near power transmission grid which in some areas does not exist. Therefore, the cost of investment in new infrastructures that allow an electric traction is too high compared with other alternatives. This argument is sometimes associated with the further limitation of the extension in tunnels due to the overhead lines. These, are the main arguments against the consideration of electric traction.

Nowadays there are still several standards for electrical railway systems mainly due to historical
reasons mentioned before. Despite the gradual acceptance of 25 kV at the power transmission grid frequency, in Europe some countries use DC systems and others such as Germany, use 15 kV at 16.7 Hz to supply their locomotives. The Fig. 1.1 shows several systems adopted by European countries.

![Figure 1.1 - Electrification Railway systems adopted as main standard in European countries](image)

A traction substation receives the electric power from the power transmission grid or an exclusive power distribution grid and converts it to an adequate voltage to supply the locomotives and trams. Therefore, the traction substations have to be spaced along the rail to provide enough power to the locomotives that are crossing the section. In Central Europe there are numerous locomotives crossing several countries and are therefore supplied by different traction systems. For this reason locomotives are equipped with electronic and mechanical devices that can adjust the type of supply to their engines.

This situation can be observed in the region around Basel, Switzerland. In Fig. 1.2, several systems used by locomotives in the Basel region can be seen and consequently the need for several substations and other infrastructures that can provide the trains with the adequate supply.

In the stations, the locomotives change their supply in zones called neutral sections. Just before the train enters in this section on-board equipment such as the traction motors, compressors, blowers are switched off. Previously, the trains used to routinely drop their pantographs for all neutral sections but this is no longer standard practice as trains often do not have to stop but only reduce their speed in neutral sections.

---

1 High speed lines in France, Spain, Italy, United Kingdom, the Netherlands, Belgium and Turkey operate with 25 kV.
The solution proposed in this work aims to present a Power Electronic Transformer for Electric Traction (PETET), which combines MCs and one High Frequency Transformer (HFT), and is able to adapt the supply to the characteristics of the train crossing that section. The proposed system can be seen in Fig. 1.3.

Figure 1.2 - Railway tracks around the city of Basel (2004) [Stati07]

Figure 1.3 - Power Electronic Transformer for Electric Traction
Therefore, due to several standards adopted for electric railway systems, the use of this system will reduce the costs of new infrastructures since it is adaptable to different standards. The advantages of the MC mentioned before, cover the need of extra devices that are usually implemented in AC electrification nowadays. The use of reactive power compensators in substations to compensate losses in the rail network and old locomotives with low power factors [Raim12], is no longer necessary since the MC provides controllable input power factor.

The PETET developed in this work can have other applications such as in the power distribution substations, since there are common aspects that will be mentioned further later in this thesis.

1.3 Contents

The thesis is organized into 6 chapters, references and appendixes.

Chapter 1 provides an overall understanding of railway transportation including historical overview and its context in Europe. The MC is introduced and its main advantages are presented. The motivation and the main goal of the developed system are discussed. Also, the structure of the thesis is presented.

Chapter 2 characterizes all the major railway electrification systems standards: from AC systems to direct current and multi-level locomotives. It is extremely important to present these technical details in order to understand the utility of the developed system. Afterwards the MC is introduced with a brief summary of the progress made with this converter, while providing its major advantages when compared to conventional VSIs.

Further in Chapter 3, the proposed system, the PETET is introduced, followed by a technical explanation of the Three-Phase MC and the space vector representation. The SVM is presented in detail in order to explain the new features of the modified SVM developed in this thesis. The overall system control of the PETET is described, as well as the Single-Phase Matrix Converter (SPMC).

Chapter 4 presents the calculations of the Input and Output Filter parameters, as well as the Current and Voltage Regulators design.

Chapter 5 presents the chosen scenarios according to the railway electrification systems, and afterwards the simulation results are shown and discussed.

Chapter 6 finalizes this thesis by, drawing conclusions and giving suggestions for future work.

A set of appendixes with auxiliary information and results is also included. Appendix A presents the equations that establish the relations between the input and output of the MC. Appendix B and Appendix C present the output voltages and the input currents space vectors for each zone. Appendix D presents the calculations for the damping resistance of the input filter. Finally, Appendix E presents the nominal voltages and their permissible limits in values and duration for railway electrification systems.
Chapter 2

State of the Art

This chapter provides an overview of the existing railway electrification systems. It explains the main differences of AC and DC railway systems and provides technical details about the main standards. Furthermore, the use of multilevel locomotives and electric battery locomotives is also discussed. Additionally, this chapter provides an overview of the Matrix Converter, focusing on its development and advantages compared with other converters.
2.1 Railway Electrification Systems

Railway Electrification Systems supply electrical energy to railway locomotives so they can operate without having an on-board prime mover. The electrical power is supplied from a distribution network at specific points: suitable substations or power centrals.

[Frey12] proposed a classification for traction electric systems, which are distinguished by three main parameters:

- Voltage
- Current
  - Direct current
  - Alternating current
    - Frequency
- Contact System
  - third rail
  - overhead line

Multiple electrification systems are used throughout the world; Table 2.1 shows the characteristics of the most used.
Table 2.A - World Railway Electrification System and Electrified Distances (1996) [OuMN98]

<table>
<thead>
<tr>
<th>System Type</th>
<th>Distance (km)</th>
<th>Main Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>Less than 1.5 kV</td>
<td>5 106</td>
</tr>
<tr>
<td></td>
<td>1.5 kV to 3 kV</td>
<td>22 138</td>
</tr>
<tr>
<td></td>
<td>More than 3 kV</td>
<td>78 276</td>
</tr>
<tr>
<td>Single-phase AC</td>
<td>50 Hz or 60 Hz</td>
<td>Less than 2 kV</td>
</tr>
<tr>
<td></td>
<td>20 kV</td>
<td>3 741</td>
</tr>
<tr>
<td></td>
<td>25 kV</td>
<td>84 376</td>
</tr>
<tr>
<td></td>
<td>50 kV</td>
<td>1 173</td>
</tr>
<tr>
<td></td>
<td>25 Hz – 11 kV to 13 kV</td>
<td>1 469</td>
</tr>
<tr>
<td></td>
<td>16.7 Hz</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>11 kV</td>
<td>35 461</td>
</tr>
<tr>
<td>Three-phase AC</td>
<td>43</td>
<td>Switzerland, France</td>
</tr>
<tr>
<td>Unknown</td>
<td>3 668</td>
<td>Kazakhstan, France</td>
</tr>
<tr>
<td>Total</td>
<td>235 186</td>
<td></td>
</tr>
</tbody>
</table>

Despite several differences in the railway electrification systems, there are common technical details. The power transmission grid should be a three-phase balanced system, even though the unbalanced railway loads weaken that equilibrium. The main cause is the biphasic nature of the railway electric system from the point of view of the power transmission grid. Consequently, the existence of certain structures is common in all the systems, to protect the power transmission grid from defects, and assure the quality of the energy provided to the railway locomotives.

Traction Power Supply Systems – These systems include traction power substations, which are located along the course at planned locations. The substations are connected to the power transmission grid and their purpose is to adapt the proper voltage to supply the electric locomotives, as well as to protect the power transmission grid against faults and other electrical defects.

Traction Power Distribution Systems – These systems consist of the overhead contact system, mainly used in AC systems, whilst the DC systems usually operate with the third rail. Both systems are used to feed electrical energy to the locomotives and need transformer substations to convert the voltage to suitable levels. They also have capacitor banks to improve the power factor. Moreover, switching stations and, in some cases, autotransformers are required.

Traction Power Return Systems – These System consist of the running rails, impedance bonds, cross-bonds, overhead static wires, return conductors and the ground. They guarantee a safe path, of the current supplied to the trains, to the substation.
Loads – Loads include electrical locomotives and railbuses that receive the electrical energy through the pantograph or the third rail to their motors. The current return path is through the rail, which is connected to the ground, and in some cases also through a feeder rail.

The railway’s electrical substations play an important role in the process of supplying electrical energy to the trains. As stated above, the substations are located along the track and fed from the transmission or distribution grid. The distance between each substation, depends on various factors such as the voltage level, trains, and the surrounding electrical traffic.

Despite the increase of AC railways systems in the last decades due to the improvement of power electronic components, the DC systems are still in use in several countries, such as Italy, Belgium, and Poland. Table 2.2 presents the major advantages of both topologies.

Table 2.B - Advantages of AC and DC Railway Systems

<table>
<thead>
<tr>
<th>AC Railway Systems</th>
<th>DC Railway Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Advantages</strong></td>
<td><strong>Advantages</strong></td>
</tr>
<tr>
<td>Light Overhead Catenary – lower current intensity</td>
<td>DC train is lighter and less costly</td>
</tr>
<tr>
<td>Larger distance between Substations</td>
<td>DC motors are better suited for frequent and rapid accelerations of heavy trains</td>
</tr>
<tr>
<td>Simplicity of substations design – No need of rectifiers or rotary converters in case of the 50 Hz systems</td>
<td>Conductor rail less costly, both initially and in maintenance</td>
</tr>
<tr>
<td>Lower cost of Fixed Installations</td>
<td>No electrical interference with overhead communication lines</td>
</tr>
<tr>
<td>Higher coefficient of Adhesion¹</td>
<td></td>
</tr>
<tr>
<td>Higher Start Efficiency - the AC motors offers a more flexible and smooth start</td>
<td></td>
</tr>
</tbody>
</table>

¹The tractive effort of a locomotive is defined by the equations:

$$\text{Tractive effort} = \text{Weight on drivers} \times \text{Adhesion}$$

$$\text{Adhesion} = \text{Coefficient of friction} \times \text{Locomotive adhesion variable}$$

The friction coefficient between wheel and rail takes into consideration the conditions of the rail.

The variable to take in consideration is the “Locomotive adhesion variable”, which represents the ability of the locomotive to convert the available friction into usable friction at the rail interface. Due to advantages of speed/torque control of AC engines, the AC locomotives have natural higher efficiency reaching 90% in the modern AC locomotives. [Aria10]
2.1.1 Alternate Current Systems

2.1.1.1 Direct-fed System

The overhead contact system supplies electricity to the locomotives at 25 kV AC, 50 Hz, from substations which are located at frequent intervals, alongside the track. The feeding substations are supplied with single-phase power from traction substations strategically located 35 to 60 km away from each other depending on several factors such as the intensity of traffic and the load introduced by locomotives. Compared with DC-powered systems, which operate at lower voltages, the AC systems provides the same acceleration to the train with the need of a lower current, therefore, lower losses.

To keep the balance in the three phase grid system, phase-to-phase changeover sections are installed in the catenary system to separate sections that operate at different phases, as can be seen in Fig 2.1. Power is provided by the grid system across the different phases at adjacent substations in cyclic order. Moreover, switching stations are needed in case of a substation failure.

![Figure 2.1 - Structure of an AC Feeding Railway System](image)

The power transformer in the substations provides 25 kV in the secondary winding, with one of the terminals connected to the catenary system and the other terminal connected to the ground and to the traction return conductor. For this reason, the system is called as 1x25 kV.

Fig. 2.2 represents the electrical circuit of the 1x25 kV system with the representation of the current ($I_c$) that flows in the catenary system and returns ($I_r$) to the substation in the traction return conductor.
2.1.1.2 **Autotransformer-fed System**

Similarly to the Direct-fed System, phase breaks, feeding points and switching stations are also installed due to the reasons previously explained. However, the 1x25 kV suffers from voltage drops in the catenary, sometimes reaching the 5 kV, when the distance to the feeding substation is high. Fig. 2.3 represents a scheme of the Autotransformer-fed System that aims to solve this issue, [HySJ02]. In the substation a 50 kV is split into a dual 25 kV supply using a three winding transformer. One winding supplies 25 kV between the catenary and the rails as the 1x 25 kV systems, thus allowing the circulation of 25 kV locomotives in the autotransformer-fed system. The other winding is connected to a feeder cable parallel to the catenary. Since the feeder-to-rail and catenary-to-rail voltages are both 25 kV and in antiphase, the system earned the name 2x25 kV.

