TRANSMISSION LINE MODELING FOR REAL-TIME SIMULATIONS

Maria Isabel Silva Lafaia Simões
Instituto Superior Técnico
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
Email: maria.simoes@ist.utl.pt

Abstract—Real-time simulation of power systems transients is an important tool when there is a need to include physical elements in the system under study, rather than their mathematical models. Real-time is hard to achieve in digital simulations, where accuracy runs oppositely to processing speed. It is therefore necessary to combine parallel processing with efficient numerical techniques for model computation. Transmission lines allow parallel processing in power systems studies, by dividing large networks into smaller independent subnetworks. The need to account for frequency dependent parameters, poses a challenge on the definition of an adequate line model. The goal of this work is to establish adequate numerical techniques for approximating the propagation parameters for transmission line modeling, allowing real-time simulations.

Keywords: Real-time simulations, transmission line modeling, frequency dependent line model, RT_WB Line, optimized modal delay computation, optimized modal poles assignment.

I. INTRODUCTION

Real-time simulation of power systems transients is an important tool when there is a need to include physical elements in the system under study, rather than their mathematical models. However, the real-time is hard to achieve in digital simulations, where accuracy runs oppositely to processing speed. It is therefore necessary to combine parallel processing with efficient numerical techniques for model computation. Transmission lines play an important role for allowing parallel processing in power systems studies, by dividing large networks into smaller independent subnetworks.

The accurate representation of a transmission line requires the use of its frequency dependent parameters. This poses a challenge on the definition of an adequate line model. The goal of this work is to establish adequate numerical techniques for approximating the propagation parameters for transmission line modeling, allowing real-time simulations.

The study of existing line models provides the basis for the development of the RT_WB Line, which is a reformulation of the EMTP-RV model WB Line (based on the Universal Model [1]), in-line with the real-time simulation target. To ensure additional accuracy with reduced fitting resources, two optimizations are suggested, concerning the computation of the modal delays and the assignment of the modal poles.

The validation of the developed model consists of tests in frequency and time domain conditions and uses an application of the RT_WB Line which order of approximations is in-line with examples in real-time modeling literature, namely [8], [9]. The WB Line application computed by the EMTP-RV 2.3 is also considered and taken as a reference of accuracy.

II. EMTP-RV AND TRANSMISSION LINE MODELING

The EMTP-RV 2.3 is a specialized software for the simulation of electromagnetic, electromechanical and control systems transients in multiphase power systems. The transmission line models available in the EMTP-RV provide a summary of the evolution of line modeling.

A. EMTP-RV models review

1) CP Line – constant parameters line model: this line model is based on the work by Dommel [2]. It is based on modal analysis and each of the n line modes is characterized in terms of the corresponding characteristic admittance $Y_c$ and propagation function $H$, which are computed through a real constant transformation matrix. The model approximates the line as an ideal lossless line ($R = 0, G = 0$). Therefore, each modal function simply becomes $Y_c = \sqrt{C/L}$ and $H = e^{-j\omega\tau}$, where $C$, $L$ and $\tau = d\sqrt{LC}$ are the capacitance, inductance and propagation delay of the corresponding mode. This model can include the effect of losses by inserting constant lumped resistances in discrete points of the line.

2) FD Line – frequency dependent line model: the FD Line is based on the work by J. Marti[3]. This model is based on the characteristic impedance $Z_c$ and propagation function $H$ of each mode. These functions are computed from the line parameters in phase domain, using a real constant transformation matrix. For each line mode, the characteristic impedance and propagation function are approximated through Asymptotic Fitting[3] in the $s$-domain, as:

$$\hat{Z}_c(s) \approx k_0 + \sum_{x=1}^{N_z} \frac{k_x}{s - p_x}$$

$$\hat{H}(s) \approx \left( \sum_{y=1}^{N_h} \frac{k_y}{s - p_y} \right) e^{-s\tau_{\text{min}}}(2)$$

where $N_z$ and $N_h$ are the number of poles used to approximate the corresponding modal functions and $\tau_{\text{min}}$ is the minimum propagation delay of the mode.
3) WB Line – wide-band line model: this model is based on the Universal Line Model, introduced in a work by Gustavsen et al.[1]. The WB Line describes the line in phase domain through the frequency dependent matrices $\tilde{Y}_c$ and $\tilde{H}$. The characteristic admittance matrix $\tilde{Y}_c$ is fitted column-by-column through Vector Fitting[4]. The elements of the propagation matrix $\tilde{H}$ are all approximated with the same poles and delays, defined by the modes:

