A Semi-Empirical Model for Vortex-Induced Vibrations of Tall Circular Towers

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Abstract

The aim of this work is to study the problem of aeroelastic transversal vibrations in circular cylinder towers, due to the alternate shedding of vortices caused by a fluid flow. Displacement amplitudes for the towers are obtained using both analytical and empirical methods. The main method proposed and used for this analysis is a time-domain semi-empirical one developed by Pinheiro [12]. This method is applicable to the specific case of circular section isolated towers. It’s based on two-dimensional models in which the three-dimensional effects are accounted for through an extension to real flow conditions and incorporation of modal analysis. These models are the empirical nonlinear model proposed by Scanlan [13] and the coupled wake oscillator by Blevins [7]. A model proposed by the recent European regulation (Eurocode 1, part 1-4) [11] is also described and analyzed. The Pinheiro method is applied to specific cases extracted from the literature. Comparisons are performed with results obtained in wind tunnel tests and with other analytical models. These give good and promising compliance, demonstrating the potential of the Pinheiro method. Some future developments for the model are mentioned, with the objective of extending its use to a wider range of applications. Finally, the principal conclusions are presented, along with some suggestions and precautions to using the model. These show the Pinheiro model is a good tool that is best used with care and together with other approaches in the project of a structure.

Keywords: Vortex-induced vibrations; Across-wind vibration; Circular cylinder towers; Lock-in; Fluid-structure interaction.

1. Introduction

Over the years, as technology got better and more sophisticated materials were developed, the height and slenderness of structures have increased. These new structures are more flexible and have less damping, due to their reduced volumetric weight. Therefore, they are more susceptible to wind induced movements and projecting them, to resist this phenomenon, is an area of growing importance. Examples of these structures are high-rise buildings, chimneys and towers.

Wind is the main action that induces bending in these structures, causing vibrations that provoke various behaviors. Due to their flexibility, the structures can experience large displacement amplitudes, velocities and accelerations. The consequences of this are multiple. There could be damage in the structure, walls, facades, and in interior objects prone to movement. In metallic towers, fatigue can cause a reduction of their service life. Telecommunications towers’ signals can be disrupted due to the antennas’ vibration. People inside buildings can also feel insecurity and nausea, caused by the discomfort associated with the movement acceleration.

Wind action can be divided in two groups: aerodynamic and aeroelastic. Aerodynamic wind actions originate effects on structures that don’t depend on their movement, while effects resulting from aeroelastic actions are dependent on the vibration of structures. Belonging to this last case are the so called self-excited forces, in which the fluid flow’s conditions around the contour of the vibrating body are modified because of the establishment of an interaction between the fluid and the structure movement.

Flutter, galloping and vortex shedding are examples of aeroelastic actions. Rhythmic vortex shedding has been widely studied and analyzed. It can cause large amplitude vibrations, especially when the phenomenon of lock-in occurs, also known as vortex capturing. This happens when the vortex shedding frequency becomes close to a natural frequency of transversal vibration of the structure, causing the vortices to separate according to this last frequency, for a given range of wind velocity.

The most common case in the study of this phenomenon is that of a fixed rigid cylinder, horizontally disposed and under the action of a uniform laminar flow. This can be simulated by a two-dimensional flow where all the vortices shed with the same intensity and frequency. However, in real cases, there are parameters that make this evaluation more complex, causing three-dimensional effects that alter the flow and the vortices formation. Some of the factors that change the analysis conditions are, for example, the finite dimensions of the structure, the variation of its cross section, the wind speed variation along the height and the turbulence influence. The existence of other
structures connected to the original structure, such as stairs and viewing platforms, as well as the presence of other buildings in the nearby area, can also affect these conditions.

This work focuses on the specific analysis of circular cross-section isolated structures of chimneys and towers, under vortex-induced transversal vibrations. The wind behavior is more widely studied among these structures, making it simpler to obtain and compare results for them with good reliability. In this way we can avoid (or decrease) resorting to wind tunnel tests, which are usually more time-consuming and expensive. The wind interaction for buildings with other types of cross-sections is generally more complex. There are currently no strictly theoretical or empirical formulations allowing to obtain satisfactory results, and so wind tunnel tests are used as their primary form of modeling.

There are various proposals from different authors to model the phenomenon of vortex shedding, all having in common the fact they originate in the simple case of two-dimensional flow. Analytical, empirical, and semi-empirical models can all be employed. The latter ones stand out in the case of towers, taking into account factors that generate three-dimensional effects in the flow whenever possible. The commonly used approach is the construction of empirical models and then matching their results to reality through a suitable choice of parameters. This construction is based in in-situ measurements or wind tunnel tests and the parameters are obtained by correspondence to the most conditioning characteristics of the phenomenon.

It is one of these semi-empirical models, proposed by Pinheiro, which is mainly analyzed here. It’s essentially used in this work to obtain estimates of the maximum transversal displacement amplitudes when lock-in occurs. In wind engineering, it will often suffice to know only this parameter in order to study and design the structure.

Despite all the recent developments, namely in the field of computational fluid dynamics, there are still many difficulties in the three-dimensional modeling of the specific problem addressed here. This is due to the sheer number of parameters and resulting complexity that can affect the behavior of structures. This fact is verified when one examines the existing discrepancies between different semi-analytical models and calculation methods, suggested by diverse regulations.

