Design of timber structures: analysis method and evaluation of current codes regarding timber joints

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1 – INTRODUCTION

This study consists in the analysis of the methods and the evaluation of current regulations regarding timber joints. To do so, two kinds of timber joints are presented: glued joints and dowelled joints. Regarding glued joints, it is shown how it works. Finger joints and scarf joints, are also mentioned on this study.

Regarding dowelled joint, it is presented the demonstration of one of the expressions for the joint resistance capacity. Similar approach can be done in order to get the others expressions. All the expressions for the joint resistance are shown as they appear in Eurocode 5. Based on these expressions, the failure mode is shown. A graph that compares the joint resistance depending on the thickness of the timber element is also presented.

2 – GLUED JOINTS

In a glued joint the applied force is transmitted to the glue as shear stress as represented on part a) in Figure 1. When constant, the shear stress on the glue (τ_m) has a direct relation between the applied force (P) and the glued area, which is the average stress. This stress distribution admits a constant relative deformation among the different sections on the timber elements, as represented on part b) in Figure 1. In reality, that relative deformation is not constant, and the same happens to the sheer stress distribution, as represented on part c) in Figure 1. The peak values appear at both ends of the glue line, in opposition to the middle values, where they are lower. This conclusion was described by Volkersen in 1938 (Crocetti et al, 2011), who suggested a shear stress distribution.

![Figure 1 – Glued Joint](image)

Figure 2 shows the stress variation along the glue line, comparing it through both methods. In the example it was considered a 10kN force applied. Both elements are 45mm thick and the material is solid timber in strength class C30, with elastic modulus of 12GPa. The glue line is 400mm long (the graph origin is at the middle of the glue line), and 100mm wide. The glue thickness is 1mm and is resorcinol phenol glue. Its distortion modulus is 0.7MPa. In the graph, the average stress is represented by the continuous line, and the stress according to Volkersen by the dash line.
Finger joints and scarf joints are used as glued joints. The first relies on the execution of fingers in both timber elements, which connect. The second one consists on a cut that is done to unify both structural elements. A scarf joint is more effective than a finger joint, since that force is distributed through a bigger area. The down side about the scarf joint is that there is an enormous amount of wasted material, since it is dependent on the slope angle. Figure 3 presents a finger joint and a scarf joint.

The design of a finger joint is done according to the shear stress on the fingers. In order to assure the security of the joint, the resistance of the connection has to be bigger than the resistance of the timber. There are normal stress acting on the timber ($\sigma$) and shear stress on the glue ($\tau$). These parameters are presented in Figure 4.

A scarf joint depends on the normal stress on the timber and the shear ($\tau$) and normal ($\sigma$) stresses on the glue. The resistance of the joint is calculated through the equilibrium of horizontal and vertical forces acting on the joint. Figure 5 represents the stresses acting on the joint. Part a) presents the stresses and part b) the sum of those stresses.
Through different slopes, it is possible to obtain different stresses. There is no stresses variation along the glue line, as peaks in the ends. This means that in scarf joints, the deformation is constant. The normal and shear stresses variation are represented on Figure 6. In this graph, the stresses are represented as a function of the slope. The continuous line represents the shear stress ($\tau$) and the dash line the normal stress on the glue ($\sigma_l$).

The full demonstrations for the angle design of a scarf joint and a finger joint can be obtained on the master thesis (Hilário, 2013).

3 – DOWELED JOINTS

Johansen (Johansen, 1949) identified the possible failure modes that occur in a doweled joint and establish its load-carrying capacity. All demonstrations for the failure modes can be obtained on the master thesis (Hilário, 2013). There, the equations for the timber capacity and the dowel’s plastic moment can also be found.

One of the failure modes of doweled joints includes a plastic hinge on the dowel. On the section where the plastic hinge occurs, the plastic moment of the dowel cross section must be considered. As that moment is the highest along the dowel, there are no shear stresses at that section. Figure 7 represents that failure mode. The timber capacity is represented by $f_h$ and the dowel plastic moment by $M_y$. Equation 1 describes the joint capacity expression and the moments equilibrium to $M_y$. 

$$f_h = \frac{\sigma_l \cdot t}{\sin \alpha}$$

$$\frac{P}{t} = \frac{\tau \cdot t}{\sin \alpha}$$

Figure 5 – Equilibrium on a scarf joint

Figure 6 – Graph with scarf joint’s stress variation
Depending on the location of the plastic hinge, this equation is solved in order to $b_1$ or $b_2$. Through the relations between the thickness of the different materials ($a$, $b$ and $t$) and the relation between the timber capacities ($\beta$), the values for $b_1$ and $b_2$ are on Equation 2.

$$
F_v = \frac{f_{h1} \cdot b_1 \cdot d}{2 + \beta} \cdot \left( \sqrt{2 \cdot \beta \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot (2 + \beta) \cdot M_y}{f_{h1} \cdot d \cdot t_1^2}} - \beta \right)
$$

$$
M_y = f_{h1} \cdot d \cdot (b_1 + a_1) \left( b_2 + \frac{b_1 + a_1}{2} \right) - f_{h2} \cdot d \cdot \frac{b_2^2}{2} - f_{h1} \cdot d \cdot a_1 \left( b_2 + b_1 + \frac{3 \cdot a_1}{2} \right)
$$

(1)

With the values for $b_1$ and $b_2$, it is possible to rewrite Equation 1, as appear on Equation 3.

$$
F_v = \frac{f_{h1} \cdot d \cdot t_1}{2 + \beta} \cdot \left( \sqrt{2 \cdot \beta \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot (2 + \beta) \cdot M_y}{f_{h1} \cdot d \cdot t_1^2}} - \beta \right)
$$

$$
F_v = \frac{f_{h1} \cdot d \cdot t_2}{1 + 2 \cdot \beta} \cdot \left( \sqrt{2 \cdot \beta^2 \cdot (1 + \beta) + \frac{4 \cdot \beta \cdot (1 + 2 \cdot \beta) \cdot M_y}{f_{h1} \cdot d \cdot t_2^2}} - \beta \right)
$$

