Comparison of Space Propulsion Methods for a Manned Mission to Mars

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Abstract

In this work a comparison of different propulsion systems on a manned mission to Mars is performed. The main objective is to assess the possibility of reducing travel time considerably, since long stays in space cause severe health problems and might lead to death, keeping the mass at departure within bounds. The systems under comparison are the chemical engine (CECE), the nuclear thermal engine (NERVA), the ion engine (PPS 1350-G) and Rubbia’s “Pure Electro-Magnetic Thrust” (PEMT) concept, using a nuclear engine. A typical mission architecture is sketched, including the mass budget for the cargo and manned spacecraft, based on existent mission proposals to Mars. The trajectory of the spacecraft is determined by a numerical integration of the equation of motion, for the interplanetary phase, and by Keplerian conics approximation. We conclude that the ion engine combined with the classical chemical engine is the one which yields the shorter travel times with the lowest mass. The results obtained using the PEMT suggest it is a better solution for farther destinations than Mars.

Keywords: Manned Mission, Mars, Chemical Propulsion, Nuclear Thermal Propulsion, Electrical Propulsion, Pure Electro-Magnetic Propulsion, Interplanetary Trajectory, CECE Engine, NERVA Engine, PPS 1350-G Engine.

1 INTRODUCTION

In the present work a comparison of the performance of different propulsion systems is developed, in the context of a manned mission to Mars. With the objective of diminishing the travel time, we use a single mission architecture as baseline, while changing the propulsion system in type and size.

Mars is the selected destination since it is the next logical step in human exploration of the Solar System.

Manned missions have only gone as far as the Moon in the Apollo project. A key factor of success was that every component of the Apollo spacecraft was optimised to serve a specific function at a specific time, and to be discarded as soon as it had fulfilled its function, minimising the mass to be carried [1].

Proposals of manned missions to Mars exist, involving chemical propulsion or nuclear thermal systems. However, after several years of research and development, other types of propulsion systems (and also newer chemical engines) have been used in different missions [2–6].

One which is examined in the present work is the “Pure Electro-Magnetic Thrust” (PEMT), presented in 2002 by Carlo Rubbia [7]. In this proposal, the engine produces thrust as a Solar Sail, but uses a nuclear reactor as the power source.

The selected propulsion systems are paradigmatic of each known form of propulsion:
- The CECE - for the classical chemical propulsion;
- The NERVA - for the nuclear thermal propulsion;
- The PPS 1350-G - for the modern electrical propulsion;
- The PEMT - as the new concept propulsion.

In what concerns missions to Mars two proposals are known.

Mars Direct was developed by Robert Zubrin in 1990 and it consisted of a 25 billion American dollars (of 1990) program [8]. The total mission mass was set to be below 1,000 t and was designed to use the technology available at the time [8]. The proposal comprised two Saturn V type launchers, one bearing a 45 tonne unmanned cargo module and the other a 25 tonne human habitat module, with a crew of four astronauts. The modules were thought to use the chemical upper stage of the rockets to launch to a 180 day transfer to Mars. Upon arrival the modules were supposed to execute aerocapture. For the return the crew would use fuel produced from Mars’ atmosphere. Again a 180 day transfer was considered. In total, the mission would last 910 days.

A new proposal, dubbed “Design Reference Mission” (DRM) was developed in 1992–1993, and based on Mars Direct [9]. It was thought to comprise a crew of six astronauts and four Saturn V type launchers, bearing a cargo lander, an Earth Return Vehicle (ERV), an habitat lander and a manned habitat. Each module would use nuclear thermal rockets as propulsion system and would have a mass ranging between 60 to 70 tonnes.

One of the drawbacks of these proposals is their duration. It is known that a person in space is subjected to many hazards, among which cause loss of bone/muscle mass and a substantial probability increase of developing cancer [10–12]. Thus, a 180 day transfer is a great risk, most particularly in view of the unpredictability of the solar cycle.

After the concepts presented here, new mission proposals were developed, as the DRM 5.0 [13] and the “Austere Human Missions to Mars” [14]. These ones are variants of the DRM, with updated technology and including (or not) options as In-Situ Resource Utilization and aerocapture.

2 PROPULSION SYSTEMS

The engines to be discussed exhibit the highest specific impulse ($I_{sp}$), thrust ($F_{T}$) and thrust-to-weight ratio ($F_{T}/w$) among the numerous engines available. We use engines currently being tested and others with flight proven capabilities. For a broader discussion of propulsion systems, including a putative gravity control, the reader is referred to references [2, 15, 16].

2.1 Classical Chemical Engines

Within this class only cryogenic systems are considered due to their high thrust and $I_{sp} \approx 400$ s [2].
Chemical engines are used as impulsive ones due to their high thrust and relatively low Isp. Another reason for using chemical engines in this manner is the gravity loss, which is directly proportional to the time it takes to execute a manoeuvre, increasing the real fuel expenditure [2].

After computing the \( F_I / m \) and analysing the various engine features the selected one is the Common Extensible Cryogenic Engine (CECE) of Pratt & Whitney Rocketdyne [17]. This engine is considered an evolution of the RL 10 (in use since 1963 in the United States [18]), and it has the capability to re-start up to fifty times. The CECE has also a relatively high thrust with a low mass (see table 1).