As presented in Fig. 2.3, if considering that the load current drawn by the train is “I”, then each phase, catenary and feeder carries half of the load current “I/2”. The autotransformer forces an equal distribution of the current along the track, and the currents split and merge only in the section where the train is located. Note that the rails carry less than the full load current in opposite direction of the train, and that it is the only section where the rail carries current. Also, the catenary never conducts the full load current. The feeder provides the cancellation of inductive interference except in the section where the train is located, since it carries a current equal but in opposite to the current in the catenary.
Due to the feeder and the autotransformers, there is a substantial reduction of the return current. Therefore, it is possible to provide more power to the locomotives which is an advantage for high speed trains. The 2x25 kV systems additionally allow a higher distance between substations, lower emission of electromagnetic radiation and smaller equivalent impedance when compared to the 1x25 kV systems.

In countries where 60 Hz is the standard grid power frequency, such as the United States of America and Brazil, 25 kV at 60 Hz is adopted for electric traction.

2.1.1.3 15 kV 16.7 Hz Systems

The 15 kV 16.7 Hz systems are used in several countries in Europe from the time when those countries began high-voltage electrification at 16.7 Hz. In some regions of Germany, Austria and Switzerland the system is supplied by several plants such as nuclear power plants and hydroelectric power plants that are either dedicated to generate 110 kV at 16.7 Hz single phase, or have special generators for this purpose. The neutral is connected to a safety ground through an inductance as is common practice in the distribution power systems. Therefore, the voltage of each conductor with respect to ground is of 15 kV. At the transformer substations, the voltage decreases to 15 kV AC and then supplies the overhead line.

In Sweden, Norway and some other regions of Germany, the power is provided directly from the three-phase grid (110 kV at 50 Hz), converted by synchronous-converters or static converters to low frequency single phase and feeds the overhead line, [Dani10].

The need for a separate supply infrastructure and the lack of any technical advantages with modern traction machines and controllers has restricted the use of this system outside the original five countries.

2.1.2 Direct Current Systems

Tramways and metropolitan railway systems usually run on Direct Current. In the substations a rectifier is needed for AC-DC conversion, usually a 12 pulse rectifier featuring two sets of 6-pulse rectifiers connected in series or in parallel, thus minimizing the current harmonic distortion. The lower supply voltage of these systems, which consequently draw higher currents; result in thicker and heavier overhead line and pantograph that has to be pressed more firmly against the overhead line resulting in greater wear. The Metro, which operates with lower voltages, usually 750 V, is supplied through a thick conductor running along the track, called third rail.

Section and tie posts are sometimes used to prevent voltage drops on double tracks where substations are located apart from each other. Due to the higher current in the conductors, the substations in DC Systems are only distanced 3 to 5 km from each other, in the case of heavy suburban traffic supplied with 750 V, and 40 km to 50 km for main lines operating at higher voltages such as 1.5 kV and 3 kV.
2.1.3 Other Systems

Multi-voltage locomotives are another option to solve the several voltage standards. These locomotives are prepared to operate in AC and DC systems and with different levels of supply voltages.

The well known Train à Grande Vitesse (TGV) TransMancheSuperTrain (TMST) operates from Brussels to the south of London, and crosses different electric systems that operate a 25 kV, 50 Hz AC and 3 kV DC, both with overhead lines. For this reason, there is the need to use two pantographs, which are switched on or off when the change of system occurs, and the use of transformers and power electronic converters to adapt the supply to the traction motors.

The TGV TMST, working in AC, has a main transformer that is energized and reduces the 25 kV, before sending it to be rectified. At this point, auxiliary inverters acquire sufficient energy for the hotel electric power, and the inverters in the motor block acquire the energy needed for traction. This energy is converted into three phase AC to feed the traction motors.

When the train is running in a DC system (1.5 kV or 3 kV), the DC input is supplied directly via a different main breaker before being filtered, and then the previously mentioned inverters are used to convert the DC system to an adequate AC system, used to feed the traction motors.

There are other types of multi-voltage locomotives that can operate both at 25 kV, 50 Hz AC, or 15 kV, 16.7 Hz AC, from overhead lines. In this case, there is the need to use two transformers for each frequency.

The electric battery locomotives are another type that is being introduced in recent years replacing some diesel powered locomotives. This technology has improved but is still far from experiencing great performances, hence this type of locomotives are only being used in industrial environments such as mines, and local deliveries in towns and large industrial plants. This type of trains, with low maintenance and free from smoke, are still very limited due to the small capacity of the batteries.

The major advantage of this technology is the absence of infrastructures along the track to provide energy, as in the conventional AC and DC Systems, thus allowing a considerable reduction of costs.

In the London Underground electric battery locomotives are used for hauling engineer’s trains, as they can operate when the electric traction current is switched off.

2.2 Matrix Converter

The Matrix Converter is a power electronic converter made by several controlled semiconductor switches that directly connect each input phase to any output phase. The bi-directional switches have to commutate in the right way and sequence in order to reduce losses and produce the desired output with high quality input and output waveforms. The MC converts directly the output into a desired
magnitude and frequency, without using a DC-link, as in the conventional back-to-back converter. Furthermore, with the bidirectional switches, it is possible to provide bidirectional power flow and controllable input power factor.

The AC-AC matrix topology was first investigated in 1976 [CSYZ02], [HoLi92], but it was the work of Venturini and Alesina published in 1981 [AlVe81] that gave the MCs its current appreciation [WRCE02], [HoLi92]. However, their original modulation strategy was abandoned in favor of new solutions. The voltage transfer ratio was limited to 0.5 in the original Venturini and Alesina modulation strategy, but it was presented later [HoLi92] that the maximum voltage transfer ratio could be increased to $\sqrt{3}/2$, a value which represents intrinsic limitations to the Three-Phase MCs with balanced supply voltages.

In 1992 a new modulation strategy for matrix converters known as “indirect modulation method” [NeSc92] was developed, considering that the MC could be represented as a virtual association of a three phase rectifier and a three phase inverter connected through a virtual DC-link. This approach represented a significant step in the development of a new modulation strategy, as it was possible to apply the well-established Space Vector Modulation (SVM) techniques used in rectifiers and inverters to MCs, [HuBB92].

Later, a new modulation method [PiSi07] based on SVM was proposed, but instead of considering the “indirect method” representation of the MC, new systematic modulation strategy based on the direct power conversion process carried out by the MC was adopted. The Direct SVM method defines a systematic selection of the space vectors, which are used in the modulation process, with a compact and easy formula that controls the input power factor and the output voltages without significant addition of calculations.

Nowadays, MCs are usually defined as frequency and voltage universal converters, as they allow:

- Multiphase AC-AC conversion [HuBB92]
- Three-phase to single-phase [MBHG98]
- Single-phase to Three-phase [DoHD98]
- Single-phase to single-phase [ZuWA97]
- AC-DC [HoLi92]
- DC-AC [HoLi92] [BoCa93]

The MC has several advantages when compared with the back-to-back converter, as shown in Table 2.3. Nevertheless there are some potential disadvantages that have prevented a higher commercialization of MCs so far. Past research has mentioned those concerns, but by now solutions have been found and the MCs have developed fast in the last few years.
Table 2.C - Matrix Converter advantages and disadvantages when compared to a Back-to-back Converter

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>No intermediate DC link</td>
<td>Large number of semiconductors</td>
</tr>
<tr>
<td>Allow power regeneration</td>
<td>Limitation of the output value to:</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{3}/2 V_i$</td>
</tr>
<tr>
<td>Input current waveforms nearly sinusoidal</td>
<td>Higher probability of disturbances in output and input voltages and currents</td>
</tr>
<tr>
<td>Can be operated with nearly unitary power factor</td>
<td>More complex control system</td>
</tr>
<tr>
<td>High power density</td>
<td></td>
</tr>
<tr>
<td>Higher versatility (Converts AC-AC DC-AC, or AC-DC and for multiple input and output phases)</td>
<td></td>
</tr>
<tr>
<td>Lower weight and dimension and can work under higher temperatures</td>
<td></td>
</tr>
</tbody>
</table>

Currently MCs may be used in electrical substations to regulate Distribution Grid voltages [Alca12], [APS13], in high power applications to regulate the power flow in Transmission Grids [Mont10] [MSPJ11], in the renewable energy applications where they provide the electrical connection between the power generator and the electric grid [Fern13], in the transportation industry ranging from the aerospace sector to the railway sector. Besides the low distortion of the input/output waveforms, the lower weight and volume of MCs when compared to back-to-back structures, and the bidirectional power flow are a great advantage for the transportation sector [DPPC11], allowing regenerative braking.
Chapter 3

Power Electronic Transformer for Electric Traction

This chapter provides an overview of the Modular Power Electronic Transformer for Electric Traction, which was developed in this thesis. The Three-Phase Matrix Converter, the SVM as well as its innovative feature are designed and described in this chapter.
3.1 Introduction

The PETET (Power Electronic Transformer for Traction) was designed to be a universal AC-AC or AC-DC high power electronic transformer, thus providing a variable output voltage system, not only in magnitude but also in frequency, without a significant harmonic degradation of the input current waveforms. The proposed system is to be installed in the electric substations and has its input directly connected to the transmission or distribution grid. The proposed system output is to be connected to the overhead line of the rail.

Due to the semiconductor’s limitations and the high voltage levels used in electric traction systems, it is advisable to adopt a modular structure, in order to guarantee that each semiconductor will only support a small fraction of the maximum voltage and current values. Thus, sizing the proposed system with an adequate number of modules, it is possible to guarantee that the semiconductors maximum admissible voltage and current values are never reached.

As standard procedures and among the literature [CaZT12] [Saee08] [Silv13], one possibility was identified, in which some power electronics converter modules may be connected in parallel, thus guaranteeing that each module supports lower voltage values.

Fig. 3.1 represents the proposed approach, where modular multilevel PETET converters are connected. The voltage is equally divided by each module, and therefore each semiconductor supports lower voltages than the transmission or distribution grid voltages.

Capacitors have to be designed to support the desired voltages and an input coil is necessary to filter the input currents. Furthermore, it is necessary to use of an efficient control system for all the modules since the output voltages have to be synchronized. In these cases, it is common procedure to have one synchronous modulator for all the modules and separate current regulators, one for each module. Therefore, the modulator and the controllers developed in this thesis will be easily adaptable for several modules.

An additional advantage of this system is that it is able to guarantee a N+1 or N+2 redundancy. In case of failure in one of the power electronic modules, the defective module may be taken out of service and the other PETETs modules will support the remaining voltage.
The PETET is presented in Fig. 3.2 and it is based on one three phase High Frequency Transformer (HFT) [Silv12a] supported by power electronic converters. The input is connected with a three phase MC controlled by SVM with an innovative feature that guarantees the non-saturation of the HFT. The output consists of three Single Phase Matrix Converters with an output filter. These three MCs are used to restore the original SVM signal.
Another important feature is the electrical ground provided by this system. In addition to providing a path to ground for the current, which assures human safety, the connection of one of the service supply conductors to electrical ground stabilizes service voltage. Without the electrical ground, the service voltage may float and could become dangerously high under certain conditions. Similarly, the connection of the neutral to ground guarantees that the voltage on the neutral with respect to the adjacent earth remains at an adequate level.

### 3.2 Three-Phase Matrix Converter

The Three-Phase Matrix Converter, represented in Fig 3.3, consists of nine controlled bidirectional switches that allow a connection between two three-phase systems; the input with voltage source characteristics and the output system with current source characteristics.

Assuming ideal bidirectional switches (zero voltage drop when ON, no leakage current when OFF and null switching times) each switch can be represented mathematically by a variable $S_{kj}$ (3.1) with the value “1” if the switch is closed (ON) or “0” if the switch is open (OFF).

$$S_{kj} = \begin{cases} 1 & \text{if the switch is closed} \\ 0 & \text{if the switch is open} \end{cases} \quad (3.1)$$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad (3.2)$$

![Figure 3.3 - Three-Phase Matrix Converter](image)
Due to the input and output characteristics, it is not possible to obtain the 512 ($2^9$) states that the 9 bidirectional switches could allow. Therefore, the possible states are 27 ($3^3$) since two topological restrictions must be respected:

- To ensure continuity of the output current sources. Consequently at least one of the switches that is connected to each output phase has to be turned ON;
- To avoid the short circuit of the input phases. Consequently, it is not possible to turn ON more than one switch per arm.