2. Formulation of the Model

When a bluff body is under the action of a fluid flow with velocity \( U \), the body’s surface experiences a fluctuating pressure variation with time. These oscillating pressure fields develop on the body when a successive separation of vortices on each of its side takes place. If the body is free to move in the transverse direction to the fluid flow, the resultant periodic net forces will interact with the structure’s movement, originating the vortex shedding induced vibration.

In the following model deduction, it’s assumed that the fluid flow conditions on the cylinder are approximately two-dimensional. It’s assumed that the cylinder has a long hard surface and entire body displacements in the flow’s transverse direction. The cylinder has mechanical damping in that same direction is elastically supported and restrained from moving in the along-

wind path. The flow is laminar with uniform average velocity. Figure 1 represents these conditions.

![Figure 1. Rigid cylinder schematic with elastic supports ([7], [12]).](image)

If the cylinder is also blocked from moving in the across-wind direction, a first reasonable approximation for the force per unit length \( F \) acting on the cylinder in time \( t \) is given by:

\[
F = \frac{1}{2} \rho U^2 D C_l \sin(\alpha_1 t)
\]  

(1)

where \( \rho \) is the air’s specific mass, \( D \) is the cylinder’s dimension in the flow’s perpendicular direction (the external diameter), \( C_l \) is the lift coefficient and \( \alpha_1 = 2\pi U S/D \), with \( S \) being the Strouhal number.

However, if the cylinder can oscillate in the across-wind direction, this simple expression for the excitation force is inadequate. Let \( y \) represent the cylinder displacement in said direction. Then the movement equation is written in the following form:

\[
m\ddot{y} + c\dot{y} + k\dot{y} = F(y,\dot{y},\ddot{y},t)
\]  

(2)

where \( m \) is the cylinder mass, \( c \) the mechanical damping constant and \( k \) the elastic stiffness.

The models presented below are built from expression (2), combining the coupled wake oscillator model by Blevins with the empirical nonlinear model proposed by Scanlan, incorporating afterwards the modal analysis.

2.1 Coupled Wake Oscillator Model by Blevins

In this model, the fluid on the wake is represented as a self-excited nonlinear oscillator, coupled to the structure. This idea is due to the self-excited nature of vortex shedding, according to [7], which indicates that models have been developed in which the lateral force coefficient satisfies a Van der Pol type equation, suggesting this approach.

The model is not suitable for non-circular sections or parallel vibrations to the flow, effectively being a method to extend the available experimental data. It does not provide a rigorous approximation of the fluid-structure interaction, but is quite useful in estimating circular cylinder structures’ response to vortex-induced resonant vibrations, for Reynolds numbers between \( 10^5 \) - \( 10^7 \).

The forces acting on the cylinder are evaluated with fluid dynamics theory, through the continuity fundamental equation, using the approach of the finite control volume and the system’s momentum change. The model deduction from these assumptions is extensive and complex, being obtained based on empirical
parameters. Further discussion can be found in references [11, 7]. The self-excited fluid oscillator nonlinear coupled equation is given by:

$$\ddot{w} + K \frac{U_t}{U D} \omega_0 w = (a_1^2 - a_4^2) \frac{U}{D} \ddot{w} - a_2^2 \frac{U^2}{D^2} \dot{w} + a_3^2 \dot{w} + a_1^2 \frac{U}{D} \dot{y}$$

(3)

The cylinder movement equation assumes the following form:

$$\ddot{y} + 2 \xi_\omega \omega_0 \dot{y} + \omega_0^2 y = a_4^2 \ddot{w} + a_1^4 \frac{U}{D} \dot{y}$$

(4)

In the previous equations, we have:

$$K' = \frac{K}{a_0 + a_3}, \quad a_i^2 = \frac{a_i}{a_0 + a_3}, \quad a_i^2 = \frac{\rho D^2 a_i}{m + a_3 \rho D^2}$$

(5)(6)(7)

$$i = 3; 4$$

$$\omega_y = \sqrt{\frac{k}{m + a_3 \rho D^2}}, \quad \xi_T = \sqrt{\frac{k}{m \omega_0^2 + \xi_f}}$$

(8)(9)(10)

where $u_t$ is the vortex wake translation velocity, $\omega_0$ the vortex shedding circular frequency, $\alpha_i$ the cylinder’s natural circular frequency, $K$ a proportionality constant, $m$ the cylinder mass per unit length (including the added mass of the fluid displaced by the cylinder), $k$ the supports’ stiffness per unit length, $\dot{w}$ is a magnitude measure of the transverse oscillations of the fluid wake, $\zeta_\omega$ and $\zeta_f$ are the viscous damping of the structure, fluid and total effective damping coefficient, respectively. $a_0, a_1, a_2, a_3, a_4$ are dimensionless constants obtained experimentally.

