Simulating the impact of high-speed network in the performance of other transport operators: the HSR in the link Lisbon-Oporto

Manuel Bebiano de Brito Camacho Brando[1]

ABSTRACT

The High Speed Rail (HSR) is part of the Trans-European Transport Network and, as such, large investments have been (and will be) done in Europe since the late 1980's. Despite the current economic crisis, it is foreseeable that Portugal will pursue its HSR project in the medium to long term. The main objective of the present research is to model the strategic behavior of passenger transport operators that compete with HSR in a multi-modal corridor. As such, we analyze the potential changes in the strategies of existing operators after the (hypothetical) entry of HSR, applying our methodology to the case study of the link between Lisbon and Oporto.

Our results indicate that the game theoretical approach combined with optimization algorithm proposed here is appropriate to simulate the operators' strategic behavior in face of new competitors in a multimodal corridor. After the optimization and according to our assumptions, the HSR could potentially double the currently estimated profits, while air would try to minimize losses, possibly by collaborating with the HSR and avoid costly connection flights between Lisbon and Oporto. The HSR can become a threat to conventional rail if the current operator is not allowed to bid for the concession of HSR. Alternatively, it can collaborate with the HSR as a feeder and trying to explore new market segments in the currently over-saturated link. Buses are the main winners as they could potentially increase ticket prices, while increasing headways, and still increase their profits, despite some loss of its modal share.

Key words: High Speed Rail, Strategic Behavior, Competition and Game Theory

1 INTRODUCTION

The High Speed Rail (HSR) is a growing transport mode in Central Europe. As such, and despite the current economic conditions in Portugal, it is foreseeable that the country will follow this path. The introduction of HSR creates new opportunities in the domestic market and triggers a series of changes in the behavior of the transport system players. The main objective of the present research is to model the strategic behavior of passenger transport operators that compete with HSR in a multi-modal corridor. As such, we analyze the potential changes in the strategies of existing operators after the (hypothetical) entry of HSR, applying our methodology to the case study of the Lisbon – Oporto link. The paper focuses on the study and comparison of the behavior of each transport operator, looking to propose a methodology to forecast the changes in the strategy of these operators through a game theoretical approach combined with an optimization algorithm. The game theoretical optimization algorithm was modeled in *Matlab*.

We begin with literature review on the subject and advantages of game theory to analyze the present problem, in Section 2. Section 3 presents the numerical model developed here and its main assumptions. Section 4 describes the application of the model to the case study. The robustness of the model is analyzed in Section 5 through tests and sensitivity analyses. Finally, we end up with the discussion of the results and recommendations for future research in Section 6.

2 LITERATURE REVIEW

The competition between different operators in the transportation market is not new. Still, there is little literature regarding the competition from the operators' perspective – usually, these are systemic analysis. Furthermore, when it exists, research is mostly limited to the competition between two different modes of transport. It should be noted that some of the references found studying the competition between operators only use the behavior of the

---

[1] Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1, 1049-001 Lisboa, manuelbrando89@gmail.com

market as a reference, this is, by varying parameters, such as the price of the trip, they try to see if it is possible to increase their market share – that is, *ceteris paribus* analyses. Other references more relevant to this research seek not only to vary these parameters but also to see whether there are advantages for the operator in making such changes. The first approach will be treated as a competition with a demand-side perspective, while the second will be a competition with a supply-side perspective.

González-Savignat (2004) analyzed the possible competition between HSR and air between Madrid and Barcelona. By that time, that is, 2004, the HSR was not yet operating in the route studied. The authors modeled a hypothetical market for the HSR as if it was already available. Using a logit discrete choice model, demand was estimated, being then possible to predict and simulate various scenarios within the policy options for transport. To predict possible market responses to different stimuli, the authors tested changes in the attributes of the trips for that OD pair. The basic scenario for air remained unchanged throughout the simulation, and to facilitate comprehension only one variable changed at a time. Román, Espino, & Martín (2007) analyze the potential competition between the HSR and air transport in Madrid-Barcelona. Through a structured discrete choice model, the authors estimated the willingness to pay for different trade-offs between attributes. In this case, the HSR was also not available in the market and so its initial market share derived from improved conventional train services. To simulate this effect, the modal share of passengers was estimated between train and plane, in face of various scenarios. After this process, the HSR replaced the conventional train with a journey time reduced by 50% and higher comfort levels. Adler, Pels, & Nash (2010) developed a methodology that aimed to study the effect that investments in infrastructure can have on the balance of competition between multiple private operators. The operators chosen for this study belonged to the air and rail modes. Operators' revenues depended on market share that in turn depended on the price of the ticket, the frequency and the average travel time from a city center to the other. Costs were determined according to the size of the vehicle, measured in number of seats of the vehicle, the frequency and distance traveled. These values included infrastructure costs and fees, dependent on each scenario developed. To estimate the market shares, the authors calibrated a hierarchical logit model. After developing the model, the authors calculated elasticities for each alternative and then they calculated the corresponding Welfare function. Dobruszkes (2011) examined whether the new market equilibrium caused by the introduction of the HSR, changed the overall demand and to what extent. The main objective focused on the question of whether this new equilibrium would lead to real change or just a drop in the occupancy rate of vehicles while maintaining the same frequency by the operators. Using historical data from the airlines he studied the market before and after the HSR joined competition for the 5 OD pairs analyzed.