These two topological restrictions have to be respected; for the first one, it is necessary to guarantee the current flow for each output phase, implying that in each row of matrix S, one switch has to be turned ON. On the other hand the second condition implies that in each row of S matrix, it is not possible to have more than one switch turned ON. Therefore, the instantaneous sum of all the elements in each row of matrix (3.2) has to be “1” (3.3).

$$\sum_{j=1}^{3} S_{kj} = 1 \quad k \in \{1,2,3\}$$  

(3.3)

The equations that establish the relations between input and output currents and voltages of the MC, as well as the technical limits of MC topology are presented in Appendix A. Finally, a table with the 27 possible switching combinations and the resultant output voltages and input currents for each combination are exhibited.
### 3.2.1 Space Vector Representation

The MC must be appropriately controlled in order to supply the currents, voltages in the frequency ranges necessary to feed the load. Methodologies and sophisticated control processes must be used to guarantee the stability of the MC, not only with satisfactory static and dynamic performance but also with low sensitivity against load or line instabilities.

In this work, a fixed frequency Space Vector Modulation (SVM) based approach is chosen. However, as the output of the matrix converter is to be directly connected to the high frequency transformer, some adjustments have to be made to guarantee that the high frequency transformer will not saturate.

With the use of Concordia/Clarke transformation (3.4) in Table A.1 (Appendix A) for the 27 possible switches combinations, it is possible to represent the output voltages and the input currents in $\alpha\beta$ coordinates, Table 3.1.

\[
C = \sqrt{\frac{2}{3}} \begin{bmatrix}
1 & 0 & 1 \\
\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}}
\end{bmatrix}
\]  

(3.4)

Table 3.1 is structured in three different groups:
- Group 1 represents the rotating vectors, with fixed magnitude and variable phase.
- Group 2 represents the vectors with variable magnitude and fixed phase.
- Group 3 represents the null vectors, each depending entirely on one input phase.

The vectors of Group 2 will be used in the MC control since its direction in the $\alpha\beta$ plane is known and consequently simplifies the vector selection process. Nevertheless, the vectors are dependent on the instantaneous values of the MC input voltages and output currents. Therefore, the magnitude and the direction of the output voltage vectors will be determined by the instantaneous value of the input voltages, and the input current vectors will be determined by the instantaneous values of the output currents. Since the input voltages are known, it is then necessary to divide the complex plane $\alpha\beta$ into six zones, whereby each represents a different space vector selection. For each zone it is possible to determine the space location of the vectors that have to be used, and therefore control the output voltages. Likewise, this can be done to control the input current since the output currents are known and therefore it is possible to divide the $\alpha\beta$ complex plane into six zones and determine the space location of the vectors to be used in the input current controller. The space vectors map can be seen in Appendix B and Appendix C.
| Group | State | Name | $V_A$ | $V_B$ | $V_C$ | $V_{AB}$ | $V_{BC}$ | $V_{CA}$ | $I_a$ | $I_b$ | $I_c$ | $|V_{oab}|$ | $\delta_o$ | $|I_{oab}|$ | $\mu_i$ |
|-------|-------|------|-------|-------|-------|----------|----------|----------|-------|-------|-------|------------|------------|------------|--------|
| I     | 1     | 1g   | $V_a$ | $V_b$ | $V_c$ | $V_{ab}$ | $V_{bc}$ | $V_{ca}$ | $I_a$ | $I_b$ | $I_c$ | $V_i$       | $\delta_i$ | $\sqrt{3} I_a$ | $\mu_0$ |
|       | 2     | 2g   | $V_a$ | $V_b$ | $V_c$ | $-V_{ca}$ | $-V_{bc}$ | $-V_{ab}$ | $I_a$ | $I_c$ | $I_b$ | $-V_i$     | $-\delta_{i+4\pi/3}$ | $\sqrt{3} I_a$ | $-\mu_0$ |
|       | 3     | 3g   | $V_a$ | $V_b$ | $V_c$ | $-V_{ab}$ | $-V_{ca}$ | $-V_{bc}$ | $I_b$ | $I_c$ | $I_a$ | $-V_i$     | $-\delta_i$  | $\sqrt{3} I_a$ | $-\mu_{o+2\pi/3}$ |
|       | 4     | 4g   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $V_{ca}$ | $V_{ab}$ | $I_c$ | $I_b$ | $I_a$ | $V_i$       | $\delta_{i+4\pi/3}$ | $\sqrt{3} I_a$ | $\mu_{o+2\pi/3}$ |
|       | 5     | 5g   | $V_a$ | $V_b$ | $V_c$ | $V_{ca}$ | $V_{bc}$ | $V_{ab}$ | $I_a$ | $I_c$ | $I_b$ | $V_i$       | $\delta_{i+2\pi/3}$ | $\sqrt{3} I_a$ | $\mu_{o+4\pi/3}$ |
|       | 6     | 6g   | $V_a$ | $V_b$ | $V_c$ | $-V_{bc}$ | $-V_{ab}$ | $-V_{ca}$ | $I_b$ | $I_a$ | $I_c$ | $-V_i$     | $\delta_{i+2\pi/3}$ | $\sqrt{3} I_a$ | $-\mu_{o+4\pi/3}$ |
|       | 7     | +1   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $0$      | $-V_{ab}$ | $0$      | $-I_a$ | $I_b$ | $I_c$ | $0$         | $0$         | $\sqrt{3} I_a$ | $-\pi/6$ |
|       | 8     | -1   | $V_a$ | $V_b$ | $V_c$ | $-V_{ab}$ | $0$      | $V_{bc}$  | $-I_c$  | $I_a$ | $0$   | $I_b$ | $-\sqrt{3} I_{ab}$ | $0$         | $-\sqrt{3} I_a$ | $-\pi/6$ |
|       | 9     | +2   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $0$      | $-V_{ab}$ | $0$      | $I_c$ | $-I_a$ | $I_b$ | $\sqrt{3} I_{bc}$ | $0$         | $\sqrt{3} I_a$  | $\pi/2$  |
|       | 10    | -2   | $V_a$ | $V_b$ | $V_c$ | $-V_{bc}$ | $0$      | $V_{ab}$  | $I_c$   | $I_a$ | $0$   | $I_b$ | $-\sqrt{3} I_{bc}$ | $0$         | $-\sqrt{3} I_a$ | $\pi/2$  |
|       | 11    | +3   | $V_a$ | $V_b$ | $V_c$ | $V_{ca}$ | $0$      | $-V_{ca}$ | $0$      | $I_b$ | $-I_c$ | $I_a$ | $\sqrt{3} I_{ca}$ | $0$         | $\sqrt{3} I_a$  | $7\pi/6$ |
|       | 12    | -3   | $V_a$ | $V_b$ | $V_c$ | $-V_{ca}$ | $0$      | $V_{ca}$  | $I_a$   | $I_c$ | $I_b$ | $-\sqrt{3} I_{ca}$ | $0$         | $-\sqrt{3} I_a$ | $7\pi/6$ |
|       | 13    | +4   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $0$      | $-V_{ab}$ | $I_b$   | $I_c$ | $I_a$ | $\sqrt{3} I_{bc}$ | $2\pi/3$    | $\sqrt{3} I_a$  | $-\pi/6$ |
|       | 14    | -4   | $V_a$ | $V_b$ | $V_c$ | $-V_{bc}$ | $0$      | $V_{bc}$  | $I_c$   | $I_a$ | $I_b$ | $-\sqrt{3} I_{bc}$ | $2\pi/3$    | $-\sqrt{3} I_a$ | $-\pi/6$ |
|       | 15    | +5   | $V_a$ | $V_b$ | $V_c$ | $-V_{bc}$ | $V_{bc}$ | $0$      | $0$     | $I_b$ | $-I_c$ | $I_a$ | $2\sqrt{3} I_{bc}$ | $2\pi/3$    | $\sqrt{3} I_a$  | $\pi/2$  |
|       | 16    | -5   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $-V_{bc}$ | $0$      | $0$     | $-I_b$| $I_c$ | $I_a$ | $-\sqrt{3} I_{bc}$ | $2\pi/3$    | $-\sqrt{3} I_a$ | $\pi/2$  |
|       | 17    | +6   | $V_a$ | $V_b$ | $V_c$ | $-V_{bc}$ | $V_{bc}$ | $0$      | $0$     | $I_b$ | $I_c$ | $I_a$ | $\sqrt{3} I_{bc}$ | $2\pi/3$    | $\sqrt{3} I_a$  | $7\pi/6$ |
|       | 18    | -6   | $V_a$ | $V_b$ | $V_c$ | $V_{bc}$ | $-V_{bc}$ | $0$      | $0$     | $I_b$ | $-I_c$ | $I_a$ | $-\sqrt{3} I_{bc}$ | $2\pi/3$    | $-\sqrt{3} I_a$ | $7\pi/6$ |
|       | 19    | +7   | $V_a$ | $V_b$ | $V_c$ | $0$      | $-V_{ab}$ | $I_c$   | $I_a$   | $0$   | $-I_b$ | $I_c$ | $\sqrt{3} I_{ab}$ | $4\pi/3$    | $\sqrt{3} I_a$  | $-\pi/6$ |
|       | 20    | -7   | $V_a$ | $V_b$ | $V_c$ | $0$      | $V_{ab}$  | $-I_c$  | $I_a$   | $0$   | $I_b$ | $0$   | $\sqrt{3} I_{ab}$ | $4\pi/3$    | $-\sqrt{3} I_a$ | $-\pi/6$ |
|       | 21    | +8   | $V_a$ | $V_b$ | $V_c$ | $0$      | $-V_{bc}$ | $V_{bc}$ | $0$     | $I_c$ | $-I_c$ | $I_a$ | $\sqrt{3} I_{bc}$ | $4\pi/3$    | $\sqrt{3} I_a$  | $\pi/2$  |
|       | 22    | -8   | $V_a$ | $V_b$ | $V_c$ | $0$      | $V_{bc}$  | $-V_{bc}$ | $0$      | $-I_b$| $I_a$ | $I_c$ | $\sqrt{3} I_{bc}$ | $4\pi/3$    | $-\sqrt{3} I_a$ | $\pi/2$  |
|       | 23    | +9   | $V_a$ | $V_b$ | $V_c$ | $0$      | $-V_{bc}$ | $V_{bc}$ | $0$     | $I_a$ | $I_c$ | $I_b$ | $\sqrt{3} I_{bc}$ | $4\pi/3$    | $\sqrt{3} I_a$  | $7\pi/6$ |
|       | 24    | -9   | $V_a$ | $V_b$ | $V_c$ | $0$      | $V_{bc}$  | $-V_{bc}$ | $0$      | $I_c$ | $-I_a$ | $I_b$ | $\sqrt{3} I_{bc}$ | $4\pi/3$    | $-\sqrt{3} I_a$ | $7\pi/6$ |
| III   | 25    | Za   | $V_a$ | $V_b$ | $V_c$ | $0$      | $0$      | $0$      | $0$     | $0$   | $0$   | $0$   | $-\delta_0$  | $0$         | $0$         | $-\mu_0$ |
|       | 26    | Zb   | $V_a$ | $V_b$ | $V_c$ | $0$      | $0$      | $0$      | $0$     | $0$   | $0$   | $0$   | $-\delta_0$  | $0$         | $0$         | $-\mu_0$ |
|       | 27    | Zc   | $V_a$ | $V_b$ | $V_c$ | $0$      | $0$      | $0$      | $0$     | $0$   | $0$   | $0$   | $-\delta_0$  | $0$         | $0$         | $-\mu_0$ |
3.2.2 Space Vector Modulation

Based on the representation of the MC as an equivalent combination of an input virtual rectifier and an output virtual inverter connected by a virtual DC-link, Fig. 3.4 [NeSc92], it is possible to synthesize the output voltages from the input voltages, and to synthesize the input current from the output currents. The virtual decoupling between the output voltage controller and the input current controller allows the use of well-established PWM (Pulse Width Modulation) approaches used in the control of rectifiers and inverters.

