Competitive Analysis Solutions for Flat Slabs and Slab with Beams
Deformability and Cost

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ABSTRACT

In building structures, reinforced concrete slabs are important structural elements in its cost, having its thickness a significant impact in the global economy of the construction. In terms of the structural behavior characteristics for service limit state, the thickness has a particular relevance on the deformation. Recognizing the need to discuss the deformability of structural solutions of building floors, the present work was undertaken. Along this study, the validity of several simplifications is discussed and a methodology based on the Global Coefficient is proposed. A comparative study has been carried out, in which reinforced concrete solutions of flat slabs and slabs supported on beams with 8.0m and 7.0m span were taken into account. Furthermore, this analysis was based on the qualitative characteristics of deformation and cost value for the different situations.

The design of slabs and subsequent calculation of deformations were based on the elastic values obtained by the finite element program SAP2000. The real evaluation of the deformations and verification of its allowable values were established on the assumptions of the NP EN 1992-1.

It was concluded that for the central panels with the same volume of concrete per square meter, the deformation and the cost for the two solutions are equivalent. Concerning the quality of deformations on peripheral panels, the solution with beams gains advantage, especially in the beams alignment. In order to approximate the quality of deformation of these two solutions, bands in alignments between columns were considered, being however a more expensive solution.

Key words: Deformation, Flat Slab, Slab with Beams, Cost

1 Introdution

This paper aims to study the performance between different structural typologies used in buildings, in particular flat slabs with capitals and / or bands and slabs supported by beams of 7 and 8 meters span, with different boundary conditions. Comparisons are made using the quantities of materials and quality of deformation, based on the criteria in the regulation NP EN 1992-1.

Firstly, it is presented a state of art in order to clarify the nonlinear behavior of reinforced concrete in service, short and long term, as well as to understand and review methodologies for the evaluation of deformations, adopted by Europe, United States and Brazil and the processes of adapting these methodologies to the case of slabs (analysis in two directions). A programmable methodology identical to Method Coefficient Global was also proposed, in which the effects of concrete creep and cracking were taken into consideration.

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2 Deformation

The deformations of structures should be put in context on service state verification, since it is not a matter related to the resistant capacity, except in situations in which second order effects are generated.

There are three main reasons for its limitation of the deformation, damage to non-structural elements, esthetic matters and loss of utility. The NP EN 1991-1 (2010) considers a maximum absolute value of L/250, due to esthetic matters, and L/500 in order to prevent damage on non-structural elements.

To carry out the evaluation of the deformation in concrete structures, it is necessary to look at the nonlinear effects in concrete due to cracking and temporal phenomena such as creep and shrinkage. According to the Virtual Work equation, one may conclude that the deformation is a function of the curvatures of the entire element.

$$a = \int_0^L \left(\frac{1}{r}.\overline{M}\right) dx \tag{2.1}$$

Figure 2.1 represents the curvatures for initial and long time:

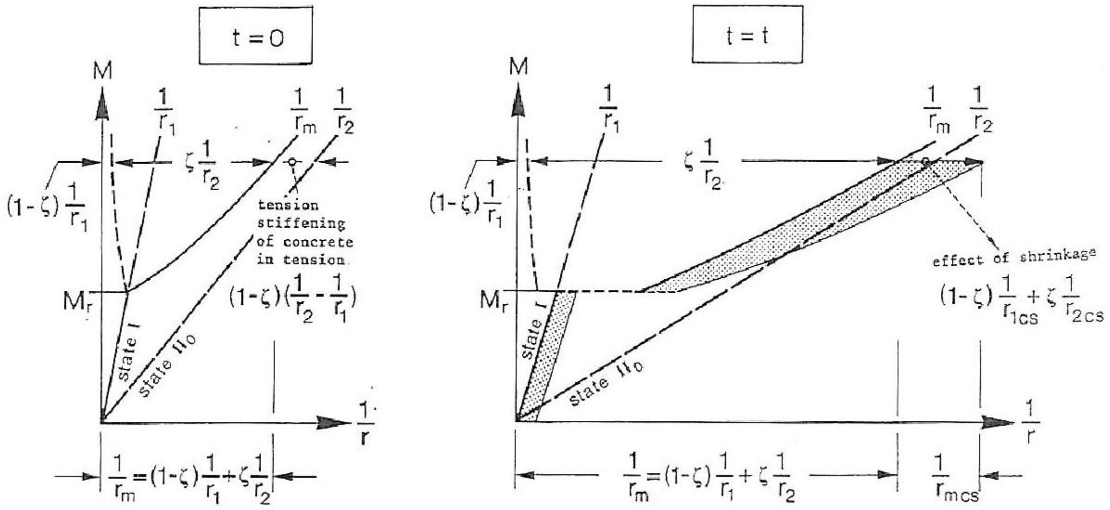


Figure 2.1 Moment-curvature diagrams for pure bending

The factor ζ indicates the level of cracking of the piece, weighting the valuation of curvatures between the states I and II. In a long term, cracking is responsible for a significant increase of curvatures.

Favre et al (1985) introduced the Method of Global Coefficients and Brandson (1977 apud Ghali, Favre and Eldbadry (2002)) Equivalent Inertia. In the present essay, a method that considers the same bases of the Method Coefficient Global has been developed, as an alternative to the abacuses.

2.1 Formal Global Coefficient

Favre et al. (1985) presented the Method of Global Coefficient where the valuation of the increment of deformation is made by a single global coefficient that represents the phenomena due to permanent loads (cracking and creep). This coefficient, long-term (Kt) and short-term (K0), depend on the creep coef. (ϕ) (long-term), amounts of reinforcement (αρ), relationship between cracking moment and determining section (Mcr / MD) and the reinforcement position (h / d).

Ghali, Favre and Eldbadry (2002) reported that the positioning of the reinforcement expressed by h / d, varies by the power of 3 making the consultation to a single abacus h / d = 1.

Thus, the final long and short term deformation will be given by 2.2 and 2.3, respectively:

$$\mathrm{a_t} = \eta \cdot \mathrm{k_t} \cdot (\mathrm{h/d})^3 \cdot \mathrm{a_c} \tag{2.2}$$

$$\mathrm{a_0} = \mathrm{k_0} \cdot (\mathrm{h/d})^3 \cdot \mathrm{a_c} \tag{2.3}$$

The variable η is a factor that considers the compression of reinforcement.

2.2 Adapte Global Coefficient

This dissertation proposes a coefficient based on the stiffness of the section, which can dispense the consultation of abacuses, allowing an equivalent calculation of the final deformation using the Method of Global Coefficient presented by Favre et. al. (1985).

Thus, starting with the elastic deformation, the coefficient is calculated by the following expression:

$$\begin{aligned}\mathrm{a_0} &= \left[(1-\zeta)\cdot\frac{\mathrm{E_{c0}} \times \mathrm{I_c}}{\mathrm{E_{c0}} \times \mathrm{I_I}} + \zeta\cdot\frac{\mathrm{E_{c0}} \times \mathrm{I_c}}{\mathrm{E_{c0}} \times \mathrm{I_{II}}}\right]\cdot \mathrm{a_c} = [(1-\zeta)\cdot \mathrm{K_{01}} + \zeta\cdot \mathrm{K_{02}}]\cdot \mathrm{a_c} \\ &= \mathrm{K_0}\cdot \mathrm{a_c}\end{aligned} \tag{2.4}$$

The modulus of elasticity and inertia will be affected due to a "decrease" of the rigidity of concrete due to creep. Thus, the long-term deformation is given by:

$$\begin{aligned}\mathrm{a_t} &= \left[(1-\zeta)\cdot\frac{\mathrm{E_{c0}} \times \mathrm{I_c}}{\mathrm{E_{c,eff}} \times \mathrm{I_{I,eff}}} + \zeta\cdot\frac{\mathrm{E_{c0}} \times \mathrm{I_c}}{\mathrm{E_{c,eff}} \times \mathrm{I_{II,eff}}}\right]\cdot \mathrm{a_c} = [(1-\zeta)\cdot \mathrm{K_{t1}} + \zeta\cdot \mathrm{K_{t2}}]\cdot \mathrm{a_c} \\ &= \mathrm{K_t} \times \mathrm{a_c}\end{aligned} \tag{2.5}$$

The figure 2.2 illustrates, according to the state I or II in analysis, the meaning of the different K coefficients.