### 2.2 Nuclear Thermal Engines

Nuclear engines have been developed since the 1940s [2]. These engines can also be used as impulsive ones, due to their high thrust and \( I_{sp} = 900 \text{s} \).

A quantity defining the nuclear fuel performance is the burnup [19]. The specific burnup is the energy released per unit mass.

One of the last nuclear engines to be developed within the Nuclear Engine for Rocket Vehicle Application (NERVA) program was the NERVA II [20], whose main features are shown in table 1.

### 2.3 Modern Electrical Engines

Electric propulsion overcomes the chemical engines limitations by separating the working fluid from the energy source. Common sources of energy are solar, nuclear power generation and radioisotope thermal generators (RTG) [3].

The thrust produced is very small, when compared to chemical engines, limited by the power supplied to the engine. However, they have greater \( I_{sp} \) allowing the engine to be running for longer periods of time with less fuel. The electrical engine is treated as a non-impulsive engine.

The PPS 1350-G is the selected engine, due to its higher thrust-to-weight ratio and for having flight proven capabilities. The main characteristics are again shown in table 1.

The selected power source is the solar photovoltaic, since nuclear power systems represent a large increase of the engine mass and the RTG technology can only achieve specific powers of 5 W/kg and is still being perfected [3]. The size of the solar array depends on several parameters and the equation for its mass is derived from reference [22].

### 2.4 Nuclear Pure Electro-Magnetic Thrust

The PEMT engine has a nuclear reactor which, instead of expelling a heated working fluid to create thrust, uses photons to produce thrust [7].

The thermal energy of a nuclear reactor is used to heat a radiator, which emits electromagnetic radiation (photons). The radiator is in front of a reflecting surface that reflects the radiation. The momentum transferred to the reflecting surface by the hitting photons, with the momentum of the radiator, produces the thrust. A non focusing reflecting conical structure is added to increase the thrust of the engine (see figure 1) [7].

A photon with a frequency \( \nu \), moving at the speed of light \( c \), has an energy \( (E_p = h \nu = c P_p) \) and a momentum \( (P_p = \frac{h \nu}{c}) \), where \( h \) is Planck’s constant [7]. Using a parallel beam \( (P = \sum P_i) \) and Newton’s 2nd law

\[
F = \frac{d}{dt} (\sum P_i) = \frac{1}{c} \frac{dE}{dt} = \frac{W}{c},
\]

for the case of a radiator with power \( W \) [7]. From the Stefan-Boltzmann law one can relate the power and the temperature \( (T) \) for a surface \( (S) \), \( W = \Phi \tau S = \varepsilon \sigma T^4 S \).

We have selected a NERVA reactor for our main engine, and resized it, assuming it can produce 22% more energy with an increase in mass of 26% (see table 1). Although this engine has an \( I_{sp} \) far greater than the ones presented so far, this value does not quantify fuel expenditure.

To ensure a small dimension for the radiator one can produce 20N for a radiator temperature of about 3,300 K. Among the materials that can withstand this high temperature without melting, carbon nanotubes are the lightest [7].

For the reflector, one needs to guarantee that it has a maximum reflecting capability for the wavelength the radiator is emitting (for our radiator, solving Planck’s equation, as shown in reference [7], we have \( T = 3,300 \text{K} \rightarrow \lambda_{max} = 0.88 \mu m \)). The reflector selected should be capable of reflecting visible and infrared radiation. The current technology for solar sails uses composite booms (on the support structure) and Aluminized Mylar sails (or carbon fibre sail substrate) [6]. The Mylar sail has the advantage of also reflecting infrared radiation.

### 2.5 Fuel Calculation

#### Impulsive Engines

When an engine can be treated as impulsive, one can use the Tsiolkovsky rocket equation for a velocity change \( (\Delta V) \). From that one obtains the propellant mass, \( M_P \). A complete mission trajectory requires \( N \) manoeuvres and it is necessary to compute the last manoeuvre first and work backwards, since the fuel mass depends on the initial mass.

The \( \Delta V \) used in the Tsiolkovsky equation is considered to be applied instantaneously. However, there is always a period of time associated with a manoeuvre and thus a spatial distribution of the burn. For safety reasons we introduce a loss factor of 10% for a manoeuvre longer than five minutes, 5% for a manoeuvre between one and five minutes, and for a manoeuvre requiring less than one minute we assume a 1% loss.

#### Non-Impulsive Engines

For non-impulsive systems the fuel spent is computed simultaneously with the trajectory using \( m = -\frac{F_I}{g_0 I_{sp}} \), which is integrated to extract the required...
fuel, using the same integration method as the trajectory calculations.

**Nuclear Pure Electro-Magnetic Thrust Engine** The fuel spent here is the fuel burned in a nuclear reaction (since it only expels photons).

With the fractional burnup of the reactor ($\beta \approx 0.04$ [19, 23]) and the maximum theoretical burnup of a $^{235}U$ ($950\text{GW}\cdot\text{d/}\text{t}$ [19]), the specific burnup can be written as $s_b = 950 \times \beta$. Using the operating time of the engine ($t_{on}$) and assuming that the engine works with a power ($W$), we have the total burnup, $B = W \times t_{on}$. Combining these characteristics one obtains the required uranium mass, $M_U = \frac{B}{s_b} = \frac{W \times t_{on}}{950\beta}$.

