Estimation and Control of a Tilt-Quadrotor Attitude

Estanislao Cantos Mateos
Mechanical Engineering Department, Instituto Superior Técnico, Lisboa,
E-mail: est8ani@gmail.com

Abstract - The aim of the present work is to continue the development of the prototype ALIV3, focusing on the estimation and control of the attitude. First the differences in motion and configurations of a tilt-quadrotor in relation with a common quadrotor are explained. The mass properties and actuators of the prototype are measured through tests specially developed. In the identification of the motors the influence of the temperature and the battery discharge were considered. The Tilt-Quadrotor is equipped with 3-axes accelerometer, 3-axes gyroscope, 3-axes compass and a barometer, all included in the IMU shield of the Ardupilot Mega 1 (APM 1). These sensors are modeled using real sensor measurements to get a better approximation to the real case. The model of the system has been linearized around hovering. Then the model is implemented in Simulink. First a 12 states LQR controller was developed to achieve the stabilization of the ALIV3. Because the model was not either fully controllable or observable, it was developed a 6 states LQR controller. The model is simulated using ideal sensor in the continuous and discrete case. Once the controller was tuned, it was simulated the model with estimation feedback introducing the Extended Kalman Filter (EKF). The final results suggest that the LQR and the EKF combined could stabilize the ALIV3.

Index Terms – Quadrotor, Tilting rotor, Extended Kalman Filter, Linear-Quadratic-Regulator, Identification.

I. INTRODUCTION

A Tilt-quadrotor is a fusion of the quadrotor and tiltrotor concepts, enabling it to move in all six degrees of freedom with the advantage of maintaining its central core leveled. This possibility results from add a tilting movement in two opposed rotors while the other two rotors remain fixed.

Quadrotor aircrafts were seen as possible solution in vertical takeoff and landing (VTOL) and in torque-induced control problems in the early of 20th century. Tiltrotors were developed during the medium of the 20th century to fully accomplish the VTOL and also the cruise flight.

The concept of Unmanned Aerial Vehicles (UAV) has bolstered the interest in quadrotors. They are nowadays under exhaustive investigation and a large number of quadrotors was introduced since 2004 for military and civil use. The Tilt-quadrotor seeks to increase their applications, adding the advantage of maintaining a payload almost perpendicular to gravity and independent of its motion.

In every quadrotor project there are two major aspects to be accounted for, the platform project and the aircraft control. Once the platform project, the ALIV3, was developed by Fernandes [6], this work seeks to guarantee a trustworthy of the platform and progress in the tilt-quadrotor control. The first step of the control development is the stabilization of the device, and this is the main objective of this work. The stabilization is studied by simulation in Matlab/Simulink.

The ALIV3 has 4 rotors in order to produce lifting forces and some sensors to estimate the attitude. The available sensors are: three axes ADXL335 accelerometer, two axes Invensense IDG-500 and one axis Invensense ISZ-500 gyroscope, three axes Honeywell HMC5843 magnetometer and a Bosch BMP085 barometer.

To obtain an adequate controller and estimator, the model of the tilt-quadrotor is derived comprising the model of the sensors and the model of the actuators. To provide more realism to the simulation, experimental tests are realized to determine the necessary parameters. Special structures for motors characteristic and moment of inertia measurements have been designed and built.

The obtained model is linearized around the hover situation. The controllability and observability of the 12 states is studied, resulting that only 6 states are fully controllable and observable. Then, a 6 states LQR controller is obtained with estimation feedback for stabilization purposes. The chosen estimator is an Extended Kalman Filter. The simulation suggests that the control and estimation methods could be implemented on the platform.

II. MODEL OF THE TILT-QUADROTOR

A. ALIV3 Platform

The ALIV3 platform consists in a structure with a center core and four arms. Two arms are fixed, in which extremes the fixed rotors are located, and the other two arms are swivel, in which extremes the tilting rotors are
placed. The two tilting rotors swivel by the action of four servos, two for each rotor. Two of the four servos are located in the center core and they are responsible of the pitch movement of the rotors. The others two servos are located in the swivel arms in an antisymmetric position to not displace the center of gravity from the geometric center as happened with the first version of the ALIV1 [6]. In the center core are also located the battery, the ArduPilot, the Power Distribution Board (PDB), the Electronic Speed Controllers (ESCs) and the landing gear is attached.