The maximum displacement amplitudes in the resonance condition (lock-in range) can be expressed in terms of a very useful variable, formed by the product of the mass ratio with the damping factor. This variable is the reduced damping parameter, also known as the Scruton number:

$$\delta_r = \frac{2m(2\pi \xi_\omega)}{\rho D^2} = 4m \pi \xi_\omega$$

(11)

Therefore, the maximum displacement amplitudes $A_y$ for a cylindrical structure in the natural vibration mode $j$, in resonance, are given by:

$$A_y = \frac{\alpha_4}{2 \pi^2 S^2 \mu_\omega \xi_\omega} \left[ a_1 - a_4 + \frac{a_2}{\pi^2 a_2 S \mu_\omega \xi_\omega} \right]$$

(12)

with $\mu_\omega = \frac{4m \pi \xi_\omega}{\rho D^2}$

Experimental results indicate values for $a_4 = 0.44$; $a_2 = 0.2 e a_3 = 0.38$. The above expression can be rewritten as:

$$A_y = 0.077 \sqrt{0.3 + \frac{0.72}{(1.9 + \delta_r) S}} D$$

(13)

where $\gamma$ is a geometrical parameter dependent on the modal shape $\phi$, which varies with height $z$ over the cylinder length $l$:

$$\gamma = \phi_{max}(z/l) \left( \int_0^l \phi^2(z) dz \right)^{1/2} \left( \int_0^l \phi^4(z) dz \right)^{1/2}$$

(14)

Figure 2 shows the theoretical results of the model in comparison with experimental results.

**2.2 Empirical Nonlinear Model by R. H. Scanlan**

If the analysis’ objectives are modest, an elementary model may be employed in the investigation of vortex shedding induced vibrations. Often, it is sufficient to just identify a structure’s maximum deflections. Thus, there is interest in having a model that can replicate some of the main experimental results observed near the lock-in range.

To build such a model, it’s assumed that the aerodynamic excitation, aerodynamic damping and aerodynamic stiffness have to be supplied to a linear mechanical oscillator. The model gains its nonlinearity by adding a nonlinear aerodynamic cubic term. The subsequent formulation is obtained, in which the fluid’s motion equation when subjected to forces caused by vortex shedding is given by:

$$m[\ddot{y} + 2 \xi_\omega \omega_0 \dot{y} + \omega_0^2 y] =$$

$$= \frac{1}{2} \rho u^2 (D) \left[ H_2(K) \left( 1 - \varepsilon \frac{y^2}{D^2} \right) \dot{y} + H_2(K) y + \frac{1}{2} \xi_\omega \omega_0 \dot{y} \right]$$

(15)

where $H_2, C_l$ and $\alpha$ are experimentally obtained adjustment parameters. $H_2$ is related to the linear aerodynamic damping and $H_2$ with the aerodynamic stiffness, $\varepsilon$ is the nonlinear aerodynamic damping parameter, also measured experimentally.

The aerodynamic damping and stiffness are defined as aerodynamic forces, expressed by the product of flow dependent constants with the displacement $y$ and its velocity derivative $\dot{y}$.

Note that, as the lock-in implies that the mechanical oscillator’s natural frequency controls the whole mechanical mechanical system, $\omega_0$ is, therefore, this natural frequency.

Some experimentally observed vortex shedding effects, such as the mentioned nonlinearity and self-limiting amplitudes, are considered in this model. In lock-in, $C_l$ and $H_2$ can be neglected because they are small when compared with the aerodynamic damping parameter. $H_1$ and $\varepsilon$, in turn, are determined from the
experimental observation of two resonant response amplitudes \(Ay_1\) and \(Ay_2\), for two damping values \(\xi_1\) and \(\xi_2\).

To evaluate these parameters, one relies on the fact that for steady state response amplitudes the average energy dissipation per cycle is zero. Then, we have the following expression:

\[
\int_0^T \left[ 4m\xi_\omega - \rho UD_1 \left(1 - \frac{\omega^2}{D_1^2}\right) \right] y^2 \, dt = 0, \tag{16}
\]

with \(\omega = \frac{2\pi}{T}\), \(T = \text{natural period}\)

Assuming that the time dependent displacement response \(y\) is harmonic:

\[
y(t) = Ay\cos(\omega t) \tag{17}
\]

Evaluating the integrals in which the terms dependent on the square of velocity and its product with the square of displacement appear explicitly:

\[
\int_0^T y^2 \, dt = \frac{\omega Ay^2 \pi}{4}, \quad \int_0^T y^2 \, dy = \frac{\omega Ay^4 \pi}{4} \tag{18-19}
\]

Thus, the previous zero energy dissipation equation can be written as:

\[
(\xi_1^2 + 4D_1^2)\rho UD_1 + 16m\xi_\omega D_1 = 0 \tag{20}
\]

The displacement amplitude value \(Ay\) is then given by:

\[
Ay = 2 \left( \frac{\rho UD_1 - 4m\xi_\omega D_1^2}{\rho UD_1 \xi_\omega} \right)^{1/2} \tag{21}
\]

Substituting the experimentally obtained \(Ay_1\), \(Ay_2\) and \(\xi_1\), \(\xi_2\) in the above equation, we subsequently have the experimental parameters, which can be used to predict a structure’s response under similar conditions:

\[
H_1 = \frac{8m\xi_\omega(\xi_2Ay_1^2 - \xi_1Ay_2^2)}{D^2 \rho(Ay_1^2 - Ay_2^2)}, \quad \varepsilon = \frac{4(\xi_1 - \xi_2)D^2}{\xi_3 Ay_2^2 - \xi_2 Ay_1^2} \tag{22-23}
\]

This model proves to be useful in evaluating responses in wind tunnel tests of scale models.