(2)

(3)

When $\beta=1$, both equations are the same. If the timber thicknesses are also the same, the joint capacity is given by Equation 4.

$$
F_v = \frac{f_{h1} \cdot d \cdot t}{3} \cdot \left( 2 \cdot \sqrt{1 + \frac{3 \cdot M_y}{f_{h1} \cdot d \cdot t^2}} - 1 \right)
$$

(4)

The different types of doweled joints failure modes are presented next. Together with schemes, the joint capacity expression that appears on Eurocode 5 are presented with a graph comparing the different kinds of failure modes. The expressions don’t include the rope effect.

The graphs refer to a doweled joint capacity as a function of the thickness of the timber element. It is considered solid timber in strength class C30, whose characteristic density is 380kg/m$^3$. One timber member is 50mm thick on timber-to-timber joints, with a thickness variation of the other. The dowel is a screw with a 12mm diameter and the ultimate steel strength of 600MPa, and a pre-drilled hole. The thick line represents the joint capacity, which is the lowest of the capacities of the failure modes.

The failure modes of a timber-to-timber joint with single shear are presented on Figure 8. Table 1 presents the joint’s resistance capacity and the graph is on Figure 9.
The failure modes of a timber-to-timber joint with double shear are presented on Figure 10. Table 2 presents the joint’s resistance capacity and the graph is on Figure 11.
Table 2 – Joint capacity

\[ F_v = f_{h1} \cdot t_1 \cdot d \]  
(1)

\[ F_v = 0,5 \cdot \beta \cdot f_{h1} \cdot t_2 \cdot d \]  
(2)

\[ F_v = 1,05 \cdot \frac{f_{h1} \cdot d \cdot t_3}{2 + \beta} \cdot \left( \frac{2 \cdot \beta \cdot (1 + \beta) + 4 \cdot \beta \cdot (2 + \beta) \cdot M_y}{f_{h1} \cdot d \cdot t_1^2} - \beta \right) \]  
(3)

\[ F_v = 1,15 \cdot \sqrt{\frac{4 \cdot \beta \cdot M_y \cdot f_{h1} \cdot d}{1 + \beta}} \]  
(4)

The failure modes of a steel-to-timber joint with single shear for a thick plate are presented on Figure 12. Table 3 presents the joint’s resistance capacity and the graph is on Figure 13.
The failure modes of a steel-to-timber joint with single shear for a thin plate are presented on Figure 14. Table 4 presents the joint’s resistance capacity and the graph is on Figure 15.

Table 4 – Joint capacity

<table>
<thead>
<tr>
<th>Failure mode 1</th>
<th>Failure mode 2</th>
<th>Failure mode 3</th>
<th>Joint resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_v = 0.4 \cdot f_h \cdot d \cdot t$</td>
<td>$F_v = 1.15 \cdot \sqrt{2 \cdot M_y \cdot f_h \cdot d}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 15 - Graph with the joint capacity for a steel-to-timber joint with single shear (thin plate)
The failure modes of a steel-to-timber joint with double shear with two thick plates are presented on Figure 16. Table 5 presents the joint’s resistance capacity and the graph is on Figure 17.

\[ F_v = 0.5 \cdot f_{th} \cdot t \cdot d \]  
\[ F_v = 2.3 \cdot \sqrt{M_y \cdot f_{th} \cdot d} \]

Table 5 – Joint capacity

The failure modes of a steel-to-timber joint with double shear with two thin plates are presented on Figure 18. Table 6 presents the joint’s resistance capacity and the graph is on Figure 19.
Table 6 – Joint capacity

\[ F_v = 0.5 \cdot f_h \cdot t \cdot d \]  
(1)

\[ F_v = 1.15 \cdot \sqrt{2 \cdot M_y \cdot f_h \cdot d} \]  
(2)

Figure 19 - Graph with the joint capacity for a steel-to-timber joint with double shear (two thin plates)

The failure modes of a steel-to-timber joint with double shear with one plate are presented on Figure 20. Table 7 presents the joint’s resistance capacity and the graph is on Figure 21.

Figure 20 - Failure modes

Table 7 – Joint capacity

\[ F_v = f_h \cdot t \cdot d \]  
(1)

\[ F_v = f_h \cdot d \cdot t \cdot \left( \sqrt{2 + \frac{4 \cdot M_y}{f_h \cdot d \cdot t^2}} - 1 \right) \]  
(2)

\[ F_v = 2.3 \cdot \sqrt{M_y \cdot f_h \cdot d} \]  
(3)
Figure 21 - Graph with the joint capacity for a steel-to-timber joint with double shear (one plate)

Usually, there is more than one dowel in the connection in order to increase the joint resistance capacity, creating a group effect. This group effect has influence on the behavior of the timber joint, due to stresses accumulation on the timber elements. To avoid that excessive stresses it is necessary to keep a minimum spacing between dowel type connectors and between the dowel and the end of the timber elements. The joint has also to be analyzed considering the group of connectors.

More information on the group effect can be found on the master thesis (Hilário, 2013).

4 – Final Remarks

Finger joints and scarf joints are many times used as a glued joint in timber structures. Although scarf joints are more effective, finger joints are many times used due to its efficiency as the wasted material in those joints is much less.

Regarding doweled joints, it was demonstrated the expression for the resistant capacity of one failure mode. For all the failure modes according to Eurocode 5, the expressions for the joint resistance were presented with a graph that compares the joint resistance depending on the thickness of the timber element.

5 – References


