The Kerr black hole hypothesis: a review of methods and results

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While a general, powerful framework for testing and constraining gravity theories in the weak field, slow-moving regime exists for several decades (the parametrized post-Newtonian formalism), an analogous experimental and theoretical framework to test the strong-field and strong-curvature regime of gravity is still being developed. One of the most remarkable strong-field predictions of Einstein’s General Theory of Relativity is the Kerr black hole hypothesis. It states that all astrophysical black holes in isolation are described by the Kerr solution, and therefore completely defined by just two parameters: their mass and angular momentum. In this paper we discuss a family of parametrically deformed Kerr spacetimes and study the relative importance of different order parameters. We construct a generalization of this spacetime and study the importance of the different types of parameters, and argue that neither of the metrics can be mapped to rotating black hole solutions in alternative theories of gravity even in the limit of small parameters and slow rotation.

INTRODUCTION

The existence of black holes (BH) in the Universe has become a scientific paradigm. After Einstein’s publication of his General Theory of Relativity in 1916, it took less than two months for Karl Schwarzschild to discover its only static (vacuum) BH solution. However rotating BHs remained elusive until Roy Kerr discovered one such exact solution, in 1963. In the decade that followed the discovery of Kerr’s solution, the work of many people has revealed many of its special properties of integrability, separability and uniqueness. Specially important are the uniqueness (or no-hair) theorems that Israel, Hawking, Carter, Robinson and others [1–4] have proved (under different mathematical assumptions): the unique end-state of gravitational collapse in a stationary, axisymmetric, rotating, asymptotically flat, vacuum spacetime, if we require that there be no closed timelike curves and that singularities are always hidden behind an event horizon, is the Kerr metric. Except for very short transient periods such as mergers, BHs in the universe are expected to satisfy the conditions of the theorem above to a very high degree of precision, and all astrophysical BHs are thought to be described solely by the Kerr solution and its two parameters, in what has been called the Kerr black hole hypothesis.

General relativity has passed all experimental tests so far: from the classical tests of Mercury’s perihelion precession, light’s deflection by the Sun and light’s gravitational redshift to binary pulsar systems, among others. However even these latter do not probe the regime of strong gravity. Taking $M$, $R$ and $v/c$ as a system’s characteristic mass, length and velocity, respectively, one can characterize the strength of the gravitational field [5, 6] by its dimensionless compactness $C = \frac{GM}{Rc^2}$ and spacetime curvature $\xi = \frac{GM}{Rc^2}$, where $G$ is Newton’s constant and $c$ the speed of light in vacuum. For a body orbiting the surface of the Sun or for a binary pulsar system $C \sim 10^{-6}$, $v/c \sim 10^{-3}$ and $\xi \sim 10^{-28} cm^{-2}$, while on the surface of a neutron star or event horizon of a stellar mass BH one has $C \sim 0.1 - 1$ and $\xi \sim 10^{-13} cm^{-2}$, and $v/c \sim 0.4$ prior to merger.

While Einstein’s Equivalence Principle, tested at least at the level of 1 part in $10^{13}$ [7], makes life very hard for non-metric theories of gravity [8], dozens of alternative metric theories have since been proposed - even as late back as Gunnar Nordström’s in 1913. Most of these are by now ruled out by Solar System experiments, binary pulsars systems and the Parametrized Post Newtonian (PPN) framework [9], but many alternative theories that include GR as a special case remain only constrained, and do predict qualitative and quantitative differences from GR in the strong field regime.

So far however no general consistent framework to test strong field gravity has been developed, and current approaches have been divided in two kinds [10]: a top-down and bottom-up approach. In the top-down case, one modifies and parametrizes the action, and studies how these deviations can be constrained by observations (something which can involve tremendous amount of work for one single alternative theory). In the bottom-up approach one adopts a phenomenological parametrization of the observations and spacetime geometry and infers how these should modify the underlying theory, while aiming for generic tests of gravity theories such as those of Lorentz and parity violation, variable $G$ and massive graviton, and polarization modes of gravitational waves [6, 11].

