Simulation of 2D non-isothermal flows in slits using lattice Boltzmann method

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Abstract: The present work aims to study the simulation of 2D non-isothermal flows through the benchmark problem Poiseuille flow using the lattice Boltzmann method. The study is divided in two parts: the simulation of the isothermal counterpart of this flow with the most popular boundary condition schemes: bounceback, Zou He and periodic boundary conditions; and the simulation of the internal energy field through a DDF model and the counter-slip energy boundary condition scheme. The lattice model used is a D2Q9. Both Dirichlet and Neumann boundary conditions are implemented for the non-isothermal model. The results for the isothermal model are found to be in good agreement with the ones present in the literature to the extent of their existence. For the results obtained in non-tested conditions in the literature, problems of solution convergence were found. For the non-isothermal model, the results were validated in two different ways. For the condition of constant temperature at the wall (Dirichlet), the results were compared with the ones obtained with software Fluent and were found to be in satisfactory agreement. For imposed heat flux at the wall, excellent agreement between the numerical and analytical solution were found except for the corners of the domain near the exit of the flow.

Keywords: numerical methods, lattice Boltzmann method, thermal lattice Boltzmann method, boundary conditions, 2D Poiseuille flow, heat transfer

1. Introduction

The lattice Boltzmann method (LBM) is a numerical tool that has become increasingly popular for solving problems regarding fluid flows. Unlike conventional numerical methods, which obtain the values for macroscopic quantities by discretely solving balances for these same macroscopic quantities – like Navier-Stokes equations -, the LBM is based on solving, through the Boltzmann equation, the evolution of a single entity – the particle distribution function \( f(x, \xi, t) \) [1]. The macroscopic variables are later obtained through the velocity moments of this distribution function.

The main advantages often associated with LBM are[2][3]: the convection operator (or streaming step) is linear; there is an explicit equation for pressure; complex boundary conditions can be formulated through simple rules; and it is a numerical method extremely suited for parallel computing.

The LBM was born from LGA, a boolean kinetic based numerical method to solve partial differential equations of mass and momentum balance, and in which concepts the LBM is based [4]. The first appearance from the LBM dates to 1988, with the work of McNamara and Zanetti [5], who introduced the use of the Boltzmann equation and the particle distribution function in the LGA structure and concept, avoiding some of its limitation, like statistical noise. The introduction of the BGK collision model was also a critical step in the development of the LBM [6] since it contributed to give a simple formulation for the collision step of the method.

Boundary conditions for the isothermal models started with the bounceback[7,8], which is still the most popular boundary condition scheme to date for its implementation simplicity. Other early schemes also enjoy significant popularity like the extrapolation scheme [3], non-equilibrium extrapolation scheme [9], Zou He scheme [10] and counter-slip velocity scheme [11]. Periodic boundary conditions are also regularly used for their advantages related to avoiding compressibility errors. More recently, other schemes were proposed for modeling more complex geometries.

The models for solving isothermal flows were early developed to be able to solve non-isothermal flows as well. Currently, the models for solving non-isothermal flows are classified in three categories[12]: Multispeed(MS), double distribution function (DDF) and hybrid.

The DDF models are the most widely used among LBM developers, and are based on solving two separate distribution functions, one for the hydrodynamic part and another for the energy part. These models have several advantages, namely better stability and inclusion of viscous dissipation and compression work [13]. Although the isothermal models are very well established and popular, its thermal counterpart does not enjoy the same level of success, namely because the lack of proper modeling of boundary conditions,
particularly Neumann boundary conditions[12]. Many schemes have been proposed for thermal boundary conditions, most of them based on the schemes created for solving velocity boundary conditions[14,15].

In this work, the boundary condition scheme of internal energy counter-slip, proposed by d’Orazio and Succi[16] and d’Orazio et al.[17,18], is implemented for simulating both imposed temperature and heat flux boundary conditions. This scheme is reported in the literature to be the most accurate for straight boundaries for both Dirichlet and Neumann boundary conditions[13], although the authors report, for the tests with heat flux boundary condition, a minimum error of 12% and not good agreement with analytical and numerical profiles near the walls[17].

Both isothermal and non-isothermal models were implemented in this work to solve the Poiseuille flow between two parallel and infinite walls. Velocity, pressure and periodic boundary conditions were tested for the isothermal model, using bounceback and Zou He schemes. For implementing constant temperature and flux boundary conditions to solve the non-isothermal Poiseuille flow, the internal energy counter-slip scheme was used, as mentioned.

