Dispersion Compensation in Optical Fiber Systems

Luís Miguel Pinto Correia de Carvalho Marques
Electrical and Computer Engineering Department
IST
Av. Rovisco Pais 1, 1049-001 Lisbon, Portugal

Abstract

This work aims to understand the time dispersion in optical communication systems and to find its solution.

We start by presenting a brief introduction regarding dispersion and its constituents for a single-mode fiber.

We derive the pulse propagation equation, in the linear regime, and show the influence and consequences of the dispersive effects, such as the group velocity dispersion and the higher-order dispersion, in different pulses.

In order to avoid dispersive effects on the pulse transmission, in the linear regime, two dispersion management schemes are presented: compensation scheme based in dispersion compensation fibers (DCFs) and compensation dispersion based in fiber Bragg gratings (FGBs).

Finally, the influence of the nonlinear effects in pulse propagation of optical fiber systems is presented and analyzed. Under special circumstances it is possible the propagation of solitons. To conclude, we’ll discuss the most used and common dispersion management in solitons systems, so that the jitter effect is minimised.

1 - Introduction

Since the beginnings of time, there has always been an enormous need to establish long distance communications. The increase of telecommunication services and their massification has led to an improvement in the capacity of telecommunication networks [9]. Narinder Kapany in partnership with Harold Hopkins created the first glass fiber in order to guide light and images. This innovator light and image transmission system was based on the studies of John Tyndall, who used a water full recipient with a small hole to prove that the light could propagate through the recipient and able to leave with the water. The first optical fiber showed superior losses of 1000 dB/km, making it unpractical in telecommunications [3]. The advance of glass quality and manufacturing process of optical fiber that allowed to achieve an absolute minimum in the third window attenuation, and the development of optical amplifiers, solved the problem about fiber losses. However, nowadays, problems about dispersion and nonlinear effects in fiber remained unresolved [7].

We are now moving towards the fifth generation of optical communication systems and the expectations for this new generation are quite high. Several alternative approaches that are being tested are likely to solve the dispersion problem, such as: the upgrading of current systems introducing dispersion compensation schemes (e.g. DCFs); the use of dispersion-management, changing the way of designing conventional linear systems; solitons systems that take advantage of the fiber nonlinearity [3].

When it comes to long distance optical communication systems and high transmission rate, the nonlinear effects can take on a big role in the degradation of systems performance. Solitons systems allow to compensate the dispersion and
nonlinear effects simultaneously - manifestation was observed for the first time by John Scott Russell [6-9]. Solitons propagate in optical fiber, according to nonlinear Schrödinger equation in which the amplitude of the pulse envelope has the hyperbolic secant shape.

2 - Pulse Propagation in Linear Regime

The time dispersion is one of the effects of optical fiber, responsible for the degradation of the information transported. The first step of this work is to understand and quantify this manifestation, so that it can be possible to reduce the dispersion.

2.1 - Time Dispersion in Single-Mode Fibers

The main advantage of single-mode fibers is that intermodal dispersion is absent because the energy of the injected pulse is transported by a single mode [1]. However, in single-mode fibers still exists a dispersion source, entitled group velocity dispersion (GVD). The GVD results of different spectral components of the pulse travelling at slightly different group velocities due to the variation of the refractive index of the core and cladding with the frequency [10]. Total dispersion has two contributions: material dispersion \( D_M \) and waveguide dispersion \( D_W \). Material dispersion occurs because the refractive index of cladding changes with the wavelength and waveguide dispersion due to light-confinement problem in the fiber core, where the light is also propagated through the fiber cladding that travel at different velocities. These contributions are given by [3]

\[
D_M = M_2 \frac{\partial^2 N_2}{\partial \lambda^2} = -\frac{\lambda}{c} \frac{d^2 n_2}{d \lambda^2} \quad (2.1)
\]

\[
D_W = M_2 \Delta \frac{d(vb)}{dv} - \frac{N_2^2 \Delta}{n_2 \lambda c} \left[ v \frac{d^2(vb)}{dv^2} \right] \quad (2.2)
\]

where \( c \) is the velocity of light, \( n_2 \) is the refractive index of the cladding material, \( N_2 \) is the group index of the cladding, \( \nu \) is the normalized frequency parameter and \( b \) is the normalized propagation constant.