$$\tilde{Y}_{c_{ij}}(\omega) \approx k_0 + \sum_{x=1}^{N_y} \frac{k_x}{j\omega - p_x}$$ (3)

$$\tilde{H}_{ij}(\omega) \approx \sum_{k=1}^{n} \left( \sum_{m=1}^{N_k} \frac{c_{mk_{ij}}}{j\omega - p_{mk}} \right) e^{-j\omega\tau_k}$$ (4)

where:

- $n$ is the number of line modes,
- $N_y$ is the number of poles to fit the $j^{th}$ column of $\tilde{Y}_c$,
- $N_k$ is the number of poles defined by the $k^{th}$ mode
- $\tau_k$ is the minimum delay defined by the $k^{th}$ mode.

The poles of $\tilde{Y}_c$ are generally real, whereas those of $\tilde{H}$ may be real or complex. The modal poles and delays that approximate $\tilde{H}$ are obtained by applying Vector Fitting[4] to each modal propagation function. The residues $c_{mk_{ij}}$ are computed from a set of samples of $\tilde{H}$, by solving a linear least squares problem.

B. EMTP-RV line models testing

This section presents the major conclusions of a set of tests to compare the line models provided by the EMTP-RV 2.3. The line represented by the models throughout these tests is described in appendix.

1) Frequency scans: The first test consists of frequency scans in short-circuit and open-end conditions, plotted in figures 1 and 2.

As regards the short-circuit scan, all line models generate inaccurate values for low frequencies. However, the approximations are more accurate for higher frequencies, except for the CP Line which is generally very inaccurate. An exception is observed near the 1 kHz, for which all models agree. This is the frequency used by EMTP-RV to compute the transformation matrix of all models and the parameters of the CP Line. Concerning the open-end condition, the FD Line and the WB Line provide very accurate results for all the range observed – 0.1 Hz to 1 MHz. The CP Line is also very accurate, except for frequencies beyond the 2 kHz.

2) Line energization and single-phase short-circuit: This test concerns two common transient conditions: line energization and single phase short-circuit. Figure 11 illustrates the circuit used, where the line described in appendix is connected through ideal switches to a three-phase ideal source of 1V peak voltage and 50 Hz. The three phases are connected to the source simultaneously at $t = 20$ milliseconds. After the transient of line energization, a short-circuit occurs at $t = 180$ milliseconds in phase 3 of the line. The voltage at the receiving end of phase 1 is observed. The results of this test are plotted in figures 3 and 4, which represent only the CP Line and the WB Line approximations. This is done for simplicity, given the FD Line generates practically the same results as the WB Line.
The time responses according to the FD Line and the WB Line show very good agreement for both transient conditions. The CP Line, however, generates inaccurate results, specially in approximating the energization transient, by clearly neglecting the distortion phenomena. The attenuation is another inaccurate aspect in the time response of the CP Line.

3) Current induced by phase coupling: This is a test in which all models show significantly different results. Consider figure 14, where the transmission line is short-circuited at all terminals except at the sending end of phase 1, which is connected to a DC voltage source of 1V, at time $t = 1$ ms, by an ideal switch. The current induced at the sending end of phase 3, $i_{k3}(t)$, is observed. The results are plotted in figures 5 and 6. The FD Line and the WB Line agree only for the very initial period of the transient. An important aspect in this time response is that the induced current must decline to zero in steady-state, due to extinction of the coupling phenomena. The FD Line and, particularly, the CP Line are very inaccurate in approximating this behavior. Therefore, the WB Line is the only model of the test providing results physically acceptable.