On the experimental side, the next decades promise a second golden age of general relativity. The first de-
rection of gravitational waves is around the corner, and will potentially be achieved in the next 5-10 years with the LIGO/VIRGO Earth-based detectors [12]; in parallel, the space-based detector LISA (to launch perhaps before 2030) will open the field of millihertz gravitational wave astronomy. In the last few years, X-ray observations of accretions discs already provided measurements of the spin of stellar mass and supermassive BHs (e.g., [13–15]), to name one example of strong field phenomena that increasingly accurate observations in the electromagnetic spectrum have the potential to deliver.

Among the special properties of the Kerr spacetime is its multipole moment structure, given by [16–18]:

$$M_t = M_i + iS_l = M(ia)^l,$$  

(1)

where $M_0$ is the mass and $S_1 = aM$ is the angular momentum (in units where $G = c = 1$). This relation has been a key part in proposals to test the Kerr BH hypothesis. Because setting the values of $M$ and $a$ locks those of all other multipole moments, having independent measurements of three different moments is enough to perform a null-hypothesis test. While the first multipole moment, the mass $M$, can be measured from Newtonian far-field observations, sufficiently accurate measurements of the other moments require either probing the central region extremely closely, as with accretion discs tests, or very clean and long observations such as extreme mass ratio inspirals.

The spacetimes proposed so far to test the Kerr BH hypothesis include the Manko-Novikov metric [19, 20], the quasi-Kerr solution [21] and the so-called bumpy BHs [22–24]. In this work we study an alternative spacetime put forward by Johannsen and Psaltis [25]. We describe this solution and its properties in the next section, and in following we study the relative importance of the higher order parameters. In the last two sections we construct a generalization of this metric and study the importance of the different types of parameters and, finally, we argue that neither of these two spacetimes can be mapped to rotating BH solutions in alternative theories of gravity, even in the limit of small parameters and slow rotation.

**JOHANNSEN-PSALTIS SPACETIMES**

An alternative to the several approaches of the bumpy BH formalism listed above was recently developed by Johannsen and Psaltis [25]. The authors found the need for it because of the pathologies present in the other metric perturbations which they find unsuitable for tests involving observations of the images of inner accretion flows, X-ray observations of relativistically broadened iron lines or of the continuum spectra of accretion disks, for which good behaviour very close to the event horizon is crucial. The approach taken was to perturb the Schwarzschild metric and obtain a rotating metric using the Newman-Janis algorithm. The difference is that they neither impose Einstein’s equations as Glampedakis & Babak [21] and Vigeland & Hughes [23], nor the existence of an approximate Carter constant as Vigeland, Stein & Yunes [24]. Johannsen and Psaltis perturb the Schwarzschild metric

$$ds^2 = -f[1 + h(r)]dt^2 + f^{-1}[1 + h(r)]dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$  

(2)

as in [26] (although in this case both component perturbations are the same), where the perturbation is chosen to be a series of inverse powers of $r$:

$$h(r) \equiv \sum_{k=0}^{\infty} \epsilon_k \left(\frac{Mr}{r}\right)^k$$  

(3)

Then the Newman-Janis procedure [27–29] is applied and one arrives at the modified Kerr metric

$$ds^2 = -[1 + h(r, \theta)] \left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr \sin^2 \theta}{\Sigma} \left[1 + h(r, \theta)\right]dt d\phi + \frac{\Sigma[1 + h(r, \theta)]}{\Delta + a^2 \sin^2 \theta h(r, \theta)} dr^2 + \Sigma d\theta^2 + \Sigma \sin^2 \theta d\phi^2$$  

(4)

where

$$h(r, \theta) \equiv \sum_{k=0}^{\infty} \left(\epsilon_{2k} + \epsilon_{2k+1}\right) \left(\frac{M^2}{\Sigma}\right)^k$$  