A nuclear reactor is usually loaded with uranium dioxide ($UO_2$). The mass of uranium dioxide to produce a certain power, during a certain period of time, is extracted from the mass of uranium required,

$$M_{UO_2} = \frac{M_U \times A_{UO_2}(U)}{A_{U}(U)} \quad (2)$$

Since the reaction normally involves $^{235}U$ atoms, and these only represent 0.72% of the uranium in natural state, one uses an enriched material, which in this case is 80 wt% $^{235}U$ (almost considered weapons-grade [24], but feasible [25]).

### 3 MISSION TO MARS

Considering an architecture similar to the DRM proposal, we construct a mission with four astronauts, and aim to minimising both time and mission mass.

The manned spacecraft is comprised of a human habitation module, the propulsion system and a transport capsule. The first two involve what we call the mother ship. The human habitation module is an extrapolation from Mars Direct and DLR proposals, with the guidelines of reference [26], and houses the crew during transit to and from Mars. The transport capsule carries the crew from Earth to the mother ship and then to Mars (upon arrival).

The unmanned spacecraft contains the cargo module (which is also an extrapolation from previous proposals), the corresponding propulsion system and a Mars ascent vehicle (MAV). The MAV is the vehicle used by the astronauts to leave Mars and rendezvous with the mother ship. It is also used for landing on Earth. Although its dry mass is equal to the transport capsule, it has also a fuel mass to lift off from Mars.

#### 3.1 Mission Architecture & Mission Timeline

We use this base architecture for comparison of the different propulsion systems. Considering that the objective is to minimise travel time, the cargo and crew are sent separately from each other. This allows to send with the cargo the required fuel for the manned return trip, lowering substantially the initial mass of the manned spacecraft, and not wasting fuel to send the cargo at high speed as well (which would lead to a smaller payload mass, so to keep the total mass in acceptable ranges).

**Parking Orbit & Departure** The first step is the launch from Earth to a parking orbit (the launch itself is not considered here as different configurations are required). It is usual to use an initial parking orbit on missions to other celestial bodies [1, 27, 28]. In this orbit the interplanetary injection manoeuvre would be more effective [29]. A priori we do not know the time taken for the assembly and thus the orbit used is higher than the common ones ([1, 27, 28]) and follows the example of the International Space Station [30] (perigliss altitude $r_p = 500 \times 10^3\text{ m}$ and eccentricity 0).

After being fully assembled, and tested, the spacecraft would raise the orbital apogee before executing the interplanetary injection (launch orbit with eccentricity 0.95). This is a common manoeuvre whose primary objective is to lower the $\Delta V$ loss [3, 27, 28]. Only before executing the EtM injection would the crew rendezvous with the mother ship (carried by a launcher in the transport capsule).

**Interplanetary Transfer** For the interplanetary phase the tested scenarios are the coast trajectory and the powered trajectory. Aiming to minimise travel time and mass we use a simplified procedure with three parameters:

- Direction and module of the spacecraft velocity at departure - needed when a shorter transfer is intended;
- Direction and intensity of the thrust - for a curvilinear trajectory, if the thrust is not parallel to the velocity, we can change the velocity direction possibly reaching an outer point faster;
- Whether acceleration is continuous or braking is performed before arriving to the target planet, during the interplanetary phase - it may minimise the capture fuel requirements since non-impulsive engines have a higher $I_{sp}$.

**Capture on Mars** The capture manoeuvre on Mars is performed using the classical single impulse brake (since aerobraking and continuous thrust brake need long periods of execution [3, 7, 31], and also since aerobraking is still a matter of research only performed in smaller spacecrafts [32]). The capture orbit was selected based on previous missions ([27, 33, 34]). It has a perigliss altitude $r_p = 300 \times 10^3\text{ m}$ and eccentricity 0.9. The mother ship would remain at this orbit until return, so to save fuel (by not lowering the entire spacecraft, and raise it again for the departure). On this
capture orbit the fuel for the return is waiting for a subsequent rendezvous. Meanwhile the crew (inside the transport capsule) would intercept the cargo before landing on Mars, at a lower circular orbit (periapsis altitude \( r_p = 200 \times 10^3 \) m), where it leaves the engine used for the lowering manoeuvre. On this lower orbit the crew can use the atmosphere to brake and land.

The mission timeline for the manned Earth to Mars mission phase is displayed in figure 2.

![Figure 2: Sketch of the mission timeline for the Earth to Mars phase. Legend: Diamond shapes represent single moments. Rectangles are mission phases. Squares represent actions. Oval shapes are variable parameters (and underline terms their options).](image)

**Return Trajectory** After completion of the ground mission, the crew enters the MAV and leaves the planet’s surface. An orbital decay is assumed and the engine left in orbit has now a lower periapsis altitude \( r_p = 180 \times 10^3 \) m. After raising the orbit, it would meet the mother ship prior to the MtE injection manoeuvre. The capture on Earth is similar to the one on Mars (the entire spacecraft is captured, since propulsion systems with nuclear material should be carefully disposed of or possibly re-utilised). Subsequently, the crew enters the MAV, detaches from the mother ship, lowers the periapsis and circularises the orbit \( (r_p = 180 \times 10^3 \) m) before landing on Earth.