C. Notes on the EMTP-RV models performance

The tests have showed that the FD Line and the WB Line are adequate models to simulate typical transient conditions, such as line energization and simple short-circuit or open-end conditions. On the other hand, the CP Line provides inaccurate results for all the tests, and should therefore be avoided in transient studies. Though the FD Line and the WB Line provide very similar results in many conditions, there are some critical tests in which only the WB Line succeeds in providing results physically acceptable, as the test of induced currents shows. Furthermore, the WB Line uses a total of 11 + 21 = 32 poles to fit $\bar{Y}_c$ and $\bar{H}$ compared to the 51 + 69 = 120 poles used by the FD Line to fit $\bar{Z}_c$ and $\bar{H}$. Therefore, the WB Line presents a great improvement in efficiency, by obtaining better results with less resources. The efficiency of the WB Line is reinforced by the fact of being a phase-domain model – there is no need to convert from phase to modal quantities, and vice-versa, at each simulation step.

III. WIDE-BAND MODEL FOR REAL-TIME SIMULATIONS

The developed line model $RT_{WB}$ Line is a reformulation of the EMTP-RV model $WB$ Line (based on the universal model [1]), in-line with the real-time simulation target. Therefore, it is a phase domain model based on the rational approximations of $\bar{Y}_c$ and $\bar{H}$ using the scheme described through equations (3) and (4). The following text presents two optimization procedures, concerning the computation of the modal delays and the assignment of the modal poles, which are introduced in the developed model to ensure additional accuracy with reduced fitting resources.

A. Optimal modal delay identification

Consider the propagation function of the $k^{th}$ line mode:

$$\tilde{H}^k(\omega) = e^{-(\alpha_k(\omega) d + j\omega \tau_k(\omega))}$$  (5)

where $\tau_k(\omega)$ is the propagation delay of the $k^{th}$ mode. Each function $\tilde{H}^k(\omega)$ may be approximated by a rational function.
and a constant time delay factor \( e^{-j\omega \tau'_k} \):

\[
\bar{H}^k(\omega) = \bar{H}^k(\omega)e^{-j\omega \tau'_k} \approx \left( \sum_{n=1}^{N_k} \frac{c_n}{j\omega - p_n} \right) e^{-j\omega \tau'_k} \quad (6)
\]

where \( N_k \) is the number of poles used to fit the \( k \)th mode. The extraction of a constant propagation delay allows a more accurate fitting for the same number of poles.

The constant delay \( \tau'_k \) may be computed so that \( \bar{H}^k(\omega) \) becomes approximately a minimum phase shift function. According to [10], this is done through:

\[
\tau'_k \approx \tau_k(\omega) + \frac{1}{\omega} \left( \frac{\pi}{2} \frac{d}{d\ln \omega} \left| \bar{H}^k(\omega) \right| \right)_{\omega = \omega_0} \quad (7)
\]

computed for a frequency \( \omega \) such that \( |\bar{H}^k(\omega)| = 0.1 \).

However, according to [10], the computed delay may not correspond to the most accurate fitting of \( \bar{H}^k(\omega) \). This motivates the construction of a routine to compute the modal delays leading to the most accurate approximation as defined in (4). Tests involving different lines have showed that a good estimation for \( \tau'_k \) can be found in the interval from \( 0.9 \times \tau'_k \) to \( 1.1 \times \tau'_k \), where \( \tau'_k \) is given by (7). Tests have further shown that generally it is enough to search with an iteration of 1% of the base modal delay.

Though the accuracy in the approximated modes is not directly related to the phase domain results [10], tests have showed that, for certain orders of the approximating modal functions, the use of optimized modal delays leads to improved accuracy both in the modal and phase domain, when compared to the approximating functions obtained by simply using the lossless modal delays. Nevertheless, it must be noted that the improvements introduced by this process vary with the particular line and with the order of the model.

### B. Optimal modal poles assignment

The developed model approximates the propagation matrix using the poles and delays defined by the modes, as expressed in (4). In order to respect the pre-defined order of the model, the sum of the poles assigned to each mode, \( N_k \), must be equal to the maximum number of poles allowed for \( \bar{H} \), that is, \( \sum N_k = N_{\text{max}} \). Practical tests have showed that assigning an equal number of poles to each mode generally does not lead to the most accurate fitting of \( \bar{H} \). Therefore, it is advantageous to optimize the number of poles assigned to each modal propagation function. This is done by trying all the possible distributions of the number of poles among the modes, with the requirement that the total number of poles must respect the pre-defined order of the model. The following numerical example illustrates and justifies this procedure.

#### 1) Numerical example: Consider the line described in appendix. Table I shows the decrease in the average approximation error thanks to optimizing the number of poles assigned to each mode, for several model orders.