![Figure 3.4 - Model of the virtual Matrix Converter, with two conversion stages, used to synthesize the SVM approach](image)

The PWM approach assumes that the MC is fed by a symmetric and balanced three-phase system of line-to-neutral and line-to-line voltages (3.5), with root mean square (RMS) value $V_i$ and angular frequency $\omega_i$.

\[
\begin{bmatrix}
V_{AB}(t) \\
V_{BC}(t) \\
V_{CA}(t)
\end{bmatrix} = \sqrt{2} \sqrt{3} V_i 
\begin{bmatrix}
\cos(\omega_i t + \frac{\pi}{6}) \\
\cos(\omega_i t + \frac{\pi}{6} - \frac{2\pi}{3}) \\
\cos(\omega_i t + \frac{\pi}{6} - \frac{4\pi}{3})
\end{bmatrix}
\]  

(3.5)

The aim is to ensure that the line-to-line output voltages of the MC (3.5) follow a sinusoidal waveform of a line-to-line output voltage reference (3.6), with RMS value $V_o$ and frequency $\omega_o$. 
Assuming the input and output filters of the MC are ideal, the input and output currents can be approximated by their first harmonic. In these conditions, it is possible to define the output currents as sinusoidal waveforms with RMS value $I_o$, frequency $\omega_o$ and phase $\phi_o$, (3.7).

$$
\begin{bmatrix}
I_A \\
I_B \\
I_C
\end{bmatrix} \approx \sqrt{2} I_o \begin{bmatrix}
\cos(\omega_o t + \phi_o) \\
\cos(\omega_o t + \phi_o - \frac{2\pi}{3}) \\
\cos(\omega_o t + \phi_o - \frac{4\pi}{3})
\end{bmatrix}
$$

(3.7)

The aim is to guarantee that the input currents follow sinusoidal reference waveforms (3.8), with RMS value $I_i$, angular frequency $\omega_i$ and phase $\phi_i$.

$$
\begin{bmatrix}
i_{a_{\text{ref}}}(t) \\
i_{b_{\text{ref}}}(t) \\
i_{c_{\text{ref}}}(t)
\end{bmatrix} = \sqrt{2} I_i \begin{bmatrix}
\cos(\omega_i t + \phi_i) \\
\cos(\omega_i t + \phi_i - \frac{2\pi}{3}) \\
\cos(\omega_i t + \phi_i - \frac{4\pi}{3})
\end{bmatrix}
$$

(3.8)

Applying the Concordia/Clarke transformation to (3.6) and (3.8) it is possible to simplify the analysis of the three-phase system into a two coordinates system (3.9) and (3.10).

$$
\begin{bmatrix}
V_{a_{\text{ref}}}(t) \\
V_{b_{\text{ref}}}(t)
\end{bmatrix} = 3 V_o \begin{bmatrix}
\cos(\omega_o t + \frac{\pi}{6}) \\
\sin(\omega_o t + \frac{\pi}{6})
\end{bmatrix}
$$

(3.9)

$$
\begin{bmatrix}
i_{a_{\text{ref}}}(t) \\
i_{b_{\text{ref}}}(t)
\end{bmatrix} = \sqrt{3} I_i \begin{bmatrix}
\cos(\omega_i t + \phi_i) \\
\sin(\omega_i t + \phi_i)
\end{bmatrix}
$$

(3.10)

The well-established SVM method is indicated for PWM control in inverters, since it allows a high power transfer rate with low harmonic distortions, [Rash11].
In MCs, the objectives of SVM are:

- To synthetize the input currents of the rectifier \(i_a, i_b, i_c\) through the current of the intermediate DC link \(I_{DC}\).
- To synthetize the output voltage of the inverter \(V_A, V_B, V_C\) through the voltage in the intermediate DC link \(V_{DC}\).

However, it is necessary to take into account that the virtual rectifier inverter association has no intermediate filtering stage, which results in a time variant \(V_{DC}\) voltage and current \(I_{DC}\).

The rectifier has to generate a voltage \(V_{DC}\) with constant mean value and at the same time, has to guarantee sinusoidal input currents with controllable power factor. This last condition is achieved by adjusting the phase \(\phi_i\) (3.8) between input voltage and the respective input current.

### 3.2.2.1 Rectifier Stage Modulation

In the rectifier stage there are nine possible switching combinations which guarantee the current continuity in the DC link. Applying \(a\beta\) transformation to the currents that result from these nine combinations, Table 3.2, allow the establishment of nine space vectors. The nine combinations can be divided into six non-zero input currents which are active vectors \(I_1\) to \(I_6\) and three zero input currents which are zero vectors \(I_0, I_7\) and \(I_8\), (Fig. 3.5).
Table 3.B - Rectifier space vectors for the possible switching combinations

| Vector | $S_{r11}$ | $S_{r12}$ | $S_{r13}$ | $S_{r21}$ | $S_{r22}$ | $S_{r23}$ | $i_a$ | $i_b$ | $i_c$ | $||i_{a\theta}(t)||$ | $\delta_i$ | $V_{DC}$ |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-------|-------|-------|-----------------|----------|--------|
| $I_1$  | 1         | 0         | 0         | 0         | 0         | 1         | $I_{DC}$ | 0     | $-I_{DC}$ | $\sqrt{2} I_{DC}$ | $\pi/6$  | $-V_{CA}$ |
| $I_2$  | 0         | 1         | 0         | 0         | 0         | 1         | 0     | $I_{DC}$ | $-I_{DC}$ | $\sqrt{2} I_{DC}$ | $\pi/2$  | $V_{BC}$ |
| $I_3$  | 0         | 1         | 0         | 1         | 0         | 0         | $-I_{DC}$ | $I_{DC}$ | 0     | $\sqrt{2} I_{DC}$ | $5\pi/6$ | $-V_{AB}$ |
| $I_4$  | 0         | 0         | 1         | 1         | 0         | 0         | $-I_{DC}$ | 0     | $I_{DC}$ | $\sqrt{2} I_{DC}$ | $-5\pi/6$ | $V_{CA}$ |
| $I_5$  | 0         | 0         | 1         | 0         | 1         | 0         | 0     | $-I_{DC}$ | $I_{DC}$ | $\sqrt{2} I_{DC}$ | $3\pi/2$ | $-V_{BC}$ |
| $I_6$  | 1         | 0         | 0         | 0         | 1         | 0         | $I_{DC}$ | $-I_{DC}$ | 0     | $\sqrt{2} I_{DC}$ | $-\pi/6$ | $V_{AB}$ |
| $I_7$  | 1         | 0         | 0         | 1         | 0         | 0         | 0     | 0     | 0     | 0              | -0       | -0     |
| $I_8$  | 0         | 1         | 0         | 0         | 1         | 0         | 0     | 0     | 0     | 0              | -0       | -0     |
| $I_9$  | 0         | 0         | 1         | 0         | 0         | 1         | 0     | 0     | 0     | 0              | -0       | -0     |

Therefore, knowing the location of the desired input current in the $q\beta$ plane, it is possible to synthetize it, as a combination of the adjacent space vectors, (Fig. 3.6).
Assuming \( N_i \in \{1, ..., 6\} \) represents the number of the sector, where the input current reference vector is located and considering \( \theta_i \) as the phase related to the sector where the vector is located, it is possible (3.11) to relate the \( \theta_i \) to the instantaneous phase \( \varphi_i = \omega_i t + \phi_i \) (3.10) of the input current vector.

\[
\theta_i = \varphi_i - \frac{\pi}{3} (N_i - 1) + \frac{\pi}{6} \quad N_i \in \{1, ..., 6\}
\]

Based on the representation of Fig. 3.6 it is possible to synthetize the input current reference vector using trigonometric relations. It is assumed that the adjacent vectors \( I_1 \sim I_6 \) are \( I_5 \), \( I_r \) and zero vectors \( I_7 \), \( I_8 \) and \( I_9 \) with the respective duty cycles \( d_5 \) (for \( I_5 \)), \( d_r \) (for \( I_r \)) and \( d_0 \) (for one of the zero vectors). Considering that the switching frequency is much higher than the input frequency \( f_s \gg f_i \), it is possible to define the reference vector \( I_{refab} \) as (3.12) for each commutation period.

\[
I_{refab} = I_r d_r + I_5 d_5 + I_0 d_0
\]

The duty cycles \( d_5 \), \( d_r \) and \( d_0 \) (3.13) can be calculated by using a trigonometric analysis applied to the vectors presented in Fig. 2.16 c), [HuBo95].
\[
\begin{align*}
  d_y &= \frac{T_y}{T_s} = m_c \sin \left( \frac{\pi}{3} - \theta_i \right) \\
  d_\delta &= \frac{T_\delta}{T_s} = m_c \sin \theta_i \\
  d_\phi &= \frac{T_\phi}{T_s} = 1 - d_y - d_\delta
\end{align*}
\]  

(3.13)

The variable \( m_c \) (3.14) is the current modulation index that relates the magnitude of the input current \( I_{\text{imax}} \) with the current in the DC intermediate stage \( I_{DC} \).

\[
m_c = \frac{I_{\text{imax}}}{I_{DC}}
\]  

(3.14)

The mean value of the intermediate stage voltage \( V_{\text{DC}} \) can be calculated assuming that the input power \( P_{\text{in}} \) is equal to the instantaneous power in the intermediate stage \( P_{\text{DC}} \) and to the output power \( P_{\text{out}} \). The equation is valid in case of no power losses (which occur if ideal switches are considered) and assuming the voltages and currents are approximately equal to their respective first harmonics.

\[
P_{\text{DC}} = P_{\text{in}} = P_{\text{out}}
\]  

(3.15)

It is possible now to reach the equation (3.16) by calculating the \( P_{\text{DC}} \) (3.15) based on the \( V_{\text{DC}} \) and current \( I_{DC} \); while the input power \( P_{\text{in}} \) is calculated taking into account the line-to-neutral voltage \( V_{\text{imax}} \) and the input current of the rectifier \( I_{\text{imax}} \).

\[
V_{\text{DC}} I_{DC} = \frac{1}{2} V_{\text{imax}} I_{\text{imax}} \cos(\phi_i)
\]  

(3.16)

The intermediate stage voltage \( V_{\text{DC}} \) (3.17), which is calculated from (3.16) depends on three parameters: the magnitude of the line-to-neutral input voltage \( V_{\text{imax}} \) or the line-to-line \( V_{\text{icmax}} \); the current modulation index \( m_c \) and the phase \( \phi_i \) between input current and input voltage.

\[
V_{\text{DC}} = \frac{3}{2} V_{\text{imax}} I_{\text{imax}} \frac{I_{\text{imax}}}{I_{DC}} \cos(\phi_i) = \frac{3}{2} V_{\text{imax}} m_c \cos(\phi_i) = \frac{\sqrt{3}}{2} V_{\text{icmax}} m_c \cos(\phi_i)
\]  

(3.17)
Notice that the voltage $V_{DC}$ is constant in steady state, with a maximum value that equals $\sqrt{3}/2$ of the line-to-line input peak voltage, and it is achieved when the displacement factor of the input current is zero. As the phase shift increases, the voltage in the intermediate stage decreases.

### 3.2.2.2 Inverter Stage Modulation

In the inverter stage (Fig 3.7), there are eight possible switching combinations and their magnitude is directly related to the available voltage $V_{DC}$ in the intermediate stage.