### 2.3 Semi-Empirical Model by Pinheiro

This model builds on the previous two models, by extending their two-dimensional conditions to a three-dimensional case. It uses the concept of vortex cells to divide the variable section tower structure, under the action of variable profile wind speed \(U(z)\), in sections in which vortex shedding occurs at a certain frequency (Figure 3).

![Figure 3. Scheme of the Pinheiro analytical model for vortex shedding induced vibrations in slender towers: (a) vortex cells in variable section tower under wind action with profile speed \(U(z)\); (b) constant circular section structure under uniform velocity action (2D model); (c) generalized aeroelastic force \(F_\alpha\) associated with vibration mode \(j\) \[12\].]

As a result, there is a \(f_j\) frequency in each cell \(i\), associated to diameter \(D_i\) and velocity \(U_i\) average values. They are related to the section’s Strouhal number via the following equation:

\[
S = \frac{f_i D_i}{U_i} \tag{24}
\]

Through Scanlan’s two-dimensional model, the aeroelastic force \(F(z)\) in each section is given by:

\[
F(z,t) = \frac{1}{2} \rho U_i^2 (D_i) \left[ H_1 \left(1 - \frac{\varepsilon y^2}{D_i^2} \right) \frac{\dot{y}}{U_i} + C_\alpha \sin(\omega_0 t + \alpha) \right] \tag{25}
\]

where \(y\) is the structure’s displacement in function of time. It is considered that each section is inserted in an infinite rigid structure with constant section \(D_0\), under the action of laminar wind \(U_0\), as illustrated by Figure 3.b.

The theoretical results of the coupled wake oscillator model by Blevins are used to determine the vibration amplitudes \(Ay\) in terms of the Scruton number, through which \(H_1\) and \(\varepsilon\) are calculated.

The \(F(z)\) forces per unit length in each section \(i\) are then combined and transformed in generalized forces, obtaining the tower structure’s motion equation in terms of the \(Y_j\) amplitude of its \(j\) vibration mode:

\[
M_j \left( \ddot{Y}_j + 2\xi_j \omega_0 \dot{Y}_j + \omega_0^2 Y_j \right) = Fm_j(H_3, \varepsilon) \tag{26}
\]

where \(M_j, \xi_j, \omega_0\) are, respectively, the structure’s mass, damping and natural frequency associated with the \(j\) natural vibration mode. \(Fm_j\) is the modal force, function of the \(\phi_j(z)\) modal shape and the \(F(z)\) force per unit length. The \(Fm_j\) modal force, in turn, is expressed by:

\[
Fm_j = \int \phi_j(z) F(z) \, dz \tag{27}
\]

#### 2.3.1 Evaluating the Parameters \(H_j\) and \(\varepsilon\)

As mentioned above, the determination of the parameters \(H_j\) and \(\varepsilon\) is made based on two amplitudes \(Ay\) for two different damping values \(\xi\) in each cell with a particular wind speed \(U_i\). This is shown in Figure 3, which portrays the characteristic response of a constant section rigid cylinder (two-dimensional model in Figure 3.b).

Since the Strouhal number’s typical value for circular sections under subcritical regime is approximately \(S = 0.2\), it follows then that the reduced speed is \(U_i = 5 \left(1 / S = 1 / 0.2 = 5\right)\). It’s for this value that the maximum amplitudes \(Ay_1\) and \(Ay_2\) are determined.

Therefore, in agreement with the coupled wake oscillator model, we calculate \(Ay_1\) and \(Ay_2\) in each vortex cell by using the equations which describe the model’s theoretical results, plotted in Figure 1 as a function of the Scruton number:

\[
Ay_n = 0.07 \left[ \frac{0.3 + 0.72}{(1.9 + \delta)S} \right] D_{cell}, \quad \delta = \frac{4m_{cell}\pi x_n}{\rho D_{cell}^2}, \tag{28}
\]

with \(n = 1\) or \(2\).
Taking into account experimental results given in reference [7], linear responses were admitted in the intervals $4.0 < U_r < 5.0$ and $5.0 < U_r < 6.0$, since the studied models can only give the maximum values. A useful simplification is shown in Figure 4. Consequently, to derive other amplitude values beyond the maximum one, for reduced speeds other than 5, we need to do merely a linear interpolation between the already known values.

Figure 4. Linearized curves of the amplitude as a function of reduced speed for different values of $U_r$ used in determining the parameters $H_1$ and $\varepsilon$ [12].

The lock-in range is defined assuming a 20% variation around the vortex shedding’s critical frequency. Outside this range, it’s considered that the rigid cylinder has minor displacements yielded by the following force:

$$ F = \frac{1}{2} \rho U^2 D L \text{sen}(\omega_s t + \alpha), \quad \text{with} \quad \omega_s = 2\pi f_s = 2\pi \frac{SU}{D} \quad (29) $$

Nevertheless, another simplification is adopted to disregard the results for reduced speeds of less than 4 or greater than 6. Although in Figure 4 this consideration is not fully explicit, it’s used since the calculated amplitudes are very small and therefore irrelevant for this study.