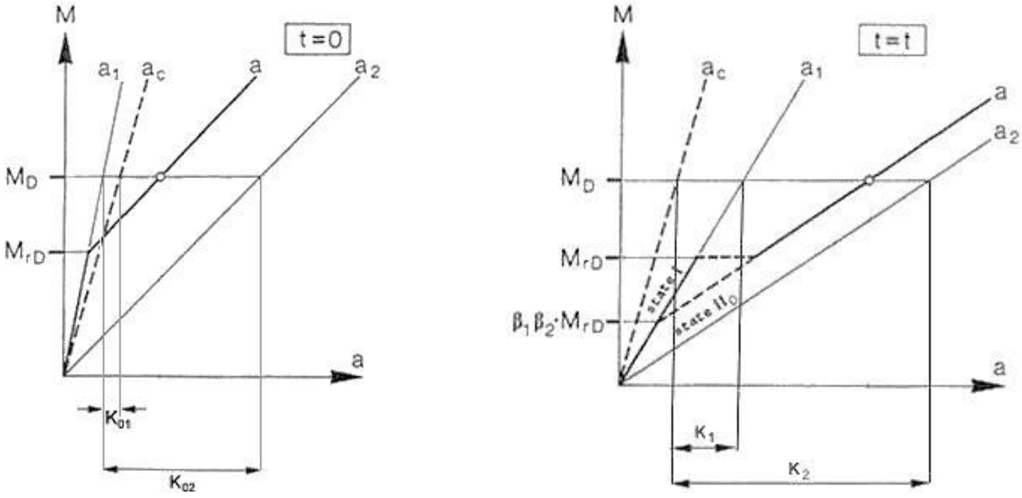


Figure 2.2 increment of deformation due to K factor

2.3 Equivalente Inertia

This is the method adopted by ABNT-NBR6118 (2004) regulation and United States by ACI-435(2003) regulation.

Ghali, Favre and Eldbadry (2002, p. 236) state that "An analogous approach for estimation of the deflection due to load (...) is based on calculation of an effective moment of inertia to be assumed constant over the full length of the member . "

Brandson (1977 apud GHALI, FAVRE, ELDBADRY, 2002) presents the following equation to calculate the equivalent inertia

$$\mathrm{I_e} = \left(\frac{\mathrm{M_r}}{\mathrm{M}}\right)^3 \cdot \mathrm{I_I} + \left[1 - \left(\frac{\mathrm{M_r}}{\mathrm{M}}\right)^3\right] \cdot \mathrm{I_{II}} \leq \mathrm{I_c} \tag{2.6}$$

Equivalent inertia is close to the state I or II according to the magnitude of loads.

In the short term and based on the elastic deflection value, the deformation is:

$$\mathrm{a}_0 = \mathrm{a}_\mathrm{c} \times (\mathrm{I}_\mathrm{c}/\mathrm{I}_\mathrm{e}) \tag{2.7}$$

In the long term, the deflection is given by

$$\mathrm{a}_\mathrm{t} = \mathrm{a}_0 + \alpha_\mathrm{f} \cdot \mathrm{a}_0 = (1 + \alpha_\mathrm{f}) \cdot \mathrm{a}_0 \tag{2.8}$$

In which α_f is a factor that takes into account the creep and shrinkage.

2.4 Method to Evaluate de Deformation in Slabs

According Favre et al (1985), the calculation of deformations from the Global Coefficient and Equivalent Inertia can be generalized to slabs through two procedures, the orthogonal bands method and the direct method.

In the orthogonal method, the slab is divided into a set of orthogonal strips. The deformation is calculated through a "path" defined by the bands. Working with relative deflections and a coefficient for each band, at final, it would have been reached a deformation for each of these bands. The final deformation at the center of the slab will be the sum of all relative deformation for each band, of the chosen path.

It is shown by Figure 2.3 the relative deformation and the chosen path, column to point B and C, and point B and C to point A

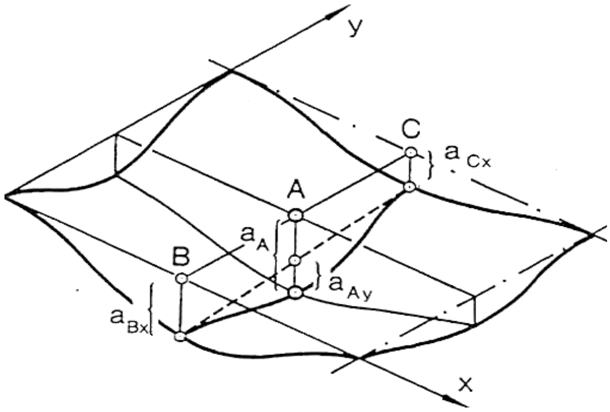


Figure 2.3 – Example of orthogonal strips

The direct method is based only on two sections, the first at the column and the second at the center (point A), calibrating a generic global coefficient for the whole slab. The long or short term deflection results on is the result of multiplying the coefficient by the elastic deformation.