![Inverter Stage Modulation Diagram](image)

**Figure 3.7 - Inverter stage of the virtual Matrix Converter**

**Table 3.C - Inverter space vectors for the possible switch combinations**

<table>
<thead>
<tr>
<th>Vector</th>
<th>$S_{i11}$</th>
<th>$S_{i12}$</th>
<th>$S_{i21}$</th>
<th>$S_{i22}$</th>
<th>$S_{i31}$</th>
<th>$S_{i32}$</th>
<th>$V_A$</th>
<th>$V_B$</th>
<th>$V_C$</th>
<th>$V_{AB}$</th>
<th>$V_{BC}$</th>
<th>$V_{CA}$</th>
<th>$V_o$</th>
<th>$\delta_o$</th>
<th>$I_{DC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$V_o$</td>
<td>$V_C$</td>
<td>$V_o$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>$\sqrt{3}V_{DC}$</td>
<td>$\pi/6$</td>
<td>$i_a$</td>
</tr>
<tr>
<td>$V_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$V_o$</td>
<td>$V_o$</td>
<td>$V_C$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>$\sqrt{3}V_{DC}$</td>
<td>$\pi/2$</td>
<td>-$i_c$</td>
</tr>
<tr>
<td>$V_3$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$V_C$</td>
<td>$V_C$</td>
<td>$V_o$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$\sqrt{3}V_{DC}$</td>
<td>$5\pi/6$</td>
<td>$i_a$</td>
</tr>
<tr>
<td>$V_4$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$V_C$</td>
<td>$V_o$</td>
<td>$V_C$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>$\sqrt{3}V_{DC}$</td>
<td>-$5\pi/6$</td>
<td>-$i_a$</td>
</tr>
<tr>
<td>$V_5$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$V_C$</td>
<td>$V_C$</td>
<td>$V_o$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>$\sqrt{3}V_{DC}$</td>
<td>$3\pi/2$</td>
<td>$i_c$</td>
</tr>
<tr>
<td>$V_6$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$V_o$</td>
<td>$V_C$</td>
<td>$V_o$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>$-\pi/6$</td>
<td>-$i_a$</td>
</tr>
<tr>
<td>$V_7$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$V_8$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The eight space vectors can be seen in Table 3.3. The reference vector $V_{oref\alpha\beta}$ of the line-to-line output voltage (3.18), where $V_{oc}$ is the RMS value and $V_{ocmax}$ is the line-to-line voltage magnitude, describes a circular path in the $\alpha\beta$ plane and is synthesized using the space vectors represented in Fig. 3.8 b).

$$V_{oref\alpha\beta}(t) = \sqrt{3} V_{oc} e^{j\omega t} = \frac{\sqrt{3}}{2} V_{oc_{max}} e^{j\omega t}$$ \hspace{1cm} (3.18)

Therefore, the maximum of the output voltage reference (3.19) is the maximum of the available voltage in the intermediate stage $V_{DC}$. Thus, the limitation of the output voltage of the rectifier-inverter association model is imposed by the rectifier.

$$V_{oc_{max}} = V_{DC}$$ \hspace{1cm} (3.19)

To determine the modulation function, a similar procedure to that used in the rectifier is considered, where $N_{v} \in \{1, ..., 6\}$ represents the number of the sector where the line-to-line reference voltage vector is located. Considering $\theta_v$ (3.20) as the phase related to the sector where the vector is located, it is possible to relate $\theta_v$ with the instantaneous phase $\varphi_v = \omega_v t + \frac{\pi}{6}$ (3.9) of the line-to-line output voltage.

$$\theta_v = \varphi_v - \frac{\pi}{3} (N_{v} - 1) + \frac{\pi}{6} \hspace{1cm} N_{v} \in \{1, ..., 6\}$$ \hspace{1cm} (3.20)
Figure 3.8 - a) Line-to-line output voltage sectors; b) Space location of vectors $V_0$ to $V_7$, defining 6 sectors in the $\alpha\beta$ plane; c) Representation of the synthesis process of $V_{oref\alpha\beta}$ using the space vectors adjacent to the sector where the reference vector is located.

The vector $V_{oref\alpha\beta}$ can be obtained using the adjacent space vector $V_\alpha$, $V_\beta$ and $V_0$ represented in Fig 3.8 c) and the duty-cycles associated to the vectors $d_\alpha$, $d_\beta$ and $d_0$. Considering that the switching frequency is much higher than the input frequency $f_o >> f_s$, it is possible to synthesize the reference vector $V_{oref\alpha\beta}$ (3.21) for each switching period.

$$V_{oref\alpha\beta} \approx V_\alpha d_\alpha + V_\beta d_\beta + V_0 d_0$$  \hfill (3.21)

The duty cycles $d_\alpha$, $d_\beta$ and $d_0$ (3.22) can be calculated by using a trigonometric analysis to the vectors presented in the Fig. 3.8 c), [HuBo95].

$$\begin{align*}
  d_\alpha &= \frac{T_\alpha}{T_s} = m_v \sin \left( \frac{\pi}{3} - \theta_v \right) \\
  d_\beta &= \frac{T_\beta}{T_s} = m_v \sin (\theta_v) \\
  d_0 &= \frac{T_0}{T_s} = 1 - d_\alpha - d_\beta
\end{align*}$$ \hfill (3.22)

The constant $m_v$ (3.23) is the modulation index that relates the maximum value of the output voltage $V_{o,max} = \sqrt{2} V_o$ with the intermediate stage voltage $V_{DC}$. 

$$m_v = \frac{V_{o,max}}{\sqrt{2} V_o}$$ \hfill (3.23)
The current mean value $I_{DC}$ in the intermediate stage can be calculated considering the approximations used in the rectifier case, and once again assuming no power losses, (3.24).

$$m_v = \frac{V_{omax}}{V_{DC}} \quad (3.23)$$

$$P_{DC} = P_{out}$$

$$V_{DC} I_{DC} = 3 \cdot \frac{1}{2} \cdot V_{omax} \cdot I_{omax} \cdot \cos(\phi_o) \quad (3.24)$$

The intermediate stage current $I_{DC}$ (3.25), which is directly calculated from (3.23) depends on three parameters: the magnitude output current $I_{omax}$, the modulation index $m_v$ and the phase $\phi_o$ between the output voltage and output current.

$$I_{DC} = 3 \cdot \frac{1}{2} \cdot \frac{V_{omax}}{V_{DC}} \cdot I_{omax} \cdot \cos(\phi_o)$$

$$= 3 \cdot \frac{1}{2} \cdot m_v \cdot I_{omax} \cdot \cos(\phi_o) \quad (3.25)$$

As stated before, considering that the switching frequency is much higher than the input and output frequency ($f_s >> f_i, f_s >> f_o$) from the rectifier and inverter model, it is possible to assume that in a commutation period the mean values of $V_{DC}$ voltage and $I_{DC}$ current in the intermediate stage are constant. Therefore, it is possible to apply simultaneously the output voltage modulation and the input current modulation in the rectifier and inverter model of Fig. 3.4.

Since the rectifier stage needs two non-zero vectors to realize the input current modulation and the inverter stage needs also two non-zero vectors to realize the output voltage modulation, the modulation function will need five state-space vectors; four non-zero and one zero vector. To modulate the input currents and output voltages, the duty-cycles (3.26) of each selected vector are obtained multiplying of the duty cycles calculated for the rectifier and inverter, [HuBo95].
In (3.26) there are two modulation indexes: the voltage index $m_v$, and the current index $m_c$, whose product $m_c \times m_v$ represent the gain of the transfer ratio of the MC. To guarantee that the MC follows the voltage and input references defined in [Niel96], the voltage modulation index (3.26) must be redefined as (3.27), and replaced in (3.17) and in (3.23) with $m_c = 1$ [HuBo95], so the dependence on the fictitious voltage $V_{DC}$ in the intermediate stage, can be eliminated.

$$
\begin{align*}
  d_f d_a &= m_c m_v \sin \left( \frac{\pi}{3} - \theta_i \right) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
  d_f d_\beta &= m_c m_v \sin \left( \frac{\pi}{3} - \theta_i \right) \sin(\theta_v) \\
  d_\delta d_a &= m_c m_v \sin(\theta_i) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
  d_\delta d_\beta &= m_c m_v \sin(\theta_i) \sin(\theta_v) \\
  d_0 &= 1 - d_f d_a - d_f d_\beta - d_\delta d_a - d_\delta d_\beta
\end{align*}
$$

(3.26)

In (3.26) there are two modulation indexes: the voltage index $m_v$, and the current index $m_c$, whose product $m_c \times m_v$ represent the gain of the transfer ratio of the MC. To guarantee that the MC follows the voltage and input references defined in [Niel96], the voltage modulation index (3.26) must be redefined as (3.27), and replaced in (3.17) and in (3.23) with $m_c = 1$ [HuBo95], so the dependence on the fictitious voltage $V_{DC}$ in the intermediate stage, can be eliminated.

$$
m_v = \frac{V_o}{\frac{3}{2} V_{max} m_c \cos(\phi_o)}
$$

(3.27)

After defining the duty-cycles of the MC SVM (3.26), it is then necessary to determine the vectors that participate in the modulation process. The duty cycles determine the space vectors to control the MC; however, the order in which they are selected is not yet established and can be decided under several criteria. The selection of the vector in each instant depends not only on the sector location of the line-to-line output voltage reference but also on the sector location of the input current reference. Following these conditions it is possible to identify the vectors, which should be used in the modulation process, in Table 3.4.

Based on the sector of the line-to-line output reference voltage, and on the sector of the input reference current, it is, it is possible to choose one vector in Table 3.4 during a certain time frame in order to control output voltages and input currents.
The duty cycles used in SVM are calculated based on the output voltage and input current references (3.26). To know the time interval during which the corresponding state space vectors are applied to the converter, a sawtooth high frequency \((f_s >> f_i, f_s >> f_o)\) carrier waveform is compared to these \((3.26)\) duty cycles. In Fig. 3.9 it can be seen that the different time intervals in which each vector is selected, are the regions that define which component of the output voltage reference \(\alpha\beta\) and input current reference \(\gamma\delta\) are needed to represent the respective reference vectors (3.12), (3.18).
The selection of the vector that specifies the commutations of the three-phase matrix converter switches is not only based on the regions of Fig. 3.9, but also in Table 3.4, as the vectors are different for every location of the input currents and output voltages. For each current and voltage sector, Table 3.4 presents the four vectors that will be used in the modulation process, thus producing the $\alpha$ or $\beta$, and $\Upsilon$ or $\delta$ components necessary to follow the reference voltage and current vectors. The selected vector is finally sent to the three-phase matrix converter to command the switches state.

A summary of this process can be seen using one example shown in Fig. 3.10. The components $d\delta d\beta$ are selected during $T_{d\delta d\beta}$, and with Table 3.4, which receives information about the location of the input current and output voltage, it is possible to select the vector "-2". The sequence continues in order to represent also the reference vectors (3.12), (3.18) by the components $d\Upsilon$ and $d\alpha$.

Figure 3.9 - Modulation process used to select the space vectors and the time interval when they are applied.

Figure 3.10 - Selection scheme for the SVM vectors
The modulator will produce an output voltage waveform similar to the one represented in Figure 3.11 (considering only one of the input voltages).

![Output voltage waveform similar to the one obtained with the modulator (and considering only one input voltage, for simplicity in the representation)](image)

Fig. 3.11 shows that the mean value of the voltage during each switching period is not zero, which means that for transformers that operate at higher frequencies (1 kHz) the voltage is continuously increasing during one half of the grid period or decreasing during the other half of the grid period. Consequently, this is a problem for HFTs as they can easily saturate. To avoid this problem, a new strategy is developed in this thesis in order to ensure the non-saturation of the HFT.

### 3.2.3 Modified Space Vector Modulation

The conventional SVM presented in the previous chapter is improved in order to produce an output voltage with an average value equal to zero in each switching period. Therefore, the duty-cycles of Fig. 3.9 should be divided in two time frames; the first one corresponding to the vector selected by SVM, and the second one to the vector that produces a symmetric voltage (3.28). Note that $T_{das\beta}$ is variable since the duty cycles are not constant.

$$H_{sb}(t) = \begin{cases} 
SVM \text{ Vector} & 0 \leq T < T_{das\beta}/2 \\
-1 \times SVM \text{ Vector} & T_{das\beta}/2 \leq T < T_{das\beta}
\end{cases} \quad (3.28)$$
As a result, in the modified modulation process, four more duty cycle signals are created (3.29), when compared to the original SVM (3.26).

\[
\begin{align*}
    d_y d_{a+} &= \frac{m}{2} \sin \left( \frac{\pi}{3} - \theta_i \right) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
    d_y d_{a-} &= \frac{m}{2} \sin \left( \frac{\pi}{3} - \theta_i \right) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
    d_y d_{b+} &= \frac{m}{2} \sin \left( \frac{\pi}{3} - \theta_i \right) \sin(\theta_v) \\
    d_y d_{b-} &= \frac{m}{2} \sin \left( \frac{\pi}{3} - \theta_i \right) \sin(\theta_v) \\
    d_{\delta} d_{a+} &= \frac{m}{2} \sin(\theta_i) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
    d_{\delta} d_{a-} &= \frac{m}{2} \sin(\theta_i) \sin \left( \frac{\pi}{3} - \theta_v \right) \\
    d_{\delta} d_{b+} &= \frac{m}{2} \sin(\theta_i) \sin(\theta_v) \\
    d_{\delta} d_{b-} &= \frac{m}{2} \sin(\theta_i) \sin(\theta_v) \\
    d_0 &= 1 - d_y d_a - d_y d_b - d_{\delta} d_a - d_{\delta} d_b
\end{align*}
\] (3.29)

Due to the nomenclature of the vectors established in Table 3.1, the vector that produces a symmetric voltage corresponds to the vector named with a symmetric number. Therefore, in the example of Fig 3.10 the three phase MC is controlled by \( H_{3y}(t) \) (3.28), whereby the vector selected for \( 0 \leq T < T_{d5d5}/2 \) is the vector “-2” and for \( T_{d5d5}/2 \leq T < T_{d5d5} \) the selected vector is “+2”.