2. Isothermal flows

2.1 Model for isothermal flows

The isothermal LBM model is derived directly from the Boltzmann equation,

\[ D_t f + \mathbf{a} \cdot \nabla f = \Omega_t \]

by discretizing it. The result of this discretization is the lattice Boltzmann equation (LBE),

\[ f_i(x + \mathbf{x}_i \delta t, t + \delta t) - f_i(x, t) = \delta t \Omega_t f_i + \delta t \mathbf{f}_i \]

with \( F_i \) being an external force expression

\[ F_i = \omega_i \xi_i \frac{\mathbf{x}_i}{c_s^2} \cdot \mathbf{a} \]

and \( \Omega_t \) being the collision operator. The collision operator used is the BGK, already mentioned,

\[ \Omega_t f_i = \frac{1}{\tau_f} (f_i - f_i^{eq}) \]

where \( \tau_f \) is the relaxation time. The equation for calculating the discrete form of equilibrium distribution function \( f_i^{eq} \) is derived from the continuous Maxwell-Boltzmann equilibrium distribution function

\[ f_i^{eq} = \frac{\rho}{(2\pi R T)^{D/2}} \exp \left( -\frac{\mathbf{\xi}_i \cdot \mathbf{u}}{2RT} \right) \]

and takes the following form for a D2Q9 lattice model, used in this work,

\[ f_i^{eq} = w_i \rho \left( 1 + \frac{3 (\mathbf{\xi}_i \cdot \mathbf{u})^2}{c^2} + \frac{9 (\mathbf{\xi}_i \cdot \mathbf{u})^2}{2c^4} - 3 u^2 \right) \]

2.2 Boundary conditions for isothermal flows

The boundary condition schemes used for solving the isothermal models are the bounceback and Zou He schemes; periodic boundary conditions are also used.
Bounceback

The bounceback scheme is the oldest and, simultaneously, the most widely used scheme for simulating boundary conditions. This scheme is based on the idea that a particle that hits the wall is reflected back in the opposite direction, positioning the wall in the middle of two nodes. For the specific bounceback scheme used in this work – the halfway scheme – the nodes where the boundary condition is implemented are the fluid nodes, and the wall is in the middle of these nodes and a line of nodes that are inside the wall. The equation for this scheme is

\[
f_{i}(\mathbf{x},t + \delta t) = f_{i}(\mathbf{x},t) - 2\alpha_{i} \frac{\xi_{i} \cdot (\rho u_{F})}{c_{s}^{2}}
\]

which for a no-slip boundary condition is reduced to

\[
f_{i}(\mathbf{x},t + \delta t) = f_{i}(\mathbf{x},t)
\]

where \( \bar{\alpha} \) has the opposite direction of \( \alpha \). This scheme is formally a scheme with first order precision. The only possibility for annulling second order errors is by attributing a special value to the relaxation time, called in the literature as magic relaxation time.

Zou He

For the Zou He scheme, the boundary between the fluid and the wall overlaps with the domain nodes. It makes use of the Eq.(8) and Eq.(9), which when extended for solving, for example, the undetermined populations on the south boundary of figure 3 - \( f_{2}, f_{5} \) and \( f_{6} \), have the form

\[
f_{2} + f_{5} + f_{6} = \rho - (f_{0} + f_{1} + f_{3} + f_{4} + f_{7} + f_{8}) \quad 14
\]

\[
f_{5} - f_{6} = \rho u_{x} - (f_{1} - f_{3} - f_{7} + f_{8}) \quad 17
\]

\[
f_{2} + f_{5} + f_{6} = \rho u_{y} + (f_{4} + f_{5} + f_{6}) \quad 18
\]

These equations allow one to obtain an equation to solve the density or the velocity according to which is imposed. For example, if the density is imposed, the resulting equation to determine density is

\[
\rho = \frac{1}{1 - u_{y}} (f_{0} + f_{1} + f_{3} + 2(f_{4} + f_{7} + f_{8})) \quad 19
\]

To solve the values for these populations, one extra equation is needed, which results from the idea of the bounceback of non-equilibrium in the direction normal to the wall, and can be translated as

\[
f_{2}^{\text{eq}} = f_{4}^{\text{eq}} \iff f_{2} - f_{2}^{\text{eq}} = f_{4} - f_{4}^{\text{eq}} \quad 20
\]