We observe that \( D_M \) has a positive slope, \( D_W \) has a negative slope and \( D \) is zero near 1312 nm. Since the waveguide contribution \( D_W \) depends on fiber parameters such as the core radius \( a \) and the index difference \( \Delta \), it's possible to design the fiber so that the total dispersion equal to zero [10]. Then, the total dispersion equation can be rewrite as [3]

\[
D = -\frac{2\pi c}{\lambda^2} \beta_2 = \frac{\lambda S_D}{4} \left[ 1 - \left( \frac{\lambda_{zd}}{\lambda} \right)^4 \right] \quad (2.3)
\]

where \( \beta_2 \) is known as the group-velocity dispersion coefficient, \( \lambda_{zd} \) is the zero-dispersion wavelength and \( S_D \) is the higher-order dispersion at the position where the higher-order dispersion is nil. The higher-order dispersion is given by [3]

\[
S = \frac{\partial D}{\partial \lambda} = \frac{4\pi c}{\lambda^3} \beta_2 + \left( \frac{2\pi c}{\lambda^2} \right)^2 \beta_3 = \frac{S_D}{4} \left[ 1 + 3 \left( \frac{\lambda_{zd}}{\lambda} \right)^4 \right] \quad (2.4)
\]

where \( \beta_3 \) is known as the higher-order dispersion coefficient.

2.2 - Pulse Propagation Equation

The pulse propagation equation in the linear regime is derived in order to determine the shape
of the pulse at the output of the communication link. Assuming that there’s no change in refractive index and \( A(0,t) \) is the pulse envelope at entrance of the fiber, \( z = 0 \), the electric field is, in that point, linearly polarized in the direction of \( x \), given by:

\[
E(x,y,0,t) = \hat{x} F(x,y) B(0,t)
\] (2.5)

We can prove that each frequency component of the optical field propagates inside the single mode fiber as [1]

\[
\hat{E}(x,y,z,\omega) = \hat{x} F(x,y) \hat{B}(z,\omega)
\] (2.6)

where \( \hat{B}(z,\omega) = \hat{B}(0,\omega) e^{i\beta_z} \). Being \( \omega_0 \) the carrier frequency, applying the properties of Fourier transform, the amplitude in the time domain can be written as

\[
B(z,t) = A(z,t) e^{i[\beta_0 z - \omega_0 t]}
\] (2.7)

in which \( A(z,t) \) is pulse envelope along the fiber and \( \beta(\omega) = \beta(\omega_0 + \Omega) = \sum_{m=0}^\infty \beta_m / m! \) where \( \beta_m = d^m \beta / d\omega^m \). To calculate the basic propagation equation is useful to determine \( A(z,t) \) in terms of \( A(0,t) \). The following equation is defined as

\[
\frac{\partial A}{\partial z} = i \sum_{m=1}^\infty \frac{\beta_m}{m!} A_m(z,t) - \frac{\alpha}{2} A(z,t)
\] (2.8)

Being \( A_m(z,t) = -i^{(m-2)} d^m A / dt^m \) therefore if the fourth order and greater (\( m \geq 4 \)) propagation terms and the attenuation constant are ignored, we can reach

\[
\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} = 0
\] (2.9)

This differential equation can be rewritten through the following normalized variables:

\[
\zeta = \frac{x}{L_D}, \tau = \frac{t - \beta_1 z}{\tau_0}, L_D = \frac{\tau_0^2}{|\beta_2|}
\] (2.10)

Applying these new normalized variables in the differential equation, we get

\[
\frac{\partial A}{\partial \zeta} + \frac{1}{2} \text{sgn}(\beta_z) \frac{\partial^2 A}{\partial \tau^2} - \kappa \frac{\partial^3 A}{\partial \tau^3} = 0
\] (2.11)

\[
\text{sgn}(\beta_z) = \frac{\beta_2}{|\beta_2|}, \kappa = \frac{\beta_3}{6|\beta_2|\tau_0}
\] (2.12)

Introducing the new variable \( \xi \), designated normalized frequency, so that \( \xi = \Omega \tau_0 = (\omega - \omega_0)\tau_0 \), we have

\[
\tilde{A}(\zeta,\xi) = \tilde{A}(0,\xi) e^{i[\Omega (\zeta + \kappa \xi^2) + |\beta_2|\xi^2 / 2]}
\] (2.13)