Regarding the analysis of competition between HSR and conventional train, Hsu & Chung (1997) developed an analytical model to calculate market shares between the HSR and conventional rail (CR). With this model, the authors wanted to assess whether the operators behave strictly as direct competitors or if CR could feed and supplement the services of the HSR. Instead of using the usual logit models, this new approach encompassed both the choice of the mode and the choice of the route framed in a scenario of relations between key variables. Hsu, Lee, & Liao (2010) resorted to game theory to describe the result from competition or cooperation in the railway market in Taiwan. In this study, the authors used the OD pair Taipei-Kaohsiung to develop the model. With the assumption that, in the short-term, the price turns out to be the main tool used by operators in their strategies, the problem was analyzed in this paper as a model of competition based on price-cost rigid structure and fixed characteristics of the products. It is assumed that the best way to solve the problem is using the Nash equilibrium with an heuristic.

Finally, Shyr & Hung (2010) evaluated the potential impact of HSR in the overall Social Welfare, by comparing HSR and all remaining modes. Beyond studying the new price regulation that should be applied, they studied as well the new frequencies that competing services should practice and also which would be the effect of a coalition of airlines in this context and how their profits would be divided. For this analysis, the authors defined a discrete choice model and the equations of payoffs for each operator. After using the Nash equilibrium to solve the problem of new frequencies and prices, they then applied the concepts of cooperative strategies for how the profits would be divided between the airlines. The case study was the pair OD Taipei-Kaohsiung.

3 THE MODEL

3.1 Game theoretical approach to the problem

Game theory is a study of strategic decision-making and was the approach chosen to solve this problem. A game is composed by 3 major parts the players, the strategies and the payoffs. The players will be the operators in the corridor, 4 of them each one representing one mode (1 for bus, 1 for air, 1 for train and 1 for HSR). To this players will be added a fifth element, the private transportation. The car will be introduced in this model not as a player because he doesn't have the same degrees of liberty as the other modes hence he will not try to maximize his payoffs. The need to introduce this element it's related to the fact that he represents a huge market share in the link so if we want a more accurate perception of the viability of the project it needs to be included. The strategies are the paths that each operator may choose. Since the model it's based on 4 variables, ticket price, access time, travel time and headway the player's strategies are limited. The access time can't be changed by the operator since he is attached to an infrastructure and we also assumed that the travel time it's already optimized. So we assumed that the operator as two variables he can change and for each one he has 3 courses of action, he can increase, decrease, or maintain the ticket price and the headway. So we can say that he has 9 possible strategies.

Table 1 – Possible strategies of transport operators and respective changes in attributes

Possible Strategies	Ticket Price	Travel Time	Access Time	Headway
1	–	–	–	–
2	↘	–	–	↗
3	↘	–	–	–
4	–	–	–	↗
5	↘	–	–	↘
6	↗	–	–	↘
7	–	–	–	↘
8	↗	–	–	↗
9	↗	–	–	–

The payoff represents the result of the strategy the player chose together with the strategy of the other player. In our case the payoff is represented by a profit function, so to know all the payoffs possible for each player was necessary to build a model. This model it's divided in 2 parts. The first one it's to insert the HSR in the market and build a base scenario. The second one it's an optimization cycle, which will determine what's the best payoff that each player can get.

