Evolution of Fairness Under N-Person Ultimatum Games

MSc Thesis
Extended Summary

Fernando P. Santos*
*Departamento de Engenharia Informática and INESC-ID
IST-Taguspark, Universidade Técnica de Lisboa,
2744-016 Porto Salvo, Portugal
fernando.pedro@ist.utl.pt

***Abstract*—Fairness and cooperation are behavioral traces that, even if apparently irrational, are widespread across human societies. Ultimatum Game is one common metaphor from Game Theory that allows the study of these behaviors. Although it is vast the literature that addresses this game in a form of pairwise interaction, it is not clear what would be the effects of playing it within a group. Yet, some market exchanges, summits, political treaties or various referendum activities are often developed in a context that involve proposals being subject of group suffrage. In this context, we propose a N-Person version of the Ultimatum Game. With this new framework we studied the mechanisms that promote fairness among groups, resorting to computational simulations of multiagent populations, where each agent acts and learns using the processes proposed by Evolutionary Game Theory. We concluded that the number of needed persons to accept a proposal has a huge influence on the emergence of fair proposals and on the selective pressure of punishment: if more people is required to accept a proposal, offered values tend to rise. We also verified the multiple points of convergence that may be obtained, when we assume that the population is organized in complex structures. In general, we concluded that an uncertainty leads to fairness.**

I. Introduction

A model is a simplification of reality. In this context, modeling human behavior is to make assumptions about what we believe that happens in reality, based on some evidence. Many disciplines, such as philosophy, psychology or political science, have special concerns in inquiring what are good assumptions to make about human nature. Computer science, in particular, has a special interest in this field, namely in branches of artificial intelligence or affective computing. Classical economics, on the other hand assumes that human behavior is taken for granted and summarized by a deterministic rule: the theory of **rational** choice. This theory defines human decision-making in simple terms, suggesting that people always measure the benefits, always subtract the costs and always act in accordance with the maximization of returns. Common sense may confirm this assumption. However, laboratory experiments demonstrate an apparent *irrationality* of humans, usually preferring fair strategies, thus not following the selfish ones [1] [2] [3]. Across societies, division of labor [4], the creation of welfare states [5] and international treaties are also examples of cooperation among humans. Looking at the apparent rationality of selfishness and the origin of cooperation, the major problem we are addressing in this paper can be summarized by the question: **What mechanisms explain the emergence of fairness and cooperation?**

The importance of this problem is enormous, and its answers may have an explanatory and orienting purpose. Explanatory as they may help to explain the animal evolution and how we, humans, end up having such a complex behavior. Orienting, because knowing how cooperation emerges may be a requisite to know how cooperation can be maintained or even fostered.

Following a widely accepted notation, we assume that cooperation happens when an individual concedes a benefit to another, incurring in a cost to himself [6]. Those two concepts, cooperation and fairness, have been modeled in the context of different game metaphors. The academic field where this study is performed is called **Game Theory**, and it suggests the use of interesting interaction paradigms as, for example, the **Ultimatum Game**. This game was first proposed by Guth, Schmittberger and Schwarze [7] and its simplicity contrasts with the surprising experimental results derived from it. In this game, an agent in the role of Proposer is given a quantity of something (let's say, money). Then, he must decide an amount to give to other agent, the Responder. The Responder, knowing the offer and the total amount possessed by the Proposer will decide to accept or reject the offer. Acceptance will guarantee to each agent a payoff equivalent to the division proposed. Rejection will let both agents with nothing. Anonymity is imposed between individuals, so reputation is not a concern. The predicted outcome of this game consists in a very low offer from the Proposer and the unconditional acceptance of the Responder. Each one is trying to maximize the payoff: the Proposer saving the biggest slice to himself and the Responder trying to earn something, in alternative to nothing. Yet, as we will see better, this is not what really happens during experimental situations.

In this game, complexity can be introduced, if we allow agents to observe the behavior and success of each other and if that observation enables social learning. **Evolutionary Game Theory** (EGT) [8] is the framework that allows to introduce complexity in traditional game theory. Using it, we associate payoffs with fitness in the previous game theoretical models, and the idea of Game Theory can be used to understand evolution. Agents with more payoff will be considered more fit and so, will be able to reproduce often. Reproduction can

be understood, not only in terms of generating new agents, but also in reproducing ideas and making other agents to imitate one's behavior. Now, the goal is not to predict how agents will behave when interacting with each other, but rather how the behavior of agents will change during evolution. As such, we will not have a pair of agents anymore, but a population of them interacting successively.

Using such framework, we propose a new group bargaining game, consisting in a multiplayer generalization of the traditional Ultimatum Game. We named it N-Person Ultimatum Game. Using this game as framework, we were able to study the effect of different parameters in the possible evolution of fairness. Directly related with the imitation process, we study the effect of selection strength and mutation values. Related with the game, we study the effect of varying the number of rejections that must occur in the group to overall reject a proposal. We also study the effect of playing the game in different population structures (well-mixed, regular and scale-free). This paper is organized as follows:

In the next Section we will present key concepts and related works, useful to understand the fundamentals and motivation of our proposal. In Section III, we formalize the N-Person Ultimatum Game and derive some analytical conclusions. In Section IV, we show the results of the computational simulation using the N-Person Ultimatum Game, in three different population topologies. To finalize, in Section V we draw some concluding remarks and provide some insights into future work.

II. Related Work

As stated in the previous Section, our goal with this work is the study of the mechanisms that promote fairness, in a context of Ultimatum Game played by N agents. Using other game metaphors, there are some mechanisms that already propose solutions to the conundrum of the evolution of fairness and cooperation. In this section, we present some of these works.

A. Mechanisms that favor cooperation

Cooperation happens. Yet, to simulate and explain its emergence, EGT is not enough. Some other assumptions have to be made, so cooperation can beat defection in the struggle to be a stable strategy. Nowak summarizes five of the mechanisms that allow the individuals to forgo their selfish pretensions [6]. They are kin selection, direct reciprocity, indirect reciprocity, structured populations and group selection. Each of these mechanisms, plus strong reciprocity [2], are explained with more detail next. Notice that these mechanisms are studied in the context of Prisoner's Dilemma, as this is the common game to model the strategic decision of one agent to cooperate or defect. The payoff matrix of this game is summarized in Table I. The rational choice is to Defect, since it is the strategy that leads to the best outcome.

	Cooperate	**Defect**
Cooperate	benefit-cost	-cost
Defect	benefit	0

Table I: Payoffs of Prisoner's Dilemma corresponding to the player performing the column decisions.