The ALIV3 platform has suffered an upgrade in the structure. The fixed arm was built in carbon fiber and it had a bad performance to torsion and flexion efforts. It had also problems with resonance when the motors were turned on. So it was replaced by another one in aluminium. The swivel arms have been counterbalancing and the upper central core have been modified to harbor the ArduPilot.

B. Coordinate System

Two frames are defined to describe the ALIV’s motion: an inertial frame, the North East Down (NED) centered in O, and a body frame, the Aircraft-Body-Centered (ABC) centered in Oc, both shown in Figure 2. Vectors expressed in the inertial frame are marked with the superscript $^I$ and vectors expressed in the body fixed frame have the superscript $^B$.

The position of the tilt-quadrotor, denoted $p^I = [X; Y; Z]^T$, corresponds to the displacement of Oc relative to O. The rotation of the ABC frame relatively to the NED frame defines the attitude of the aircraft. In aeronautic literature Euler angles are normally used: roll ($\phi$), pitch ($\theta$), and yaw ($\psi$). Then the attitude is described by $\Phi = [\phi; \theta; \psi]^T$.

C. Performance of a Tilt-Quadrotor

In a standard quadrotor motor M1 and M3 have a clockwise rotation whilst motors M2 and M4 have a counter-clockwise rotation. However, the presence of two tilting rotors in the ALIV3 is only possible in a hover situation. In forward motion (Figure 3), if motors M2 swivels a pitch angle $\theta_2$ and M4 swivels a pitch angle $\theta_4$ and they rotate in the same direction, the moment $Q_2 \sin(\theta_2)$ created in the x-axis by rotor 2 is not counteracted by the moment $Q_4 \sin(\theta_4)$ created in the x-axis by rotor 4. In fact, the total moment created is increased. So, it is compulsory that rotors 2 and 4 rotate in opposite directions in order to cancel the resulting moment. To cancel the moment in the z-axis, rotors 1 and 3 also have to rotate in opposite directions. In lateral motion a similar behaviour takes place. So, the presence of two tilting rotors makes compulsory that M2 and M3 have a clockwise rotation whilst M1 and M4 have a counter-clockwise rotation.

Some important conclusions from the performance study are:

a) Yaw motion can only be performed by tilting rotors 2 and 4 an opposite pitch angle $\theta_1$. Perform the yaw motion without tilting the rotors is not possible because the equilibrium equations impose a null yaw acceleration.

b) A standard rebalance without tilting any rotor is proved to be possible.

c) A rebalance by tilting rotors is also possible, but it is a manoeuvre very complicated. It is better to try by a standard form.

d) When the ALIV platform is not levelled, but $T_1 = T_3$, $T_2 = T_4$, $\phi_2 = \phi_4$, $\theta_2 = \theta_4$ and it supports its own weight, its attitude can be stabilized.

The first approach to the stabilization and attitude control has been accomplished without tilting any rotor, so problems associated to the yaw control will appears. In
following works must be considerate also the movement of the rotors.

\section*{D. Dynamics and Kinematics of a Tilt-Quadrotor}

The dynamics and kinematics of a Tilt-Quadrotor can be summarized in equations (1)-(4) in accordance to \cite{8}:

\begin{align}
\dot{P}^i &= S^i V^B 
\end{align}

\begin{align}
\dot{\Phi}^B &= T \Omega^B 
\end{align}

\begin{align}
\dot{\Omega}^B &= -\Omega^B \times \left( \Omega^B \right) + \mathbf{M}^B 
\end{align}

\begin{align}
m\dot{V}^B &= F^B + mS g^i - \Omega^B \times m V^B 
\end{align}

Equation (1) defines the relation between the linear movement of the quadrotor as derivative of the position in the inertial frame and the linear movement in the body frame. The matrix \(S\) is the rotation matrix which expresses a vector from the inertial frame to the body frame. Equation (2) expresses the kinematic relation between the derivatives of the Euler angles and the angular velocities rates. The matrix \(T\) is written in equation (5):

\begin{align}
T &= \begin{bmatrix}
tan \theta sin \phi & tan \theta cos \phi \\
0 & cos \phi \\
0 & sin \phi & -sin \phi \\
0 & cos \phi & sin \phi 
\end{bmatrix}
\end{align}