Having these equations, and using Eq.(6) as well, one can obtain three expressions that solve the three undetermined populations

\[
f_{2} = f_{4} + \frac{2}{3}\rho u_{y} \quad 21
\]

\[
f_{5} = f_{7} - \frac{1}{2}(f_{1} - f_{3}) + \frac{1}{2}\rho u_{x} + \frac{1}{6}\rho u_{y} \quad 22
\]

\[
f_{6} = f_{8} + \frac{1}{2}(f_{1} - f_{3}) - \frac{1}{2}\rho u_{x} + \frac{1}{6}\rho u_{y} \quad 23
\]

This scheme can be used to impose either pressure or velocity boundary conditions and has precision of second order.

Periodic boundary conditions

These boundary conditions arise from the existence, or not, of the second term in the right side of Eq(2), which acts as if the flow is driven by a body force, and it’s very advantageous in the sense that is extremely easy to implement and, as stated before, produces no compressibility errors, which is extremely useful for validating LBM models.

3. Non-isothermal flows

3.1 Models for non-isothermal flows

For the non-isothermal model, one has to solve the evolution of two distribution functions. The new distribution function – the internal energy distribution function \( g \) – is derived from \( f \) (the one already presented):
Substituting $g$ in the Boltzmann equation, Eq. (1), the evolution equation for the internal energy distribution function is

$$D_t g = - \frac{g - g^{eq}}{\tau_g} - fZ$$

The discrete model for solving non-isothermal flows will be based on a discretization of Eq. (25), and a new discretization of Eq. (1). Eq. (2) is not suited for this model because the collision operator $\Omega_i$ was considered constant in time, which in the thermal model introduces a second order truncation error (He, et al., 1998).

The resulting discrete equations for the non-isothermal model are

$$\tilde{f}_i(x + \xi_i\delta t, t + \delta t) - \tilde{f}_i(x, t)$$

$$= \frac{\delta t}{\tau_f + 0.5\delta t} \left( \tilde{f}_i(x, t) - f_i^{eq}(x, t) \right) + \frac{\tau_f \delta t}{\tau_f + 0.5\delta t} F_i$$

$$\tilde{g}_i(x + \xi_i\delta t, t + \delta t) - \tilde{g}_i(x, t)$$

$$= \frac{\delta t}{\tau_g + 0.5\delta t} \left( \tilde{g}_i(x, t) - g_i^{eq}(x, t) \right) + \frac{\tau_g \delta t}{\tau_g + 0.5\delta t} f_i(x, t)Z_i(x, t)$$

where

$$\tilde{f}_i = f_i + \frac{\delta t}{2\tau_f} (f_i - f_i^{eq}) - \frac{\delta t}{2} F_i$$

$$\tilde{g}_i = g_i + \frac{\delta t}{2\tau_g} (g_i - g_i^{eq}) + \frac{\delta t}{2} f_i Z_i$$

which are two new variables necessary for avoiding implicitness in the model. The equilibrium distribution function for $g$ is obtained by introducing Eq. (5) in Eq. (24), and the discrete form for the resulting equation takes form according to each population it is calculated for.

$$g_0^{eq} = \frac{4}{3} \rho e \left[ -1.5 \frac{u^2}{c^2} \right]$$

$$g_1^{eq} = \frac{1}{9} \rho e \left[ 1.5 + 6 \frac{\xi_i \cdot u}{c^2} + 4.5 \frac{(\xi_i \cdot u)^2}{c^4} - 1.5 \frac{u^2}{c^2} \right]$$

To calculate the macroscopic variables, accounting for the change of variables made in Eq. (28) and Eq. (29), the equations are

$$\rho = \sum_i \tilde{f}_i$$

$$\rho u = \sum_i \frac{\xi_i \tilde{f}_i}{2} + \frac{\rho \alpha \delta t}{2}$$

$$\rho e = \sum_i g_i - \frac{\delta t}{2} \sum_i f_i Z_i$$

$$q = \left( \sum_i \xi_i \tilde{g}_i - \rho e u - \frac{\delta t}{2} \sum_i \xi f_i Z_i \right) \frac{\tau_g}{\tau_g + 0.5\delta t}$$