Comparing the two methods, the results showed, on average, the direct method underestimates the deflection by about 20%.

3 Competitive Analysis for Slab With Beams and Flat Slabs

Based on the Adapted Global Coefficient and the Method of Orthogonal Band, slab supported by beams and flat slab with different boundary conditions were analyzed. As is shown in Figure 3.1, the slabs were divided in multiple panels. The first panel corresponding to the central panel, the panel with one free edge corresponds to the number 2, and panel 3 relates to the corner panel. Moreover, a 7 and 8 meter span has been considered.

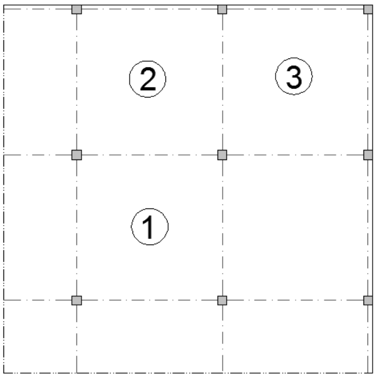


Figure 3.1 – Peripheral panels

Values of deflections corresponding to panel 1 would be inferior, since there exist a greater deformation in panels 2 and 3. In addition The deformation of panels 2 and 3 attenuates the deformation of panel 1. In this study, for the central panel, a panel will be evaluated simulating the continuity of the conditions.

Thus, the next figure shows the panel considered in evaluation of panel 1.

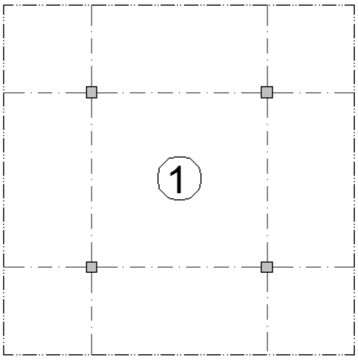


Figure 3.2 – Central panel

Between the three solutions considered, F22 represent the flat slab with a 22 centimeters thickness with capitals and peripheral bands with a thickness of 35 centimeters. In order to decrease the value of deflection, another solution, F22*, has been considered, in a 3 meter band was added in comparison with F22, connecting the capitals and peripheral band. Both solutions have capitals/bands 35 centimeters thick. V20 representes a slab supported by beams with 20 centimeters thick and beams with 75x30cm cross section.

The competitive study was based on two main parameters, quantity of materials and quality of deformation of each solution.

3.1 Amount of Materials

In terms of amount of materials, the solutions are compared by the equivalent thickness and the rate of reinforcement. The equivalent thickness is a factor that gives immediate idea of the volume of concrete in the panel. On the other hand, the reinforcement ratio allows the comparison of the amount of reinforcement in each panel and solution.