Equation (3) is the dynamic equation associated to moments. It contains the input moments developed by the four propellers, \(\mathbf{M}^B = [M_x, M_y, M_z]^T\). The inertia matrix \(I\) is diagonal due to the symmetry of the tilt-quadrotor. Equation (4) is just the second law of Newton. \(F^B = [0; 0; \sum_{i=1}^{4} F_i]^T\) are the forces created by the four propellers, \(g^i = [0, 0, g_0]^T\) is the gravity vector and \(g_0 = 9.81 \text{ m/s}^2\) is the gravity constant. Equation (6) shows how the moments and forces are calculated considering \(K_T\) and \(K_T M\) two constant characteristic of the propellers and \(d\) the distance from the rotors to the center of gravity:

\begin{align}
F_i &= K_T a_i^2 
\end{align}

\begin{align}
M_x &= (F_2 - F_4)d 
\end{align}

\begin{align}
M_y &= (F_1 - F_3)d \quad \text{(6.b)}
\end{align}

\begin{align}
M_z &= \frac{k_q}{K_T} (-F_1 + F_2 + F_3 - F_4) 
\end{align}

\section*{E. Model of the Sensors}

Sensors allow us to estimate the state variables. The sensor measurements are affected by noise and bias, which are the principal source of imprecision. The model of the gyroscopes, accelerometer, compass and barometer is presented:

1) \textbf{Gyroscopes}: The Invensense IDG-500 sensor is used to measure angular velocities corresponding to roll and pitch rates, \(P\) and \(Q\), and the Invensense ISZ-500 sensor is used for yaw rate, \(R\). Both present the same dynamics and governing equations. According to \cite{4} and \cite{7}, the gyroscopes are mostly affected in two ways: a stochastic Gaussian noise component, \(\mu_g\), and a slowly time-varying non-stochastic bias, \(b_g(t)\). Therefore, Gyroscopes measurement can be written as:

\begin{align}
\tilde{\Omega}^B = \Omega^B + \mu_g + b_g
\end{align}

When the tilt-quadrotor is flying, the four motors are turned on and they cause undesired noise corrupting the measurements. Practical measurements of noise have been realized. The spectral powers obtained are: \(\mu_g = [0.2023, 0.0799, 0.1009]^T \text{ (}^\circ/\text{s})^2/\text{Hz}\).

2) \textbf{Accelerometer}: The accelerometer is used to measure the direction of the gravity vector, \(g^i\). Since it is always pointing down in the NED frame with an intensity of \(g_0 = 9.81 \text{ m/s}^2\), through the accelerometer measurement vector \(\tilde{a}^B\) the pitch and roll angles can be obtained. The accelerometer is not only sensitive to the gravity but also to accelerations due to its movements, \(a^B\). According to \cite{1}, one can write \(\tilde{a}^B = g^i + a^B\). Like in the gyroscopes, accelerometers are also affected by a Gaussian noise and a bias, but the bias can be eliminated with a calibration routine. Introducing them in the expression before, the accelerometer measurement can be written as:

\begin{align}
\tilde{a}^B = g^i + a^B + \mu_a + b_a
\end{align}

where \(a^B\) is the pitch and roll, and \(\mu_a\) and \(b_a\) are the Gaussian noise and a bias, respectively. Practical measurements of noise have been realized. The spectral powers obtained are: \(\mu_a = [0.0212, 0.0130, 0.0186]^T \text{ (m/s}^2)^2/\text{Hz}\).