(5)

and one has the usual Kerr functions

$$\Sigma \equiv r^2 + a^2 \cos^2 \theta, \quad \Delta \equiv r^2 - 2Mr + a^2$$  

(6)

Constraints on the function $h(r, \theta)$ were then analysed on the basis that the spacetime is asymptotically flat, that it agrees with the observational weak-field constraints on deviations from the Kerr metric and that the Einstein equivalence principle is valid, but Einstein’s equations are not imposed.
### TABLE I: Inner accretion disc edges and Killing horizon topology for the Johannsen-Psaltis spacetime.

<table>
<thead>
<tr>
<th>Spin parameter</th>
<th>Deviation parameter</th>
<th>Killing horizon topology</th>
<th>ISCO</th>
<th>Vertical instability</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a \leq 0 )</td>
<td>( \epsilon_3 &lt; \epsilon_3^{\text{bound}} )</td>
<td>Spherical</td>
<td>Yes</td>
<td>Inside of ISCO</td>
</tr>
<tr>
<td>( a &gt; 0 )</td>
<td>( \epsilon_3 &gt; \epsilon_3^{\text{bound}} )</td>
<td>Disjoint</td>
<td>Yes</td>
<td>Inside of ISCO</td>
</tr>
</tbody>
</table>

Asymptotic flatness requires \( \epsilon_0 = \epsilon_1 = 0 \), and through the parameterized post-Newtonian (PPN) approach we find an observational constraint on \( \epsilon_2 \). In this framework, asymptotic flatness is expressed as

\[
d s^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega
\]

where

\[
A(r) = 1 - \frac{2M}{r} + 2(\beta - \gamma) \frac{M^2}{r^2},
\]

\[
B(r) = 1 + 2\gamma \frac{M}{r}
\]

and where the PPN parameters \( \beta \) and \( \gamma \) equal one in general relativity.

The best current constraint on \( \beta \) is from the Lunar Laser Ranging experiment [31]:

\[
|\beta - 1| \leq 2.3 \times 10^{-4},
\]

which implies

\[
|\epsilon_2| \leq 4.6 \times 10^{-4},
\]

since it is found that the asymptotic form of this modified Kerr metric identifies

\[
\epsilon_2 = 2(\beta - 1),
\]

\[
\gamma = 1.
\]

With this constraint, Johannsen and Psaltis chose to set all \( \epsilon_n \) to zero, except for \( \epsilon_3 \), the first unconstrained parameter. The function \( h(r, \theta) \) therefore is now given by

\[
h(r, \theta) = \epsilon_3 \frac{M^3 r}{\Sigma^2}
\]

The properties of the Johannsen-Psaltis metric that have been studied by several authors [25, 30, 32, 33] include the existence of closed event horizons and the structure and dependence of the inner disc edge instabilities. Surprisingly it was found that the parameter space is divided in two disjoint regions: one where the inner disc edge instability is radial and there is a closed spherical topology Killing horizon, and a second one where the inner disc edge instability is vertical and the Killing horizon is disjoint. This is shown in figure 1. There is however a third, very thin, region separating the two regions above where the Killing horizon has spherical topology and where the inner edge instability is vertical.

Liu, Chen and Jing [34] investigated the properties of the ergosphere and energy extraction by the Penrose process for this metric, while constraining the extra parameter \( \epsilon_3 \) so that the spacetime maintains a closed event horizon. They have found that the ergosphere is sensitive to \( \epsilon \) and becomes wider with its increase, and also that for \( a \leq M \) the maximum efficiency of energy extraction by the Penrose process can be as much as 20.7% larger than for the Kerr BH. Furthermore, because for the JP metric

![FIG. 1: ISCO type and location for the Johannsen-Psaltis spacetime. The red curve divides the parameter space in two regions. For the region marked in blue the accretion disc inner edge is radially unstable, and for the region in green it is vertically unstable. (Figure 6 of [30]).](image-url)
where \( a > M \) is allowed for some range, the maximum efficiency can exceed in 60% the Kerr case. Konoplya and Zhidenko [35] found that scalar, electromagnetic and Dirac fields are stable in the Johannsen-Psaltis spacetime for \( a = 0 \), but the rotating case has not yet been studied due to the non-separability of the equations.