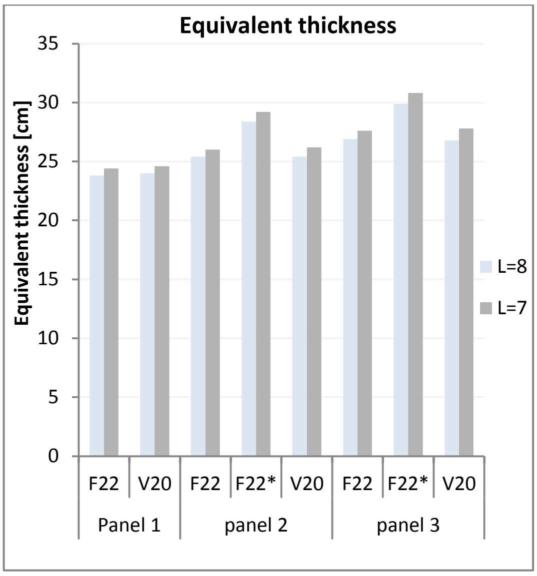


Figura 3.1 – Equivalent thickness

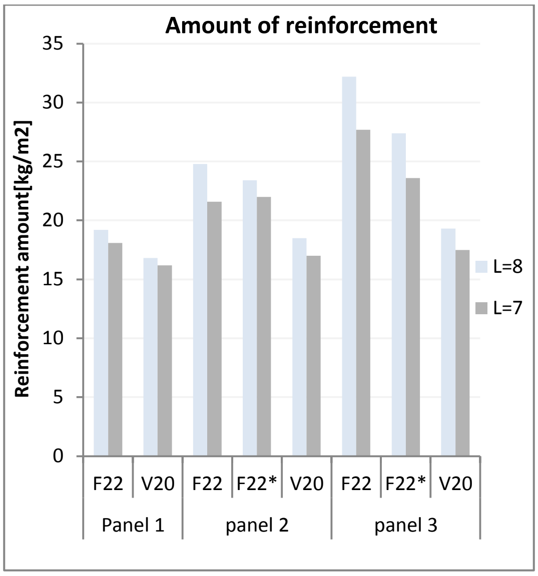


Figura 3.2 – Amount of reinforcement

Note that for the central panel (1), the two solutions exhibit equivalent values for the global thickness. By increasing the span, the equivalent thickness increases as well, since the dimensions of elements such as capitals / bands and beams haven't been changed. The slab with beams requires a lower rate of reinforcement since the beam allows a greater lever arm between the concrete compression zone and the tensioned zone which requires a lower amount of reinforcement. For a shorter span, both solutions require less reinforcement ratio as the moments of the external actions are lower.

The final cost, also measuring the quantities of formwork, is shown in the following figure:

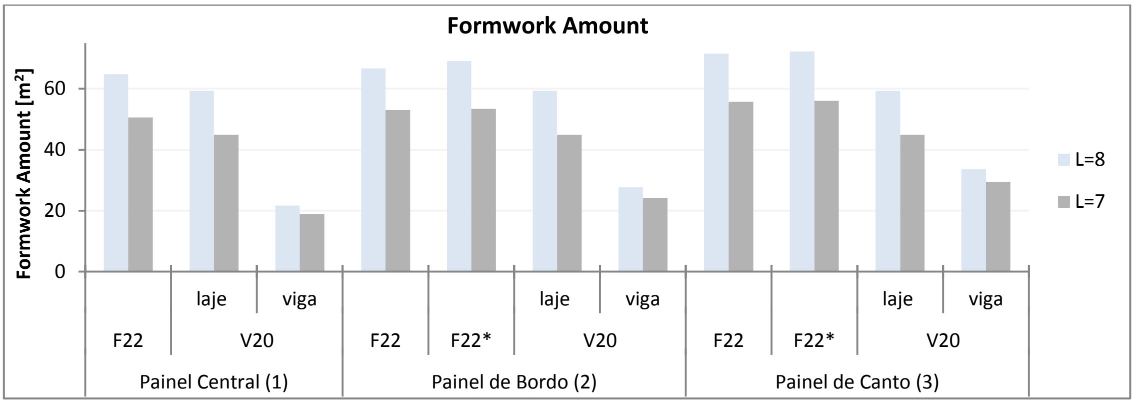


Figure 3.3 – Formwork Amount

For slabs supported by beams, it will be necessary to separately analyze the quantity of formwork for beams and slabs as these elements have different costs. It should be noted that the beamed solution shows a greater amount of area formwork relative to the flat slab solution.

3.2 Direct control of deformation

The European regulation NP EN 1992-1 (2010) defines that for appealing reasons, absolute deformation should be limited to L/250 and L/500 due to damage limitation, where L is the span between pillars. In the case of slabs, there are two possible spans to consider, the "orthogonal" span (L) and the diagonal of the panel (L_2) as it is shown by the following figure.

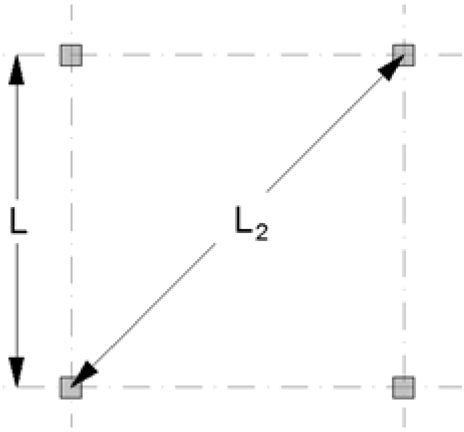


Figure 3.4 - Considered spans

The deformations in the center of each panel are shown by Figure 3.1, for each solution with spans of 7 and 8 meters.