Now, it’s easy to determine the spectral pulse value, at any position in the fiber, just by following the next steps

1 - \( \tilde{A}(0,\xi) = \text{FFT}[A(0,t)] \);
2 - \( \tilde{A}(\zeta,\xi) = \tilde{A}(0,\xi) e^{i[\Omega (\zeta + \kappa \xi^2) + |\beta_2|\xi^2 / 2] \xi} \);
3 - \( A(\zeta,\tau) = \text{IFFT}[\tilde{A}(\zeta,\xi)] \);
4 - \( A(z,t) \).

### 2.3 – Evolution of Gaussian Pulse

In this section we are going to simulate the propagation of the Gaussian pulse inside the optical fiber. For an initially chirped Gaussian pulse, the incident field can be written as

\[
A(0,t) = e^{-\left(\frac{1 + C}{2}\zeta^2\tau_0^2\right)}
\] (2.14)

where \( C \) is the chirp parameter. In order to better understand the dispersion effects we will simulate, for different chirp values, the propagation of Gaussian pulse along the optical fiber, in anomalous region.

![Figure 2.2 – Evolution of the Gaussian pulse along the optical fiber for \( C = 0 \).](image-url)
Figure 2.3 – Evolution of the Gaussian pulse along the optical fiber for \( C = 2 \).

Figure 2.4 – Evolution of the Gaussian pulse along the optical fiber for \( C = -2 \).

As a result of dispersion effect, pulses will suffer a time broadening, which can induce ISI that limits the bit-rate of a communication link. With the increase of fiber length, this effect is more accentuated. We conclude that, for all cases, there is a reduction in amplitude along the fiber due to the broadening of the pulse caused by group-velocity dispersion. We observe that for \( C < 0 \) the chirped Gaussian pulse broadens monotonically at a rate faster than in the absence of frequency chirp, \( C = 0 \). For \( C > 0 \), the pulse width initially decreases and becomes minimum at a distance \( z_{\text{min}} = |C|L_D/1 + C^2 \). The reason is related to the fact that the dispersion-induced chirp counteracts to the input chirp. After this minimum value, the dispersion-induced chirp adds to the input chirp and the chirped Gaussian pulse broadens monotonically, and for long distances we obtain worse results than with the unchirped Gaussian pulse [2].

2.4 - Higher-order Dispersion

In this section, we are going to study the higher-order dispersion effects on the Gaussian pulse propagation. Although the contribution of GVD dominates in most cases of practical interest, it’s sometimes necessary to include the higher-order dispersion governed by \( \beta_3 \) [2]. There are two cases where we can’t neglect the higher-order dispersion coefficient: for ultra-short pulses where the spectrum is very large, and when the pulse wavelength nearly agrees with the zero-dispersion wavelength \( \lambda_{ZD} (\beta_2 = 0) \). We inspect that higher-order dispersion effects are more obvious when the pulse width has lower values. Figure 2.5 shows the pulse shapes at \( z = 5L_D \) for an initially unchirped Gaussian pulse for two cases: \( \beta_2 = 0 \), and \( L_D = L_D' \). When \( \beta_2 = 0 \) there are strong oscillations, with intensity dropping rapidly to zero between successive oscillations. For \( L_D = L_D' \), we can see that the pulse is distorted, and it becomes asymmetric with an oscillatory structure near one of its edges, depending on the signal of \( \beta_3 \).

Figure 2.5 – Higher-order dispersion effects in pulse shapes at \( z = 5L_D \) of an initially Gaussian pulse with \( \tau_0 = 1 \) ps, for \( \beta_2 = 0 \) and for \( L_D = L_D' \).

Then, we examine the higher-order dispersion effects in a chirp Gaussian pulse for \( \beta_2 = 0 \) and \( \beta_3 = 0.1 \) ps\(^3\)/km. The \( \beta_3 \) parameter is affected by a \( C^2 \) factor, so for symmetric values of \( C \) the evolution of chirp Gaussian pulses are similar [5]. In figure 2.6, we noticed that, the
greater the $C$ value, the more significant are the effects of the higher-order dispersion.