Fig. 3.12 presents the modified modulation process with the extra four duty cycle signals (3.29). It can be seen that the intervals are now divided in two (except for \( d_0 \)); the first is defined by the original SVM as in Fig 3.9, and the second interval is created by the extra duty cycle signals from the modified SVM.
Figure 3.12 - Modified modulation process used to select the space vectors and the time interval when they are applied

This process demands a higher switching frequency but provides a zero mean value for the MC output voltage. For the same example as the one represented in Fig.3.11, the modified SVM would produce a waveform similar to the one represented in Fig 3.13.

Figure 3.13 - Output Voltage waveform obtained with the modified SVM
3.3 Single-Phase Matrix Converter

The Single-Phase Matrix Converter consists of four controlled bidirectional switches making a 2x2 matrix (3.30) that allows the connection between two single-phase systems; the input with voltage source characteristics and the output system with characteristics of current source as shown in Fig 3.14.

![Diagram of Single-Phase Matrix Converter](image)

Figure 3.14 - Single-Phase Matrix Converter

Assuming ideal bidirectional switches (zero voltage drop when ON, no leakage current when OFF and null switching times) each switch can be represented mathematically by a variable $s_{kj}$ (3.1) with the value “1” if the switch is closed (ON) or “0” if the switch is open (OFF). The, it is possible to represent the converter as a matrix (3.30)

$$
S_{10} = \begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
$$

(3.30)

Similarly to the Three-Phase Matrix Converter the same topological restraints apply in the single phase matrix converter. Therefore, there are four possible states, which are described in Table 3.5 with the respective correlations between the electrical variable combinations.
Table 3.E - Possible switching combinations for a Single-Phase Matrix Converter

<table>
<thead>
<tr>
<th>State</th>
<th>S_{11}</th>
<th>S_{12}</th>
<th>S_{21}</th>
<th>S_{22}</th>
<th>V_A</th>
<th>V_B</th>
<th>I_A</th>
<th>I_B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>V_a</td>
<td>V_b</td>
<td>I_A</td>
<td>I_B</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>V_b</td>
<td>V_a</td>
<td>I_B</td>
<td>I_A</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>V_a</td>
<td>V_a</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>V_b</td>
<td>V_b</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Since the non-saturation of the transformer is assured by the modified SVM strategy used in the Three-Phase Matrix Converter, the purpose of the SPMC is to restore the output voltage applied to the load into the voltage synthesized by the original SVM.

Considering Table 3.5, the SPMC is able to produce at its output the same voltage applied at the input or its symmetric voltage. In the current example, \( H_{10}(t) \) determines the state “1” for the SPMCs, during \( 0 \leq T < T_{a\delta a\beta}/2 \) and the state “-1” for \( T_{a\delta a\beta}/2 \leq T < T_{a\delta a\beta} \) as indicated in (3.31).

\[
H_{10}(t) = \begin{cases} 
\text{State “1”} & 0 \leq T < T_{a\delta a\beta}/2 \\
\text{State “-1”} & T_{a\delta a\beta}/2 \leq T < T_{a\delta a\beta}
\end{cases}
\]  

(3.31)

As an example of the modulation process, a sequence of vectors is presented in Fig. 3.15. According to the conventional SVM approach, vector -2 should be applied during time interval \( T_{a\delta a\beta} \). However, to avoid the HFT saturation, it is necessary to use the modified SVM. Then:

- In the first half of the considered interval, where the vector is “-2”, the SPMCs reproduces the input voltage at the output.
- In the second half of the considered interval, the three phase MC modified SVM changes the vector to “+2”. Consequently, there is the need to restore the vector selected by the original SVM. This can be done guaranteeing that the SPMCs are in the state “-1” (3.31), thus assuring that the voltage applied to their output, is symmetric to their input voltage.
Modified SVM

Vectors applied to Three Phase Matrix Converter

1. \[ +4 \rightarrow +1 \rightarrow -1 \rightarrow +7 \rightarrow -2 \rightarrow +2 \rightarrow +1 \rightarrow 0 \rightarrow -3 \rightarrow -3 \]

\[ Td\delta d\beta /2 \rightarrow Td\delta d\beta /2 \]

waveform with an average value equal to zero

SPMCs states

Original SVM

Output of SPMCs with the original vectors

2. \[ +1 \rightarrow -1 \]

\[ Td\delta d\beta /2 \rightarrow Td\delta d\beta /2 \]

waveform of the original SVM restored

3. \[ -4 \rightarrow +1 \rightarrow -7 \rightarrow -2 \rightarrow +1 \rightarrow 0 \rightarrow +3 \]

\[ Td\delta d\beta /2 \rightarrow Td\delta d\beta /2 \]

Fig 3.15 shows the process of the selection of a sequence of vectors. In the first phase, the modified SVM selects the vectors that determine the switches state of the Three Phase Matrix Converter. Therefore, it is possible to see in Fig 3.15 that after a new vector is applied, the next is its symmetric. The time interval in which both vectors are applied is the same.

The second phase presents the possible states of the SPMC. As explained before, the SPMCs restore the original SVM vector by applying in the output, the input voltage or its symmetric. Finally, the third phase shows the original vectors determined by the conventional SVM, which in practice are restored in the load voltage.

The developed SVM approach ensures that the final output is the one that would be obtained by the conventional SVM method, while avoiding the saturation of the HFT.
Chapter 4

Filters Sizing and Controllers Design

This chapter provides the calculation of the Input and Output Filter parameters, as well as the Current and Voltage Regulators design.
4.1 Input filter

The implemented filter LC is a second order low pass filter with a damping resistance connected in parallel to the coil. This topology was chosen to minimize losses as shown in [Pint03].

In order to simplify the design of the input filter, the analysis will be performed based on the equivalent single-phase represented in Fig 4.1.

Knowing the maximum voltage and the minimum current in the output of the filter, it is possible to determine the value of the capacitance for the single phase equivalent, (4.1), [Silv12b]. The value of the capacitance depends also on the input angular frequency ($\omega_i$) and the input power factor ($P_f$).

$$C_{fin} = \frac{I_{min}}{\omega_i V_{o max}} \tan(\cos^{-1}(P_f))$$  (4.1)

In order to calculate the coil it is necessary to establish the cutoff frequency of the input filter. The cutoff frequency of the filter should be one decade above the input frequency ($f_i$) and one decade below the switching frequency ($f_s$), i.e., $\omega_i < \omega_c < \omega_s$. [Mont10], [Pint03], [PiSi11]. Then the self-induction value of the coil is given by (4.2).
An important factor to take into consideration in the input filter design is the calculation of the damping resistance. The damping resistance, which is connected parallel to the coil, aims to reduce oscillations from created by the semiconductors switching, thereby guaranteeing that the system does not become unstable.

In order to calculate $r_p$, some assumptions were taken:

It was assumed that there is a constant power in the output of the overall system. Consequently the incremental resistance $r_i$ is negative.

\[
\frac{\partial P_{OMC}}{\partial t} = 0 = \frac{\partial V_{OMC}}{\partial i_{OMC}} = -\frac{V_{OMC}}{i_{OMC}} = -R_o
\]  

(4.3)

The negative incremental resistance $r_i$, can be calculated as a function of the input and output voltage of the MC, (4.4), [Silv12b].

\[
r_i = \frac{\partial V_o}{\partial t} = \frac{\partial}{\partial t} \left( \frac{p_o}{\eta_{in}} \right) = -\frac{p_o}{\eta_{in}}
\]

(4.4)

\[
r_i = -R_o \eta \frac{V_o^2}{V_{OMC}^2}
\]

(4.5)

Knowing that the maximum transfer ratio of the Three Phase Matrix Converter is $v_{OMCe}$, and replacing $\frac{V_o}{V_{OMC}}$ by the maximum voltage transfer ratio of the Three Phase Matrix Converter, it is possible to simplify (4.5) into (4.6).

\[
r_i = -\frac{4}{3} R_o \eta
\]

(4.6)
Knowing the value of $r_i$ and the filter impedance $Z_i$, it is possible to obtain $r_p$ (4.7) as presented in Appendix D, where $\xi$ is the damping factor of the filter.

$$Z_f = \frac{L_i}{\sqrt{C_i}}$$

$$r_p = \frac{r_i Z_f}{2 \xi r_i - Z_f}$$

(4.7)

Table 4.1 presents the values of the parameters obtained for the input filter. These values will be used later in the simulations.

Table 4.A - Input filter values

<p>| Values of the input filter ($r_p$||L damping) |
|------------------------------------------------|</p>
<table>
<thead>
<tr>
<th>L(mH)</th>
<th>C($\mu$F)</th>
<th>$r_p$(\Omega)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>31.5</td>
<td>2.95</td>
</tr>
</tbody>
</table>

### 4.2 Output Filter

The implemented LC filter is a second order filter, which aims to minimize the high frequency current harmonics that are present in the MC output current. Consequently, the current that flows to the overhead line of the rail, is a clean current from the standing point of harmonic content. In Appendix E the nominal voltages and their permissible limits in values and duration for the main electric traction systems are presented.

In order to simplify the design of the output filter, the analysis will be performed based on the equivalent single-phase represented in Fig. 4.2.
To calculate the parameters of the filter, it is necessary to know in advance the RMS value of the rated current in the filter (4.8).

\[ i_{out} = \frac{S_{NT}}{3V_{primary}} \frac{1}{\bar{n}} \]  

(4.8)

Taking into consideration the high frequency transformer, it is possible to calculate the equivalent value of the output resistance, \( r_{out} \) (4.9).

\[ r_{out} = \frac{S_{NT}}{3i_{output\_filter}^2} \]  

(4.9)

With the value of the equivalent resistance \( r_{out} \), it is possible to obtain the characteristic impedance of the output filter output, \( Z_{of} \) (4.10), wherein \( \xi \) is the damping factor of the filter with a value of 0.7.

\[ Z_{of} = \frac{r_{out}}{2 \xi} \]  

(4.10)

In order to calculate the coil and the capacitance, it is necessary to consider the cutoff frequency of the output filter. The cutoff frequency of the filter has to be one decade below the switching frequency (\( f_s \)) and one decade above the grid frequency (\( f_i \)), \( f_i < f_c < f_s \), i.e., \( \omega_i < \omega_c < \omega_s \). [Mont10], [Pint03]. The capacitance value is given by (4.11).
\[ C_{\text{out}} = \frac{1}{\omega_c Z_{af}} \] \hspace{1cm} (4.11)

Finally, the self-induction value of the coil is obtained from (4.12)

\[ L_{\text{out}} = \frac{Z_{af}}{\omega_c} \] \hspace{1cm} (4.12)

Table 4.2 presents the values of the parameters obtained for the output filter. The values are used afterwards in the simulations.

| Values of the output filter (LC) |
|-------|-------|-------|
| L(mH) | C (\(\mu F\)) | R(\(\Omega\)) |
| 5     | 5      | 3      |

### 4.3 Output Current Regulator

Considering that the MC is feeding an equivalent three phase RL load, the output currents, in dq coordinates are given by (4.13)

\[
\begin{align*}
\frac{di_{\text{d}d}}{dt} &= \frac{R}{L} i_{\text{d}d} + \frac{1}{L} H_{\text{d}} \\
\frac{di_{\text{dq}}}{dt} &= \frac{R}{L} i_{\text{dq}} + \frac{1}{L} H_{\text{q}}
\end{align*}
\] \hspace{1cm} (4.13)

The commands voltages \(H_{\text{d}q}\) in (4.14), guarantee that the output currents, \(i_{\text{d}d}\) and \(i_{\text{dq}}\) follow their references.
\[
\begin{align*}
H_d &= \omega L_i + V_d \\
H_q &= \omega L_i + V_q
\end{align*}
\] (4.14)

The current regulator block diagram is represented in Fig. 4.3, where \(I_{\text{odqref}}\) is the reference current and \(I_{\text{odq}}\) the load current. Both are multiplied by \(\alpha_i\), the current sensor gain, and the difference between the two currents, i.e., the current error is applied to the controller \(C_i(s)\). This controller generates the modulating voltage used by the SVM.