A mission trade tree is displayed in figure 3, where we can see which propulsion system is used in each mission segment. When a coast transfer is selected no propulsion system is used during the interplanetary phase.

**Cargo Mission** The first step is the launch to an Earth parking orbit. Here a long assembly is not expected and the parking orbit has a lower periapsis and a zero eccentricity (periapsis altitude \( r_p = 200 \times 10^3 \) m). After checking every system it would execute the EtM injection (again raising the orbit to an eccentricity of 0.95 and executing a classical single impulse departure).

For the interplanetary phase a slower trajectory is intended, with savings on departure and capture. The option of using a powered transfer is also tested, to lower even further the departure and capture fuel requirements.

Upon arrival on Mars it would execute the capture to a highly elliptical orbit (as the manned spacecraft). The cargo does not perform the MtE trajectory and all the spacecraft, apart from the fuel for the manned return trajectory, would be lowered to a circular orbit \( (r_p = 200 \times 10^3 \) m), where it remains until the crew arrives.

### 3.2 Mass Budget

The mass budget for our mission is comprised of:

- **Mass of four crew members** \( M_{Crew}^\text{Total} = 0.4 t \);
- **Spacecraft Structural Mass** - habitat module (\( M_{Hab} = 31.52 t \)) and transport capsule (\( M_{TC} = 5.8 t \)) for the manned spacecraft, cargo module (\( M_{Cargo} = 42.11 t \)) and MAV (\( M_{MAV} = 18 t \)) for the unmanned;
- **Propulsion System (\( M_{PS} \))**:  
  - Departure and Capture Engines (\( M_{En}^{DC} \));  
  - Interplanetary Engines (\( M_{En}^{IP} \));  
  - Solar Panels (\( M_{SP} \)) - for the electrical engines;  
  - Radiator & Reflectors (\( M_{RR} \)) - for Rubbia’s concept engine;
- **Propellants for every manoeuvre** \( M_{P}^T = M_{P}^D + M_{P}^L + M_{P}^{HT} + M_{P}^{MT} \);
- **Tanks** \( (M_{T}^L = M_{P}^D + M_{P}^L + M_{P}^{HT} + M_{P}^{MT}) \).

This results in a dry mass of \( M_{Dry} = 37.7 t + M_{PS} \), where the tanks are not included since they are discarded after each manoeuvre (following Apollo’s example), and a total mass of \( M_{Total} = M_{Dry} + M_{PS} \), for the manned spacecraft.

While the crew, the modules (habitat and cargo) and the transport capsules are constants, the others are problem variables, depending on the mission options (propulsion system selected and number of engines). The propellants and tanks also depend on the execution time of the manoeuvres.

In the unmanned spacecraft the dry mass is \( M_{Dry} = 60.11 t + M_{PS} + M_{En}^{IP} \) and the total mass \( M_{Total} = M_{Dry} + M_{PS} + M_{En}^{IP} \). The fuel payload entry \( (M_{PP}) \) represents the propellants and tanks needed for the manned return trajectory. Due to this entry, the cargo mission can only be computed after the manned mission calculations.

### 4 TRAJECTORY PROBLEM & SOLUTION

The objective here is to calculate the trajectory between two planets and obtain the time of flight and correspond-
ing $\Delta V$s.

### 4.1 Equations of Motion

In general the spacecraft is subjected to gravity ($\vec{F}_g = -\frac{mG}{r^2}\vec{u}_r$, where $\vec{u}_r$ is its direction, $\mu = GM$, $G$ is Newton’s constant and $M$ the relevant gravitational mass) and thrust ($\vec{F}_T = F_T\vec{u}_{F_T}$, where $\vec{u}_{F_T}$ is its direction).

With the spacecraft position ($\vec{R}$), velocity ($\vec{v}$), acceleration ($\vec{a}$) and force ($\vec{F}$) written in Cartesian form we have

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

with $r = \sqrt{x^2 + y^2 + z^2}$.

When only gravity is present, equation (3) can be simplified yielding the well known trajectory equation of the Keplerian orbits [35]. When a spacecraft has a continuous thrust, equation (3) cannot be simplified and a numerical integration is performed. The numerical method used is the Runge-Kutta-Fehlberg 7th-order formula with 8th-order error estimate, commonly employed in astronomy [36].

A simplification of our problem concerns the motion of the planets. It is assumed that both planets move in coplanar circular orbits. By assuming a circular motion, the departure date, and hence its location, does not affect the trajectory since the velocity of both planet and distance between them are constant. The only requirement on the planets position is the phase angle between them, so to reach the target planet. This allows for a standard procedure for the departure location between all the different trajectories. Having both orbits at the same plane simplifies the problem to two dimensions, reducing the number of unknowns to six.

When using a continuous thrust and performing trajectory optimisation, it is usual to include a control parameter for the thrust direction [37]. In this work we do not intend to optimise the trajectory, but to compare the propulsion systems. We use two options for the thrust direction:

1. The thrust has a constant angle with the velocity vector, $\vec{F}_T = F_T \times \left( R'(\theta_{F_T}) \times \frac{\vec{v}}{||\vec{v}||}\right)$.
2. The thrust has a constant angle relative to the Sun, $\vec{F}_T = F_T \times \left( R'(\theta_{F_T}) \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.