3) \textbf{Magnetometer}: The compass is designed to detect the magnetic North direction, written \(N^i = [1; 0; 0]^T\). However, the measurements are also corrupted by a Gaussian noise and a bias. Therefore, the sensor measurement is:

\begin{align}
\tilde{N}^B = SN^i + \mu_m + b_m
\end{align}

For a near hover situation, it can be assumed that the compass is held horizontally \((\theta = \phi = 0)\). Then, the model can be further simplified to:
\[ \bar{N}^B = \begin{bmatrix} \cos \psi \\ -\sin \psi \\ 0 \end{bmatrix} + \mu_m + b_m \]  

(10)

The measurements of the compass are affected by soft iron and hard iron distortions. According to [2], hard iron distortions are caused by the presence of magnet fields. They produce a constant additive error regardless of the orientation. Soft iron distortions are similar to hard iron distortions but the error varies with the orientation. As a consequence, the hard iron distortions can be included in a constant bias term, but the soft iron effect cannot be easily accounted for and will be neglected. Practical measurement of noise has been realized. The spectral power obtained is: \( p_m = 0.0500 \text{ (deg)}^2/\text{Hz} \)

4) **Barometer**: The barometer is a sensor used to obtain the altitude by means of pressure. This barometer can also measure temperature. Referring to the model of the standard atmosphere it is possible to compute the data to obtain the altitude:

\[
Z = -44330 \cdot \left(1 - \left(\frac{p}{p_0}\right)^{0.165}\right) 
\]

(11)

where \( p_0 = 101325 \text{Pa} \) is the pressure at sea level. The barometer measurements are affected by a noise term. So, the sensor measurement is:

\[
\bar{p} = p + \mu_p 
\]

(12)

where \( \bar{p} \) is the sensor measurement and \( \mu_p \) is the Gaussian measurement noise. The readings are not affected by the sensor position in the frame. Practical measurement of noise has been realized. The spectral power obtained is: \( \mu_p = 0.4269 \text{ (Pa)}^2/\text{Hz} \)

**F. Model of the Actuators**

Actuators are employed to produce forces on the system. Through these forces we bring the tilt-quadrotor to a desired state. In this case the actuators are the motor and propeller sets.

1) **Propellers**: The friction forces of the air in contact with the propellers, the flapping of the blades and the ground effect will be neglected. According to the Blade Momentum Theory, the thrust, \( T_i \) and the moment, \( Q_i \) created by each propeller are proportional to the squared angular velocity of the blade. They can be written as :

\[
T_i = K_T \omega_i^2 \\
Q_i = K_Q \omega_i^2
\]

(13)

(14)

where \( K_T = \rho \pi r^4 C_T \) is a constant that relates the thrust to the square of the angular velocity of the propellers, \( K_Q = \rho \pi r^4 C_p \) is a constant that relates the moment to the square of the angular velocity of the propellers. \( C_T \) is the thrust coefficient of the propeller and \( C_p \) is the power coefficient of the propeller.

2) **Motors**: The motors mounted in the ALIV3 are brushless. They need a speed controller that receives a PWM signal from the arduino board associated to an angular velocity. The PWM signal is a square digital signal, a common method to provide an analog signal with digital means. Once the speed controller receives the PWM signal, it turns the PWM signal into a triphasic signal to feed the brushless motor.

The behavior of the motor is a consequence of an interaction between an electric and mechanical dynamics. However the influence of the electrical dynamics of the motor is considerably faster than the mechanical dynamics of the motor with the propellers. Then, it is possible to say that in the permanent regime the angular speed of rotation is proportional to the PWM signal. Therefore, the model of the motors can be simplified to a first-order system corresponding to the mechanical dynamics [1]:

\[
\frac{\omega_{\text{LI}}}{\text{PWM}_I} = \frac{k_i}{Ts+1} 
\]

(15)

where \( \tau \) is the time constant of the motor and \( k_i \) is its dc gain. The actuator set considered, contains some nonlinearities since the PWM signal takes discrete values.