![Diagram of the current regulator block diagram](image)

Figure 4.3 - Output current regulator block diagram

To design the current regulator, it is possible to represent the three phase matrix converter as a first order transfer function with a delay \(T_d\). This transfer function is designated by \(G(s)\), and given by (4.15).

\[
G(s) = \frac{1}{1 + sT_d}
\] (4.15)

The average delay of the system (MC + SVM) response \(T_d\), is usually considered as half the switching period of the MC, \(T_c\) (4.16).

\[
T_d = \frac{T_c}{2}
\] (4.16)

However, in the proposed system the average delay is considered to be higher, since a transformer, more than one MC and input and output filters also exist in this system. Therefore, to guarantee the whole system stability, a delay in the order of the grid period is considered.
$C_i(s)$ is a Proportional-Integral (PI) Controller, which ensures a second order closed chain dynamics (4.17). This compensator ensures a null static error and an acceptable rise time.

$$C_i(s) = \frac{1 + sT_z}{sT_p} \quad (4.17)$$

To calculate the parameters $T_z$ and $T_p$, it is considered that the zero of $C_i(s)$ cancels the lowest frequency pole, introduced by the output LR filter. $T_z$ is given by (4.18) where $R_{out}$ is the sum of the coil intern resistance with the load resistance.

$$T_z = \frac{L_{out}}{R_{out}} \quad (4.18)$$

The value of $T_p$ is calculated by (4.19), where $\alpha_i$ is the current gain and $T_d$ is the average delay of the system.

$$T_p = \frac{\alpha_i T_D}{R_{out}} \quad (4.19)$$

According to the $H_{dq}$ command variables, the modulation voltages are given by (4.20). The calculation of $V_{od}$ and $V_{oq}$ as functions of $H_{sq}$ allows the decoupling of the d and q control actions.

$$\begin{cases} V_{od} = H_d - \omega L i_{oq} \\ V_{oq} = H_q + \omega L i_{od} \end{cases} \quad (4.20)$$

The block diagram corresponding to the system control current in dq coordinates with controller, modulator, MC and output filter is shown in Fig. 4.4.
Table 4.3 presents the values of the parameters obtained for the current regulator (PI controller).

Table 4.C - Load current regulator values

<table>
<thead>
<tr>
<th>$T_d(s)$</th>
<th>$K_i$</th>
<th>$K_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>$3 \times 10^{-5}$</td>
<td>500</td>
</tr>
</tbody>
</table>

These values will be further used in the simulations.
4.4 Output Voltage Regulator

The voltage regulator is shown in Fig. 4.5, where the capacitor ($C_{\text{load}}$) designed in (4.11) was changed to the SPMCs output, next to the load.

![Diagram](image)

Figure 4.5 - Load voltage regulator

The voltage regulator has to ensure that the load voltage, which is the same as the capacitor voltage, (4.21) remains within the standardized values (Appendix E).

\[
V_{\text{load}} = \frac{i_c}{sC_{\text{fout}}} \quad (4.21)
\]

From Fig. 4.5, the output capacitors currents are given as in (4.22), in $\alpha\beta$ coordinates and are related to the load currents and to the converter output currents.

\[
\begin{align*}
C_{\text{fout}} \frac{\partial v_c}{\partial t} &= i_{\text{load}_a} - i_{\text{system}_a} \\
C_{\text{fout}} \frac{\partial v_c}{\partial t} &= i_{\text{load}_\beta} - i_{\text{system}_\beta}
\end{align*} \quad (4.22)
\]
Equations (4.22) can be then written in the canonical form (4.23):

\[
\begin{align*}
\frac{\partial v_{\alpha}}{\partial t} &= \frac{v_{\text{load}_d}}{RC_{\text{out}}} - \frac{1}{C_{\text{out}}} i_{\text{system}_\alpha} \\
\frac{\partial v_{\beta}}{\partial t} &= \frac{v_{\text{load}_\beta}}{RC_{\text{out}}} - \frac{1}{C_{\text{out}}} i_{\text{system}_\beta}
\end{align*}
\] (4.23)

Applying the Clarke/Park transformation to (4.23), the system equations are obtained (4.24).

\[
\begin{align*}
\frac{\partial v_{cd}}{\partial t} &= \frac{v_{cd}}{RC_{\text{out}}} - \frac{1}{C_{\text{out}}} i_{\text{system}_d} - \omega v_{cq} \\
\frac{\partial v_{cq}}{\partial t} &= \frac{v_{cq}}{RC_{\text{out}}} - \frac{1}{C_{\text{out}}} i_{\text{system}_q} - \omega v_{cd}
\end{align*}
\] (4.24)

In the design of the controller (Fig. 4.6), it is considered that the load current \(i_{\text{load}}\) is a disturbance of the system [PSSF11], [APS13]. As the current output of the matrix converters is controlled, it is also possible to consider that the matrix converters, filters and transformer leakage inductances can be represented by the current source \(i_{\text{system}}\).

Based on Figure 4.6 and in 4.24, it is possible to obtain the voltage regulator block diagram, wherein the block \(\frac{1}{sT_d+1}\) represents the matrix converter controlled by current, [PiSG06].

---

Figure 4.6 - Simplified scheme used on the load voltage regulator design
From Fig. 4.7, the voltage response to the disturbance introduced by the load is obtained by (4.25).

\[
\frac{V_{load}}{I_{load}(s)} = \frac{1}{sC_{fout}} \left( 1 + \frac{\alpha_v}{\alpha_l} \left( K_p + \frac{K_i}{s} \right) \frac{1}{sT_d + 1} \right)
\]  
(4.25)

Simplifying (4.25) it is possible to obtain the transfer function in the canonical form (4.26).

\[
\frac{V_{load}}{I_{load}(s)} = \frac{s \frac{\alpha_v}{T_dC_{fout}\alpha_l} (T_d s + 1)}{s^3 + \frac{1}{T_d} s^2 + \frac{K_p \alpha_v}{T_dC_{fout}\alpha_l} s + \frac{K_i \alpha_v}{T_dC_{fout}\alpha_l}}
\]  
(4.26)

To determine the PI controller parameters, the denominator of (4.26) is compared to the third order ITAE polynomial (4.27).

\[
P_3(s) = s^3 + 1.75w_o s^2 + 2.15w_o^2 s + w_o^3
\]  
(4.27)

Equating the terms in the polynomial s (4.25) to the denominator of (4.26) it is possible to determine the values of the proportional and integral gains of the PI compensator (4.28).
By solving (4.28), the proportional gain $K_p$ and the integral gain $K_i$ are obtained:

\[
\begin{align*}
K_p &= \frac{2.15 C_{\text{fout}} \alpha_i}{\alpha_v T_d (1.75)^2} \\
K_i &= \frac{C_{\text{fout}} \alpha_i}{\alpha_v T_d (1.75)^2}
\end{align*}
\]

Table 4.4 presents the values of the parameters obtained in the voltage regulator design (PI controller).

<table>
<thead>
<tr>
<th>Values of the load voltage regulator</th>
</tr>
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<tbody>
<tr>
<td>$T_d$(s)</td>
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<tr>
<td>0.0005</td>
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Chapter 5

Obtained Results

The results are obtained for two scenarios and are followed by several comments.
5.1 Introduction

Fig. 5.1 presents the PETET block diagram, which was simulated using MATLAB/Simulink software.

The developed system was implemented in MATLAB/SIMULINK software in order to evaluate its performance.

The simulations presented aim to confirm the adequate operation of this system in several operation scenarios. These scenarios are intended to simulate different electric traction systems with different voltage levels and frequencies, as described in Chapter 2.

5.2 Scenario 1 - 50Hz operation

The first scenario aims to determine the performance of the PETET in a 1x25 System (25 kV at 50 Hz). Due to the semiconductor’s limitations described above, the simulation was designed for a single PETET module, to provide a small fraction of 25 kV in the output.
Table 4.4 displays the data used for the simulations. The voltage values in Table 4.4 designate RMS line-to-neutral values.

Table 5.A - Data Simulation for the first scenario

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRMS\textsubscript{in}</td>
<td>2 kV</td>
</tr>
<tr>
<td>ω\textsubscript{in}</td>
<td>2π × 50 rad s\textsuperscript{-1}</td>
</tr>
<tr>
<td>V\textsubscript{RMSref}</td>
<td>500 V</td>
</tr>
<tr>
<td>ω\textsubscript{out}</td>
<td>2π × 50 rad s\textsuperscript{-1}</td>
</tr>
<tr>
<td>Load</td>
<td>Resistance</td>
</tr>
</tbody>
</table>

Fig. 5.2 presents the line-to-neutral load voltages, which are close to their sinusoidal waveform references, with a fundamental frequency of 50 Hz. Despite the small disturbances created by the MCs switching and the HFT, the waveforms are nearly sinusoidal. The same can be seen in Fig. 5.3, where the load currents are shown.

Figure 5.2 - Line-to-neutral load voltages
Fig. 5.4 presents one phase of the load voltage and its reference. As can be seen, both are very similar apart from some natural perturbations in the load voltage caused by the MCs switching.

Fig. 5.5 presents one of the line-to-line Three-Phase MC output voltages, applied to the HFT input. The switching frequency of the MC is around 5 kHz, and the MC provides the maximum input voltage according to the SVM method.
Fig. 5.6 presents the input current in one phase of the HFT. Since the transformer operates at 1 kHz, the 50 Hz component is not considered for the HFT saturation issue, as the average value of the current is nearly equal to zero in each switching period.

With a closer zoom of Fig. 5.6, it can be seen in Fig. 5.7 that the average value of the current is zero in each switching period, since the applied voltage changes from maximum to minimum and vice versa for identical time intervals. Consequently, the non-saturation of the HFT is assured.

The MC by converting AC-AC using SVM seeks also a sinusoidal input current. Fig. 5.8 presents the input current filter by the input filter. The matrix converter currents contain high frequency harmonics, which are created by the MCs switching at high frequencies. However the currents drawn from the grid have an approximately sinusoidal waveform, as the filter reduces the high frequency harmonic content.
Fig 5.9 presents one phase of the input current and line-to-neutral input voltage of the Three-Phase MC. The slightly leading current indicates that the system injects reactive power. With higher load system currents, the input power factor will be closer to 1.

Fig 5.10 presents one phase of the line-to-neutral load voltage and the voltage regulator performance. At t=2s, the magnitude of the output voltage reference is decreased with a good response of the voltage regulator as shown in Fig. 5.10 and Fig. 5.11.
The same situation is possible to observe with the errors of the components $v_d$ and $v_q$, in Fig 5.11. At $t=2s$, when the voltage reference is decreased, it is easily noted that there is a high peak for a small time instant, but the controller quickly guarantees that the steady-state is reached, and the output voltages are equal to their reference values.

Figure 5.11 – $v_d$ and $v_q$ voltage error.
Another scenario was tested with the increase of the load at t=1.8s. As expected the current regulator decreased the injected current, in order to maintain the same voltage as it can see in Fig. 5.12 and Fig. 5.13.

Figure 5.12 – One phase of load current.

Figure 5.13 – One phase of line-to-neutral load voltage
5.3 Scenario 2 – 16.7Hz operation

The second scenario aims to determine the performance of the PETET in a 15 kV, 16.7 Hz System. Table 5.2 displays the data used for the simulations. The voltage values in Table 5.2 designate RMS line-to-neutral values. In these simulations, the self-inductance value of the output filter coil was increased 2 times, since the desired output frequency is 3 times lower.

Table 5.2 - Data Simulation for the second scenario

<table>
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</tr>
<tr>
<td>(\omega_{in})</td>
<td>(2\pi \times 50 \text{ rad s}^{-1})</td>
</tr>
<tr>
<td>VRMS_ref</td>
<td>500 V</td>
</tr>
<tr>
<td>(\omega_{out})</td>
<td>(2\pi \times 16.7 \text{ rad s}^{-1})</td>
</tr>
<tr>
<td>Load</td>
<td>Resistance</td>
</tr>
</tbody>
</table>

Fig. 5.14 presents the line-to-neutral load voltages, which are close to their sinusoidal waveform references, with fundamental frequency of 16.7 Hz. Despite the small disturbances created by the MCs switching and the HFT, the waveforms are nearly sinusoidal.