3.2 Methodology

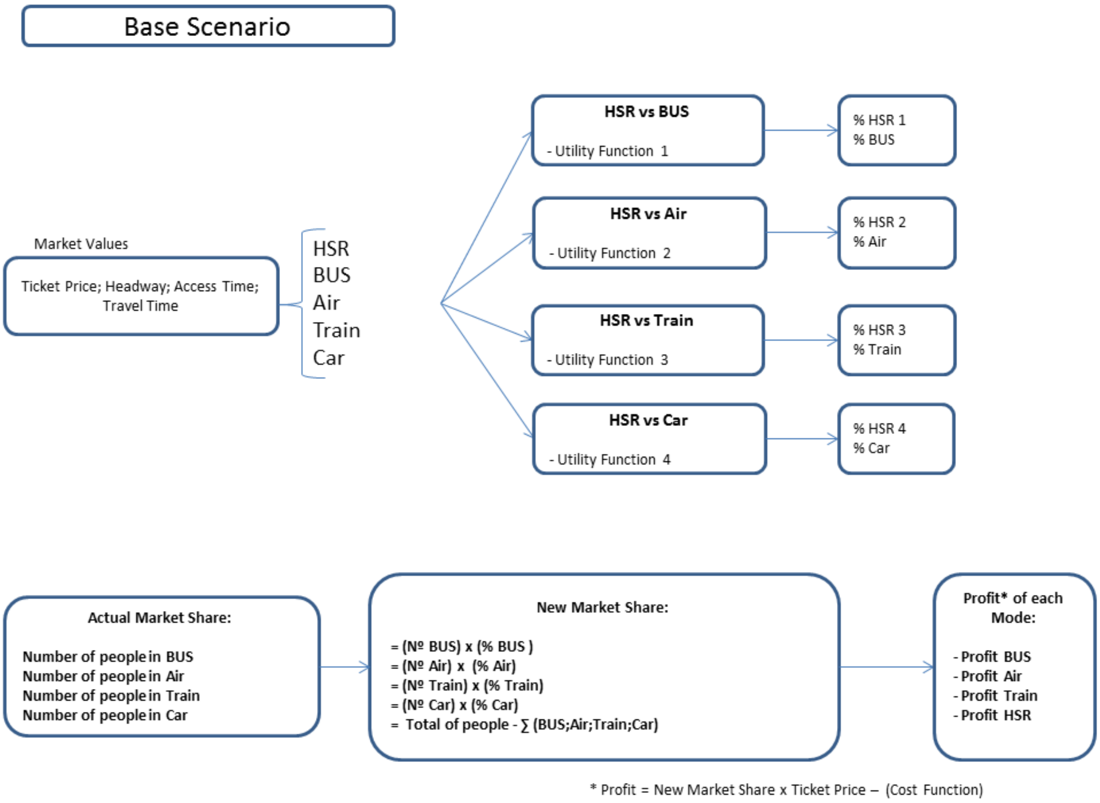

Figure 1 – Methodology used to analyze the strategic changes from operators

As explained above, this paper will try to maximize the payoffs for each player by using their profit functions. However, the HSR doesn't exist in the actual market. So the objective in building this base scenario it's to create a hypothetical market where the HSR already exists and as a profit. So the first step it's to see how many people from the other modes are willing to change for HSR getting that way a market share for this mode. So we start to define values for all the variables and for all the modes, then using the utility functions, from a binomial logit mode we can estimate new market shares. The second step will be to define for each mode, with the new market shares, the profit functions so that we can initialize the optimization cycle. The profit function, as it was made by other authors (Hsu, Lee, & Liao (2010) e Adler, Pels, & Nash (2010), among others), will be divided in two parts, one representing the revenues and other representing the costs. The part that represents the revenues will only depend on the ticket price, market share and total of people travelling by that mode. The part representing the costs will depend on the cost that each player as when operating his vehicles. The costs will be represented by cost functions for each mode gathered from other papers.

3.2.1 The profit functions

The profit functions have a major importance in this problem because they represent the payoffs we want to maximize. The objective of this exercise it's just to study one link so many of the complexities associated to the profit functions of companies of this size will be disregarded. So the profit function is:

$$L_k = MS_k \times P \times Pr_k - TC_k$$

where, MS_k is the modal share of the mode k (%),
P is the total passengers traveling between the link,
Pr_k is the ticket price of the mode k (€), e
TC_k is the total costs of the mode k, estimated by the cost functions (€).

So to get the values for the profit functions we need to know the cost per trip for each mode. Before we analyze each cost function used by the different modes we will present a set of parameters that are common to all.