Figure 2.6 – Higher-order dispersion effects in pulse shapes at $z = 50$ km of an initially Gaussian pulse with $\tau_0 = 1$ ps, for $C = 2$, for $C = 1$, and $C = 0$.

3 - Dispersion Compensation in Linear Regime

Because of big degradation imposed by dispersion, it becomes necessary to develop techniques and dispersion compensating optical devices, which are able to minimize the effects caused by the dispersion. There are several technological possibilities to compensate this phenomenon, and in this article we address the dispersion compensation fibers (DCFs) and fiber Bragg gratings (FBGs).

3.1 - Dispersion-Compensating Fibers

The use of DCF provides an all-optical technique that is capable of compensating the fiber dispersion completely if the average optical power is kept low enough that the nonlinear effects inside optical fibers are negligible [1]. This technique combines segments of optical fiber with different characteristic to reduce the average dispersion of the entire fiber link to zero. Each optical pulse propagates through two fiber segments, the second of which is the DCF, and at the end of this fiber link we have [2]

$$ A(L, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(0, \omega) \cdot e^{-i(\beta_{21}L_1 + \beta_{22}L_2)\omega^2 + i(\beta_3L_1 + \beta_3L_2)\omega^3 - io\omega} d\omega \quad (3.1) $$

where $L = L_1 + L_2$ and $\beta_{2j}, \beta_{3j}$ are the GVD and higher-order dispersion parameters for the fiber segment of length $L_j$ ($j = 1,2$). Considering $\beta_{3j} = 0$, the condition for dispersion compensation can be written as

$$ \beta_{21}L_1 + \beta_{22}L_2 = 0 \quad (3.2) $$

We verify that the fiber length of DCF must be such as to satisfy

$$ L_2 = -\frac{\beta_{21}}{\beta_{22}}L_1 \quad (3.3) $$

To minimize costs is typical to use DCFs with large negative value of $\beta_{22}$ so that $L_2$ segment can be as small as possible. The main disadvantages of DCFs are the high production costs, exhibit relatively high losses and possibility of the occurrence of nonlinear phenomena. Considering $\tau_0 = 100$ ps, $\beta_{21} = -20$ ps$^2$/km, $\beta_{22} = 2500$ ps$^2$/km and $L_1 = 2500$ km, by the equation (3.3) we obtain $L_2 = 20$ km. The evolution of unchirped Gaussian pulse is shown in figure 3.1. The dispersion of unchirped Gaussian pulse is fully recovered and the amplitude of the pulse at the output of the DCF is the same as at the input of single-mode fiber.

Figure 3.1 – Evolution of unchirped Gaussian pulse in single-mode fiber with $L_1$ length; Evolution of unchirped Gaussian pulse in the DCF with $L_2$ length.

3.2 – Fiber Bragg Gratings

The appearance of the FBGs permitted the development of various applications due to its properties, versatility and variety of controllable parameters, which may format in various ways.
their spectral characteristics [18]. In telecommunication’s area, FBGs can be used in different points of a transmission system. In general, a FBG is formed by a set of elements spaced at a certain distance [14]. These segments of optical fiber reflect certain wavelengths, which satisfy the resonance condition, and transmit all other wavelengths. This is possible thanks to the introduction of periodically or aperiodically variations of the refractive index along the length, acting as a dielectric mirror for a specific wavelength [11]. Thus a FBG acts as an optical reflection filter because of the existence of a stop band, the frequency region in which most of the incident light is reflected back [1]. This stop band is centred at the Bragg wavelength, given by

\[ \lambda_B = 2\bar{n}\Lambda \]  

(3.4)

where \( \Lambda \) is the grating period and \( \bar{n} \) is the average mode index. When the resonance condition is verified, there is a maximum reflectivity for the wavelength that satisfies the Bragg condition. To analyze the behaviour of such structures, we can use the coupled-mode equations that describe the coupling between the forward- and backward-propagating waves at a given frequency \( \omega \) [16].