$Z_i$ can be expressed as

$$Z_i = (\xi_i - u(x, t)) \cdot [u(x + \xi_i\delta t, t + \delta t) - u(x, t)]$$

A Chapman-Enskog expansion may also be performed to Eq. (25) to obtain the energy equation

$$D_t e = \nabla \cdot (k \nabla e) + \phi - p \nabla \cdot u$$

as well as the relations between $\tau_f$ and $\nu$, and $\tau_g$ and $\alpha$.

$$\nu = \tau_f RT$$

$$\alpha = 2\tau_g RT$$

3.2 Boundary conditions for non-isothermal flows

The boundary condition scheme used for solving imposed temperature and heat flux boundary conditions is the counter-slip internal energy. The basic equation for this is, (41.a is generic and 41.b and 41.c for the cases of populations 4 and 7)

$$g_i = \rho (e + e')$$

[corresponding form for equilibrium]

$$g_4 = \rho (e + e') \frac{1}{9} \left[ 1.5 + 1.5 \frac{\xi_4 \cdot u}{c^2} + 4.5 \frac{(\xi_4 \cdot u)^2}{c^4} 
- 1.5 \frac{u^2}{c^2} \right]$$
which means that the undetermined populations are the result of its equilibrium form plus a parameter \( e' \) that adjusts its value so that the imposition of the macroscopic variable in the node is satisfied.

To solve the undetermined populations, one will use either Eq.(35) or Eq.(36), depending on the type of boundary condition one wants to solve. If, for example, one wants to impose constant temperature in a boundary, Eq.(35) is used to determine the value of \( e' \), which will be used in Eq.(41) to find the unknown populations. The expressions to determine the values of \( e' \) in the cases present on the simulated problems are:

- For a boundary with no-slip condition
  \[
  \rho e' = 2\rho e + 1.5\delta t \sum_i f_i |Z_i - 3G|
  \]

- For a boundary with imposed velocity
  \[
  \rho e' = \frac{\rho e \left( 4 - \frac{3u_y}{c} - \frac{3u_y^2}{c^2} \right) + 3\delta t \sum_i f_i |Z_i - 6G|}{2 + 3 \frac{u_y}{c} + 3 \frac{u_y^2}{c^2}}
  \]

- For the corners
  \[
  \rho e' = \frac{\rho e \left( 5 - 4 \frac{u_y}{c} - \frac{5u_y^2}{c^2} \right) + 6\delta t \sum_i f_i |Z_i - 12G|}{7 + 4 \frac{u_y}{c} + 5 \frac{u_y^2}{c^2}}
  \]

where \( G \) represents the known populations.

To solve the unknown populations for the case of imposed heat flux, one has to use Eq.(36), instead of Eq.(35). In this case, the equations to determine the unknown populations at a wall (with a no-slip condition, since is the simulated situation) for the specific example of the populations at the north wall of figure 3, are the following

\[
g_{4,7,8} = 3[r. p. c. e] \left[ \sum_{3,5,6} \frac{g_i - \tau_g}{\delta t} + \frac{0.5\delta t \eta_y}{\tau_g} \right] - \frac{1}{2} \sum_i f_i |Z_i|
\]

4.1 Results for isothermal flow

In this work, for simulating the isothermal flows, the various boundary conditions schemes were combined in different simulations. These combinations consisted of alternate the usage of Zou He or bounceback for simulating the no-slip boundary condition, and the periodic, pressure and velocity boundary conditions for inlet and outlet.

The results shown here are the value of the error to the analytical solution, order of convergence with domain discretization and graphical display of pressure and velocity fields. Error value is calculated through the following \( L_2 \) norm

\[
\|L_2\|_\infty = \sqrt{\sum_i (u_i^{num} - u_i^{analytic})^2}
\]

The analytical solution for the velocity field when the flow is fully developed is

\[
u_{x}(y) = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - h^2)
\]

For the pressure and periodic boundary conditions, the velocity field is developed in the whole domain, as shown in figure 4.

Fig.4- Velocity field for fully developed flow in all the domain.

With periodic boundary conditions and Zou He scheme at the wall, machine accuracy was achieved for all simulated Reynolds numbers and relaxation times which didn’t make the solution unstable.
Numerical and analytical velocity profiles at the exit of the domain.

Fig. 6 – Order of convergence for simulations with bounceback and periodic boundary conditions, for Re=20, for \( \tau_f \) with values of 0.6 and 0.9. \( Y \) is the number of nodes in \( y \) direction.