One of the first mechanism proposed was **Kin Selection**. This may be defined as the cooperation between relatives. It was an idea firstly proposed by Hamilton [9] and may be summarized as follows: an individual will cooperate with his relative because the benefit of the second also favors the first. This relationship occurs if we assume that the purpose of an individual is the safeguard and propagation of his genes. A related individual will carry genes of that individual with a given probability (this probability is called relatedness), so the payoff of the *relate* will contribute to the payoff of the individual, proportionally to their relatedness. This explanation is convincing, yet insufficient. Indeed, is easy to observe cooperation among people that are not related genetically. The role of repeated interactions was then studied in the context of **Direct Reciprocity**.

To reciprocate means to answer with the strategy that previously someone used against us. Trivers was pioneer in the study of reciprocity as the basis for cooperation [10], and later, Axelrod [11] promoted a tournament of Prisoner's Dilemma games. The conclusion was that Tit-for-Tat (TFT) is the winning strategy. TFT only states that an agent should first cooperate and then use the strategy that his opponent used in last round. This leads to the conclusion that reciprocity can be seen as an emergent phenomenon, leading agents with more payoff and, transposing EGT language, more fitness. However, TFT may be considered too simplistic. What would happen if one player decided, by mistake or on purpose, defect in one play? A cycle of defections will occur forever. Regarding this problem of a mistake infecting all the population, and knowing that a population of unconditional cooperators can easily be invaded by defectors, Nowak and Sigmund [12] proposed Win-Stay Lose-Shift as the winning strategy in the context of repeated Prisoner's Dilemma and, this time, using an EGT framework. Direct reciprocity extended the kin selection theory as a promoter of cooperation, but it is still not enough as it is not unusual to observe cooperation even if you never played before with the same person.

This is how **Indirect Reciprocity** comes into scene. People may cooperate just to improve their reputation, maybe hoping that a good prestige will have future benefits. Cooperation may also occur with persons that one will never meet or interact again. Nowak and Sigmund [13] modeled indirect reciprocity using, again, EGT. The model proposed considers that all the agents in a population have an *image* accessible by all other agents. Such *image* is the result of past actions: if the agent cooperates, that *image* is improved; if agent defects, the image downsizes. It is shown that, if the strategy used by one agent is also based on the other's image, cooperation dominates the population. The evidence of indirect reciprocity can be visible in the existence of social norms. Ohtsuki and Iwasa studied this issue [14] [15]. They introduced social norms as being the assignment of a reputation (good or bad) to an agent given his own reputation, the reputation of the opponent and the action performed (cooperate or defect). They proved that only eight social norms are able to guarantee high (and stable) levels of cooperation. Using reputation in order to define social norms was also studied by Pacheco, Santos and Chalub [16]. In this research, *stern-judging* was defined as the norm that promotes cooperation, in scenarios where individuals have reputation and act accordingly to it. This norm (*stern-judging*), one of the *leading-8 norms* proposed by Ohtsuki and Iwasa [14], states

that you will be good if you help good people and refuse to help bad people. This means that if you are good, you will encourage other people to help you. Otherwise you will be punished.

Until this point, the models reviewed assume that all individuals can freely interact with any other individual, with equal probability. This kind of populations is called well-mixed. Yet, model them this way is ignoring the fact that usually people interact only with a limited number of close persons. Populations in real world are highly **structured**, and the emergence of cooperation may be affected by that structure. Evolution in **networked populations** is a generalization of spatial reciprocity, initially proposed by Nowak and May [17], one of the most famous cellular automata discussed in the literature. In this study, the Prisoner's Dilemma was played as done traditionally, without reputation or memory. However, agents were organized in a square lattice and each one of them was only able to interact with adjacent neighbors. The dynamics of the game employed by Nowak and May was simple and deterministic. After each round, the payoff of each agent was the sum of the payoff earned during plays with the adjacent agents. Each agent will then update his behavior accordingly with the payoffs earned by the neighbors: the best one will be imitated. Although the dynamics of this game seem simple, the result was the emergence of chaotic patterns, showing the non-predictable conversion between cooperative and defective agents. The authors argued that spatial structure seem to be crucial to the emergence of cooperation. Yet, it is hard to believe that each person connects with an exact number of neighbors. The structures that model this simple arrangements are called regular graphs. However, recent advances in network theory [18] [19], allow us to be aware that populations are organized in complex interaction structures, ranging from random-like graphs to scale-free networks. **Scale-free graphs**, studied by Barabasi, Albert and Jong [19], are peculiar, having the number of edges by each node defined by a power law. Later, Santos, Rodrigues and Pacheco [20] developed a model where, using a set of agents playing Prisoner's Dilemma and structured in a scale-free graph, was concluded that cooperators gain with this kind of structure, being cooperation a strategy that dominates the population. Structure in population was then used to enlarge the mechanisms presented in previous sections (direct and indirect reciprocity). Pacheco, Traulsen, Ohtsuki and Nowak [21] combined direct reciprocity and structured populations to conclude that the variation in connections between the agents even changes the rules of the game, modifying the values of the payoff matrix. Similarly, it has been shown how different dilemmas can change the social structure, creating sufficient conditions for cooperation to thrieve [22]. Later, Vukov, Santos and Pacheco [23] created a model where each agent was able to access the previous play of the opponent, as seen in indirect reciprocity. Those agents were organized in a regular graph. The authors concluded that the combination of the reasoning about the past actions of the opponent (cognition) with the organization of the agents in a regular graph (spatial structure) enables cooperation to emerge.

The idea that cooperation may not only create successful individuals but also better **groups** was not yet presented. The rational is straightforward: if a group of cooperators has more fitness than a different one, the group itself will be able to reproduce and grow quickly. The other group may even disappear. One of the latest studies of this group dynamic in the context of EGT was done by Traulsen and Nowak [24]. In this study, each agent belongs to a group and will reproduce with a probability directly proportional to payoff. The offspring will remain in the same group. When the size of the group reaches a threshold, the group will split in two groups. The authors concluded that, if the benefit/cost ratio is sufficient, cooperation will spread.

The previous mechanisms do not explain all possible cooperative behaviors. Experimental evidence suggests that people also cooperate (or punish defectors) when gains are not expected, even in the future. These situations, not considered before, state that the players are not genetically related, will not encounter again, will not care about reputation as plays are anonymous and do not belong to a common group. Yet, they cooperate. The predisposition to cooperate in such scenarios is studied in a new area of research called **strong reciprocity**.