The frequency is a variable that it's determined by one of two components, the headway or the maximum people that a vehicle can transport, these components will affect the fleet size and hence the cost functions. So to know what will be the size of the fleet necessary to determine the costs we start defining the maximum trips one vehicle can make per day.

$$fm_k = \frac{TT_{O,k}}{(TV_k + T_{bat})}$$

Where, fm_k is the maximum trips one vehicle can make per day of the mode *k*,
$TT_{O,k}$ is the total daily operation time of the mode *k*,
TV_k is the time of one travel in one way (minutes), and
T_{bat} is the additional time needed to prepare the vehicle for the next trip (minutes).

Now knowing the maximum trips per day one vehicle can make, it's necessary to know how many people they can transport. So using the formula below

$$Vg_k = fm_k \times 0.8Lt_k$$

Where Vg_k is the maximum number of people transported by one vehicle of mode k, and
Lt_k is the maximum capacity of one vehicle/composition of mode k.

We assumed that the operator would buy a new vehicle when its occupancy rate was above 80%. So has mentioned above the needs buy more vehicles to the fleet will depend on one of two criteria:

- The daily demand exceeds Vg_k
- The headway its inferior to the $(TV_k + T_{bat})$

So then we can present the cost functions for each mode.

3.2.2 Cost functions of each mode

The cost function used for HSR and Train is based on de Rus and Nash (2007).

$$c_{AVF} = Rs\ (\sum_k \frac{f_{kr}\, S_{kr}}{2(450)}) + \sum_k (\propto_k^{oc} + \propto_k^{ac})(2f_{kr}GCD_{ij})$$

Where, *RS* is the fixed cost of buying an amortized train with 450 seats,
α_k^{oc} corresponds to the operational cost of the train by kilometer,
α_k^{ac} corresponds to the access cost to the infrastructure,
f_{kr} corresponds to frequency, and
GCD_{ij} is the Great Circle Distance

After some simplifications the result was:

$$c_{HSR} = mc_{HSR} + (co_{HSR} + ca_{HSR}) \times d$$

Where, c_{HSR} correspond to the cost of making one trip with HSR (€/trip),
mc_{HSR} correspond to the cost of each composition of HSR or train in one trip (€/trip),
co_{HSR} corresponds to the unitary operational cost of HSR or train (€/km),
ca_{HSR} corresponds access cost to the infrastructure for HSR or train (€/km), and
d is the distance that HSR or train as to make (km).

To get the value for mc_{HSR} we use the follow equation

$$mc_{HSR} = \frac{MC_{HSR} \times \left\lceil \frac{f_{HSR}}{(fm_{HSR})} \right\rceil}{f_{HSR}}$$

Where, MC_{HSR} correspond the estimation cost of HSR or conventional train in one day (€),

f_{HSR} is daily frequency of HSR or conventional train (trips/day),
fm_{HSR} is the maximum trips one vehicle can make per day.

This expression allows us to know how many trains will be needed per day, and the average number of trips they do. As we have seen before the fleet size will depend on one of two factors, the capacity or the headways. This dependence it's represented in the following expression:

$$f_{HSR} = max\left(\frac{TT_O}{T_{i,HAR}}; \frac{Pax}{Lt_{HAR} \times 0{,}8}\right)$$

Where, TT_O is the operation hours considered in this study (min),
$T_{i,HSR}$ is the headway defined for HSR or conventional train (min/composition),
Pax is the total demand in one way per day (passengers/day),
Lt_{HSR} is the capacity of one HSR or conventional train (passengers/composition).

This way we get the average cost for one trip, than multiplying by 365 days, by 2 to consider both ways and the frequency, we get the final equation as follows:

$$TC_{HSR} = c_{HSR} \times f_{HSR} \times 2 \times 365$$

In the Bus case we couldn't find any functions in the literature that fitted the Portuguese case of interurban transportation. Hence, we proposed a new formulation based on the information provided in André (2006).

$$c_{BUS} = mc_{BUS} + (co_{BUS} + Port) * d$$

Where, c_{BUS} is the unitary cost of making one trip with a BUS (€/trip),
mc_{BUS} is the rolling stock cost in one trip for a bus(€/trip),
co_{BUS} is the unitary operational cost for a bus (€/km),
$Port$ is the unitary cost for tolls (€/km),
d is the distance a bus as to make (km).

The main difference between this expression and the one used to HSR is the fact that the cost of the rolling stock is already calculated for one trip, so we don't need to use the process described above. The operational costs can be divided as shown below:

$$co_{BUS} = fc_{BUS} + m_{BUS} + rh_{BUS}$$

Where, fc_{BUS} is the unitary consume of fuel (€/km),
m_{BUS} is the unitary maintenance cost (€/km),
rh_{BUS} is the unitary cost with personnel, this is the driver(€/km).