### 3.2.1 – Uniform Fiber Bragg Gratings

A fiber Bragg grating is designated as uniform when its spatial properties are constant throughout its length. Along the uniform fiber Bragg grating, the two waves are given by:

\[ \frac{\partial A_f}{\partial z} = i\delta A_f + i\kappa_g A_b \]  

(3.5)

\[ -\frac{\partial A_b}{\partial z} = i\delta A_b + i\kappa_g A_f \]  

(3.6)

where \( A_b \) and \( A_f \) are the spectral amplitudes of two waves, \( \delta \) is the detuning from the Bragg wavelength and \( \kappa_g \) is the coupling coefficient. Solving analytically the coupled-mode equations the reflection coefficient, \( r_g \), and its phase, \( \phi_g \), are given by

\[ r_g = \frac{i\kappa_g \sin(q_g L_g)}{q_g \cos(q_g L_g) - i\delta \sin(q_g L_g)} \]  

(3.7)

\[ \phi_g = -\arctan \left( \frac{\text{Im}(r_g)}{\text{Re}(r_g)} \right) \]  

(3.8)

where \( q_g^2 = \delta^2 - \kappa_g^2 \) and \( L_g \) is the FBG length.

![Figure 3.2 - Magnitude of the reflectivity as function of \( \delta L_g \)](image)

Figure 3.2, shows the reflectivity for \( \kappa_g L_g = 2 \) and \( \kappa_g L_g = 4 \), and we observe that, in stop band, the greater \( \kappa_g L_g \) value, the more the reflectivity approaches 100%. However, we verify that the presence of secondary maximums in reflectivity is due to the appearance of multiple reflections in the boarder of the FBG (as a Fabry-Pérot cavity) [13]. The problem can be solved by using apodisation techniques [12].

From the reflective signal phase, the group-delay is written as

\[ \tau_g = \frac{d\phi_g}{d\omega} = -\frac{\lambda^2}{2\pi c} \frac{d\phi_g}{d\lambda} \]  

(3.9)

thus the grating-induced dispersion is given by

\[ D_g = \frac{d\tau_g}{d\lambda} = -\frac{2\pi c}{\lambda^2} \frac{d^2\phi_g}{d\omega^2} = -\frac{2\pi c}{\lambda^2} \beta_{2g} \]  

(3.10)

where \( \beta_{2g} \) is the dispersion coefficient.
Figure 3.3 - Phase of the reflectivity as function of $\delta L_g$ for a uniform FBG with $\kappa_g L_g = 2$ and $\kappa_g L_g = 4$.

Figure 3.4 – Time delay as function of wavelength for a uniform FBG with $\kappa_g L_g = 2$ and $\kappa_g L_g = 4$.

We observe that, in stop band, the phase variation is nearly linear, so that region will correspond to a minimum group-delay value and consequently the dispersion value is lower. We are able to state that the grating-induced dispersion exists only outside the stop band and the higher the $\kappa_g L_g$ product, the higher will be the dispersion value. High dispersion values are greater, with the group delay varying rapidly, which is caused by multiple wavelength reflections before exiting the grating.

Another important parameter is the bandwidth of the grating. The bandwidth depends on $L_g$ and $\kappa_g$. The smaller the length of the grating, the greater the width of the stop band, but consequently leads to a lower maximum reflectivity. We found that the uniform FBG have a reduced bandwidth, not allowing to be used at a high rate [1].

3.2.2 – Chirped Fiber Gratings

Chirped fiber gratings (CFBG) allow compensating the dispersion for high rates. These permit the variation of Bragg wavelength condition throughout their length by varying the physical grating period $\Lambda$ or by changing the effective mode index $\bar{n}$ along $z$ [1]. Considering a linear grating period variation, given by

$$\Lambda(z) = \Lambda_0 + C_\Lambda z$$

(3.11)

where $\Lambda_0$ is CFBG period in one of their ends and $C_\Lambda$ is the aperiodicity coefficient. Hence, it is possible to obtain linear aperiodicity causing an increase in the Bragg wavelength and a consequent shift of the centre of the stop band to progressively lower frequencies as the period increases. Different frequency components of an incident optical pulse are reflected at different points, depending on where the Bragg condition is satisfied [1]. So the high-frequency components of the pulse are the first to be reflected, and as a result of this all components of the pulse leave the grating at the same time [5]. This situation corresponds to anomalous GVD. The dispersion in a CFBG, where the optical period varies linearly along its length, is given by [12]

$$D_g = \frac{2\bar{n} L_g}{c \Delta \lambda}$$

(3.12)

where $\Delta \lambda = 2\bar{n} L_g C_\Lambda$ designates the difference between the spectral components reflected at the ends of CFBG. By replacing we obtain
We noticed that a Bragg grating with linear aperiodicity is independent of their length, only changing with the aperiodicity coefficient. Therefore, by using a CFBG with a length in the order of dozens of centimetres is possible to compensate the GVD imposed by a single-mode fiber with an approximately length in the hundreds of kilometres [1].

\[ D_g = \frac{1}{c C_A} \]  

\[ (3.13) \]

Figure 3.6 – Reflection and delay for linearly CFBG with \( L_g = 25 \) mm.