B. Strong Reciprocity

Strong reciprocity tries to deal with the punishment of defectors in a genuine altruistic fashion. Gintis, Bowels, Boyd and Fehr presented this idea [2], where the cost to punish an individual is spent without expecting future recover. The authors call it *altruistic punishment*. It is said that altruistic punishment, when combined with group selection, is capable of maintain high levels of cooperation in large groups [25]. Recently, Vukov, Pinheiro, Santos and Pacheco investigated the role of altruistic punishment in the context of Prisoner's Dilemma and Stag Hunt Game, considering that the games are played in scale-free networks [26]. In this case, altruistic punishment does not always explain the emergence of cooperation. The works in this area, traditionally, have as basis the Ultimatum Game, in order to explain and point out the existence of strong reciprocity in humans. Also the punishment of free-riders in Public Goods Games, in experimental cases, proves that strong reciprocity among persons must be taken into account, in the journey to explain the emergence of cooperation. The related work with Ultimatum Game and Public Goods Games is presented in the next sections.

C. Fairness and Punishment in Ultimatum game

Some authors seek to explain cooperation in the Ultimatum Game (presented in Section I) as the result of empathic behavior. In one simple sentence, empathy means to want for the other what you would want for yourself. Page and Nowak [27] modeled a population where some empathic players were introduced, studying next if their behavior infects the remaining population or if empathy is a characteristic that will disappear over time. The authors concluded that the introduction of empathic players leads to fair outcomes, as the values proposed and accepted increase considerably. Nowak, Page and Sigmund [28] also introduced the concept of reputation in Ultimatum Game, presented previously in this document, in the context of indirect reciprocity. The conclusion was that reputation would lead to the emergence of fairness. Note that this study does not satisfy the anonymity required by the defenders of strong reciprocity [2]. The spatial structure in which agents are organized was also taken into consideration, in other studies: Page, Nowak and Sigmund [29] concluded

that when the agents are arranged in a structure, only interacting with direct neighbors, the proposed values and acceptance thresholds will increase, as clusters of fairer agents may start to emerge. More recently, and again using the concept of empathy, Iranzo, Flora, Moreno and Sanchez [30] derived that empathy emerges spontaneously, if the agents are organized in a graph, only interacting with direct neighbors and if the role of proposer and receiver changes randomly between plays. Recently, Rand, Tarnita, Ohtsuki and Nowak justified that, when using stochastic game theory with mutations simulating possible mistakes made by the players, the offers and the expectation values increase considerably [31]. The reasoning is that mistakes will lead to an unpredictable and heterogeneous populations. Against this kind of populations, the best strategy is not to offer small quantities, as in the traditional Ultimatum Game.

D. Cooperation in N-Person Games

In the previous section, all the mechanisms that try to explain cooperation among individuals share a characteristic: the assumption that interactions only occur between a pair of individuals. Again, daily life poses situations where a more than two agents interact, as noticed in group hunting, common resource management or agreements to secure from external threads. Usually, N-Person Games are a generalization of Prisoner's Dilemma with N agents, being in this case called **Public Goods Game**. To play this game, each individual gives a quantity to a pile. The pile is multiplied by a factor and is divided by all the players. One of the problem of these situations is free-riding, happening when one agent decides not to contribute to the pile (defection). Note that this is the rational action to perform, since the final division of the pile will also guarantee benefit to the individuals that did not donate anything. In spite of that, cooperation seems to prevail in these games, as experimental examples suggest [2]. Some work has been made in this field, seeking to explain why cooperation emerges in this scenarios. Two possible options are considered: the punishment of defectors and the voluntarism in entering the game. Those possibilities were studied analytically by Hauert, Haiden and Sigmund [32]. The authors proved that using those assumptions (possibility to punish defectors and optional participation on the game), the emergence of cooperation is a possibility. More recently, F. Santos, M. Santos and Pacheco [33] concluded that the cooperation in Public Goods Games may emerge as outcome of social diversity and of the amount contributed by each individual. In this study, the agents are organized in a scale-free network and play only with direct neighbors. This means that one agent will participate in a number of games proportional to the number of connections he has. The social diversity mentioned has to do with this diversification in the number of possible connections.

E. Analysis of related work

As we have seen, mechanisms that explain cooperation were motif of a lot of research. Prisoner's Dilemma, Ultimatum Game and Public Goods Game are common game metaphors to study how self-interest is deployed by cooperation. However, we assume the importance of doing this study in the context of a multilayer version of Ultimatum Game. Some market exchanges, summits, political treaties or various referendum activities are often developed in a context that involve proposals being subject of group suffrage. In this context, we propose a N-Person version of the Ultimatum Game and we contribute with a study of possible mechanisms that enhance fairer proposals. This proposal is detailed in the next section.

III. Model

In this section, we present a proposal of the N-Person Ultimatum Game. In this generalization of the 2-person game, a Proposer will make an offer to a group. The individuals of that group will then accept or reject the offer. In the main version of the game (*with suffrage*), a proposal will be accepted if the number of individual rejections remain below a constant threshold (M). In the alternative version (*with responder competition*), created specially to validate the experimental work made by Fischbacher, Fong and Fehr [3], a proposal will be accepted if at least one person in the group accepts it. If more than one agent accepts, the offer will be randomly attributed to one of the acceptors. Next, we formally define both games and draw some analytical conclusions.

A. The N-Person Ultimatum Game

The average payoff of an agent in a group is given by the payoff earned when she is the Proposer and the average payoff earned when each of her co-players make one proposal. The expression stands as an average over all possible games with one giver and $N-1$ responders

$$\Pi_i = \frac{1}{N}((1-p_i)a_i + p_1 a_1 + .. + p_k a_k) \tag{1}$$

where N is the number of players in the group, Π_i is the average payoff of agent i, p_i is the offer of agent i and a_k can assume values 1 or 0, if the proposal of agent k is accepted or rejected, respectively (this function will be detailed next). Agent i plays with k opponents, numbered from 1 to k, and $N = k+1$. The proposal of one agent is accepted if the total number of rejections in the group remains below M.If $N = 2$ and $M = 1$, the game played is the traditional two person Ultimatum Game. Notice that the maximum payoff is obtained when p_i is the smallest possible and $p_1 ... p_k$ (the opponents offers) are maximized. Therefore, the pressure to **free-ride**, offering less and expecting that other will contribute, should be high. As said before, two version of this game were simulated. The first version, the one where each agent has vote power to possibly reject the game, was studied deeply. What changes between the two instances of the game is the implementation of the acceptation flag (a_k). Both versions are formalized next.