Here $R'(\theta_{F_T}) = \begin{bmatrix} \cos(\theta_{F_T}) & -\sin(\theta_{F_T}) \\ \sin(\theta_{F_T}) & \cos(\theta_{F_T}) \end{bmatrix}$ is a rotation matrix and $||\vec{v}|| = \sqrt{x^2 + y^2}$ is the norm of the velocity vector.

By separating the trajectory in three phases (departure, interplanetary phase and arrival), we use the “patched conics” approximation [35].

For all orbital transfers (apart from the departure, capture and interplanetary transfer) we use a two impulse manoeuvre (if we need to change the periapsis radius) or a simple one impulse manoeuvre on the periapsis (if we want to maintain the periapsis radius).

### 4.2 Departure & Capture

For the departure and capture phase we use chemical or nuclear thermal engines and the impulsive approximation.

The spacecraft to escape the departure planet it must have a certain velocity at the frontier of the sphere of influence (SOI), the hyperbolic excess velocity ($V_{\infty}^D$). Considering the spacecraft is in a launch orbit around the planet we must give a $\Delta V_D$ to enter the escape hyperbola. With the periapsis radius ($r_p$) and eccentricity ($e'$) of the launch orbit, we have for a certain planet with a certain gravitational parameter ($\mu$),

$$\Delta V_D = \sqrt{(V_{\infty}^D)^2 - \frac{2\mu}{r_p}} - \sqrt{\frac{\mu}{r_p} (1 + e')}.$$  \hspace{1cm} (4)

For the capture the $\Delta V_C$ is the velocity change given to the spacecraft to leave the hyperbola (where now $V_{\infty}^C$ is the velocity when entering the planet’s SOI).

### 4.3 Interplanetary Transfers

The interplanetary phase is computed with a direct application of the numerical scheme.

The velocity of the spacecraft at the beginning ($V_{\infty}^D$) is computed as a vector addition of the departure planet’s mean orbital velocity ($V_1$) and the velocity of the spacecraft at the end of the departure phase ($V_{\infty}^D$), which can be oriented in any direction [38]. In this work the direction of the velocity is determined with respect to the planet’s velocity.

The vector addition to obtain $V_{\infty}^D$ is

$$V_{\infty}^D = V_\infty^D + V_1 = V_\infty^D \times \left(R'(\theta_V) \times \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) + \begin{bmatrix} 0 \\ V_1 \end{bmatrix},$$  \hspace{1cm} (5)

where $R'(\theta_V)$ is the rotation matrix and $\theta_V$ the angle between $V_{\infty}^D$ and $V_1$, which in our work is always a vertical vector at the spacecraft departure location.

Using the velocity at the end of the interplanetary journey ($V_{\infty}^A$), and performing a vector subtraction with the arrival planet’s mean orbital velocity ($V_2$), we compute $V_{\infty}^C$ for the capture phase. From a geometrical construction we see that the direction of the velocity vector of the arrival planet depends on the arrival location $x_T$ and $y_T$. The velocity for the capture phase is thus given by

$$V_{\infty}^C = V_\infty^A - V_2 = V_\infty^A - \begin{bmatrix} V_2 \times \sin(\alpha) \times a \\ V_2 \times \cos(\alpha) \times b \end{bmatrix},$$  \hspace{1cm} (6)

where $a = \frac{\arctan\left(\frac{y_T}{x_T}\right)}{\pi}$ and $a$ & $b$ are constants alternating between $-1$ and $1$ depending on the quadrant.

Two conditions are used as stopping criteria of the numerical method:

1. If the distance of the spacecraft to the target planet is smaller than that of the planet’s SOI dimension.
2. If the distance of the spacecraft to the Sun is greater than that of the planet, for an Earth to Mars transfer, smaller in case of a Mars to Earth transfer.

After reaching the target planet we compute the initial phase angle between the planets ($\phi_0$) using heliocentric angles. If $\phi_0 > 0$ the arrival planet must be ahead of the departure one and vice versa, if $\phi_0 < 0$.

An important quantity is the waiting time on Mars ($t_w$). The waiting time can only be determined after having the TOF for the Earth and Mars transfers by:

$$\phi^{ELM}_0 = \theta_\odot - \theta^f_\oplus$$ in the Earth to Mars trajectory
$$\phi^{MLE}_0 = \theta_\odot - \theta^f_\oplus$$ in the Mars to Earth trajectory

$$\Phi = \phi^{MLE}_0 - \phi^{ELM}_0 \Rightarrow t_w = \frac{2\pi N}{n_M - n_E},$$  \hspace{1cm} (7)

where $N = 0, 1, 2, \ldots$ to give a positive time.
5 RESULTS & DISCUSSION

To compute the various mission scenarios we consider the following parameters:

1. Propulsion System:
   - Engine Type - chemical (CECE), nuclear thermal (NERVA), electrical (PPS 1350-G) and/or Rubbia’s concept (PEMT);
   - Number of Engines and Mass of the System - apart from fuel tanks and propellants.

2. Characteristic Energy ($C_3 = (V_C^3)^2$):
   - Absolute value;
   - Vector direction ($\theta_V$).

3. Thrust Direction ($\theta_{F_T}$);

4. Brake (during the interplanetary phase):
   - Location - starting at the middle of the transfer, progressively braking closer to the target planet;
   - Thrust direction - a search for the optimum direction for the thrust when braking.