<table>
<thead>
<tr>
<th>Total poles</th>
<th>Distribution of poles</th>
<th>Improvement on the approximation error</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>[2 – 4 – 1]</td>
<td>-25%</td>
</tr>
<tr>
<td>9</td>
<td>[4 – 4 – 1]</td>
<td>-6%</td>
</tr>
<tr>
<td>12</td>
<td>[5 – 0 – 7]</td>
<td>-55%</td>
</tr>
<tr>
<td>15</td>
<td>[6 – 1 – 9]</td>
<td>-43%</td>
</tr>
<tr>
<td>18</td>
<td>[8 – 1 – 9]</td>
<td>-15%</td>
</tr>
</tbody>
</table>

As table I shows, the optimized assignment of modal poles has a positive impact on the approximating propagation matrix, with a reduction of at least 6% in relation to the error obtained with equal distribution of modal poles, for most of the orders tested for the RT_WB Line. A deeper look to table I shows that, for a low number of approximating poles, the optimized distribution tends to assign more poles to modes 1 and 2, whereas for a higher approximating order, modes 1 and 3 are preferred. Nevertheless, only in exceptional cases one mode is neglected, being assigned zero fitting poles, as showed in table I for a total of 12 poles. Therefore, there is not an explicit tendency of optimal pole distribution that allows to define a single strategy that works both for low and high orders of approximation. As a consequence, in order to achieve the optimal approximation of the propagation matrix \( \bar{H} \), the developed procedure tries all the possible assignments of modal poles. This is not the most efficient process, but it certainly reaches the most accurate result, based on the average error of the fitting matrix \( \bar{H} \).

### IV. RT_WB LINE MODEL VALIDATION

The validation of the developed model consists of frequency and time domain simulations in the EMTP-RV 2.3 environment, using an application of the RT_WB Line, which performance is compared to that of the WB Line, computed by the EMTP-RV and taken as a reference of accuracy. The application of the WB Line uses \( 21 + 11 = 32 \) poles to fit \( \bar{H} \) and \( \bar{Y}_c \), whereas the RT_WB Line application uses only \( 9 + 9 = 18 \) poles, respectively. The total number of poles used by this application is in-line with the examples in real-time line modeling literature, namely [8]; [9].

#### A. Frequency response – short-circuited and open-ended line

This test evaluates the accuracy of the line models by comparing their frequency responses with the expected behavior, according to analytical expressions computed with the exact line functions, which are rewritten, respectively, as:

\[
\bar{I}_{k, \text{short}} = \bar{Y}_c (I - \bar{H}^2)^{-1} (I + \bar{H}^2) \bar{V}_{\text{source}} \quad (8)
\]

\[
\bar{V}_{\text{m,open}} = (I - \bar{H}^2)^{-1} \bar{H} 2\bar{V}_{\text{source}} \quad (9)
\]

where \( \bar{V}_{\text{source}} \) is the phasor representing the 1V–RMS, three-phase symmetrical source feeding the line. Given the symmetry of the problem, it is sufficient to analyze one phase of the line (phase 1 was chosen).
Figure 7 plots the relative error of the approximating short-circuit responses according to the WB Line and RT_WB Line applications. The approximating responses, specially in what concerns the developed model, are inaccurate for the range of frequencies up to 50 Hz. However, for higher frequencies, both models provide accurate approximations, except when approximating the various peaks in the expected frequency response. Figure 8 shows a zoom of the relative errors of the approximating responses from 700 Hz to 10 kHz, which includes the switching transients frequencies. For this range, the RT_WB Line is more accurate than the WB Line, despite using lower order approximations.

Fig. 7: Relative error of the short-circuit frequency response (0.1 Hz - 1 MHz) – WB Line (bold) and RT_WB Line (thin). Current at the sending end of phase 1.

Fig. 8: Detailed relative error of the short-circuit frequency response (700 Hz - 10 kHz) – WB Line (bold) and RT_WB Line (thin). Current at the sending end of phase 1.

Figure 9 plots the relative errors of the approximating open-end responses, and it shows that both the WB Line and the RT_WB Line applications are very accurate for low frequencies. However, the errors tend to be higher for the approximating peaks in the open-end response. Figure 10 shows a zoom into the relative errors of the open-end responses, from 700 Hz to 10 kHz. As verified in the short-circuit scan, the RT_WB Line is more accurate than the WB Line for this range of frequencies.