B. With Suffrage

The N-Person Ultimatum Game with suffrage aims at simulating a common political scenario where a proposal is suggested to a group. The group will then vote it, and acceptation depends upon a maximum number of objectors. If the proposal of agent *i* is accepted, a_i will acquire value 1, and will be 0 otherwise:

$$a_i = \begin{cases} 1, & \text{if } \sum_{j=1}^{N-1} \Theta(q_j - p_i) < M \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

where $\theta(x)$ is the Unit Step function, assuming the value 0 when $x < 0$ and value 1 when $x \geq 0$. In this case, a game will be accepted if the number of rejections remains below M. Notice that, an agent will reject a proposal (p) if it stands lower than his expectation threshold (q).

C. With Responder Competition

As said before, a new acceptation function was created in order to model the experimental work done by Fischbacher, Fong and Fehr [3]. In that work, the authors concluded that when the proposal is made to a group of competing responders, the values of the offers have a pressure to get lower. The reason is the human risk-aversion. If a person knows that her proposal will have more than one responder, and if the necessary condition to have the proposal accepted is one positive vote, that person will probably be less generous. The risk of having the proposal rejected is lower and so, she can try to maximize the gains, offering less.

In this version of the game, the acceptation function will have a stochastic parameter defining in what conditions a responder will get a proposal. If more than one responders accept an offer, that offer is randomly assigned to one of them:

$$a_i = \begin{cases} 1, & \text{if } p_i \geq q_j \wedge rand \leq \frac{1}{\sum_{j=1}^{N-1} \Theta(p_i - q_j)} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

where $rand$ is a random value between 0 and 1. This means that an agent will get an offer if two conditions are met. First, she must accept that offer. Secondly, she will get it with a probability that depends upon the number of acceptors.

D. Example and Predicted Outcome

Lets illustrate the game with a simple example. Suppose that Sleepy, Fair and Wicked get together to play N-Person Ultimatum Game. In this game, to simplify the setup, the total amount to divide is 10 and the agents can offer integer values from 1 to 10. **Sleepy** will offer 1 and expect 1, as he does not care for fairness neither punishment. **Fair** will offer a value of 4, near the egalitarian division, and accept everything equal or above 1. **Wicked** will offer 1 but will be very exigent and punitive, only accepting offers above or equal to 4.

It is possible to calculate the payoff earned when playing the Suffrage version of the N-Person Ultimatum Game, given two different values of M (the number of rejections needed to overall reject an offer). Those payoffs are summarized in Table II. In the case when $M = 2$, Sleepy will earn 14 of payoff, resulting from the combination of the remaining of his proposal (10-1=9), plus the offer of Fair (4) and the offer of Wicked (1). All the proposals are accepted, as only Wicked is rejecting proposals below 4 and the number of rejections to derail a proposal is 2. When $M = 1$, Sleepy will only earn 5, resulting from the accepted proposal of Fair (4) and the accepted proposal of Wicked (1). Notice that his proposal (1) is rejected by Wicked, as it is only needed one vote against to derail an offer. It is important to notice how a simple increment in M changes the relative success of strategies. Also, remember that in benefit of simplification, we are omitting the division of all average payoff by $\frac{1}{N}$, division that would not affect the relative success of the strategies.

M	Sleepy	Fair	Wicked
2	14	8	14
1	5	7	13

Table II: Payoffs of agents playing an example of N-Person Ultimatum Game.

E. Analytical properties of the mini-game

It is important to study analytically some properties of the game being proposed. However, one should remember that the use of differential equations to study the dynamics of evolution is only possible under the assumption that the population is infinite and well-mixed. To simplify our analysis, we also consider a reduced version of Ultimatum Game, usually called mini-game and described by Sigmund [34]. The use of this simplified version is necessary, due the need to constrain the number of possible strategies. In the mini-game version of the Ultimatum Game, instead of the strategies in the space $(p, q) \in [0, 1] \times [0, 1]$, we will have just 4 strategies (G_1, G_2. G_3, G_4).

Those strategies are given by two possible choices of p and q values, (h)igh and (l)ow, with the imposition that $0 < l < h < 1$. G_1, the **pro-social** strategy, is defined by $p = h$ and $q = h$. G_2, the **paradoxical** strategy, is given by $p = l$ and $q = h$. G_3 is the **mild** strategy, where $p = h$ and $q = l$. Finally, G_4 is the **asocial** strategy, where $p = l$ and $q = l$. We perform an analysis assuming that only three possible strategies co-habit in the population. This comparison requires the definition of 12 payoff functions, to evaluate the behavior of the agents in the 4 different scenarios $(\{G_1, G_2, G_3\}, \{G_1, G_2, G_4\}, \{G_1, G_3, G_4\}, \{G_2, G_3, G_4\})$.

Three different fitness functions must be used, regarding each of the three strategies in use, in each scenario. As an example, lets consider the fitness function of an agent employing the strategy G_a:

$$f_a(x_a, x_b) = \sum_{\substack{j=0 \\ k=0}}^{N-1} H(j, k, x_a, x_b) \Pi_a^{a,b,c}(j+1, k)$$
$$H(j, k, x, y) = \frac{N-1!}{j!k!(N-1-j-k)!} x^k y^j (1-x-y)^{N-1-j-k} \quad (4)$$

where x_a is the rate of agents of type G_a in the population, N is the size of the group, k is the number of elements of type G_a in the group (and j is the number of G_b) and $\Pi_a^{a,b,c}(j, k)$ is the payoff of an agent using the strategy G_a against agents using G_b and G_c, in a group with j agents using G_a, k agents using G_b, (and consequently, $N - k - j$ agents using G_c). Notice that the fitness functions correspond to the sum of the payoff earned in every possible group compositions, regarding the number of agents using each strategy. The groups are sampled using an Hypergeometric distribution, represented by the function H.

As an example, consider only the payoff function of agent using strategy G_1, used when studying the $\{G_1, G_2, G_3\}$ case:

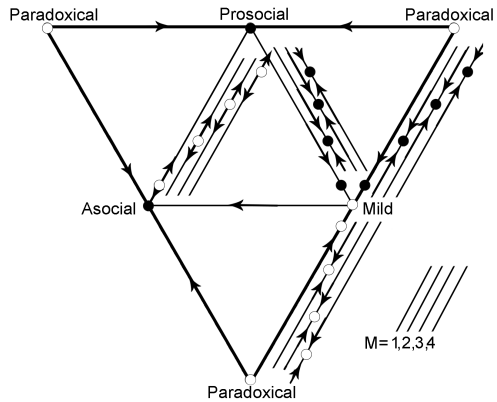

Figure 1: Simplex that represents the effect of changing M, in the evolution of strategy use. Going from left to right, the different edges represent the changes in fixed points when M increases. Circles represent rest points and white circles represent unstable rest points

$$\begin{aligned}\Pi_1^{1,2,3}(j,k) = \frac{1}{N}((1-h)+(k-1)h+ \\ \theta(M-k-j)kl+ \\ \theta(M-k-j-1)(N-k-j)l)\end{aligned} \tag{5}$$

where M is the number of agents that reject the game, below which the proposal is still accepted. $\theta(x)$ is the Unit Step function, assuming the value 0 when $x < 0$ and value 1 when $x \geq 0$. Figure 1 is a summary of what changes in the evolutionary dynamics when M is changed. Circles represent rest points and white circles represent unstable rest points [34]. Each point in the interior of the Simplex faces is a possible combination of rates of use of each strategy, in the population. We conclude some interesting properties, as M increases: **1)** an higher rate of population tends to adopt a Paradoxical strategy, instead of a Mild Strategy. **2)** It is easier to reach a point where the population could be attracted to a fixed point where everyone is Asocial, instead of Prosocial. Going from Mild to Paradoxical represents an increase in the values of q and a decrease in the values of p. Going from Prosocial to Asocial means a decrease in both p and q values. So, our principal analytical conclusion is that **increasing M will have a positive effect on q and a negative effect on p.**

IV. Evaluation

In the fields of Game Theory and Behavioral Economics, computer simulations are an important complement to analytical demonstrations and and to the laboratory experiments. This section is used to present the results concerning the computer simulations of the game. First, we will briefly explain how we proceeded in these simulations. Next, some plots are presented and their results are explained.

A. Methods

In the proposed computer simulations, an agent is defined by a p and a q value, representing the strategy used in the N-Person Ultimatum Game. At each step, one focal agent and one second agent are randomly selected from the population. The fitness of both agents is calculated. In the case of well-mixed populations, where each agent is free to interact with any other agent, the fitness corresponds to an average payoff, taken over 3000 rounds of the game, always with random opponents. In the case of structured populations, fitness is an accumulated payoff, resulting from the sum of $K+1$ games, where K is the number of neighbors of one agent. After having calculated the fitness of focal and second agent, the focal agent will update his p and q values, as a result of a learning process, or cultural adaptation. To achieve this, the focal agent will copy the second agent, with a probability given by

$$\frac{1}{1+e^{\beta(f_x-f_y)}} \tag{6}$$

where f_x is the fitness of focal agent and f_y is the fitness of second agent [35]. The constant β is the intensity of selection. If this value is 0, imitation will occur randomly. As it increases, the imitation will depend more upon the fitness difference. When imitation occurs, a random value is added to the p and q values being imitated. This mutation is located between $-\epsilon$ and ϵ. This process of updating the strategy will occur Z times, as Z is the size of the population. After this, we say that one generation elapses. In the well-mixed scenario, we measure evolution during 10^4 generations. In the structured cases, we wait 10^5 generations. This set of generations will define one run. The stationary state of one run is calculated using the time-average of the last 10^3 generations. Some of our results resort to an ensemble-average over all the runs. In other results, the stationary state after each run is represented.

B. Results

Here, we present the results of the simulations performed with the game. They are divided regarding what parameters we are studying. First, we present the effects of varying selection strength (β) and mutation space (ϵ). Next, is evidenced the impacts of changing M. Different population structures are also tested, namely, regular networks and scale-free networks. Lastly we present the outcomes resulting from the variation in the number of responders, regarding the version of the game with competing responders.

1) Selection and Innovation: The constants β and ϵ are not inoffensive, in the evolution of the average p and q values. Figure 2 represents this effect, showing the stationary average values of p and q in the population, averaged over 100 runs. As one may observe, an increase in β has a negative effect of p and q. On the other hand, if we increase ϵ, the average values of p and q in the population will also increase. The results in Figure 2 refer to simulations in well mixed populations, with $N = 5$ and $M = 1$, where N is the size of the groups and M the number of rejections below which the proposal is still accepted.

2) Rejection Threshold: When a proposal is made to a group, the number of necessary positive votes to approve it depends upon the rules imposed in the game. In our simulations, we tested different values of M. Remember that this M is the minimum number of rejections in the group, so that the proposal is not accepted.

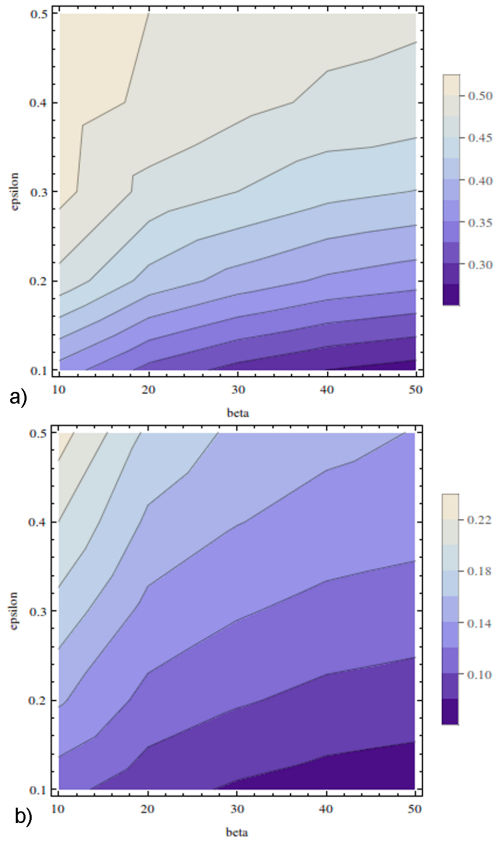

Figure 2: Result of varying β and ϵ in well-mixed populations. a) refer to the values of p and b) refer to the values of q. Both plots represent values as an ensemble average over runs, of stationary states. Clearer zones represent higher values.

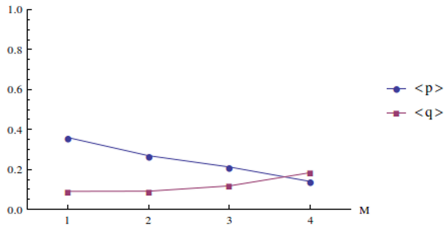

Figure 3: Result of varying M in well-mixed populations. Each point corresponds to an ensemble average over 100 runs, of the stationary values of p and q.

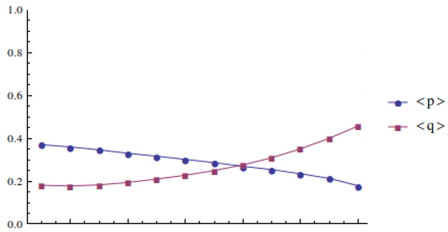

Figure 4: Result of varying M in regular populations of degree 12.