Obtaining this way a very similar expression, as the one used for HSR and conventional train:

$TC_{BUS} = c_{BUS} \times f_{BUS} \times 2 \times 365$, where f_{BUS} is the daily frequency for bus.

The cost function used for air was proposed by Swan and Adler (2006).

$$c_{AIR} = 0{,}019 \times (d + 722) \times (Lt_{AIR} + 104) \times 2{,}2$$

Where c_{AIR} corresponds the unit cost of producing one single 'average' flight (€/trip);
'0,019', '722' e *'104'* are calibration parameters,
'2,2' is a conversion factor suggested by Swan and Adler (2006),
d represents the distance traveled by the aircraft,
Lt_{AIR} Is the number of seats of the airplane.

This formulation above gives us the cost for one trip, so using the same principles as before.

$TC_{AIR} = c_{AIR} \times f_{AIR} \times 2 \times 365$, where f_{AIR} is the daily frequency of airplanes.

3.3. Simulating the strategic behavior of operators: cyclic optimization algorithm

We proposed an optimization algorithm to simulate strategic behavior of the operators towards HSR. It's built based on the idea of all the payoffs being confronted and then the player will choose the one that gives him its maximum profit. This payoff will depend on four variables, the ticket price and headway the player defines, and the ticket price and headway the other player defines. In the HSR case it will not just depend on four variables instead it will depend of 8, the 2 that he can control and the other 6 the other operators define.

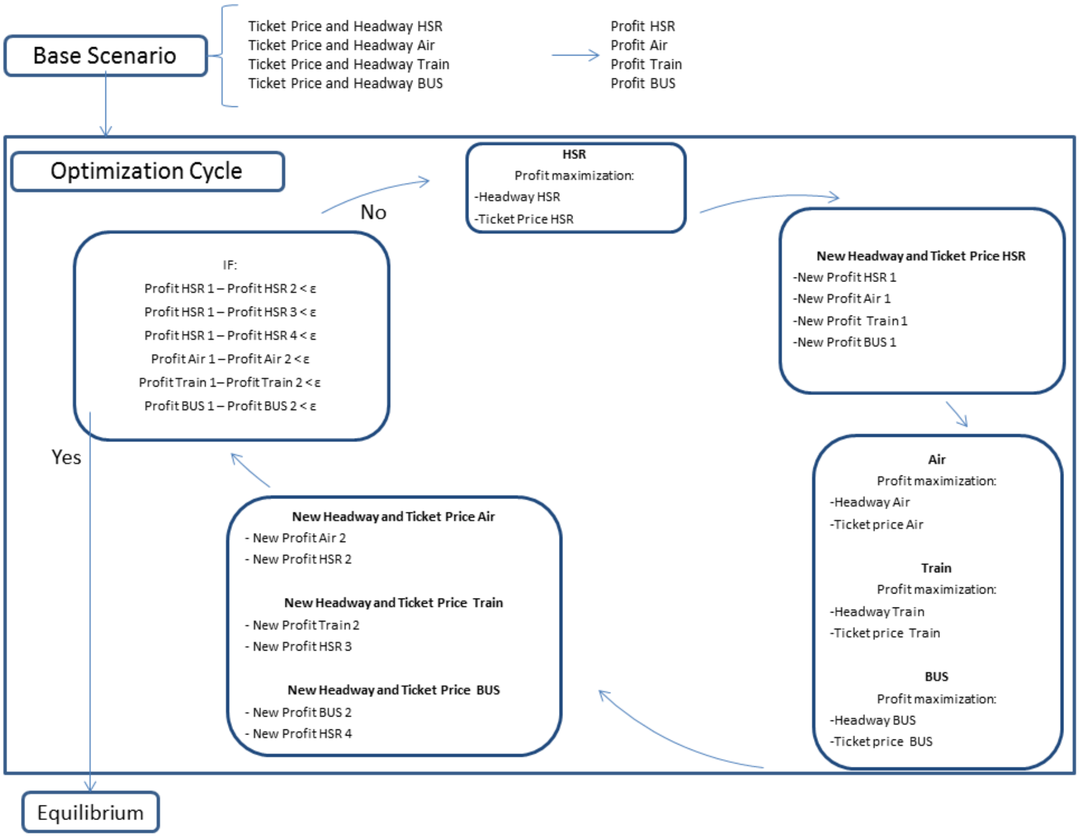

Figure 2 – Cyclic optimization loop to simulate the operators' strategic behavior towards HSR