We observe that the CFBG usually has a wider bandwidth than the bandwidth of a FBG, because in this type of gratings, the Bragg condition occurs on a larger number of spectral components [5].

4 – Dispersion Compensation in Nonlinear Regime

For higher optical power of the input signal or for longer transmission distances the nonlinear effects of optical fibers cannot be neglected. We verify that the increase of the intensity causes a variation in refractive index of the optical fiber [7]. In nonlinear regime, the dependency of the refractive index with the field intensity causes an interesting manifestation designated as self-phase modulation (SPM), a phenomenon that leads to spectral broadening of optical pulses [2]. SPM and GVD will limit the performance of optical fiber communication systems. However, in anomalous-

dispersion region, a fascinating manifestation of the fiber nonlinearity occurs as a result of a balance between the dispersive and nonlinear effects, allowing the propagation of optical solitons.

4.1 - Optical Solitons

The mathematical description of solitons employs the nonlinear Schrödinger (NLS) equation, which is given by [4]

\[ \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial t^2} + |u|^2 u = 0 \]  

\[ (4.1) \]

Through the IST method (inverse scattering transform) it is possible to define the soliton order \( N \) as follows

\[ u(0,\tau) = N \text{sech}(\tau) \]  

\[ (4.2) \]

The first-order soliton \((N = 1)\) corresponds to the fundamental soliton. In figure 4.1 we demonstrate that the fundamental soliton shape doesn’t change on propagation, because SPM compensates the GVD effects. As opposed to what occurs with the fundamental soliton, the shape of higher-order solitons changes, figure 4.2, but nonetheless shows a periodic evolution, recovering its original shape at \( \xi_0 = \pi/2 \), or in real units

\[ z_0 = \frac{\pi}{2} L_D = \frac{\pi \tau_0^2}{2 |\beta_2|} \]  

\[ (4.3) \]

In the case of third-order soliton, SPM dominates initially, but GVD soon catches up and leads to pulse contraction as seen in figure 4.2 [2].

Figure 4.1 – Fundamental soliton.
However, the optical fibers have losses, and due to this, the balance between SPM and GVD is lost. It is necessary to introduce amplifiers to ensure that the solitons maintain their width transmission and that these will create phase fluctuations. These fluctuations cause random time fluctuations, introducing jitter, or usually named Gordon-Haus jitter, which contributes to the degradation of the system [6].

4.1.1 – Intrapulse Raman Scattering

When it comes to high bit rate solitons, systems that require a small width pulse, Raman scattering must be taken into account. Due to the reduced width of the soliton, the spectrum is wider, whereby higher frequencies transfer energy to lower frequencies, through the Raman-induced frequency shift [6]. The equation that describes the propagation of solitons in an optical fiber, without ignoring Raman effects, is given by

$$\frac{\partial u}{\partial \lambda} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u - n_r u \frac{\partial |u|^2}{\partial \tau} = 0 \quad (4.4)$$

where $\tau_r = T_{R}/n_0$ is the Raman effect coefficient and $T_R$ is the Raman term, typically $3$ fs in third window. For chirp parameter $C = 0$, the Raman-induced frequency shift $\Omega_p$ (RIFS) grows linearly with distance as [2]

$$\Omega_p(z) = -\frac{8T_R P_0}{15\tau_0} z = -\frac{8T_R |\beta_2|}{15\tau_0} z \quad (4.5)$$

where we used the condition $N^2 = \gamma P_0T_R^2/|\beta_2| = 1$. The negative sign shows that the frequency is reduced, i.e., the soliton spectrum shifts towards longer wavelengths [2]. The expression shows that the RIFS is relevant only for ultra-short pulse, since the smaller the pulse width, the higher the RIFS.