**G. Tilt-Quadrotor Simulator**

Figure 4 show the implementation of the complete model of the system in Simulink. There are 5 main blocks, where are implemented the motors, the dynamics and kinematic equation, the sensors, the estimator and finally the controller.
The CG is measured by a graphic procedure. It consists in hanging the ALIV3 by a string from different points. Firstly the CG is measured in the horizontal plane and later in the vertical plane. This string draws a line upon the plane we are measuring. So, the CG is placed where these lines intersects. The result of this procedure confirms that the CG is placed in the geometric center and at height of 161mm from the base.

The inertia matrix is considered diagonal since the prototype is symmetric about the x-axis and about the y-axis. To measure the moments of inertia, an oscillation method is used. It considers a linear system made by a rigid body constrained in such way that its only possible motion is a rotation about an axis fixed in space. Then the body is subject to an elastic restoring torque that makes the quadrotor oscillate with a period that depends on the inertia of the quadrotor, but also on the stiffness and on the damping of the system:

$$T = 2\pi \frac{I}{\sqrt{k \cdot \zeta^2}}$$ \hspace{1cm} (16)

where $k$ is the stiffness of the system, $I$ is the inertia and $\zeta = c/2\sqrt{k}$ with $c$ the damping coefficient of the system.

To constrain the movements and allow the oscillation a special structure has been designed and built for any quadrotor of similar dimensions (Figure 6). The measured moments of inertia are:

- $I_x = 0.0367 \text{ kg} \cdot \text{m}^2$
- $I_y = 0.0262 \text{ kg} \cdot \text{m}^2$
- $I_z = 0.0504 \text{ kg} \cdot \text{m}^2$

B. Actuators Identification

One important part of this work is the identification of the actuators, in particular the motor identification.

1) Motors: Identify the motors implies to find the characteristic curve that relates the PWM signal sent from the ArduPilot and the angular velocity of the motor’s rotor in the permanent regimen and the time constant of the motors.
variable considered is the \( \text{RPM/V} \) since the Voltage control the battery discharge.

![Fig. 8. Temperature Test](image)

Figure 9 shows the curves \( \text{PWM} - \text{RPM/V} \) for the 4 motor-ESC sets. From this curve and the battery voltage, it is possible to estimate the RPMs of each motor for a PWM signal. In fact, the average of the relative errors of this estimation around the operation point is lower than 1%. These curves can be considered as linear, as the model suggested. The regression lines of these lines are:

\[
\begin{align*}
\text{(RPM/V)}_1 &= 0.5272 \cdot \text{PWM} - 364.17; R^2 = 0.9987 \\
\text{(RPM/V)}_2 &= 0.5003 \cdot \text{PWM} - 331.63; R^2 = 0.9976 \\
\text{(RPM/V)}_3 &= 0.5180 \cdot \text{PWM} - 341.19; R^2 = 0.9932 \\
\text{(RPM/V)}_4 &= 0.5250 \cdot \text{PWM} - 360.84; R^2 = 0.9986
\end{align*}
\]

Therefore, Equation (15) is transformed to:

\[
\frac{\omega}{\text{PWM}} = \frac{k_l}{\tau_s + 1} \tag{17}
\]

From the regression lines it is possible to obtain the dc gain and the dead zone of each motor. The time constant has been considered the same as in Fernandes’s work [6].

It has been also studied what happened when the motor-ESC set is providing the BEC to the PDB. The results show that the average of the relative differences in of the RPMs in both situations is 0.82%. Therefore, the BEC influence can be neglected.

![Fig. 9. Curve RPM/V-PWM](image)

2) Propellers: Propeller identification can be summarized in determining the thrust coefficient and the power coefficient. The results are a \( C_T = 0.0140 \) and a \( C_p = 0.0033 \) which leads to a \( K_T = 1.40 \cdot 10^{-5} \text{gr} \cdot \text{s}^2 \) and a \( K_Q = 4.2 \cdot 10^{-7} \text{gr} \cdot \text{m} \cdot \text{s}^2 \).