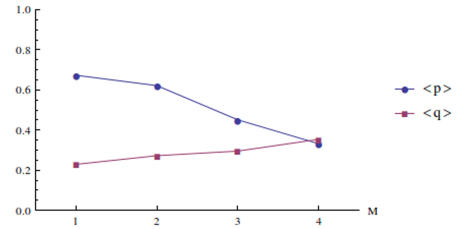

Figure 5: Result of varying M in scale-free populations of average degree 4.

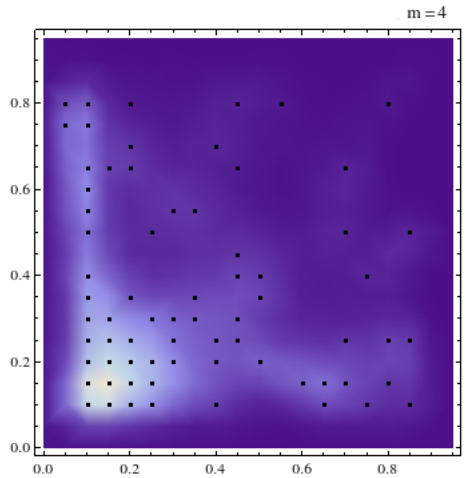

Figure 6: The plot represents the final position of each agent in what concerns to final values of p and q, in the end of generations and considering 100 different runs, when $M = 3$. The more clear the area, more agents end up there. The black dots represent the mode of the positions, in the end of each run.

The results of varying M in well-mixed populations are presented in Figure 3. Further, in Figure 4, it is possible to observe the effect of using different M in regular populations, when each agent has 12 neighbors (degree of 12). It is worth to notice that only makes sense to vary M from 1 to $N - 1$. If $M \leq 0$, all the proposals would be rejected and if $M \geq N$, all proposals would be accepted. Figure 5 represents the effect of varying M in scale-free populations. Note that, in all those plots, β and ϵ were fixed in, respectively, 50 and 0.2.

3) Population Structure: Looking to what happens in the scale-free distribution of the population, we represent the $p \times q$ area where each agent ends up, after 10^5 generations. The more clear the area, more agents end up there. Note that we use a granularity of 0.05, meaning that the values are in a discrete space and rounded to the nearest multiple of 0.05. In Figure 6, each of those black dots represents the mode of the distribution of agents, at the end of each run.

4) Responder Competition: To finalize this exposition of results, we present here the effects of changing the number of responders, when a proposal is made to a group and the responders are competing to acquire that proposal. The main conclusion of Fischbacher, Fong and Fehr [3] was that increasing the number of responders will induce the proposer

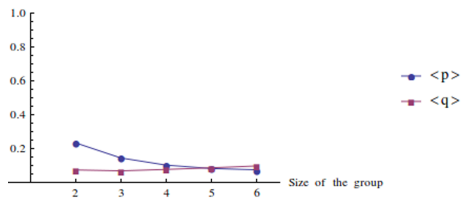

Figure 7: Result of varying the number of responders when they are competing.

to offer less, thus, pressing the values of p to decrease. Those conclusions were formulated based on a set of experimental tests with real persons in a laboratory. In Figure 7, we present the results achieved using as basis our computer simulation, in the context of N-Person Ultimatum Game with competing responders.

C. Discussion

From Figure 2, we can conclude that high values of ϵ and low values of β promote an increase in the values offered by the agents and also an increase in the values expected. We may thus confirm that, against a population whose expectation values are not easily predicted, to offer more is safer and guarantees that one's proposal will be accepted. In fact, one may say that **uncertainty, in this context, seem to promote fairness**.

One parameter that must be considered (and probably, from which results the most important outcome of this work) is M. As said before, M represents the number of agents that should vote against one proposal, in order to overall reject it. As our results show, in Figures 3, 4 and 5, a **decrease in** M **promotes fairer proposals** and lower propensity to punish. The effect of this parameters in real world scenarios is rather noticeable. Imagine that you are playing in the role of proposer and facing two different scenarios. In the first, all the members of the group for which you are making the proposal must accept your terms, so your proposal is accepted. In the second, just one member is needed, so your proposal is taken on. To which of these groups you will make an higher proposal? One may say that the more rigorous group, introduced in first place, will benefit from an higher proposal since it will be harder to get it approved. The simulations show precisely this. Also, note that this achievement is in consonance with the analytical predictions summarized in Figure 1. The increase in the average values of q may simply be due to a decreased selective importance. This happens because, if we increase M, the capacity of one individual to reject a game will become weakened. The required values of some individuals (q) may thus freely increase, and that evolution may not affect the number of games accepted, as long as other agents in the group accept the proposals.

It is also considerable the effect of playing the game with populations that are structured in a scale-free network. As Figure 6 allows us to verify, there are many possible convergence points (multiple equilibrium), at the end of each run. In spite of this, the average values of p are higher, when compared with the ones verified in a well-mixed population or in a populations structured in a regular topology.

To conclude this Evaluation chapter, it is also worth to compare our results with the ones obtained experimentally by Fischbacher, Fong and Fehr [3]. As you may understand from Figure 7, the values of p slightly decrease as the number of elements in one group increase. Notice that, if the number of elements in one group increase, it means that we are adding more responders. Fischbacher et al. supported that people have risk-aversion, and this risk-aversion is the cause of high offers when playing Ultimatum Game with just one responder. However, using the paradigm of *homo economicus* described in the introductory chapter of this work, it is predictable that a responder always accepts one offer. Knowing this, a proposer always offers very small quantities. The experimental work done by Fischbacher et al. proves that the proposer does not feel that the responder will always accept any offer. The reasoning is similar to the one drawn in the context of varying M: if the proposer perceives that it is hard to get his proposal accepted, he will propose more. In this case, a proposal is conceived to be hard to accept if the group of responders is smaller. It is reasonable. If there are a lot of competing responders and if just one needs to accept the proposal, the more responders, the more the chances of having one agent accepting the game.

V. Conclusion

In the present work, we took the challenge of contributing to the many efforts that try to provide a justification for cooperation and fairness. We identified Game Theory and Ultimatum Game as precious tools, in order to study how persons behave when faced with the choice between payoff maximization or fair motivations. In this context, and thanks to recent works that address the differences between two-person games and n-person (described in Section II-D), we decided to study a multiplayer version of Ultimatum Game. Therefore, the question that motivated this work was *"What mechanisms explain the emergence of fairness, in N-Person Ultimatum Game?"*. Seeking to answer this, we designed N-Person Ultimatum Game and performed a set of computational simulations with multiagent systems, to observe the evolution of individual behavior when the agents successively interact and learn. We tested three different aspects: the effects of selection strength and mutations, the effects of changing the rules concerning the the number of necessary acceptors and the effects of playing the game in different network topologies. In this chapter, we conclude with the summary of our contributions, with some final remarks and finally, giving some insights about future work.