Fig. 9: Relative error of the open-end frequency response (100 Hz - 1 MHz) – WB Line (bold) and RT_WB Line(thin). Voltage at the receiving end of phase 1.

Fig. 10: Detailed relative error of the open-end frequency response (700 Hz - 10 kHz) – WB Line (bold) and RT_WB Line (thin). Voltage at the receiving end of phase 1.

B. Line energization and single phase short-circuit

This test evaluates how the models represent the behavior of the line under typical transient conditions. Figure 11 illustrates the circuit used, where the line is connected by ideal switches to a three phase symmetrical source of 1 V peak voltage and 50 Hz. The line energization occurs at \( t = 20 \) ms, with the closure of the switches connecting the line to the source. After reaching steady-state, a new transient is originated, at \( t = 180 \) ms, by closing the receiving end switch (short-circuit on phase 3). The voltage at the receiving end of phase 1 is observed. The results of this test are plotted in figures 12, which concerns the energization transient, and 13 regarding the short-circuit of phase 3.

Both figures show very good agreement between the two models performance. A difference is perceived only in the energization condition, where the approximating line response according to the RT_WB Line denotes a slight weaker attenuation of the voltage at the receiving end of phase 1.
Fig. 11: Circuit used to study the response to line energization followed by single-phase short-circuit, according to the *WB Line* and to the *RT_WB Line* applications.

![Circuit diagram](image1)

Fig. 12: Line energization: voltage at the receiving end of phase 1 (*WB Line* – bold; *RT_WB Line* – thin).

![Voltage plot](image2)

Fig. 13: Line energization: voltage at the receiving end of phase 1(*WB Line* – bold; *RT_WB Line* – thin).

![Voltage plot](image3)

**C. Current induced by phase coupling**

This is a test in which generally all model applications generate considerably different results. Figure 14 illustrates the circuit used, where the line is short-circuited at all terminals, except at the sending end of phase 1, which is connected to a 1V–DC voltage source by an ideal switch, 1 millisecond after the simulation start. The current at the sending end of phase 3 is observed.

![Circuit diagram](image4)

The simulation results for these conditions are illustrated by the plots of figures 15 and 16. As figures show, the time responses according to the two models agree only for the very initial period of the transient. Furthermore, the response according to the *RT_WB Line* presents two undesired peaks, as plotted in figure 16. The induced current in steady-state is another important aspect, which approximation is more accurately computed using the *WB Line*.

![Current plot](image5)

Fig. 14: Circuit used to study the current induced in phase 3 by phase coupling, according to the *WB Line* and to the *RT_WB Line* applications.

![Current plot](image6)

Fig. 15: Current at the sending end of phase 3, induced by energization of phase 1, at the first 20 miliseconds (*WB Line* – bold; *RT_WB Line* – thin).

![Current plot](image7)

Fig. 16: Current at the sending end of phase 3, induced by energization of phase 1, at the first second of simulation (*WB Line* – bold; *RT_WB Line* – thin).

Though this test is not relevant for switching transients studies, it must be noted that the inaccuracy of the results is a consequence of the low order used for the approximating line functions of the *RT_WB Line* application. To demonstrate this, consider including in the test another application of the developed model, which order of the approximating functions $\vec{H}$ and $\vec{Y}_c$ is the same as that used by the *WB Line* application, generated by the EMTP-RV. The new results are plotted in figures 17, 18 and 19.
Fig. 17: Current at the sending end of phase 3, induced by energization of phase 1, at the first 20 milliseconds (WB Line – bold; RT_WB Line applications: low order – thin; high order – dashed).

Fig. 18: Current at the sending end of phase 3, induced by energization of phase 1, at the first second of simulation (WB Line – bold; RT_WB Line applications: low order – thin; high order – dashed).

Fig. 19: Current at the sending end of phase 3, induced by energization of phase 1, at the first 50 second of simulation (WB Line – bold; RT_WB Line applications: low order – thin; high order – dashed).

Figure 17 shows that it is possible to reach a good agreement with the WB Line time responses, by using the higher order application of the developed model. Furthermore, the accentuated peaks in the approximating response of the low order RT_WB Line application are practically eliminated when using the higher order approach.