For periodic boundary conditions and bounce back on the wall, three relaxation times were tested: 0.6, 0.9 and the magic number for the relaxation time in this flow, which is \( \frac{20 + \sqrt{20Re}}{32} \approx 1.08 \); from here on, when the magic value for relaxation time is mentioned, it refers to this particular value. The results achieved for this flow in terms of the value of the error are displayed in table 1. The same results were achieved for different Reynolds number.

<table>
<thead>
<tr>
<th>Re</th>
<th>( \tau )</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>Convergence order</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.6</td>
<td>1.48</td>
<td>0.37</td>
<td>0.09</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>0.93</td>
<td>0.23</td>
<td>0.06</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>magic</td>
<td>Machine accuracy</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1- Error(%) for periodic boundary condition and bounceback at the wall.

For the cases of imposed pressure at inlet and outlet, some problems were found with the solution convergence, and manifested in different ways for bounceback or Zou He schemes applied to the wall.

Figure 7 illustrates what happens in this case with the bounceback scheme applied to the wall, where a problem of conversion can be found at the exit of the domain.

With the Zou He scheme applied at the wall, the convergence problem was found near the inlet instead of the outlet. One example of very poor convergence at the inlet is the one represented on figure 8 for a flow with Re=20 and the magic number referred before \( \frac{20 + \sqrt{20Re}}{32} \).

For the simulations with velocity boundary conditions at the inlet and outlet, the form of velocity field expected can be seen in figure 9. One can see, since at the inlet a uniform profile of velocity is imposed, that there is a region of flow development near the inlet.
Fig. 9- Velocity field for flows with velocity inlet and outlet boundary conditions.

The error results, for one Re=20, obtained for the case with the bounceback scheme applied to the wall, are shown in table 2, which imply that the error has a higher value for higher relaxation time. The order of convergence for these simulations is around 2; the lowest order of convergence registered for this boundary condition combination was 1.72.

<table>
<thead>
<tr>
<th>Re</th>
<th>τ</th>
<th>10</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.6</td>
<td>0.53</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>3.03</td>
<td>0.93</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>magic</td>
<td>6.72</td>
<td>2.01</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table 2- Error(%) in simulations with bounceback and velocity boundary conditions for inlet and outlet.

For the case of Zou He boundary conditions placed in all the boundaries, higher errors were obtained and also a (surprising) order of convergence of about 1. Table 3 shows the values of the error for Re=20 for different relaxation times. Figure 9 shows a plot with the order of convergence for these results.

<table>
<thead>
<tr>
<th>Re</th>
<th>τ</th>
<th>10</th>
<th>20</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.6</td>
<td>8.46</td>
<td>4.00</td>
<td>1.95</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>7.34</td>
<td>3.48</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>magic</td>
<td>5.65</td>
<td>2.81</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Table 3- Error(%) in simulations with Zou He scheme at the wall, and also at inlet and outlet imposing velocity boundary conditions.

4.2 Results for non-isothermal flows

Two different boundary conditions were simulated for the non-isothermal flows: constant heat flux and constant temperature at the wall.

To validate the simulations with heat flux boundary condition, the numerical results were compared with the analytical solution, deduced in this work for the thermally developed region of the flow. The equation that determines analytically the temperature field is

\[
T(x,y) = \frac{3}{2h^3k} \left( \frac{h^2 y^2}{2} - \frac{y^4}{12} - \frac{5h^4}{12} \right) + \frac{hq''}{k} \left( \frac{17}{35} + \frac{2q''}{mc_p} \right) x + T_{m,i}
\]

The validation of the simulations for flows with constant temperature at the wall was performed by comparing its results with equivalent simulations performed with software Fluent.

The results for the flows throughout all the simulations performed with imposed heat flux are in excellent agreement with the theoretical results at the entire domain, except for the nodes in the corners at the outlet, where the error is larger. Figure 11 shows the result for the error calculated in the thermally developed part of the flow. This figure is representative of the results of all the simulations performed.
Fig.11 – value of local error in thermally developed region for flows with imposed heat flux at wall.

Figures 12 and 13 also evidence the problem of the correct modeling of the corners at the outlet, with figure 12 showing the evolution of average and surface temperatures, and figure 13 shows the evolution of nondimensional temperature.

Fig.12- Simulation results for the evolution of the average temperature and surface temperature for flows with imposed heat flux.