Since we used binomial logit, modeled through the discrete choice model provided by Steer Davies Gleave (2006), HSR is the only mode that can compete directly with all others. So we are in a presence of 3 games (HSR vs. Air, HSR vs. Bus, and HSR vs. Train) where the HSR tries to maximize the total payoff of the 3 games and the other modes will only try to maximize their payoff when competing with HSR. To guarantee convergence and so that the players will not choose values that cannot be supported by the infrastructure it's necessary to limit the variance of the ticket price and headway. So for every mode was define a maximum and a minimum for each of these two variables. As we can see the cycle starts with the maximization of one player defining the values for his variables. This will make a change in the modal share for both players, changing also the profit. Then it's the second players turn to maximize their payoff, he then changes the values in his variables and creates a new set of profits. Then if the profits obtained by the first player and the second player don't differ more than a certain value we can assume that we have arrived to the equilibrium, that way we can finish the cycle. The value defined was 10.000€ because we are talking about payoffs in the order of millions so this represents only 1% change.

4 MODEL RESULTS AND ANALYSIS

As it was said before a lot of values had to be assumed in the cost functions. The table below resumes them:

Table 2 – Assumptions for modeling

	HSR	Conventional Train	Bus	Air
Capacity (places)	300	318	46	150
Cost of vehicles (‘000 €)	10.500	20.500	-	-
Unit average vehicle cost per trip (€/trip)	-	-	62,5	-
Lifetime (years)	40	40	-	-
Amortization cost (€/day)	720	1404	-	-
Operation cost (€/km)	9,4	12,57	0,35	-
Cost of infrastructure access (€/km)	1,35	10	-	-
Tolls (€/km)	-	-	0,12	-
Distance between Lisbon and Oporto using different infrastructures (km)	336	297	303	279

Using the above values to feed the model we obtain the following values for the base scenario and for the final one (after optimization).

Table 3 – Optimized demand for each operator

		HSR		Air		Conventional Train		Bus	
		(a. v.)	(%)	(a. v.)	(%)	(a. v.)	(%)	(a. v.)	(%)
Profit (k€)	BS	- 26.156,0	-253%	- 73.341,0	-51%	- 31.984,0	-92%	5.338,7	47%
	FS	39.977,0		- 35.863,0		- 2.528,4		7.857,5	
Modal share (‘000 pax)	BS	1.223,6	186%	8,0	439%	657,7	-42%	683,7	-30%
	FS	3.496,0		43,0		383,7		477,2	
Average ticket price (€)	BS	49	-24%	100	-76%	30	-33%	18	22%
	FS	37		24		20		22	
Headway (min)	BS	60	-5%	120	100%	60	300%	30	103%
	FS	57		240		240		61	

Notes: Base scenario - BS; Final scenario (FS); (a.v.) – Absolut value.

And the respective market shares.

Table 4 – Final market shares after optimization of individual strategies

	Initial	Base	Final	Variation between base and final scenarios
Air	2,8%	0,1%	0,6%	439,3%
Conventional trains	12,5%	9,1%	5,3%	-41,7%
Bus	9,7%	9,5%	6,6%	-30,2%
Private car	75,0%	64,4%	38,9%	-39,6%
HSR	-	17,0%	48,6%	185,7%

As we can see from the tables above this new distribution benefits every player because some increase their profits while others decreased their losses. The values show that HSR was the mode that has more profits and gathered a market share around the 50% of all passengers. In order to do that they maintained their headway and decreased the ticket price. The airplane obtained 35 million euros of loss and a very low market share. Their strategy was to increase the headway to the maximum possible and decrease the price. The conventional rail presented a similar strategy, the results were a total of 400 thousand people traveling by this mode and losses rounding the 2.5 million euros. The bus adopted a curious strategy has they increased slightly the price and the headway. They lost a little of their initial market share, despite that they increased their profits.

The main conclusions that can be drawn from this optimization are that HSR with a price decrease to values close to the ones used by today's conventional train can assimilate a large percentage of the public transportation market and still a large volume of passengers from individual transportation. The airplane with the entry into operation of the HSR loses any chance of competition and as such he just tries to minimize their losses by changing the number of flights to a minimum and put the lowest price possible. These results are consistent with the published literature that refers to air as profitable only for distances greater than 800 km and unbeatable from 1500 km. Regarding the conventional train, they try to maintain operability lowering its price and increasing their headway this way they can keep some of their market share but still can't achieve net profit. One possible solution would be to conventional rail work as feeder to the HSR. The bus mode is the less affected by HSR and this may be due to the fact that his market segment is a little different, people that usually go by bus value much more the price instead of the time.