As a result, with the above data, the parameters $H_1$ and $\varepsilon$ are obtained by the following expressions, which were already written above in another form:

$$ H_{1,\text{cell}} = \frac{8 m_{\text{cell}} \pi S}{\rho D^2_{\text{cell}}} \left( \frac{A y_1^2 - A y_2^2}{A y_1^2 - A y_2^2} \right), \quad (30) $$

$$ \varepsilon_{\text{cell}} = \frac{4 D^2_{\text{cell}} (\xi_1 - \xi_2)}{A y_1^2 - A y_2^2} \quad (31) $$

Next, the simple case of a constant section tower under constant wind speed along the height is shown. It’s utilized to demonstrate the more general case of a variable section circular tower under the effect of a variable wind profile that this model covers. The effect of the spatial correlation is also considered in this model.

2.3.2 Constant Section Tower under the Action of Constant Wind Speed along the Height

The two-dimensional model explained above, which is applied to an infinite length structure (Figure 5.a), is extended to a cantilevered tower like the one shown in Figure 5.b, by incorporating the modal shape. It’s considered as a simplification that the vortex shedding occurs in a single frequency, which implies the existence of only one vortex cell.

Figure 5. Incorporation of the modal shape (b) to the rigid model (a) and corresponding discretization (c) [12].

On any point $z$ of the tower, the $y$ displacement can be calculated through modal superposition by means of the following equalities:

$$ y(z, t) = \phi(z) \dot{Y}(t) \quad \therefore \quad y(z, t) = \phi(z) \ddot{Y}(t) \quad (32)(33) $$

where $Y$ is the time dependent amplitude of the modal shape $\phi(z)$.

In turn, the $F(z)$ aerelastic force per unit length of equation (25) is rewritten for a rigid cylinder of the type shown in Figure 5.a, taking into account the above equations, as follows:

$$ F(z, t) = \frac{1}{2} \rho U^2 D \left[ H_1 \left( 1 - \frac{\varepsilon \phi \ddot{Y}^2}{D^2} \right) \frac{\phi Y}{U} + \right. $$

$$ + C_L \text{sen}(\omega_s t + \alpha) \left] \right. \quad (34) $$

Developing further, we get:

$$ F(z, t) = \frac{1}{2} \rho U^2 D \left[ H_1 \left( \phi_j - \frac{\varepsilon \phi \ddot{Y}^2}{D^2} \right) \frac{\dot{Y}}{U} + \right. $$

$$ + C_L \text{sen}(\omega_s t + \alpha) \left] \right. \quad (35) $$

As the tower is evaluated in a discrete way based on finite elements rather than a continuous manner, the nodal displacement vector is given by:

$$ y = \phi Y \quad (36) $$

where $\phi$ is the eigenvectors matrix and $Y$ is the modal amplitudes vector. Thus, the nodal forces vector $F$ is given by:

$$ F = \frac{1}{2} \rho U^2 D \left[ H_1 \left( 1 - \frac{\varepsilon \phi \ddot{Y}^2}{D^2} \right) \frac{\dot{Y}}{U} + \right. $$

$$ + C_L \text{sen}(\omega_s t + \alpha) \right] L \quad (37) $$

where $L$ is the diagonal matrix that contains the influence lengths $l_k$ of each node $k$, which should multiply the force per unit length $F$, as shown in Figure 5.c. Then, the modal force is:

$$ F_m_j = \phi^T F \quad (38) $$

This force can be written in a modal summation form, finally obtaining:

$$ F_m_j = \frac{1}{2} \rho U^2 D \times $$

$$ \times \left[ H_1 \left( \sum_k \phi_{jk} l_k - \frac{\varepsilon \dot{Y}^2}{D^2} \sum_k \phi_{jk} \right) \frac{\dot{Y}}{U} + \right. $$

$$ + \sum_k C_L \phi_{jk} l_k \text{sen}(\omega_s t + \alpha) \left] \right. \quad (39) $$

Consequently, with the previous expression’s modal force, the system of equations (26) is solved in the time domain by the
Runge-Kutta method for some initial condition. The tower’s displacements on any point are then obtained by modal superposition.

2.3.3 Variable Section Tower under the Action of Variable Wind Speed along the Height

This case is virtually identical to the previous one. The only differences are that now there’s the formation of distinct vortex cells, each with its own diameter $D_i$ and wind speed $U_i$ (Figure 6). As a result, each node $k$ also has its own parameters $H_j$ and $\epsilon$.

![Figure 6. Variable section tower under variable wind speed profile](image)

The spatial correlation is also considered. This parameter is a measure of the vortex organizing effect that the structure’s oscillation causes. According to experimental results, the bigger the vibration amplitude, the better the correlation. Taking this into account, it follows that the vortex caused effects on the structure can be decreased, since they all do not separate simultaneously and in the same direction along the structure’s span.

Thus, a more accurate modeling of reality can be done by reducing the magnitude of the two-dimensional model’s aeroelastic force per unit length, to account for the spatial correlation. This is achieved by multiplying that force by the modal shape itself, normalized in absolute value.

The normalized modal shape is then given by:

$$\bar{\phi}_j = \frac{\phi_j}{\phi_{max,i}}$$  \hspace{1cm} (40)

where $\phi_{max,i}$ is the maximum value of the components of the $j$ mode’s eigenvector, in the $i$ vortex cell, and $fc$ is the correlation factor. The correlation factor determines which type of correlation is assigned to the structure. As the value of $fc$ increases, the vortices become better correlated, thereby increasing their contribution to the structure’s oscillation.