The degeneracy between the spin \( a \) and the parameter \( \epsilon_3 \) in the X-ray emission from accretion discs has been investigated in several studies, mostly by Bambi [36–39]. Both parameters have a very similar impact on the profiles, since both have a similar impact on the inner edge instability, and a pure Kerr BH with a certain spin would be hard to distinguish from a Johannsen-Psaltis spacetime with a different value of spin and a non-zero value of \( \epsilon_3 \).

## Dependence on Higher Order Parameters

In this section, we relax the requirement that all extra parameters in the JP metric except for \( \epsilon_3 \) are zero and compute the ISCO (Innermost Stable Circular Orbit) location and frequency for different combinations of non-vanishing \( \epsilon_i \). In particular we show that for any value of the spin the (coordinate-dependent) ISCO can take its Kerr value for a certain combination of at least two non-zero parameters \( \epsilon_i \), but the same is true of the invariant orbital frequency \( \Omega_\phi \). This indicates how any experimental approach based on the assumption that \( \epsilon_i = 0 \) for \( i \geq 4 \) is rendered blind to the possibility that the observed spacetime, for which a certain \( \epsilon_3 \) and spin \( a \) would be estimated, is in reality a spacetime with, for example, a much larger value of \( \epsilon_3 \) and a non-zero value of \( \epsilon_4 \).

We calculated the ISCO radius, frequency and energy as a function of \( \epsilon_3 \) and \( \epsilon_4 \), for a spin of value \( a/M = .5 \) and \( a/M = .95 \). Similar results were obtained for higher order parameters and different values of the spin. It is clear that a certain value of the ISCO can be obtained for very different values of \( \epsilon_3 \), depending on the value of \( \epsilon_4 \).

This indicates that the estimate of \( \epsilon_3 \) of an experimental approach based on the assumption that \( \epsilon_i = 0 \) for \( i \geq 4 \) is not valid, as the real value of \( \epsilon_3 \) could in fact be much larger or even have the opposite sign. This is a fundamental difference from Ryan’s approach [40] where the estimate of each of the multipole moments, such as the mass, spin, and quadrupole moment, are independent of the remaining moments.

By expanding the ISCO radius and frequency as

\[
X = X_{Kerr} + \delta X_3 \cdot \epsilon_3 + \delta X_4 \cdot \epsilon_4 + \delta X_5 \cdot \epsilon_5 ,
\]

where \( X \) denotes the quantities \( R_{ISCO} \) and \( \Omega_{ISCO} \), we computed the first-order shifts in the ISCO frequency as a function of \( a/M \), as shown in the left panel of figure 2.

The plot shows the hierarchy between different parameters: to higher orders correspond smaller shifts, which is the same result of the non-linear calculation of the previous section. Note that this is a nontrivial result because equation (3) is a large distance expansion which, in principle, is not guaranteed to converge in the strong-field region near the ISCO. Indeed, such hierarchy deteriorates in the near-extremal limit, \( a \to M \). As shown in figure 2, all linear corrections are roughly equally important in this limit.

## A Generalization of the Johannsen-Psaltis Metric

We now study a generalization of this metric, starting from the following seed metric:

\[
ds^2 = -f(1+h^t) dt^2 + f^{-1}(1+h^r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

where \( f \equiv 1-2M/r \), and which differs from the JP metric because we take \( h^t(r) \neq h^r(r) \), in general. We keep its functional form:

\[
h^t \equiv \sum_{k=0}^{\infty} \left( \frac{M}{r} \right)^k, \quad \epsilon = t, r.
\]

Defining the functions

\[
K \equiv \sqrt{\frac{1+\epsilon}{1+h^t}}, \quad H \equiv \sqrt{(1+h^r)(1+h^t)},
\]
we carry out a transformation to Eddington-Finkelstein coordinates by the implicit relation

\[
du' = dt - f^{-1} K dr, \quad d\alpha' = d\phi - \epsilon d\theta,
\]

This and the remaining procedure always reduces to the JP case when \( h^t = h^r \).