These results have two failures that can lead to some interpretation. First, with the prices charged by train and plane, probably the market share of private transport would be even lower. Their prices are lower than those practiced by car and they have better journey time. However, given the constraint of discrete choice model binomial imposed by methodological approach of the stated preferences interviews of Steer Davis Gleave (RAVE (2006)), it is impossible to estimate the passage of passengers between modes. Secondly, as almost all modes decrease or maintain their headway could probably occur the situation that passengers were forced to resort to private transport at certain times of day.

To finish this analysis we recall that the individual transport has a static behavior, they don't react to 'strategic' initiatives to face the public transport operators. This would be an exercise with a complexity analysis which is beyond the scope of this analysis, because the behavior of the IT users is not governed by the same principles of economic rationality and regulatory constraints of the public transport operators. However, we could speculate that the infrastructure manager may participate in these games as an agent with a change of its own strategy, in particular as regards prices toll set and facing a potential threat of loss of significant demand.

5 SENSITIVITY ANALYSIS

As mentioned throughout the paper there were several situations where the lack of data meant that we had to assume some values/principles. This chapter is to analyze how the assumptions may have influenced the results of the model. To study this situation were made two tests: robustness of the model (that is, whether the way the model was programmed could affect the final results); and analysis of sensitivity to some parameters of the cost functions.

To test the robustness of the model we proceed to six different trials.

1. We assumed to be random the order of the optimization model;
2. We assumed to be random the initial values of ticket price and headway in the base scenario, of course they had to be between the range defined;
3. We assumed 1+2 conditions;
4. We assumed to be more competitive the individual transport, as such the ticket price and the travel time were reduced;
5. We assumed a decrease in the $T_{bat,}$ the time needed to refuel, clean, etc;
6. We assumed larger intervals for the variables headway and ticket price.

In all the cases studied the results didn't change a lot, meaning that the model it's robust.

In the sensitivity analysis we pretended to unveil which attributes contribute in a larger scale to the variation of the final results, that is the profit functions. The attributes chosen were the price of the rolling stock, the variable costs per kilometer, the conversion factor used in the cost function of air, the total of people traveling and the initial market share of each mode. We used the Pearson's and Spearman's Rho coefficient to measure the sensibility of the output to each parameter. The HSR had a strong correlation (above 0.8) with the total of people traveling and the initial market share of car, that is expected has there are no entrances or exits of passengers in the model. Also the value of the car it's related to the fact that he is the major contributor of passengers to the system representing a total of 75% of it, so the profit will be largely influenced by it. The other modes only present strong correlations to their own initial market share that is due to the fact that we used a binomial logit so there can't be any changes to other modes.

6 CONCLUSIONS AND RECOMENDATIONS

This work has achieved its primary goal, as it was shown that game theory is a tool that can contribute to quantitative and conclusive analysis of the strategic behavior of operators of public passenger transport for different demand scenarios. The proposed approach enables us to optimize the behavior of operators and anticipate what could be the possible reactions that might be expected, from a perspective of economic rationality. Contrary to what has been done so far, game theory can anticipate the behavior of players. It manages, with limitations, to predict various scenarios in a dynamic market that has not been done so far. As such, the initial objective was fulfilled and it was shown that the game theory can help to create scenarios where the operators react in a dynamic market.

This study has some limitations, due to the use of simplifications of reality. Here are listed some of the main problems encountered:

- The main limitation of the study and prevented a more complex approach to it was the fact that the data is based on a binomial logit model, which prevented a true competitive scenario among all modes.

- Because of the limited information available (either in literature or in the information provided by the operators during interviews), the cost functions are necessarily simplified and may lead to biased results. However, this bias factor was minimized sensitivity analyzes made by the model.

- Because it is a study that only contemplates a point-to-point connection, there aren't any cooperation opportunities. If this case study were housed in a more complex network, some kind of cooperation could emerge for example the reallocation of vehicles or a service being a feeder to another.

- Another problem is linked to the model variables. The variable headway is an average value of time between consecutive services. However, the demand varies significantly throughout the day and as such the needs at peak times could be much higher than the amounts used. This is also a problem because the headways practiced by public transport modes were never less than 1 hour, which in a real situation could lead to the abandonment of this type of transport.