![Figure 4.3 – Delay of fundamental soliton because of Raman-induced frequency shift for $\tau_r = 3$ fs.](image)

We observe in figure 4.3, that the fundamental soliton spectrum is shifted to the red side causing a time delay in the soliton, which increases with distance. This shift is deterministic in nature because it will depend on characteristic parameters of the pulse, such as energy and width. In the case of second-order soliton we verify that the effect of Raman on higher-order soliton leads to the breakup of solitons into two constituents which continue to move apart with further propagation inside the fiber.

By introducing optical amplifiers, they will create fluctuations in the pulse energy, that are converted into fluctuations in the soliton frequency.
through the Raman effect, which are in turn translated into position fluctuations by GVD, contributing to worsen the jitter. One solution to fight the Raman effect is the introduction of filters after each optical amplifier which increases the signal-noise ratio, reducing the jitter, or through dispersion management techniques [6].

4.1.2 – Dispersion Managed Solitons

To fight the frequency deviations, it’s usual to use periodic dispersion maps that use fiber segments with different dispersion coefficients, figure 4.5, so that during the pulse propagation, the average value of the dispersion can be maintained within the anomalous region [6]. To reduce the jitter, the coefficient values of the dispersion should be chosen so that the average dispersion, \( \bar{D} \), given by

\[
\bar{D} = \frac{D_1 L_1 + D_2 L_2}{L_1 + L_2}
\]

(4.6)
is zero. This allows not only to elevate the peak power, which leads to an increase in the value of the signal-noise ratio, and to a reduction of jitter impact, but also to an increase of the maximum transmission distance of the soliton based systems. Considering the dispersion variation, the equation governing the solitons propagation, neglecting the higher-order terms, is defined as

\[
\frac{i}{\partial \zeta} \frac{\partial u}{\partial \zeta} + \frac{\sigma(\zeta)}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = -i \frac{\Gamma}{2} u
\]

(4.7)

where \( \sigma(\zeta) = D(\zeta) / \bar{D} \) is the normalized dispersion coefficient and \( D(\zeta) \) is the dispersion in a certain local in the fiber segment.

![Figure 4.5– Periodic dispersion map [6].](image)

As shown in figures 4.6 and 4.7, in the first segment of fiber, the pulse width initially increases because \( D_1 > \bar{D} \), and in the following segment, the pulse width decreases due to \( D_2 < \bar{D} \), recovering the initial shape at the end of the \( L_1 + L_2 \) period.

4.2 – Gaussian Pulse in nonlinear Regime

The Gaussian pulse propagation is governed by the nonlinear Schrödinger equation seen in equation (4.1). We illustrate, in figures 4.8 and 4.9, the evolution of the Gaussian pulse for \( \zeta = 8 \) and \( \zeta = 40 \).

In contrast to what occurred in the evolution of the Gaussian pulse in linear regime, we observed that for the Gaussian pulse in nonlinear regime, the broadening of pulse and consequent decrease in the amplitude were less abrupt [15]. It’s established, for short distances, a balance in the pulse characteristics, which remain unchanged during the propagation, and tend

![Figure 4.6 – Evolution of DM soliton along the first segment of map.](image)

![Figure 4.7 – Evolution of DM solitons along the second segment of map.](image)
towards the solitons shape, losing energy until it acquires the shape of the fundamental soliton. However, when we consider long distances, the amplitude and the pulse width oscillate due to the dispersion and nonlinear effects. As verified, for systems with solitons, the pulse broadens and the amplitude decreases because of the GVD effects and, later on, the pulse suffers a narrowing and, consequently, an increase of the amplitude, predominating the SPM effects over the GVD effects [15]. The Gaussian pulse can be used to describe solitons in dispersion maps periodic, where the average GVD is zero at the end of each period.

Figure 4.8 – Evolution of Gaussian pulse in nonlinear regime for $\zeta = 8$.

Figure 4.9 – Evolution of Gaussian pulse in nonlinear regime for $\zeta = 40$.