In the same way as JP, we find the contravariant form of the metric and the complex null tetrad:

\[
l^\mu = \delta^\mu_\mu
\]

\[
n^\mu = \frac{1}{H} \left( \delta^\mu_\mu - \frac{f}{2} \frac{1+h^t}{H} \delta^\mu_\phi \right),
\]

\[
m^\mu = \frac{1}{\sqrt{2}\epsilon} \left( \delta^\mu_\phi + \frac{i}{\sin \theta} \delta^\mu_\phi \right).
\]

This differs from the JP case only for \( n^\mu \).

Following the Newman-Janis procedure, we arrive at the rotated metric

\[
l^\mu = \delta^\mu_\mu
\]

\[
n^\mu = \frac{1}{H(r, \theta)} \left( \delta^\mu_\mu - \frac{f(r, \theta)}{2} \frac{1+h^t(r, \theta)}{H(r, \theta)} \delta^\mu_\phi \right)
\]

\[
m^\mu = \frac{1}{\sqrt{2}(r + i a \cos \theta)} \left[ i a \sin \theta (\delta^\mu_\mu - \delta^\mu_\phi) + \frac{1}{\sin \theta} \delta^\mu_\phi \right],
\]
where we now have
\[ g_{tt} = -f(1 + h^t), \]
\[ g_{rr} = \frac{\Sigma(1 + h^r)}{\Delta + a^2 \sin^2 \theta \theta'}, \]
\[ g_{\theta\theta} = \Sigma, \]
\[ g_{\phi\phi} = \sin^2 \theta \left[ \Sigma + a^2 \sin^2 \theta \left( 2H - f(1 + h^l) \right) \right], \]
\[ g_{\theta\phi} = -a \sin^2 \theta \left( H - f(1 + h^l) \right), \]

Finally this yields a generalized Johannsen-Psaltis (JP2) metric in the covariant form:

\[ h_{\theta\phi} = \sin^2 \theta \left[ \Sigma + a^2 \sin^2 \theta \left( 2H - f(1 + h^l) \right) \right], \]
\[ g_{\phi\phi} = -a \sin^2 \theta \left( H - f(1 + h^l) \right), \]

where for ease of notation the dependencies on \( r \) and \( \theta \) have been dropped.

Imposing asymptotic flatness requires \( \epsilon_0^t = \epsilon_0^r = \epsilon_0^\phi = 0 \). The PPN bound, which for the JP metric constrained \( |\epsilon_2^\phi| \leq 4.6 \times 10^{-4} \), for this generalized metric only constrains \( |\epsilon_2^\phi| \leq 4.6 \times 10^{-4} \), while \( \epsilon_2^\phi \) is unconstrained.

The right panel of figure 2 shows the shifts of the ISCO frequency for the JP2 metric in the small parameter limit, obtained using an analogous expansion to (15). It reveals that in the slowly rotating limit the \( \epsilon_i^r \) perturbations lead to frequency shifts of higher order than the \( \epsilon_i^r \), while the reverse is true for the fast spinning case (\( a/M \gtrsim 0.9 \)). This behaviour stems from the fact that the effective potential does not involve the \( g_{rr} \) component. In the absence of rotation this is the only component involving the \( \epsilon_i^r \) terms, and these scale with the rotation parameter in the \( g_{rr} \) and \( g_{\phi\phi} \) components (equations (34) and (33)). In particular this analysis has shown that the magnitude of the frequency shift related to \( \epsilon_2^\phi \) is larger than the one from \( \epsilon_3^\phi \) or \( \epsilon_4^\phi \) in the fast-spinning case. For the two types of parameters one finds the same hierarchy as in the JP metric: higher order terms are increasingly subdominant.