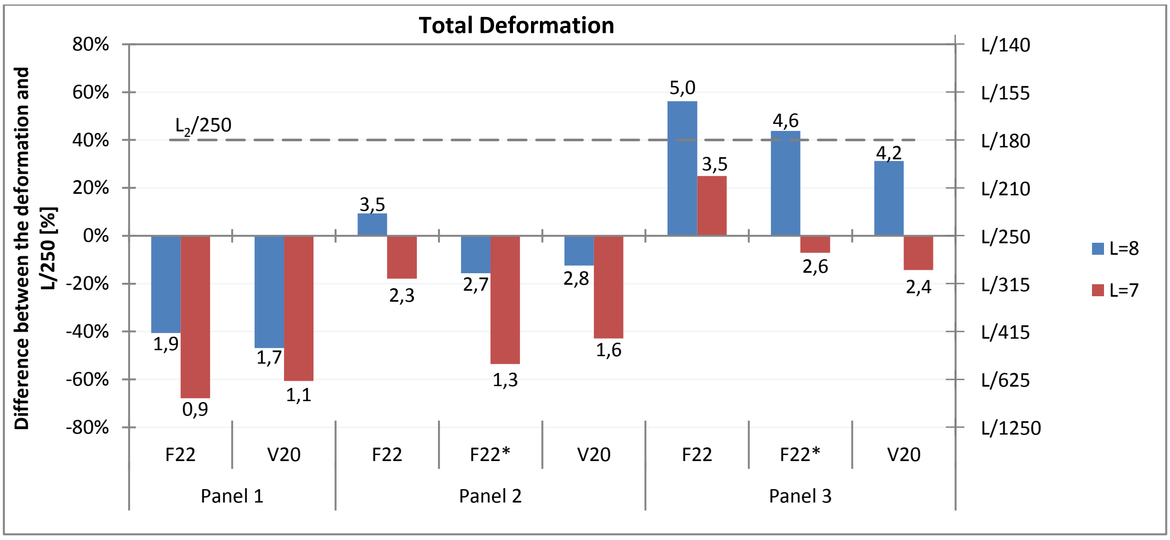


Figure 3.5 Long term deformation in centimeters

On the right axis, the difference between the limit value and the estimative for the deformation of the center panel are presented as a percentage. On the left, the vertical axis shows the deformation as a function of 7 or 8 meters span.

For the same amount of concrete, the two central panel solutions present a similar quality of deformation for long-term conditions.

As deformation depends strongly on the boundary conditions and slenderness, it is inevitable that the panels 2 and especially 3 have an higher deformation in the center of the panel. Naturally, by reducing the span and not varying the dimensions of the elements, the slenderness will decrease and therefore the deformation will decrease as well.

Figure 3.2 illustrates the elastic deformation to the center of each panel.