A. Summary of contributions

We may summarize our main proposals, conclusions and contributions in four different points. To read a detailed description of each point, it is advisable to consult the previous sections on Model and Evaluation:

1) ***Proposal of new model****:* In this work, we proposed a generalization of the Ultimatum Game for N persons. We identified a gap in existent literature, due the nonexistence of a framework that allow the study of the Ultimatum Game, when played with more than two persons. This generalization was already proposed in the context of games as Prisoner

Dilemma, and it enabled the study of mechanisms that two-person modes are not able to capture. Thus, we proposed N-Person Ultimatum Game.

2) ***Study of the effect of selection and innovation****:* With the proposed model, we studied the effects of changing the selection strength and mutation space, as was already employed in the context of traditional Ultimatum Game [31]. We concluded that the effects of the two parameters depends upon the structure used. In the case of well-mixed populations, a decrease in selection strength and an increase in mutation space, will lead to fairer offers.

3) ***Study of the effect due changes in rejection rules****:* We performed different tests using different values of M, the threshold of rejections, bellow which the game still occurs. We concluded that smaller values of M conduct to fairer offers, suggesting that the imposition of unanimity favors fairness. A big M will also have the effect of lowering the selective importance of q, in the course of evolution.

4) ***Study of the game dynamics when played in complex structures****:* Different kind of structures were used in the previous contributions. We concluded that, when using scale-free networks, there are a lot of possible convergence values. In spite of that, the average values of offers over runs were maximized when using this kind of complex structures.

B. Final remarks

In the course of this work, a considerable body of literature was analyzed. A major slice of those studies was already included in Related Work or in previous sections, but it is important to refer here some other leading works in the field of fairness and punishment. The combination of those works with some personal reflections, culminated in this section, where some final remarks and thoughts are drawn. Hopefully, they will be important guiding lights, when, a section ahead, we will resort to Future Work.

1) ***Uncertainty leads to fairness****:* In the context of this work, it is important to retain the curious idea of "perplexity, doubt, uncertainty", as fundamental characteristic of human nature. When we tested the effects of selection strength and mutation space, we were somehow modeling the unpredictability of humans. We concluded that, in the general case of well mixed populations, low selection strength and a huge mutation space conducted to fairer offers. This is reasonable: the combination of those two parameters enabled the creation of more heterogeneous and uncertain populations. This aspect was further explored, when we studied the number of necessary agents to reject one proposal. The uncertain nature of those agents, combined with rigid acceptation conditions, lead to fairer offers. In a group of doubtful agents, where it is only needed one of them to reject one proposal, the offers tended to be higher.

In both cases, we are rejecting the deterministic idea of traditional game theory. If one remember, the prediction about the used strategies in Ultimatum Game was employed using a backward reasoning: it all starts with the assumption that the Responder will always accept any offer. It is this behavior of the Responder that, when taken as granted, will induce an unfair offer by the Proposer. As we broke the deterministic behavior of the Responder, we concluded that fairer offers may emerge. Uncertainty favors fairness.

2) ***The unknown origin of punishment****:* When someone is confronted with an unfair situation, it is not unusual to feel a kind of inner rage. It is even natural to feel an impetus through the punishment of the unfair person. As we explained in Related Work, there are studies that address the role of punishment in the maintenance of cooperation and fairness. It was called altruistic punishment, and some researchers argue that it explains high rates of cooperation within groups. But those punitive agents were exogenously introduced. What, then, may be the root of this urge to punishment? Research shows that it is both a biological trace [36] and a cultural characteristic [37]. In the context of the present work, it is also importance to analyze what are the real effects of punishment. In the traditional pairwise Ultimatum Game, punishment is always directed to one Proposer by one Responder. Also, in the models where punishment is tested in the context of Public Goods Game [25], punishment always affects all the defectors in the group. In democratic institutions or other group contexts, the effectiveness of punishment may not always be one hundred percent. We model that aspect in our work. In N-Person Ultimatum Game, the effectiveness of punishment depends upon the parameter M. If M is too big, agents may not be able to punish the Proposer, if they are in the same group of a few individuals with low expectation values (non-punitive). As we argued, q looses selective importance, if M is too big. Therefore, we do not believe that the origin of higher values of q, in some cases studied in this work (Figures 3, 4 and 5), provide an accurate explanation for the emergence of punishment. The decreased selective importance of q only allow us to answer the question *"Why punishment emerged?"* with another question: *Why not?*

C. Future Work

It is hard, or impossible, to say that a work is complete. In fact, a set of possible topics can be explored, in the future. We will consider the following four as future work:

1) Experimental Validation: The results that we achieved in our computer simulations are in accordance to the risk-aversion assumption experimented by Fischbacher, Fong and Fehr [3], in the Ultimatum Game with competing responders. However, it would be illuminating and interesting to test directly the N-Person Ultimatum Game with real people, to check if the varying parameter M leads to the same conclusions we reached with simulations.

2) The impact of complex networks: As we observed, the introduction of scale-free networks allowed the existence of numerous convergence points. It would be fruitful to study deeply why this happens, for instance, analyzing if the evolution of individual agents is somehow dependent on the number of neighbors, and if the existence of near hubs favors (or affects) the emergence fairness.

3) Explanation of punishment: We argued that fairer offers emerged mainly because of the uncertain behavior of the responders. If this provides a glimpse in why fairness emerges, it does not tell us much about the origin of punishment. We only concluded that, depending of the number of rejections allowed to accept one proposal (M), the expectation thresholds of the

agents may have more or less selective pressure. A further study should address deeply the emergence of punishment within groups.

4) Mechanism Design: Going from an EGT model to the definition of real-world policies is a huge endeavor. Mechanism Design is the discipline that studies the imposition of rules in daily life games, in order to prevent agents from defect, cheat or provide erroneous information. In the Future, would be extremely beneficial to build a bridge between this kind of computation models and, for instance, real rules in the regiment of summits or treaties, in order to defy unfairness and promote solidarity.

REFERENCES

[1] H. Gintis, S. Bowles, R. Boyd, and E. Fehr, "Moral sentiments and material interests: Origins, evidence, and consequences," *Moral sentiments and material interests: The foundations of cooperation in economic life*, vol. 3, no. 8, 2005.

[2] ——, "Explaining altruistic behavior in humans," *Evolution and Human Behavior*, vol. 24, no. 3, pp. 153–172, 2003.