In the present model study, only two cases are considered, adopting for $fc$ either a unitary value or one extremely high. Therefore, for total correlation, $fc$ is infinite and $\bar{\phi}$ is a unitary vector. For partial correlation, $\bar{\phi}$ is calculated considering $fc = 1$.

Finally, the modal force expression takes the following format:

$$Fm_j = \frac{1}{2} \rho \left( \sum_{k} \bar{\phi}_{jk} \bar{\phi}_{jk}^* H_{1,i} D_k U_k - \frac{\nu^2}{D_k} \sum_{k} \bar{\phi}_{jk} \bar{\phi}_{jk}^* H_{1,i} \epsilon_i U_k \right) \dot{y}_j + \sum_{k} \bar{\phi}_{jk} C_{l,i} \bar{\phi}_{jk} D_k U_k^2 \sin(\omega_{z,i} t + \alpha)$$  \hspace{1cm} (41)

3. Vortex Shedding according to the Eurocode

The Eurocode also provides a simple method for estimating the vortex-induced vibrations in structures. Similarly to the model referenced above, if the vortex shedding frequency is equal to a structure’s natural frequency, it may experience considerable vibration displacements. This occurs when the wind speed is equivalent to a certain critical speed, which is generally an often observed wind speed.

In EC 1-4, two different methods to calculate the vortex-induced across-wind vibration amplitudes are specified - Method 1 and Method 2.

In this work, only Method 1 is applied since it is a more general method that can be used for various types of structures and mode shapes. It includes the turbulence and roughness effects and can be used for normal climatic conditions.

The maximum displacement $y_{F,max}$ is given by:

$$\frac{y_{F,max}}{b} = \frac{1}{St^2} \cdot \frac{1}{Sc} \cdot K \cdot K_W \cdot \epsilon_{lat}$$  \hspace{1cm} (42)

where $b$ is the external diameter of the circular based cylinder, $St$ the Strouhal number, $Sc$ the Scruton number, $K_W$ the effective correlation length coefficient, $K$ the modal configuration coefficient and $\epsilon_{lat}$ is the lift force coefficient.

Further explanation and analysis of these parameters can be found in EC 1-4, annex E.

It is important to note that this Eurocode proposed methodology is a very simplistic one, which does not take into account several parameters that can influence the response. As such, its results may be very distant from reality. Normally, the response amplitude estimates of this method are quite low when compared with other methods. One reason for this is its sensitivity to the damping ratio’s choice. For the same damping ratio, the Eurocode’s estimates are usually lower. This assumes even more relevance because frequently the damping ratio is not a well known parameter. Another reason is the fact that the method is based on the correlation length phenomenon. As vibrations increase, the structure’s length in which the vortex shedding forces act also increases. Thus, the Eurocode is more permissive since the vortex shedding mechanism is not distributed throughout the entire structure. It only acts on part of it, in the so called correlation length, resulting in lower amplitudes when compared with other methods where the vortices separate all over the structure. These facts can be found from references [(5), (6), (10)]. This method should therefore be applied with a careful analysis and after conducting a proper calibration, especially in relation to the damping ratio’s value.
4. Results

4.1 Overview of Examples

The examined structures in the following examples are, as stated earlier, only cylindrical structures with circular cross-section, since that is the scope of the studied model. A cylindrical rod less than 1 m tall and two chimneys with heights of around 100 m are analyzed. They are composed of different materials and the damping level is also variable. The applied wind velocity profiles are either constant or variable, the same happening for the cross-sections’ dimensions.

4.2 Example 1 – Wood Rod in Wind Tunnel

A comparison is performed between the results obtained in wind tunnel tests for a cylindrical rod of constant cross-section, according to reference [3], and the theoretical results calculated by the Pinheiro and EC 1-4 methods. Figure 7 represents the test model layout used in the wind tunnel and the cylindrical rod’s cross-section dimensions.

Figure 7. Schematic representation of the cylindrical rod in the wind tunnel and its cross-section dimensions [3].

The cylinder is made of wood with density $\rho_{\text{wood}} = 540$ kg/m$^3$. This gives a mass per unit length $m = 0.77$ kg/m. Its top is articulated through a bearing and the model’s stiffness was augmented by transversely coupling two linear springs on its free base, each with a stiffness $K = 1988$ N/m. The natural frequency is $f = 20.2$ Hz and the damping ratio is $\zeta = 0.0031$. Two wind speed profiles were applied, one being constant and the other linear. The constant profile’s velocity was successively incremented and the linear one, in which the maximum speed occurs in the cylinder’s free lower end and is given by expression (43), was also decremented. The obtained results are shown in Figure 8 and Figure 9.

$$V(z) = V_0(1 + 0.88z) \quad (43)$$

where $V_0$ is the velocity in $z = 0$.

Figure 8. Response amplitude of the cylinder under constant wind speed profile, for the first vibration mode.

Figure 9. Response amplitude of the cylinder under linear wind speed profile, for the first vibration mode.