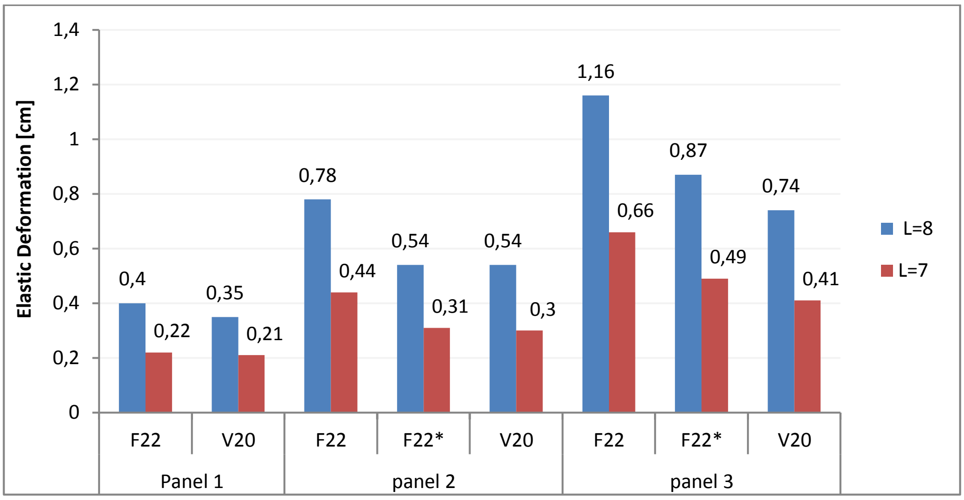


Figure 3.6 - Elastic deformation in centimeters

Comparing the two figures (3.6 and 3.7) it can be observed that, on average, the amount of deformation in the long term is five times greater than the elastic. Moreover, the larger the amount of reinforcement, the smaller the increment will be, as the reinforcement restricts the creep deformation. Thus, the slab with beams will suffer a bigger increase of deformation for the long term than the flat slab. This relationship also strongly depends on the state of cracking of the determinant sections analyzed.

Figure 3.6 depicts the increment of deformation for each panel.

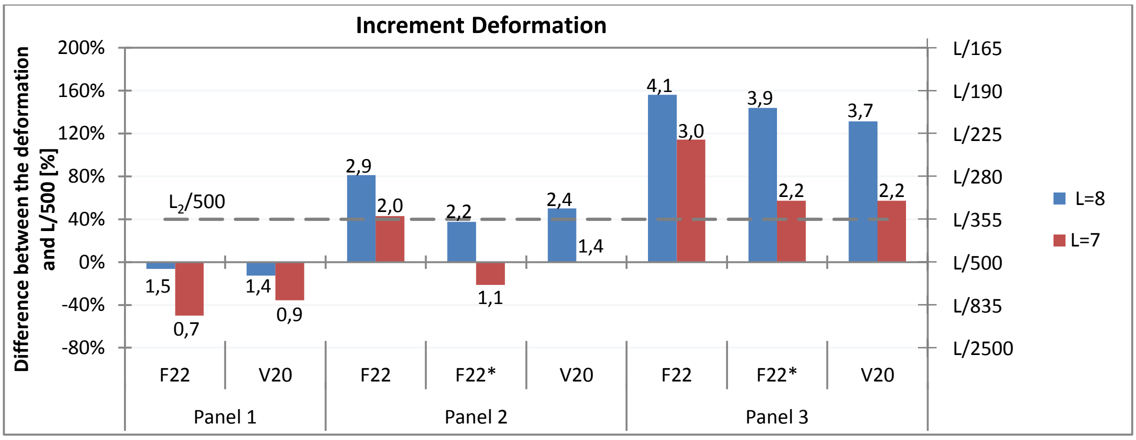


Figure 3.7 - Increment of deformation in centimeters

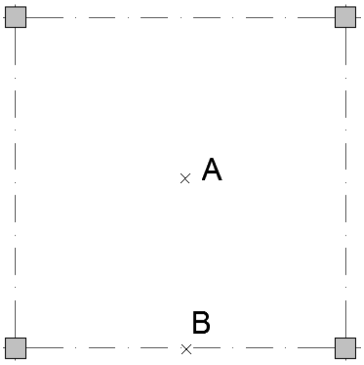
Comparing the absolute deformation (Figure 3.5) with the increments (Figure 3.7), it's concluded, in cases the total deformation is lower than L/400, the value of increment deformation is granted also by the limit value given by the NP EN 1992-1 (L / 500). Thus, for a less strict analysis would be able to verify only the limit L/400 for the total deformation.

The deformation increment exceeds the limiting value in the panel 3 for the span of 7 or 8 meters. One must bear in mind that that this value is only checked the alignment of the walls.

Note that the increase of deformation is only verified in the alignment of masonry walls. Thus, Table 1.1 shows the deformation values for the alignment of beams (which are likely to be placed walls) and the center of the 8 meters wide central panel (1).

Table 1 – Deformations and increments of deformation for point A and B

		Flat Slab	**Slab with Beams**	**Relative difference**
Total deformation (a_t)	Point B	1,4 cm	0,6 cm	57%
	Point A	1,9 cm	1,7 cm	10%
Incremental Defomation $\Delta(\mathbf{t_0} \rightarrow \mathbf{t_\infty})$	Point B	1,1 cm	0,5 cm	55%
	Point A	1,5 cm	1,5 cm	-



It should be noted that if the wall coincides with the alignment beam, to avoid damage to non-structural elements, the solution with beams has a better quality, as it is known. In a flat slab, the deformation from the column to the center of panel has a gradual increase in curvature, causing the deformation of the center panel to not be so noticeable. As such, for reasons of appearance, flat slab is a better solution.