IV. TILT-QUADROTOR FLIGHT CONTROL

The simulation of the tilt-quadrotor flight attitude control is carried out considering the ALIV3 without tilting any rotor. So the stabilization is achieved with the 4 PWM signals of the motors as input of the system.

The response of the open loop is unstable. When the system is left alone from the initial conditions set corresponding to the hovering situation, the tilt-quadrotor diverges quickly. That reflects the necessity of a control method to stabilize the attitude of the ALIV3.

A. Control Method.

The control method selected is a Linear-Quadrated-Regulator (LQR). It is an optimal controller in the sense that it minimizes the cost function, defined in equation (19). Considering the state space system of the model linearized:

\[
\begin{align*}
\dot{X} &= AX + BU & \text{Dynamic Equation} \tag{18.a} \\
Y &= AX + BU & \text{Output Equation} \tag{18.b}
\end{align*}
\]

where \( X = [V, \Omega, P, \Phi]^T \) is the state vector of the system, \( U = [\text{PWM}_1, \text{PWM}_2, \text{PWM}_3, \text{PWM}_4] \) is the input vector of the system and \( Y = [\bar{a}, \bar{\Omega}, \bar{a}, \bar{\Omega}]^T \) is the output vector of the system obtained from the sensors. The control problem consists in looking for the optimal control action \( U = -K_{lq}_{r}X \) where \( K_{lq}_{r} \) is a simple matrix gain given by \( K_{lq}_{r} = R_{lq}^{-1}B^TP \) and \( P \) is found by solving the algebraic Riccati equation. The matrix gain is chosen by the criteria of minimizing the following cost function:

\[
J_{lq} = \int_0^t \left( x^T Q_{lq} x + u^T R_{lq} u \right) dt \tag{19}
\]

where \( Q_{lq} \) and \( R_{lq} \) are the weighting matrices. They are determined by an iterative process where the initial matrices are determined by Bryson’s rule.

Therefore, the LQR controller needs a linearization of the model to find the matrix gain \( K_{lq}_{r} \). This linearization has been achieved by a computational method and tested analytically. The operation point considered is \( X_0 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \).

The system is fully controllable when all states are controllable. That implies that the rank of the matrix of controllability, \( [B | AB | \ldots | A^{n-1}B] \), is equal to the number of states [3]. The system is fully observable when the rank of the matrix observability, \( [C | CA | \ldots | CA^{n-1}]^T \),
is equal to the number of states [3]. When this study is applied to our system, there are two states that are not controllable and 4 that are not observable.

Through the Kalman decomposition [5] it is possible to separate the reachable subspace and its complement combined with the unobservable subspace and its complement. Applying the Kalman decomposition to our system, one knows that the subspace which is fully controllable and observable has a dimension of 6. The magnitudes that are not controllable are $\Psi$ and $R$, which is coherent with the conclusions of the section Performance of a Tilt-Quadrotor. The states unobservable are $X, Y, U$ and $V$ because the GPS is not considered. Therefore, in Simulation Results the system is reduced to a 6 states space system, which are $X = [W, P, Q, Z, \phi, \theta]^T$.

Figure 10 shows the response of the full 12 states in close loop with the LQR controller implemented and the set reference. The sensor measurement is considered as ideal and the weighting matrices are:

$$Q_{LQR} = \text{diag}([5, 5, 10, 100, 100, 50, 4, 4, 30, 150, 150, 150])$$

$$R_{LQR} = \text{diag}([0.01, 0.01, 0.01, 0.01])$$

One can observe that $\Psi$ does not follow the reference, but the controller gets stabilize it. When the closed loop dynamic is analyzed, all poles have a negative real part (Figure 11) what means that the system is stable.