[3] U. Fischbacher, C. M. Fong, and E. Fehr, "Fairness, errors and the power of competition," *Journal of Economic Behavior & Organization*, vol. 72, no. 1, pp. 527–545, 2009.

[4] P. Seabright, *The Company of Strangers: A Natural History of Economic Life (Revised Edition)*. Princeton University Press, 2010.

[5] S. Bowles and H. Gintis, "Reciprocity, self-interest, and the welfare state," *Nordic Journal of Political Economy*, vol. 26, no. 1, pp. 33–53, 2000.

[6] M. Nowak, "Five rules for the evolution of cooperation," *science*, vol. 314, no. 5805, pp. 1560–1563, 2006.

[7] W. Güth, R. Schmittberger, and B. Schwarze, "An experimental analysis of ultimatum bargaining," *Journal of Economic Behavior & Organization*, vol. 3, no. 4, pp. 367–388, 1982.

[8] J. Smith, *Evolution and the Theory of Games*. Cambridge university press, 1982.

[9] W. Hamilton, "The genetical evolution of social behaviour. i," *Journal of theoretical biology*, vol. 7, no. 1, pp. 1–16, 1964.

[10] R. Trivers, "The evolution of reciprocal altruism," *Quarterly review of biology*, pp. 35–57, 1971.

[11] R. Axelrod, *The evolution of cooperation: revised edition*. Basic books, 2006.

[12] M. Nowak, K. Sigmund *et al.*, "A strategy of win-stay, lose-shift that outperforms tit-for-tat in the prisoner's dilemma game," *Nature*, vol. 364, no. 6432, pp. 56–58, 1993.

[13] M. Nowak and K. Sigmund, "Evolution of indirect reciprocity by image scoring," *Nature*, vol. 393, no. 6685, pp. 573–577, 1998.

[14] H. Ohtsuki, Y. Iwasa *et al.*, "The leading eight: social norms that can maintain cooperation by indirect reciprocity," *Journal of Theoretical Biology*, vol. 239, no. 4, pp. 435–444, 2006.

[15] H. Ohtsuki and Y. Iwasa, "Global analyses of evolutionary dynamics and exhaustive search for social norms that maintain cooperation by reputation," *Journal of theoretical biology*, vol. 244, no. 3, pp. 518–531, 2007.

[16] J. Pacheco, F. Santos, and F. Chalub, "Stern-judging: A simple, successful norm which promotes cooperation under indirect reciprocity," *PLoS computational biology*, vol. 2, no. 12, p. e178, 2006.

[17] M. Nowak and R. May, "Evolutionary games and spatial chaos," *Nature*, vol. 359, no. 6398, pp. 826–829, 1992.

[18] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, p. 440–442, 1998.

[19] A. Barabási and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[20] F. C. Santos, J. F. Rodrigues, and J. M. Pacheco, "Graph topology plays a determinant role in the evolution of cooperation," *Proceedings of the Royal Society B: Biological Sciences*, vol. 273, no. 1582, pp. 51–55, 2006.

[21] J. Pacheco, A. Traulsen, H. Ohtsuki, and M. Nowak, "Repeated games and direct reciprocity under active linking," *Journal of theoretical biology*, vol. 250, no. 4, p. 723, 2008.

[22] F. Santos, J. Pacheco, and T. Lenaerts, "Cooperation prevails when individuals adjust their social ties," *PLoS Computational Biology*, vol. 2, no. 10, p. e140, 2006.

[23] J. Vukov, F. Santos, and J. Pacheco, "Incipient cognition solves the spatial reciprocity conundrum of cooperation," *PloS one*, vol. 6, no. 3, p. e17939, 2011.

[24] A. Traulsen and M. Nowak, "Evolution of cooperation by multilevel selection," *Proceedings of the National Academy of Sciences*, vol. 103, no. 29, pp. 10 952–10 955, 2006.

[25] R. Boyd, H. Gintis, S. Bowles, and P. J. Richerson, "The evolution of altruistic punishment," *Proceedings of the National Academy of Sciences*, vol. 100, no. 6, pp. 3531–3535, 2003.

[26] J. Vukov, F. L. Pinheiro, F. C. Santos, and J. M. Pacheco, "Reward from punishment does not emerge at all costs," *PLoS computational biology*, vol. 9, no. 1, p. e1002868, 2013.

[27] K. Page and M. Nowak, "Empathy leads to fairness," *Bulletin of Mathematical Biology*, vol. 64, no. 6, pp. 1101–1116, 2002.

[28] M. Nowak, K. Page, and K. Sigmund, "Fairness versus reason in the ultimatum game," *Science*, vol. 289, no. 5485, pp. 1773–1775, 2000.

[29] K. Page, M. Nowak, and K. Sigmund, "The spatial ultimatum game," *Proceedings of the Royal Society of London. Series B: Biological Sciences*, vol. 267, no. 1458, pp. 2177–2182, 2000.

[30] J. Iranzo, L. Floría, Y. Moreno, and A. Sánchez, "Empathy emerges spontaneously in the ultimatum game: Small groups and networks," *PLOS ONE*, vol. 7, no. 9, p. e43781, 2012.

[31] D. G. Rand, C. E. Tarnita, H. Ohtsuki, and M. A. Nowak, "Evolution of fairness in the one-shot anonymous ultimatum game," *Proceedings of the National Academy of Sciences*, vol. 110, no. 7, pp. 2581–2586, 2013.

[32] C. Hauert, N. Haiden, and K. Sigmund, "The dynamics of public goods," *Discrete and Continuous Dynamical Systems Series B*, vol. 4, pp. 575–588, 2004.

[33] F. Santos, M. Santos, and J. Pacheco, "Social diversity promotes the emergence of cooperation in public goods games," *Nature*, vol. 454, no. 7201, pp. 213–216, 2008.

[34] K. Sigmund, *The calculus of selfishness*. Princeton University Press, 2009.

[35] A. Traulsen, M. Nowak, and J. Pacheco, "Stochastic dynamics of invasion and fixation," *Physical Review E*, vol. 74, no. 1, p. 011909, 2006.

[36] A. G. Sanfey, J. K. Rilling, J. A. Aronson, L. E. Nystrom, and J. D. Cohen, "The neural basis of economic decision-making in the ultimatum game," *Science*, vol. 300, no. 5626, pp. 1755–1758, 2003.

[37] J. Henrich, R. Boyd, S. Bowles, C. Camerer, E. Fehr, H. Gintis, R. McElreath, M. Alvard, A. Barr, and J. Ensminger, "" economic man" in cross-cultural perspective: Behavioral experiments in 15 small-scale societies," *Behavioral and brain sciences*, vol. 28, no. 6, pp. 795–814, 2005.