3.3 Costs

The unitary cost for each material considered was respectively 0.90€/m^2, 100€/m^3 for steel and concrete, 12.65 €/m^2 of formwork for slab and 17.5 €/m^2 of formwork beam.

Warn again for the different unit prices of the formwork since the income of skilled labor are unequal.

Finally Figure 3.8 shows the cost for each solution and each panel a gap 7 to 8 meters.

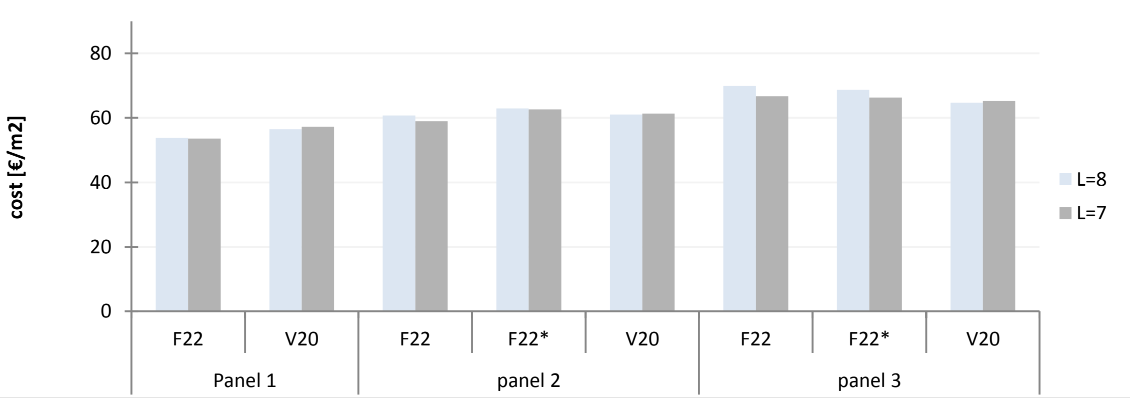


Figure 3.8 – Final cost

With the span reduction the amount of reinforcement decreases whereas the amount of concrete is increased, causing an equivalent cost between with beams and flat solution is substantially.

4 Conclusion

The work aimed to provide a broader description of the calculation methodologies for concrete slabs, flat and with beams, and especially the evaluation of its deformations.

In the case study, the "Adapted Global Coefficient Method" was compared with the method of the Formal Coefficients Global presented by Favre et al (1985), where a marginal difference is reached, so that the two methods are, in practical terms, equivalent. Furthermore, the Adapted Coefficients Global Method is also compared to the Equivalent Inertia Method, presented by Brandson (1977 apud GHALI, FAVRE, ELDBADRY, 2002), hosted by the American standard ACI-435. It comes to the conclusion that the differences between the two methods differ roughly 10% from each other.

Favre et al (1985) proposed two methodologies for the calculation of deformations in slabs, the Direct Method and the Method of Orthogonal Bands. Using the Adapted Global Coefficient Method, it is concluded that, based on this study, the Direct Method, in general, underestimates the deflection around 20% in comparison with the method of Orthogonal Bands.

The evaluation of the structure deformations, considering the various panels with different boundary conditions, allowed to observe that, in the center of the panel, creep and cracking caused an increase, in relation to the elastic values, ranging from three to seven times depending on the degree of cracking and five times in average. In not cracked elements, the deflection occurs mainly due to creep, which will reach a value, approximately, three times greater than the elastic deformation. The quantity of reinforcement is a major factor as it is restricting the increase of the deformation over time. Therefore, the greater the quantity of reinforcement is, smaller the increase of the deformation on time will be.

Regarding deformation and amount of concrete, both solutions are equivalent for central panel (1). For peripheral panels, the slab with beams (V20) has a higher performance. It was also concluded that slabs with beams, on the alignment of the beams, display a better quality of increment deformation which may result in a better solution to prevent damage on non-structural elements. In visual terms a flat slab turns out to be a better solution, because the increasing of curvature from the column until the center of the panel is more gradual. After decreasing the span from 8.0 to 7.0 meters and not modifying the structural elements, it was revealed, for the same cost, an improvement of quality of deformation around 45%.

5 References

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