**NON-MATCHING TO ALTERNATIVE THEORIES**

There are at least two analytical solutions of slowly rotating BHs in alternative theories of gravity: one found by Yunes and Pretorius [41] for Dynamical Chern-Simons (CS) Modified Gravity, and another found by Pani, Macedo, Crispino and Cardoso [42] for a class of alternative theories obtained by including all quadratic, algebraic curvature invariants generally coupled to a single scalar field. However we show that the JP metric cannot describe such solutions even when using the more general case derived above with \( h_{\theta\phi}^l \neq h_{\phi\theta}^r \).

By expanding the JP2 metric in the slow rotation approximation we show that it cannot be mapped to any of the two known slowly rotating BH spacetimes in alternative theories, and is unlikely to be matchable to any other future known solution even in the small parameter and small rotation limit. The main reason is because there is not enough freedom over different components: when attempting the match, the terms of different inverse powers of \( r \) of one metric component (e.g., \( g_{tt} \)) essentially fix half of the parameters, rendering the matching unfeasible. It is clear that if the JP2 metric cannot be matched the same if true of JP, as the latter is a particular case of the former. One has only two general perturbation functions for three a priori independent metric components.

The \( g_{tt} \) component of the JP2 metric in the small rotation and parameters limits, that is, valid to order \( \mathcal{O}(a^2, \epsilon_i, a \epsilon_i) \), is given by:

\[
 g_{tt} = -1 - \epsilon_0^t + \frac{2M}{r} + M^2 \epsilon_0^t \frac{2 \epsilon_0^t - \epsilon_1^t}{r} + M^2 \epsilon_1^t \frac{2 \epsilon_1^t - \epsilon_2^t}{r^2} - \frac{2a^2 M \cos^2 \theta}{r^3} + M^4 \epsilon_3^t \frac{2 \epsilon_3^t - 2 \epsilon_4^t}{r^4} + M^4 \epsilon_4^t \frac{2 \epsilon_4^t - 3 \epsilon_5^t}{r^5} \frac{2 \epsilon_5^t - \epsilon_6^t}{r^6} + M^6 \epsilon_6^t \frac{3 \epsilon_6^t - 2 \epsilon_7^t}{r^7} + M^6 \epsilon_7^t \frac{3 \epsilon_7^t - 8 \epsilon_8^t}{r^8} + \ldots
\]
FIG. 2: Shifts to the ISCO frequencies, $\delta \Omega_i$, in the JP (left panel) and JP2 (right panel) metrics, in the small parameter limit, as a function of the spin $a/M$. Left: $\epsilon_3$ (black), $\epsilon_4$ (red), $\epsilon_5$ (blue). Right: $\epsilon_2^r$ (dashed green), $\epsilon_3^r$ (dashed black), $\epsilon_4^r$ (dashed red), $\epsilon_3^t$ (black), $\epsilon_4^t$ (red).

$$g_{tt} = -1 + \frac{2M}{r} - a^2 \frac{2M \cos^2 \theta}{r^3} - \frac{1}{4} \alpha^2 \left( -\frac{49}{40Mr^4} + \frac{1}{3Mr^3} + \frac{26}{5r^5} + \frac{22M}{5r^3} + \frac{32M^2}{5r^6} - \frac{80M^3}{3r^7} \right),$$

(36)

and is accurate up to order $O(a^2, \alpha^2, a\alpha^2)$.