Fig.13- Isothermal lines for nondimensional temperature $\theta$

The surface temperature is supposed to be always linear and parallel to the average temperature evolution. Likewise, the nondimensional temperature is supposed to be constant throughout the thermally developed region of the domain. In figure 12 and 13, it’s clear that the expected behaviors are not verified.

The strategy used to define the populations there was to define the temperature by extrapolation, since the surface temperature should evolve linearly for this boundary condition.

Having that done, the reason for this problem to arise seems to be related with the lack of capacity, from the neighboring nodes, to absorb properly the effects of the correction performed by the parameter $e'$ at the corners.

Having this problem exposed, other results are analyzed. Table 4 is presented with the values of the error for simulations with $Pr=1$, heat flux, $q''=0.001$ and $Re=20$.

Table 4- Error(%) for simulations in flows with heat flux at the wall, $Pr=1$, $q''=0.001$ and $Re=20$ with different relaxation times an domain discretization

As it can be seen the errors are fairly low for every case. Figure 14 shows the order of convergence for the error, which is very close to second order.

Fig.14- Order of convergence for the value of the error with imposed heat flux on the wall and $Pr=1$, $q''=0.001$, $Re=20$.

Two profiles for both analytical and numerical solutions are shown in figure

Fig.15- Analytical and numerical profiles for flow with imposed flux at the wall, in the thermally developed region.

The tendencies of the error evolution by the change of some parameters were tested. It was found that the error tends to decrease with increasing Reynolds number, decreasing Prandtl number and decreasing heat flux value.

The Nusselt numbers obtained for these simulations, with heat flux imposed on the wall, were in very good agreement with theoretical value – 8.23 - since they all converged to values between 8.32 and 8.25.

The results for flows with imposed temperature at the wall were validated through comparison of the nondimensional temperature profiles obtained from Fluent and LBM computational codes.

The first result to be presented is the comparison between two profiles for a simulation with $Re=20$, $T_in=60^\circ C$, $T_{wall}=20^\circ C$, $Pr=1$ and $\tau_f=0.4$. This is the only simulation where the average error was obtained between what both computational tools provided, which was an error of 1.8%. The Nu obtained from LBM was 7.60 and the Nu obtained from Fluent was 7.69. The profiles of both solutions are shown in figure 16.
Fig. 16 – Profiles of nondimensional temperature from simulations performed in Fluent and LBM codes.

The figure such shows the nondimensional temperature field, which one can observe that it does not have the problems found when a constant heat flux was imposed since, being the isothermal lines completely parallel with each other and with the wall in all the thermally developed region.

Fig. 17 – Isothermal lines for nondimensional temperature in a flow with imposed temperature on the wall

The consequences of changing the relaxation times were also tested. Figure 18 shows that the various profiles for the different domain discretization overlap almost perfectly. Although figure 18 is based on simulations with \( \tau_g = 0.2 \), this result was attained for all the other relaxation times tested, with \( \text{Pr} = 1 \) and \( \text{Re} = 20 \).

Fig. 18 – Nondimensional temperature profiles obtained with LBM codes for different domain discretization

In terms of the effects of changing the numbers of Prandtl or Reynolds, the profiles from simulations of Fluent and LBM codes agreed better with each other when both nondimensional numbers were increased.

In what concerns the results for the Nusselt number, the values obtained from the LBM codes were all between 7.56 and 7.60. Knowing that the theoretical value for Nu in a Poiseuille flow between infinite, parallel and straight walls with constant temperature imposed on them is 7.54, the results obtained appear to be in satisfactory agreement with the theoretical prediction and agree extremely well with the results reported in the literature.

Conclusions

The LBM models for solving isothermal and non-isothermal flows were successfully implemented. The results obtained for the various boundary schemes combined in isothermal flows were overall very satisfactory, with errors of the order of machine accuracy obtained for some cases. For flows with imposed heat flux, the results were found to agree extremely well with the analytical solution for several different simulation parameters, with exception for the outlet corners, where higher errors were detected, evidencing the lack of a good strategy to determine the populations in the corners for imposed heat flux. Nusselt numbers obtained were in very good agreement with the theoretical prediction – 8.23. Values obtained were as good as 8.25. For flows with imposed temperature, results were compared with simulations performed with commercial software Fluent, and were found to be in satisfactory agreement with each other for the various parameters change simulated. The Nusselt numbers obtained are in very good agreement with the theoretical prediction and with other results reported in the literature.

References