For this work we consider a combination of two CECE engines, having thus a higher thrust with a still relatively low mass. The NERVA has such a high mass, with a high thrust, that we do not consider more than one engine in the base configuration. For the PPS 1350-G we test 10, 15, 25, 35, 45, 55, 75, 100 and 150 engines, due to its low thrust and mass (ten engines add up to 14.9 kg and a thrust of 0.9 N). For the PEMT engine, the combinations selected consider a smaller number of engines than the electrical one (due to its high mass). With a maximum of ten engines we test: 1, 2, 3, 4, 5, 6, 8, 10.

For the $C_3$ range we desire transfers with values greater than the Hohmann transfer (HT) one, as we aim to minimize travel time. However, to increase the $C_3$ value slowly, so that usual values can be found, and to test some extreme cases, an exponential rate is the selected solution, $C_3 = [\Delta V^{HT} \times (1 + 0.02)^i]^2$, with $i = 0, 1, 2, 3, \ldots, 200$.

For the velocity and thrust angles we aim, at first, for a small variation of angle so to change the velocity direction slowly, while after 45° the variation can be larger. The selected angles are thus: 0, -5, -10, -15, -20, -25, -30, -35, -40, -45, -50, -60, -70, -80. For Earth to Mars (EtM) transfers this implies all vectors are turned outwards. In Mars to Earth (MiE) transfers the vectors are turned inwards. This selection is also influenced by the number of iterations needed by the increase in the number of angles, since we have to run the mission computations for every $C_3$, angle and spacecraft configuration.

In references [39, 40] the brake usually starts at, or after, the middle of the trajectory. We start at the middle and divide the remaining distance in two, until we are close enough that the next location would be inside the planet’s SOI (where capture occurs).

5.1 Effect of Angle Variation of Velocity and Thrust

The effect of varying the initial velocity angle is initially perceived on the Mars to Earth transfer with impulsive systems. Without varying the angle, when the value of $\sqrt{C_3}$ is greater than the planet’s velocity, the spacecraft travels opposite to the planets movement. This has a strong effect on the results since the mass of the mission is increasing together with the TOF, when normally if it takes longer to reach a target the mass decreases.

We then test the effect of varying the angles on both EtM and MiE transfers for the entire $C_3$ range, and start having differences of one thousand tonnes or more. To perceive these effects better we plot the results obtained for every $\theta_V$ and $\theta_{F_T}$, for a single $C_3$. In figure 4 we can clearly see how a variation on the direction of the initial velocity (maintaining the same absolute value) can result in differences of 100 t or more. The variation of $\theta_{F_T}$ does not produce a strong effect on the mass.

A closer scrutiny of the results reveals that the cause for the mass variation is the value of the velocity at arrival. Through the change of the initial angle we induce a variation on the interplanetary TOF, and most importantly, a different spacecraft velocity vector at arrival to the target planet and thus $V_C^\infty$. This affects the arrival manoeuvre (in terms of the required propellent), and hence every manoeuvre before that. Observing that the impulsive fuel calculations use an exponential equation, we then consider the variation of $\theta_V$ and $\theta_{F_T}$ in all calculations (using the range discussed before).

By having different angles we have several results for a single $C_3$ and we have to select which $\theta_V$ and $\theta_{F_T}$ are better, for the entire trajectory. For this we use a simple algorithm, derived from the results of figure 4. Applying a certain $C_3$ we determine the first combination of angles whose trajectory reached the target planet (trajectory number X). Testing the remaining angles, for the same absolute value of velocity at departure and spacecraft configuration, we update the optimum values ($\theta_{V_{opt}}, \theta_{F_{T_{opt}}}, t_{opt}$ and $M_{opt}$) if we find a transfer, $i$, whose ($t' < t_{opt}$ or $t' < t_{ini}^X$) and $M' > M_{opt}$.

To test this algorithm we simulate the mission using 35 PPS engines for the same $C_3$ range, for the minimum time trajectories, minimum mass trajectories and the resulting optimum trajectories. The results show that the minimum mass requires a greater $C_3$ value for the same TOF of the optimum trajectory. For times of flight lower than ≈ 70 days the optimum corresponds to the minimum mass. On the remaining transfers, the optimum has lower masses than the minimum mass trajectories (apart from a few points whose maximum mass difference equals 12.3t). We note that although the $C_3$ is always greater for the minimum mass case, and $V_C^\infty$ for the optimum trajectory, the difference in $C_3$ far outweighs differences in $V_C^\infty$, making the minimum mass heavier for the same TOF. Meanwhile, the
minimum time uses smaller $C_3$ values than the optimum trajectory for the same TOF. However, since it corresponds to a faster travel it always has a $V_C^\infty$ higher, whose effect now outweighs the increase in $C_3$ in the optimum case. This increses the mass required to execute the arrival manoeuvre, and thus all masses before that.

We then compare the effect of having the thrust direction relative to the orientation of the spacecraft velocity or fixed relative to the Sun, as explained in section 4.1. The results show that the thrust relative to the Sun has a maximum mass gain of 400t, for results with TOF less than $\approx 65$ days. However, for this range of TOF, not only is the $C_3$ value greater than what is commonly used, but the minimum spacecraft mass is 3,637t. This mass is well above the recommended limit (discussed below). For the remaining results the difference between both solutions is less than $\approx 25$ t, favourable to the thrust relative to the spacecraft velocity. For this reason, henceforth, we only consider the thrust direction relative to the orientation of the spacecraft velocity.