In the EKF version, the non-linearities of the systems are approximated by a linearized version of the non-linear system model about the current estimation. Figure 12 shows the algorithm of the EKF. Because the APM board runs at a frequency $freq = 50Hz$, the system must be linearized and discretized:

$$X_{k+1} = A_dX_k + B_dU_k + w_k$$  \hspace{1cm} (20.a)

$$Y_k = C_dX_k + D_dU_k + v_k$$  \hspace{1cm} (20.b)

where $w_k$ is the process noise with covariance $Q_k$ and $v_k$ is the measurement noise with covariance $R_k$.

The improvement of the EKF comes from integrate the gyroscopes to “predict” the evolution of the attitude and correct that prediction with the information from the accelerometer and the compass. Then, instead of using the PWM signal as the input vector, one can write the system works in two steps. First it acts as a predictor: it predict the future state considering the covariance error with the help of the model of the system, the current state and the input vector. Then in the second step it corrects the predicted state and the estimated covariance error according to the measurements and the noise covariance. The Kalman filter is optimal in the sense that it minimizes the estimated covariance error.
taking the gyroscopes as input vector. However, as the system is going to be reduced to a 6 state space system, the yaw angle $\psi$ and the angular velocity about the z-axis $\Omega_z$ are removed:

$$\dot{X} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} X + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} U$$

(21)

where $X = [\phi, \theta]^T$ and $U = [P, Q]^T$. Then, matrices $C$ and $D$ have also to be adapted to the new system:

$$\dot{Y} = \begin{bmatrix} 0 & -9.81 \\ 9.81 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} U$$

(22)

where $Y = [\delta_x, \delta_y, \delta_x, \delta_y]^T$. This approach leads to a benefit in the robustness of the estimation, but only the estimation of the attitude is obtained. The output of the gyroscopes is used as direct measurement of the angular velocities. Because the dynamic equation is already linear by nature, the computations are greatly simplified and the time update can be performed with the same to $C_k = C_d$ and $D_k = D_d$.

Because the height estimation is not the main concern of this work, it has not been achieved, and ideal values of the height have been used instead. However, the vertical velocity has been estimated. The approach followed is to derive the altitude barometer measurement. A high pass filter is used with a time constant of $0.005s$ has been used.

C. Simulation Results

Because the system was not fully controllable and observable, 6 states have been removed. The state vector considered is $X = [W, P, Q, Z, \phi, \theta]$ and the output vector is $Y = [\delta_x, \delta_y, \delta_x, \delta_y, \delta_y, Z]$. Firstly, the controller is designed in the ideal case using the output vector without noise, and then, the real case is performed using the data from the estimated values.

1) Ideal Simulation: Considering the continuous model, the 6 state space linearized is obtained eliminating from the 12 states model linearized the columns and rows corresponding to the removed states. The matrix of the model linearized are:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(23.a)

$$B = \begin{bmatrix} -0.0052 & -0.0049 & -0.0051 & -0.0052 \\ 0.1134 & 0 & -0.1115 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(23.b)

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -9.81 \\ 0 & 0 & 0 & 0 & 0 & 9.81 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(23.c)

$$D = \begin{bmatrix} 0.0034 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(23.d)

The weighting matrices are:

$$Q_{lqr} = diag([50, 100, 100, 100, 250, 250])$$

$$R_{lqr} = diag([0.01, 0.01, 0.01, 0.01])$$

Figure 13 shows the response of the system to a set reference $X_{ref} = [0, 0, 0, -1, 0, 0]^T$ and initial conditions as the set reference. It can be seen that a small static error appears in the roll and in the pitch. This small error is due to the nonlinearities of the model, like the quantizers in the implementation of the motors in Simulink. It can be also appreciated that the fact of non control the position, introduce a random walk. Analyzing the closed loop dynamic, all the poles have a negative real part. This proves the stability of the system.

![Fig.13. Continuous 6 States Response](image)
very similar to the continuous model, but with a higher standard deviation. All the poles of the discrete close loop are within the unit circle, proving that the controller make the system stable (Figure 15).