For the constant wind speed, we observe that the maximum dimensionless amplitudes for the Pinheiro model are 0.44 and 0.25 for total and partial correlation, respectively, both occurring for $U_r = 5$. Comparing the results of the model with the wind tunnel test, there is good conformity between them, with slight differences in the amplitudes and the reduced speeds for which they occur. The Pinheiro lock-in range is also wider than the experimental one. These differences may be due to several factors, such as small discrepancies in the Strouhal number and natural frequencies, with the consequent effect on the critical speed, and even due to the model’s discretization in the finite element program, influencing the obtained modal shape. The total correlation case is a superior limit of the response, providing better results in this case. This indicates that the total correlation is a more true to reality hypothesis. The maximum experimental amplitude is roughly equivalent to the average of the maximum amplitudes for total and partial correlation.

The maximum dimensionless amplitude for the EC 1-4 method is 0.46, nearly the same as the one calculated for the total correlation case of Pinheiro’ model, although it occurs at a much higher reduced speed. The graphic’s shape is different from the other ones. This is because the EC 1-4 formulation consists of a simple parameter multiplication. The parameters increase with the rise of
wind speed, until reaching a certain limit from where they remain constant, and hence the amplitude becomes stable for reduced speeds greater than 6.7.

For the linear wind speed, the maximum dimensionless amplitudes are 0.43 and 0.25 for total and partial correlation, respectively, both corresponding to $U_r \approx 7$. There is still a reasonable agreement between experimental and theoretical results, especially in the total correlation case. Again, this case is a more adequate hypothesis, due essentially to the two-dimensional conditions of the wind flow, like the constant cross-section. There are no results obtained for EC 1-4 with linear speed because Method I implies that the cylinder is subject to a constant wind speed profile, since only the maximum displacements are required.

4.3 Example 2 – Reinforced Concrete Chimney in Italy

A comparison is performed between the theoretical results of a numerical method referenced in [8] and the ones calculated with the Pinheiro and EC 1-4 models. The considered chimney is shown in Figure 10.

The structure is cylindrical with a constant circular cross-section. It has a height of 100 m and the outer and inner diameters measure 6.3 m and 5.7 m, correspondingly. The density is $\rho_{concrete} = 2500$ kg/m$^3$ with a mass per unit length $m = 14137$ kg/m. The natural frequency is $f = 0.37$ Hz. A constant wind speed profile was applied, which was successively incremented, varying only the damping ratio $\zeta$. Only the total correlation case was considered, since the flow is heavily influenced by two-dimensional conditions. The results are shown in the next figures.

In this case, the lock-in range from Figure 4 was modified to be better adjusted to the numerical results. Lock-in is considered to start at $U_r = 4$ and end at $U_r = 7$, with the peak occurring at $U_r = 5.5$.

The maximum dimensionless amplitudes for the Pinheiro model are 0.0518, 0.1092 and 0.5319 for $\zeta = 0.01$, $\zeta = 0.005$ and $\zeta = 0.001$, respectively, all occurring for $U_r = 5.5$. Comparing the results, there is good agreement between them. The observed discrepancies can be explained, as was mentioned in the previous example, by the adopted simplifications and considerations for the model. We observe that as the damping ratio decreases, the differences between the amplitudes of the two methods are progressively larger. This is because the numerical method has superior self-limiting capabilities in critical undamped conditions, when compared to the Pinheiro model. The latter then produces results that can deviate more from reality for lower damping values. The EC 1-4 graphic’s shape is equivalent to Example 1, but the results are very different, corresponding to about a third of the other two methods. As was already explained, the Eurocode method is very sensitive to the structural damping $\zeta$, yielding results that can be considerably lower. It follows that this method should generally be applied with more conservative values for the damping ratios, therefore requiring a careful adjustment and calibration when comparing it with other methods.
4.4 Example 4 – Steel Chimney in Thailand

A comparison is performed between the theoretical results of the Vickery-Basu model referenced in [2], [4] and the ones calculated with the Pinheiro and EC 1-4 models. More information about the Vickery-Basu model can be found in references [12, 13]. The analyzed chimney is represented in Figure 14.

![Figure 14](image)

Figure 14. (a) Steel chimney located in Rayong, Thailand. (b) Basic dimensions of the chimney [12, 14].

The chimney is 90 m tall with an unlined cylindrical cross-section. It’s comprised of four welded together sections, with different diameters and thicknesses, whose dimensions are indicated in Figure 14.b. The steel has a density $\rho_{steel} = 7850 \text{ kg/m}^3$, resulting in an average mass per unit length $m = 1907 \text{ kg/m}$ and a total mass of about 177000 kg. The natural frequency is $f = 0.93 \text{ Hz}$. As the chimney’s cross-sections have different outer diameters, then the critical speeds are also different. Thus, to obtain the structure’s maximum response amplitude, different wind speeds were applied in each of the four sections, but remaining constant along the height of each one. As a result, for a reduced speed $U_r = 5$ in all of the sections, the considered velocities were 24.18 m/s, 19.07 m/s, 14.88 m/s and 10.23 m/s from the base to the top, respectively.

Only the maximum amplitudes obtained by the different methods are compared, since these are the most relevant results and in [2] only those values are mentioned, with no reference whatsoever to the reduced speeds for which they are obtained. Nevertheless, the amplitude graphs for the Pinheiro and EC 1-4 models are still presented. Only the total correlation case is once more considered. The results are shown in the following table and figure.