As stated in reference [8] a spacecraft with more than 1,000t is to be avoided. However, since we aim to test new propulsion systems and how fast one can travel to Mars and back, we establish a limit of 2,500t (if we do not establish a limit, masses as great as $10^{12}$t appear). To select an optimum point of all computed trajectories, for each configuration, and since we aim to minimise time and mass, we use a weighted sum of those two,

$$N_{opt} = \min \left[ \left( \frac{M_i}{\text{max}(M_i)} \times 0.25 + \frac{t_i}{\text{max}(t_i)} \times 0.75 \right) \times \text{max}(t_i) \right] \quad (8)$$

where $i$ runs between one and the maximum number of points computed and $N_{opt}$ is the iteration number with the optimum characteristics. Being the time the primary objective it has a weight of 75%.

### 5.2 Impulsive Systems - Chemical versus Nuclear Thermal

The first missions to be tested are the classical chemical and the nuclear thermal systems (options one and three of figure 3). These missions serve as the comparison cases.

Comparing CECE and NERVA solutions, we observe that NERVA is a better solution only for times of flight of $\approx 115$ to $\approx 150$ days or $\approx 95$ to $\approx 105$ days. Nevertheless, the difference between both solutions is never greater than 200t (for this range of TOF). On the return, however, the difference in mass is always favourable to the classical chemical solution.

Using all the trajectories computed for EtM and MtE, we calculate the waiting time, the total mission time and the total mission mass for each one (figure 5). In this figure we see that even if we travel faster to Mars and back we have approximately the same mission time since the waiting time is increasing. There is however a leap in waiting time for return times of flight of $\approx 125$ days. Although the same feature of increasing waiting time with decreasing TOF is found, the position of Earth and Mars is such that the waiting time is up to 400 days lower.

Nonetheless, the waiting time is not only related with the travel time, but also with $\theta_V$, i.e. we discover that even with an equal travel time, with a different $\theta_V$ the target planet would need to be in a different rendezvous location, affecting the waiting time for the return.

We then compute the cargo mission for each system using the respective optimum points (show in table 2).

The first issue to comment is that more than 90% of the mass of the missions is fuel (hereinafter fuel refers to propellants and the tanks to transport it). This is a common feature of all impulsive systems, given their low $I_{sp}$.

Analysing both missions we noticed that although the human mission total time has a difference of only one day, the NERVA system has 87t less than the CECE (mainly due to the EtM transfer). When adding up the cargo mission, this difference changes sign, meaning that the CECE total mission mass has less 163t than the NERVA. This is mainly related to the propellant payload the cargo carries (122t more for the NERVA).

For this reason we regard the chemical propulsion system as a better choice and, henceforth, is the only system we consider for departure and capture. The CECE is also a better solution due to its simplicity and for being more environmentally friendly than the NERVA [41, 42].

Comparing our mission choice and the Mars Direct we were able to save 43 days on the total time of the human mission (in particular more than 90 days in each transfer), while keeping the manned spacecraft well below the 1,000t limit advocated in Mars Direct. As expected, we could not keep the cargo mission below this limit since the fuel for the manned return trajectory is carried by it, instead of being produce on Mars.

### 5.3 Non-Impulsive Systems - Electrical Engine

Computing all transfers and analysing the results we can ascertain that the 35 engine configuration is the one with the lowest mass for the same TOF. For times of flight higher than $\approx 140$ days the difference is always lower than 15t, increasing when we decrease the TOF. On the return we observe the same behaviour, the 35 engine configuration always with lower mass. There are, however, some isolated points where the difference is favourable to other configurations, but this difference never exceeds more than 35t (apart from one point with 150 engines with 58t), and are closely related to the difference in $\theta_V$ and $\theta_{FP}$.

Following the same procedure used for the impulsive systems we compute the waiting time, the total mission time and the total mission mass. The minimal mass for the cargo
mission, in this case, is achieved using 75 PPS engines in the interplanetary phase.

As expected, the total time exhibits the same behaviour seen for the impulsive systems (the faster the travel is, the longer the waiting time on Mars before returning is). As for the impulsive systems, more than 90% of the mission mass is fuel (propellants and tanks). However, for example in the EtM transfer we have a fuel mass of 772,369 kg of which only 0.22% is for the interplanetary phase. This means that the CECE, in this mission, uses more 764,163 kg in four manoeuvres (apoapsis raising, departure, capture and orbit circularization) than 35 PPS engines working for 87 days.

Adding all the contributions and comparing with the chemical option, with coast interplanetary transfer, we find that the crew has to spend almost the same TOF than before (when we did not execute the brake). Here the minimum mass is achieved using chemical at departure and arrival, and 100 PPS engines on the interplanetary phase.