A very important aspect in this test is the correct approximation of the steady-state condition, where the induced current must decline to zero. Figure 19 shows the test results after a long simulation period, and it is possible to observe that the best approximation is the one computed using the higher order application of the RT_WB Line, though it uses the same order for the approximating line functions as the WB Line application.

V. Conclusions

The accurate representation of a transmission line requires the use of its frequency dependent parameters. This poses a challenge on the definition of an adequate line model. The goal of this work is to establish adequate numerical techniques for approximating the propagation parameters for transmission line modeling, allowing real-time simulations. The study of existing line models, namely those provided by the EMTP-RV 2.3, leads to the conclusion that the WB Line, that is, the Universal Model [1] approach, presents an increased efficiency when compared to modal domain frequency dependent models, by obtaining better results with less resources. This efficiency is reinforced by the fact that the WB Line is a phase-domain model – there is no need to convert from phase to modal quantities, and vice-versa, at each simulation step. The line model developed in this work, called RT_WB Line, is a reformulation of the WB Line, in-line with the real-time simulation target. Therefore, the RT_WB Line is a phase domain model which fits the propagation matrix $\tilde{H}$ using the poles and delays defined by the modes. To ensure additional accuracy with reduced fitting resources, two optimizations are introduced, regarding the computation of the modal delays and the assignment of the modal poles.

The validation of the developed model consists of frequency and time domain simulations in the EMTP-RV environment, using an application of the RT_WB Line, which performance is compared to that of the WB Line, computed by the EMTP-RV and taken as a reference of accuracy. The total number of poles used by the developed model application is in-line with the examples in real-time line modeling literature, namely [8], [9].

The developed model presents some weaknesses, for example in approximating the low frequencies of the short-circuit and open-end responses, plotted in figures 7 and 9, or in approximating the current induced by phase coupling, as the plots in figures 15 and 16 show. These less accurate results are not due to a problem in the model computing routine, but a consequence of the low order used for the approximating functions. This is proved by considering another application of the developed model which approximating functions have the same order as those of the WB Line application. In fact, this new application provides an approximation of the steady-state induced current which is more accurate than that of the WB Line application, as plotted in figure 19.

Furthermore, these tests for which the RT_WB Line is not particularly adequate, are not the applications in which the real-time is more relevant. On the other hand, real-time is of
utmost importance for the study of switching transients, and the tests show that the numerical techniques and optimization procedures introduced in the RTWB Line allow to produce low order applications that generate accurate simulation results in switching transient conditions. Examples of this good performance are the short-circuit and open-end frequency scans for the range of 700 Hz to 10 kHz, plotted in figures 8 and 10, respectively. The test of line energization followed by single-phase short-circuit, illustrated by the plots of figures 12 and 13, is another case of good agreement between the performance of the the WB Line and RTWB Line applications, despite the difference on the order of approximations.

APPENDIX A: FUNDAMENTALS ON MULTIPHASE TRANSMISSION LINE THEORY – A BRIEF REVIEW

A. Phase domain solution

Consider an n-phase transmission line of length d, as illustrated in figure 20.

\[
\begin{align*}
\bar{V} &= \bar{T}_V \bar{V}_m \\
\bar{I} &= \bar{T}_I \bar{I}_m
\end{align*}
\]

By applying this transformation, equations (10) and (11), as well as their time domain counterparts (12) and (13), still hold, as long as all quantities are considered in modal domain. For ordinary multi-conductor line configurations, the modes are independent and therefore each of these equations transformed to modal domain in fact represents n independent scalar equations. An additional set of convolutions must be taken in order to convert the modal voltages and currents to the natural domain of phases through:

\[
\begin{align*}
v(t) &= \bar{v}_s(t) * v^m(t) \\
i(t) &= \bar{i}_s(t) * i^m(t)
\end{align*}
\]

where \(\bar{v}_s(t)\) and \(\bar{i}_s(t)\) are the inverse Fourier transformations of the transformation matrices \(\bar{T}_V(\omega)\) and \(\bar{T}_I(\omega)\).

APPENDIX B – ELECTROMAGNETIC AND GEOMETRICAL DATA OF TRANSMISSION LINE USED IN MODEL TESTING

Fig. 21: Transmission line used throughout this dissertation.

REFERENCES