2) **Realistic Simulation:**

Finally, the simulation of the model with estimation feedback is presented. The values of $\phi$ and $\theta$ are estimated by an EKF; $P$ and $Q$ are used directly from the gyroscope; $W$ is estimated by a high pass filter. Because the concern of this work is the attitude stabilization, $Z$ is used as ideal data. All sensors are considerate to work at $freq = 50Hz$ and the high pass filter used to obtain the vertical velocity uses a cut-off frequency of $\frac{1}{0.005} rad/s$.

The weighting matrices of the controller and the estimator are:

$$Q_{LR} = diag([1, 100, 100, 400, 250, 250])$$

$$R_{LR} = diag([0.01, 0.01, 0.01, 0.01])$$

$$Q_k = diag([0.00005, 0.00005])$$

$$R_k = diag([0.0212, 0.013, 0.2024, 0.0799])$$

The weight related to $Z$ has been increased and the one related to $W$ has been decreased because the controller stabilized the ALIV3 around a null vertical velocity but with an error in $Z$.

Figure 16 shows the response of the system to a set reference $X_{ref} = [0, 0, 0, -1, 0]^T$ and initial conditions as the set reference. It can be observed that the controller achieve stabilize the tilt-quadrotor around the set reference with a maximum error in $\phi$ of $1.43^\circ$ and in $\theta$ of $1.25^\circ$. Table 1 presents the mean square error (mse), the standard deviation (std) and mean of the attitude angles $\phi$ and $\theta$:

<table>
<thead>
<tr>
<th></th>
<th>$\phi$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>mse (deg)$^2$</td>
<td>0.2983</td>
<td>0.1781</td>
</tr>
<tr>
<td>std (deg)</td>
<td>0.5386</td>
<td>0.4006</td>
</tr>
<tr>
<td>mean (deg)</td>
<td>-0.0917</td>
<td>0.1330</td>
</tr>
</tbody>
</table>

Table 1. Attitude Errors

The simulation suggests that the quadrotor attitude could be stabilized for a hover flight using the implemented control and estimation methods.

V. **CONCLUSIONS AND FUTURE WORK**

The main conclusions of this work are:

a) The structure has been improved and the electronic components checked.

b) A study of the general performances of a tilt-quadrotor have been achieved, analyzing what happens when the rotors are tilted and when a tilt-quadrotors fly like a standard quadrotor. It has been observed that the yaw motion is only possible by tilting the rotors.

c) The model of the platform has been obtained experimentally. The CG has been determined via graphic
method. The inertia matrix has also been determined experimentally by a special structure designed and constructed for its determination in any quadrotor.

d) The models of the actuators have been obtained considering the electronic speed controllers, the motors, the propellers and the battery. The behavior of the motors with the temperature and the battery discharge has been studying. All the motors have been considered to have the same time constant. The identification of the propellers and the motors has been achieved with the help of a laser based tachometer to measure the angular speed and a balance to measure the weight.

e) The model of the sensors has been obtained, measuring the experimental noise when the motors are running. For greater realism in simulations, the sensors were modeled using the noise when the motors are turned on. This implied the development of Arduino based code to read the sensors measurements and interpret them.

f) The models of the motors, sensors and quadrotors dynamics have been linearized and implemented in Simulink considering that any rotors can tilt. The linearization has been done around the hover situation by computationally method and checked by analytically method.

g) The controllability and observability of the 12 states model have been studied, resulting that the model is neither fully controllable nor observable. As the analysis of the performance indicated, the yaw motion is not controllable but it is stabilized. The fully controllable and observable model without considering the GPS is a model of 6 States.

h) Finally, a LQR controller has been developed based on the 6 observable and controllable states. It has been tuned and tested using ideal data from the sensors in the case of the continuous model and in the case of the discrete model. Then, the EKF has been implemented on the Simulink model. The LQR has been retuned and it has been tested that the LQR and the EKF combined could stabilize the ALIV3 in hover without tilting any rotor.

Respect to the future work, the solution for the stabilization problem must be implemented in the real platform to check the results. Once the roll and the pitch control will be tested, the control of the ALIV3 considering the servos should be accessed.

VI. BIBLIOGRAPHY