5 - Conclusion

We conclude that optical fibers are subjected to dispersive mechanisms. For a single-mode fiber, we observed that the second-order dispersion coefficient, designated by GVD, is responsible by the broadening that occurs in the pulse propagation inside the optical fiber. This time enlargement will cause ISI, originating interference between signals that limit the bit rate and the communication link. The total dispersion can be written as the sum of two terms: the material dispersion and the waveguide dispersion. It's then possible to design optical fibers to obtain total dispersion equal to zero, through the manipulation of the fiber parameters, like the core radius and the index difference. We simulated the Gaussian pulse propagation in an optical fiber neglecting the higher-order effects and losses, in the anomalous region. The Gaussian pulse suffered an increase in its width and consequently a decrease in its amplitude. We observed that the initial chirp parameter contributed to the broadening of the pulse. Note that for $\beta_2 c > 0$ we observed that the pulse width initially decreased until a certain point. Then the pulse width broadened, as it happened for other values of chirp. We studied the higher-order dispersion coefficient and we realized that the effects are manifested in two cases: when the GVD coefficient is nil, and with ultra-short pulses, where the spectrum is vast. Due to the large degradation imposed by the dispersion in the communication systems, it was necessary to use dispersion compensating techniques. First, we analyzed the dispersion-compensation fiber technique, combining optical fiber segments with dispersion coefficient of opposite sign to SMF transmission fiber, in order to reduce to zero the average value of the dispersion. At the end of the DCF segment, we observed that the initial shape of the pulse was fully recovered, compensating GVD effects and the higher-order dispersion.

Then we studied the Bragg gratings, and we began to analyze the uniform FBG and concluded that they act as an optical filter, reflecting the wavelengths that verify the resonance condition. We found that the greater the product of $\kappa g L_g$, the greater the reflectivity, approaching of the maximum value of 100%. In stop band, where the
maximum reflectivity occurs, the dispersion-induced is nil, ascertaining that the dispersion only exists out of the stop band, due to the multiple reflections of some wavelengths in the ends of the grating. We demonstrated that to increase the bandwidth it was necessary to reduce the grating length; however the reflectivity would be lower. The bandwidth of these devices is lower than 1 nm and it also depends on the coupled-coefficient between the propagation waves. Therefore, to compensate the dispersion was used CFBGs. Through the refractive index variation or through the grating periodicity variation, the different wavelengths are reflected in different places of the grating, where the dispersion values are estimated from the difference of paths between high and low frequencies. Therefore, it was able to compensate the dispersion with a CFBG length just in the order of dozens of centimeters to recover the initial signal that was propagated in hundreds of kilometers. CFBGs are, in fact, more advantageous than DCFs, because when using DCFs for dispersion compensation, it would be necessary a bigger fiber length.

In chapter four, we analyzed the influence of the nonlinear effects in the pulse propagation. Due to the high optical fields induced, there is a refraction index variation. This dependence of the field intensity will be accountable by the SPM effect. Under certain special circumstances there is a balance between the GVD and the SPM, allowing the propagation of solitons, which maintain their shape throughout the fiber, making them desirable to optical communications. When we considered that the optical fibers had losses, the balance between SPM and GVD was lost. It is necessary to introduce amplifiers to ensure that the solitons maintain their width transmission and that these will create phase fluctuations. These fluctuations cause random time fluctuations, which introduce Gordon-Haus jitter. We also studied the strongest effect of the higher-order, the Raman effect. We concluded that this should be accounted in ultra-short pulses, introducing frequency shifts, which led to significant time deterministic shifts. To minimize the Gordon-Haus jitter and the Raman effect, were introduced periodic dispersion maps to maintain the average dispersion value. The underlying idea is quite simple and consists of mixing fibers with positive and negative GVDs in a periodic fashion such that the average dispersion over each period is close to zero, increasing the signal-noise ratio of the communication [1]. We verified that during the first segment of fiber the pulse width initially increased, and in the following segment, the pulse width decreased, recovering the initial shape at the end of the period.

Finally we simulated the Gaussian pulse in nonlinear regime. The Gaussian pulse width was broadened and its amplitude decreased at a rate lower than the observed in Gaussian pulse in the linear regime. For short distances was established a state of equilibrium of pulse characteristics, tending towards the shape of the fundamental soliton. When we considered very large distances the amplitude and the width of the pulse would vary due to the GVD and SPM effects, as we saw in periodic dispersion maps in solitons systems.
References