Inspection of equation (35) makes it clear why matching to the JP2 metric is not possible in general, even for metrics whose terms are just inverse powers of $r$. Matching the terms proportional to $a^2 \cos^2 \theta$ is sufficient to fix almost all parameters. Each $\epsilon_i^t$ coefficient appears in 4 different terms as a linear combination with another coefficient (twice with $\epsilon_i^t - 1$ and twice with $\epsilon_i^t + 1$), rendering this into an overconstrained system. Of course, this does not exclude the existence of a coordinate transformation that could allow the matching, but this is unlikely, and one could look at metric invariants, such as curvature scalars at the horizons, to disprove the possibility.

The slowly rotating BH solution of dynamical Chern-Simons [41] is given by

$$ds^2 = ds^2_K + \frac{5}{4} \frac{\alpha^2}{\beta r^3} \left( 1 + \frac{12 M}{7 r} + \frac{27 M^2}{10 r^2} \right) \sin^2 \theta dt d\phi,$$

(37)

where $ds^2_K$ is the slowly rotating Kerr metric. Because this reduces to the Schwarzschild solution when $a = 0$ matching to the JP or JP2 is not possible, since these do not reduce to the Schwarzschild case when $a = 0$ unless all of the $\epsilon_i$ parameters are zero.

CONCLUSIONS

To attack the problem of measuring deviations from the Kerr metric, different spacetimes have been recently proposed and studied. These solutions are parametrically deformed from Kerr, that is, they possess additional parameters besides the mass and the spin. These spacetimes can be fitted to observations in order to check whether they agree with the hypothesis that these additional parameters are zero, that is, that astrophysical BHs are described by the Kerr metric of general relativity. The metrics reviewed have different strengths and drawbacks, the most promising proposals so far being the generalized bumpy BHs proposed by Vigeland, Yunes and Stein for gravitational wave tests and the Johannsen-Psaltis metric for electromagnetic spectrum tests.

The Johannsen-Psaltis spacetime possesses an infinite number of parameters $\epsilon_i$, experimentally unconstrained for $i \geq 3$. The existing studies of the this metric usually assume only one non-zero additional parameter, namely $\epsilon_3$. Here we have relaxed this condition and studied the relative importance of different order parameters. We concluded that there is a hierarchy: higher orders terms are subdominant, although in the limit $a \rightarrow M$ they are roughly equally important. This difference in magnitude however is less than one order of magnitude, and there is a strong degeneracy between parameters of similar order. Therefore if one sets all parameters to zero except, say, $\epsilon_3$ the experiment is rendered blind to the possibility that the observed spacetime is in reality a spacetime with a much larger value (or even opposite sign) of $\epsilon_3$ and a non-zero value of $\epsilon_4$, for example. This is a fundamental difference from Ryan’s approach [40] to probing the spacetime geometry with EMRIs, in which, in a post-Newtonian expansion, higher multipole moments do not affect the measurement of lower multipoles.

We constructed a generalization of the Johannsen-Psaltis metric and studied the importance of the new pa-
rameters. To each parameter of the original metric now corresponds two parameters, and the Johannsen-Psaltis metric is recovered in the limit where the two parameters in each pair are equal. We found that the PPN bound does not constrain $\epsilon_3$ unlike in the JP spacetime, and it is set to zero. For this generalized metric we computed the ISCO frequency shifts in the small $\epsilon_3$ limit, and reached the same conclusion: the parameters are hierarchized by order. However for this metric a different role is played by the different types of parameters. Which one is dominant depends of the value on the rotation parameter. In particular we showed that the magnitude of the frequency shift related to $\epsilon_3$ is larger than the one from $\epsilon_3^2$ or $\epsilon_3^3$ in the fast-spinning case. Finally, by using this generalized metric we argued that the Johannsen-Psaltis spacetimes cannot be matched to rotating BHs in alternatives theories of gravity, even in the limit of small parameters and slow rotation.


[37] Cosimo Bambi, “Probing the space-time geometry around black hole candidates with the resonance models for high-frequency QPOs and comparison with the continuum-fitting method,” JCAP 1209, 014 (2012), arXiv:1205.6348 [gr-qc].