![Figure 5.14 - Line-to-neutral load voltages](image-url)
Fig. 5.15 presents one phase of the line-to-neutral load voltage and its reference. Both are very identical apart from some natural disturbances in the load voltage caused by the MCs switching.

![Figure 5.15 - One phase of the line-to-neutral load voltage (red) and its reference (blue)](image)

Fig. 5.16 shows the load currents. As expected, they are sinusoidal.

![Figure 5.16 – Load currents](image)

Fig. 5.17 presents one phase of the input HFT current. As seen in the previous scenario test, for the 50Hz grid, the average value of the current in each switching period is equal to zero as well.

![Figure 5.17 - HFT input current](image)
Fig 5.18 presents one phase of the input current and line-to-neutral input voltage of the Three-Phase MC. The slightly leading current indicates that the system injects reactive power.

Figure 5.18 - One phase of the input current (red), one phase of line-to-neutral input voltage (blue)
Chapter 6

Conclusions

This chapter finalises this work, summarising conclusions and pointing out aspects to be developed in future work.
6.1 Conclusions

The objective of this thesis was to develop a high power high frequency modular transformer for electric traction systems, capable of providing variable magnitude and frequency voltages in the output.

A great concern of this thesis was to ensure the non-saturation of the HFT, which was solved while revising the original SVM by adding an innovative feature. Another difficulty was to adapt the PETET to higher voltage levels, which are present in electric traction systems. For that purpose, a modular structure was suggested, but not tested.

To achieve the proposed objectives, the railway electrification systems were initially studied, whereby some parameters were identified that could influence the design of the PETET.

In the identified systems, two were chosen for which the PETET was designed:

- 1x25 System (25 kV at 50 Hz)
- 15 kV at 16.7 Hz System

The PETET was tested in MATLAB/SIMULINK environment, with good results. The new SVM control of the Three-Phase Matrix Converter was successful proved, which guarantees the non-saturation of the HFT. This technique was described and later demonstrated by the input current waveform of the HFT. At the same time, the proposed modulation technique guarantees both sinusoidal input currents and output voltages. However, it was understood that for the PETET implementation in each railway electrification system, different input and output filters have to be designed as well as the HFT.

The simulations were realized using lower voltages and currents. As explained above, this decision was made to provide more truthful simulations since semiconductors have voltage limitations. Therefore, it is possible to conclude that the system developed here may have other applications, e.g., a voltage regulator for the distribution grid. The advantage of an electric grounded system is also an important advantage in power distribution systems as well due to safety human reasons. Also, this system guarantees the redundancy due to voltage margin required for any fault in the power electronic devices.

Both the voltage and the current controllers, based in PI controllers, were tested with a good performance, static error close to zero, and allowing a quick responses while ensuring system stability for various load scenarios. The power factor might not be unitary since it depends on the input filter and the load conditions.
It is possible to conclude that the work achieved the proposed objectives. As all the main features of the project were confirmed by the simulations, it can be conclude that the system developed in this thesis has conditions for being implemented in the electric traction, but also in other areas.

An alternative system control that could provide a DC output would allow the PETET to be implemented also in DC traction systems. With some modifications in the modulation process, especially in the commands of the SPMCs, the PETET could easily be adapted to AC-DC conversion.

6.2 Perspectives of future work

For future research, it is suggested to use a triangle wave carrier instead of a sawtooth wave, to minimize either the semiconductors switching frequency or the Total Harmonic Distortion (THD) of output voltages and currents.

A more detailed study of the HFT implemented in this system, would provide less distortions in the output and input waveforms. The research in the field of high frequency transformers is constantly growing, as this device provides great advantages in terms of its weight and costs, compared to conventional transformers. The design of one HFT is a complex task, which would easily provide enough work for a new thesis.

Also, an investigation of the suggested modular structure of PETETs, will certainly provide interesting results and add further conclusions to the implementation of PETETs in electric traction systems. As mentioned before, new filters have to be designed as well as an efficient control system for all the modules.

Finally, a prototype of the PETET would be a great step forward to prove the validity of this system, and an opportunity to achieve more realistic conclusions.
Annex A presents the equations that establish the relations between the input and output of the MC. A table with the 27 possible switching combinations and the output voltages and input currents obtained for each combination are exhibited.
The S matrix (3.2) represents the states of the switches and enables a mathematical relation between the line-to-neutral output voltages \( V_A, V_B, V_C \) and the line-to-neutral input voltages \( V_a, V_b, V_c \). Still, the transpose of matrix \( S \) relates the input currents \( i_a, i_b, i_c \) with the output currents (A.1). (Note: The voltage and current of the input are represented with small letters \("a,b,c"\) and the voltage and current of the output phases are represented with capital letters \("A,B,C"\)

\[
\begin{bmatrix}
V_A \\
V_B \\
V_C \\
\end{bmatrix} = S
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix} = S^T
\begin{bmatrix}
i_A \\
i_B \\
i_C \\
\end{bmatrix}
\]

(A.1)

The relationship between line-to-neutral voltage and line-to-line voltage is given by (A.2).

\[
\begin{align*}
V_{AB} &= V_A - V_B \\
V_{BC} &= V_B - V_C \\
V_{CA} &= V_C - V_A \\
\end{align} \]

(A.2)

By replacing (3.4) into (A.2), it is possible to obtain a new system of equations that relates line-to-line output voltages with the line-to-neutral input voltages of the MC. (A.3).

\[
\begin{align*}
V_{AB} &= S_{11}V_a + S_{12}V_b + S_{13}V_c - S_{21}V_a - S_{22}V_b - S_{23}V_c \\
V_{BC} &= S_{21}V_a + S_{22}V_b + S_{23}V_c - S_{31}V_a - S_{32}V_b - S_{33}V_c \\
V_{CA} &= S_{31}V_a + S_{32}V_b + S_{33}V_c - S_{11}V_a - S_{12}V_b - S_{13}V_c \\
\end{align} \]

(A.3)

From (A.6), it is possible to obtain (A.4)

\[
\begin{align*}
V_{AB} &= (S_{11} - S_{21})V_a + (S_{12} - S_{22})V_b + (S_{13} - S_{23})V_c \\
V_{BC} &= (S_{21} - S_{31})V_a + (S_{22} - S_{32})V_b + (S_{23} - S_{33})V_c \\
V_{CA} &= (S_{31} - S_{11})V_a + (S_{32} - S_{12})V_b + (S_{33} - S_{13})V_c \\
\end{align} \]

(A.4)

Rewriting (A.4) in matrix form, we obtain (A.5)
\[ S_c = \begin{bmatrix} S_{11} - S_{21} & S_{12} - S_{22} & S_{13} - S_{23} \\ S_{21} - S_{31} & S_{22} - S_{32} & S_{23} - S_{33} \\ S_{31} - S_{11} & S_{32} - S_{12} & S_{33} - S_{13} \end{bmatrix} \]

(A.5)

The \( S_c \) matrix relates the line-to-line output voltage with the line-to-neutral input voltage (A.6).

\[
\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = S_c \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}
\]

(A.6)

It is now possible to show in Table A.1 the 27 possible switching combinations and the resultant output voltages and input currents for each combination.

**Table A.1 - Possible switching combinations of the Three-Phase Matrix Converter**

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<tr>
<th>Group</th>
<th>State</th>
<th>( S_{11} )</th>
<th>( S_{12} )</th>
<th>( S_{13} )</th>
<th>( S_{21} )</th>
<th>( S_{33} )</th>
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Table A.1 (cont) - Possible switching combinations of the Three-Phase Matrix Converter

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<th>S₃₂</th>
<th>Vₐ</th>
<th>Vₚ</th>
<th>Vₖ</th>
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**Group III**

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Appendix B

Zone division of the MC input phase-to-phase voltages

The input voltage of the MC is divided into 6 zones in order to know the maximum voltage vectors at each time instant.
Figure B.1 - Space Vectors Map of the MC output voltage
Zone division of the MC output current

The output current of the MC is divided into 6 zones in order to know the maximum current vectors at each time instant.
Figure C.1 - Space Vectors Map of the MC input current
Appendix D

Damping resistance of the filter

The calculation for the damping resistance of the input filter is presented here.
Considering the transfer function of the MC input filter given by (D.1)

\[
\frac{V_o(s)}{V_i(s)} = \frac{1}{C_f L_{in}} \left( \frac{s L_{in}}{r_p} + 1 \right)
\]

\[
\frac{1}{s^2 + s \left( \frac{r_p c + r_i}{C_f r_p r_i} \right) + \frac{1}{C_f L_{in}}}
\]

The denominator of the transfer function can be written as a second order polynomial (D.2) according to the desired damping.

\[
f(s) = s^2 + 2\xi \omega_n s + \omega_n^2
\]

Comparing (D.2) to the denominator of (D.1) to ensure the desired damping factor ($\xi < 1$), the value of resistance $r_p$ is given by (D.3)

\[
2 \xi \omega_n = \frac{r_p + r_i}{C_f r_p r_i} \Rightarrow r_p = \frac{r_i}{2 \xi \omega_n C_f r_i - 1} = \frac{r_i Z_f}{2 \xi r_i - Z_f}
\]

To ensure system stability due to the introduction of the damping resistor $r_p$, it is necessary to guarantee condition (D.4).

\[
\frac{r_p + r_i}{C_f r_p r_i} > 0 \Rightarrow r_p < -r_i
\]
Appendix E

Nominal voltages and their permissible limits in values and duration

A table is presented in this Appendix with the nominal voltages and their permissible limits in values and duration for the main electric traction systems.
The characteristics of the main voltage systems (overvoltages excluded) are specified in Table E.1. The following data was established by EN 50163:2004 standard (Railway applications - Supply voltages of traction systems).

The following requirements shall be fulfilled:

a) the duration of voltages between $U_{\text{min}1}$ and $U_{\text{min}2}$ shall not exceed 2 min;

b) the duration of voltages between $U_{\text{max}1}$ and $U_{\text{max}2}$ shall not exceed 5 min;

c) the voltage of the busbar at the substation at no load condition shall be less than or equal to $U_{\text{max}1}$.

For d.c. substations it is acceptable to have this voltage at no load condition less than or equal to $U_{\text{max}2}$, knowing that when a train is present, the voltage at this train's pantograph (s) shall be in accordance with Table 1 and its requirements;

d) under normal operating conditions, voltages shall lie within the range $U_{\text{min}1} \leq U \leq U_{\text{max}2}$;

e) under abnormal operating conditions the voltages in the range $U_{\min2} \leq U \leq U_{\min1}$ in Table A.1 shall not cause any damages or failures.

NOTE 1: The use of train power limitation devices on board may limit the presence of low voltage on the overhead line (see EN 50388).

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<th>Electrification system</th>
<th>Lowest nonpermanent Voltage $U_{\min2}$</th>
<th>Lowest permanent Voltage $U_{\min1}$</th>
<th>Nominal Voltage $U_n$</th>
<th>Highest permanent Voltage $U_{\max1}$</th>
<th>Highest nonpermanent Voltage $U_{\max2}$</th>
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<td>3 600</td>
<td>3 900 c</td>
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<tr>
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</table>

*a* Future d.c. traction systems for tramways and local railways should conform with system nominal voltage of 750 V, 1 500 V or 3 000 V.

*b* Special national conditions for Belgium, see Appendix B.
f) if voltages between $U_{\text{max}1}$ and $U_{\text{max}2}$ are reached, they shall be followed by a level below or equal to $U_{\text{max}1}$, for an unspecified period.

Voltages between $U_{\text{max}1}$ and $U_{\text{max}2}$ shall only be reached for non-permanent conditions such as
– regenerative braking,
– move of voltage regulation systems such as mechanical tap changer.

g) lowest operational voltage: under abnormal operating conditions $U_{\text{min}2}$ is the lowest limit of the contact line voltage for which the rolling stock is intended to operate.

NOTE 2: Recommended values for undervoltage tripping: The setting of undervoltage relays in fixed installations or on board rolling stock should be set from 85 % to 95 % of $U_{\text{min}2}$. 
References