- Finally the car presents itself as the main mode in the connection between Lisbon and Oporto, and so its importance is vital if the HSR is profitable. In this study it was assumed that people could freely replace each mode, which does not portray reality, as well as many of these trips are related to work requirements, and so in many cases there are those who cannot give up the car.

To solve some of the limitations here presented, we leave some suggestions. To turn this model more into a more robust exercise we would require a multinomial logit model, hierarchical or mixed, because only this way we would be able to simulate a real competition among all modes of the system, with the possibility of switches between all modes and not only centralized in the HSR. As it is built, the model is not presented in its full complexity. If we want a more perfect characterization of reality we would need to include this link in a network. Finally, and once more in order to increase the complexity of the problem, it would be interesting in a future study to consider the possibility of the passengers entering or leaving the system, because that way we could build an dynamic market with different needs from time to time.

7 REFERENCES

Adler, N., Pels, E., & Nash, C. (2010). High-speed rail and air transport competition: Game engineering as tool for cost-benefit analysis. *Transportation Research Part B: Methodological*, *44*(7), 812–833. doi:10.1016/j.trb.2010.01.001

André, J. (2006). "Transporte Interurbano em Portugal. O sistema actual e os seus desafios (Volume 1)", published by IST Press, May 2006

AVEP AEIE (2007), "Estudo do modelo de exploração dos serviços de passageiros na ligação Lisboa-Madrid", elaborated by SENER for AVEP AEIE, 2007

Behrens, C., & Pels, E. a. J. H. (2009). Intermodal Competition in the London-Paris Passenger Market: High-Speed Rail and Air Transport. *SSRN Electronic Journal*. doi:10.2139/ssrn.1416663

Comissão para o Transporte Integrado (2004). "High-speed rails: international comparisons", elaborated by Steer Davies Gleave for CfIT, London

de Rus G., Nash C., (2007). In what circumstances is investment in HSR worthwhile? ITS Working Paper 590, Institute for Transport Studies University of Leeds. http://www.eprints.whiterose.ac.uk/2559/.

Dobruszkes, F. (2011). High-speed rail and air transport competition in Western Europe: A supply-oriented perspective. *Transport Policy*, *18*, 870–879. doi:10.1016/j.tranpol.2011.06.002

González-Savignat, M. (2004). Competition in Air Transport The Case of the High Speed Train, *38*(April 2003), 77–107.

Gunn, H. (2001): ''Spatial and Temporal Transferability of Relationships between Travel Demand, Trip Cost and Travel Time.'' *Transportation Research*, E, 37, 163–89.

Hsu, C.-I., & Chung, W.-M. (1997). A model for market share distribution between high-speed and conventional rail services in a transportation corridor. *The Annals of Regional Science*, *31*(2), 121–153. doi:10.1007/s001680050042

Hsu, C.-W., Lee, Y., & Liao, C.-H. (2010). Competition between high-speed and conventional rail systems: A game theoretical approach. *Expert Systems with Applications*, *37*(4), 3162–3170. doi:10.1016/j.eswa.2009.09.066

Pels, E., Nijkamp, P., & Rietveld, P. (2000) Airport and Airline Competition for Passengers Departing from a Large Metropolitan Area. *Journal of Urban Economics, 48*(1), 29-45.

RAVE (2006). "Modelo Integrado de Procura de Passageiros, Rede Ferroviária de Alta Velocidade, Inquérito de Preferências Declaradas", elaborated by Steer Davies Gleave for Rave, Lisbon

RAVE (2009). "Ligação Ferroviária de Alta Velocidade entre Lisboa e Porto. Lote A – Troço Aveiro/Vila Nova de Gaia. Estudo Prévio. Tomo 18.1- Relatório Sintese", elaborated by COBA for RAVE, Lisbon

Román, C., Espino, R., & Martín, J. C. (2007). Competition of high-speed train with air transport: The case of Madrid–Barcelona. *Journal of Air Transport Management*, *13*(5), 277–284. doi:10.1016/j.jairtraman.2007.04.009

Shyr, O. F., & Hung, M. (2010). Intermodal competition with high speed rail- a game theory approach, Journal of Marine Science and Technology,Vol. *18*(1), 32–40..

Swan, W., Adler, N., 2006. Aircraft trip cost parameters: a function of stage length and seat capacity. Transportation Research Part E 42 (2), 105–115

Straffin, P. (1993). Game Theory and Strategy, published by The Mathematical Association of America