Comparing the results of all trajectories it is found that since energy is generated through a nuclear reactor, which only burns 4% of the total nuclear material, the spacecraft is far heavier at arrival, requiring much more propellant than other systems to execute the capture manoeuvre. Obviously this has an exponential effect on previous manoeuvres. For example, in the EtM transfer we require approximately 16.6 t of nuclear material for the interplanetary phase, which is ten times more than the PPS (although it is still only \( \approx 1\% \) of the total fuel mass). Afterwards, in the PPS mission, we reach Mars with a \( V_C \) of 9,927 m/s and having already consumed all propellant and discarded the tanks (leaving it just with the dry mass and fuel for the remaining transfers). With the PEMT we have a \( V_C \) of 9,236 m/s (which, by itself, would mean a lower fuel mass for the arrival manoeuvre), but it still has 16,052 kg of nuclear material (apart from the dry mass and fuel for the remaining transfers).

### 5.4 Non-Impulsive Systems - Nuclear Pure Electro-Magnetic Thrust

The PEMT engine was expected to have a better performance in terms of TOF. The results for this system show that the one engine configuration has the smallest mass for all trajectories. On the return the behaviour is similar.

As the electrical (PPS) and chemical (CECE) systems, the PEMT total time remains approximately the same until a return TOF of 120 days. For return trips longer than 120 days the total time is reduced drastically, again due to the planetary alignment.

Braking When braking with the PEMT we have considerable changes in mass, while the TOF did not have a significant increase, contrary to the PPS. The maximum difference between the TOF of a trajectory with no brake and one with a brake is never greater than 5 days. Despite that increase in TOF we have savings of mass reaching 130 t, which increases with increasing \( C_3 \). On return we were able to decrease the TOF and the mass savings can reach 200 t.

We do not discuss here the system without executing the brake, since we have lower masses for approximately the same TOF when braking, and a new optimum point for the PEMT system is used (shown in table 2).

We have in the human mission more 1,092 t with less five days than the chemical impulsive mission (more 8841 and four days than the electrical mission). The cargo mission is well above the 2,500 t limit, although it carries less 80t than before (when we did not execute the brake). Here the minimum mass is achieved using chemical at departure and arrival, and 100 PPS engines on the interplanetary phase.

5.5 Impulsive versus Non-Impulsive Systems

The trajectory results from the classical chemical solution, 35 PPS engine configuration and one PEMT, for the Earth to Mars transfer and for several \( C_3 \) values, are shown in figure 6.
The PPS propulsion system, apart from a few points where the maximum difference in mass is never greater than 30t, is a better solution than a simple chemical departure and arrival with a coast interplanetary transfer (for equal times of flight). This behaviour is not seen in the optimum solutions because the optimum point for each system results in different TOF (a difference well above the 2 days margin). Nevertheless, it was already expected that it yields better solutions since we are adding up to an already largely optimised system (the CECE) a low mass and high Isp system (even though with low thrust).

By contrast, the PEMT has always a higher mass than all the other scenarios, for these times of flight. The main reason is the need to carry so much nuclear material to use just 4% of it.

For smaller times of flight the tendency inverts and the PEMT results in a far better solution, with mass differences above 113, 153, 267 t. However, the masses for these solutions are well above our considered limit. This behaviour suggests that for trajectories to celestial bodies farther than Mars, where a higher ∆V is needed, the PEMT engine could represent a better solution.

For a better understanding of the differences between the optimum solutions for each system, figure 7 is presented.

In figure 7 we see that the maximum fuel expenditure is during departure and capture (especially at departure) and that an increase of TOF implies a decrease of the waiting time and vice versa.

![Figure 7: Plot of the human mission mass and time for the optimum solutions of each propulsion system, with percentages of the total mass and total time, respectively.](image)

The PPS configuration has a human mission time smaller than the others, with a small increase in mass (in comparison to the chemical solution). This difference is due to the extra fuel needed at departure manoeuvres (more ≈ 100t for the EtM transfer and ≈ 90t for the MtE transfer) and the capture on the MtE transfer (≈ 9t more). In the capture manoeuvre, on the EtM transfer, we have a saving of less than ≈ 12t.

Conversely, the PEMT has a smaller gain in the mission time compared to the CECE, with a high increase in mass (more than 626 t on the EtM departure and capture, ≈ 395 t on the MtE departure and capture), for the reasons discussed above.

### 6 CONCLUSIONS AND FUTURE WORK

Our analysis has shown that the classical chemical propulsion is the best solution within the impulsive systems. In most cases, for the same TOF, it has the smallest mass. When the TOF is slightly higher than the nuclear thermal system, a small increase in mass could overcome this situation.

When we add to the chemical departure and capture an electrical system one is able to reach the target destination faster with smaller masses, for most cases and when comparing with the same TOF. This better performance is achieved since one lowers TOF without increasing C3.

The same cannot be said for the PEMT system. The principal reason for this behaviour is the fact that it uses only a low fraction of the total nuclear material fuel for propulsion. If this fraction is increased, through technological improvements, the performance of the PEMT system can be greatly improved. Nevertheless, as it stands, the PEMT concept shows a better result for higher C3 values, suggesting that it is more suitable for farther destinations than Mars.

The mission to Mars could be optimised considering a three dimensional trajectory. This optimisation would help to obtain a more accurate trajectory, but also the best thrust direction in every segment of the interplanetary phase. An algorithm for departure where the number of apoaopsis raising manoeuvres is changed, instead of the option of increasing the number of engines, can be also beneficial.

For the PPS 1350-G engine our analysis indicates that an interesting combination might involve more than ten engines. However, the interaction between them should be studied as suggested in reference [43].

### References