<table>
<thead>
<tr>
<th>Damping ($\zeta$)</th>
<th>Maximum displacement amplitudes (m)</th>
<th>Pinheiro</th>
<th>Vikery-Basu</th>
<th>EC 1-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.39</td>
<td>0.40</td>
<td>0.037</td>
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<td>0.02</td>
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<td>0.03</td>
<td>0.12</td>
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</tr>
<tr>
<td>0.05</td>
<td>0.07</td>
<td>0.08</td>
<td>0.007</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Maximum absolute displacement amplitudes for the chimney, for various damping ratios.

![Figure 15](image)

Figure 15. Response amplitude of the chimney, for various damping ratios. The graph’s triangle shapes correspond to the Pinheiro model, and the line shapes to the Eurocode model.

The maximum amplitudes are in very good agreement, except for the EC 1-4 method. This indicates the Pinheiro and Vickery-Basu models are similar in their formulation. Once again, as the damping ratio decreases, the bigger are the amplitudes. This rise in the amplitude follows an exponential growth, for reasons mentioned above, like the low self-limiting capabilities in critical undamped conditions. The EC 1-4 results are similar to the preceding examples. The amplitudes are about a tenth of the other two methods. The cause for this behavior was already explained.

5. Future Work Suggestions

The Pinheiro model has potential to be applied to more general and complex situations that better characterize the vortex shedding problem and obtain more accurate results. Some promising considerations are listed next:

- The consideration of other flow regimes beyond the subcritical one, such as the critical and supercritical, through experimental data calculated for these regimes;
- The implementation of different realistic configurations of vortex cells, since the results are quite susceptible to the length definition of the vortex cells and their transition regions;
- The consideration of an automatic modification of the Strouhal number along the structure’s height, beyond the subcritical flow’s reference value of $S = 0.2$, because it is a very sensitive parameter to the Reynolds number;
- The model’s adaptation to resonance conditions characterized by excitation frequencies that are multiple or submultiple of the natural ones, capable of provoking significant oscillation amplitudes, is also important;
- The extension of the model to other cross-section types and forms, other than the circular one, is also very interesting. This is possible by obtaining experimental data adequate to each section, proceeding in a similar manner to what was done for the cylindrical section. Following this line of reasoning, it’s also possible and desirable to extend the model to other types of structures susceptible to vortex shedding, apart from towers and chimneys;
- The phenomena of lateral vibrations in structures are not solely caused by vortex shedding. As the flow velocity changes,
different aerodynamic instability mechanisms affect the structure. The development of the model to encompass these phenomena, such as flutter and galloping, is therefore something to be considered in further investigations.

6. Conclusions

The across-wind effect in structures is a complex subject that is still far from being fully understood and studied. Therefore, tools such as wind tunnels are still regarded as the most reliable way to carry out this type of analysis. Nonetheless, this did not prevent the appearance of various types of approaches to model this phenomenon. These approaches, however, lack in application flexibility, since they are often associated with specific section types or dependent on experimental data.

A semi-empirical model developed by Pinheiro for estimating transverse displacement amplitudes was presented. It applies to circular section towers under the vortex shedding phenomenon caused by the wind action.

The model builds from various assumptions, having its origin in other similar existing models. A number of simplifying concepts and hypothesis were adopted, both in the fluid mechanics and structural dynamics, which have their consequence on the method’s accuracy and applicability extension. However, these simplifications are necessary to be able to viably study a phenomenon which, although easy to understand, is extremely complex to quantify.

Several comparisons were made between the transverse displacement amplitudes obtained with the studied model and those obtained with other methods extracted from the available literature. Both experimental results from wind tunnel tested reduced models, as well as theoretical ones from other numerical methods, were compared. In each of the studied examples there was a very good agreement between all the obtained results, despite all the difficulties and uncertainties that exist in analyzing the problems. It was necessary in some cases to carry out a calibration and adjustment of the input parameters of Pinheiro’s model. This was done, for example, by changing the predetermined lock-in range. After these slight adjustments that are expected due to the complexity of the phenomenon and unique features of each structure, the results became more consistent and reliable.

On the other hand, the comparison with the Eurocode proposed method produced unsatisfactory results. Only in Example 1 were the maximum amplitudes by both models virtually equal, which is clearly insufficient for the Eurocode method to be considered reliable. As mentioned, this method is too simplistic and should be cautiously used, preferably in a first approach to the problem and as a complement to other more reliable methods.

Despite this simplified and preliminary approach, it is concluded that the Pinheiro proposed simulation model is a useful tool in analyzing the vortex shedding phenomenon in circular towers. The model was used in an elementary form and with various limitations that are necessary to be aware of, but it simultaneously has the potential to be extended to more general and wide cases. As long as certain fundamental parameters are known, it is perfectly possible to adapt the model to more complex situations, a fact that is reinforced by the good results that were obtained. A long research path is therefore left open, aimed at developing this tool, providing it with greater precision and encompassing new theories in the phenomena description.

Various models and analysis tools have been used in predicting a structure’s behavior when subjected to vortex shedding actions, each with its qualities, disadvantages and potential. These characteristics are rarely interchangeable and should complement each other throughout the course of a building project. As such, the method studied in this work is a very advantageous instrument, but it should never be used alone. It should rather be used together and complementarily to other tools in different phases of the project, to ensure a thorough, reliable and quality analysis.

7. References