Strength Assessment of Imperfect Stiffened Panels
Using Modified Stress Strain Curves

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Jury

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‘The most useful and valuable experience is that derived from failures and not successes’

Isambard Brunel
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The objective of the present thesis is to analyse the ultimate strength of steel plates and stiffened panels accounting for residual stresses, welding sequences and corrosion degradation and to develop a fast finite element approach based on modified stress-strain curves. The new developed approach may be used to speed up the analysis of the ultimate strength of steel structural components subjected to compressive load through the material stress-strain modification by accounting for different imperfections and structural degradation outlined herein. The analysis is performed by finite element method employing commercial software. The heat source of welding is defined both as pre-stresses and a moving heat one, in order to simulate welding processes and consequently residual stresses. Three types of welding sequences are conducted. The modified stress strain curves are developed for three finite element models of structural components and accounting for different levels of residual stresses and boundary conditions. The modified material stress strain curves also account for the effect of the welding sequence, shakedown and corrosion degradation. The modified stress strain curves can be directly used for nonlinear finite element analyses of ultimate strength of steel plates and stiffened panels. This work also analyses the effect of shakedown on ultimate strength. A mathematical model is developed to account for a time dependent residual stress reduction, which is used to evaluate ultimate strength.

**Keywords:** material stress strain curve, residual stresses, shakedown, corrosion, finite element method.
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O objectivo da presente tese é analisar a resistência última de painéis reforçados e placas de aço, sujeitas a tensões residuais, na sequência de soldadura, degradação por corrosão, e ainda desenvolver uma metodologia rápida do cálculo da resistência última pelo método dos elementos finitos baseada na modificação da curva de tensão-extensão. O método proposto pode ser utilizado para acelerar o cálculo e a análise dos componentes estruturais de aço submetidos a uma carga de compressão tendo em conta as diferentes imperfeições e níveis de degradação estrutural. A análise é realizada pelo método dos elementos finitos empregando uma ferramenta comercial. A fonte de calor inerente aos processos de soldadura é definida tanto como tensão pré-definida e como uma fonte de calor em movimento, a fim de simular os processos de soldadura e consequentes tensões residuais. Três tipos de sequências de soldadura são analisados. As curvas tensão-extensão modificadas são derivadas para os três modelos de elementos finitos de componentes estruturais, tendo em conta os diferentes níveis de tensões residuais e condições de fronteira. As curvas de tensão-extensão do material também tomam em conta o efeito da sequência de soldadura, "shakedown" e degradação por corrosão. As curvas de tensão-extensão modificadas podem ser aplicadas directamente na análise não-linear de elementos finitos. Este trabalho também analisa o efeito de "shakedown" sobre a resistência última. Uma formulação matemática é desenvolvida para relacionar a variação das tensões residuais do tempo e que, por sua vez, é utilizada para avaliar a resistência última.

**Palavras-chave:** curva tensão-extensão, tensões residuais, shakedown, corrosão, método dos elementos finitos.
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{mn}$</td>
<td>Maximum vertical deflection in the mid-plate</td>
</tr>
<tr>
<td>$A_{net-50}$</td>
<td>Stiffener area without attached plating</td>
</tr>
<tr>
<td>$A_{pnet-50}$</td>
<td>Plate area</td>
</tr>
<tr>
<td>$B$</td>
<td>Strain-displacement matrix</td>
</tr>
<tr>
<td>$B_{r5}, B_{r}, B_{p}$</td>
<td>Model uncertainty factors</td>
</tr>
<tr>
<td>$C$</td>
<td>Thermal stiffness</td>
</tr>
<tr>
<td>$D$</td>
<td>Plate's flexural rigidity</td>
</tr>
<tr>
<td>$D_e, D_{ep}$</td>
<td>Elastic and plastic stiffness matrix</td>
</tr>
<tr>
<td>$E$</td>
<td>Young modulus</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Tangent modulus</td>
</tr>
<tr>
<td>$F$</td>
<td>Airy's stress function</td>
</tr>
<tr>
<td>$I_{Enet50}$</td>
<td>The stiffener moment of inertia with the effective attached plating</td>
</tr>
<tr>
<td>$K_{x}, K_{y}, K_{z}$</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>$M_x, M_y$</td>
<td>Edge strip bending moments</td>
</tr>
<tr>
<td>$N$</td>
<td>Shape function</td>
</tr>
<tr>
<td>$N_x, N_y$</td>
<td>Plate's forces per unit length in $x$ and $y$ direction</td>
</tr>
<tr>
<td>$Q$</td>
<td>Energy heat input</td>
</tr>
<tr>
<td>$Q_v$</td>
<td>The energy released or consumed</td>
</tr>
<tr>
<td>$Q_{x}, Q_y$</td>
<td>Edge shearing forces</td>
</tr>
<tr>
<td>$R$</td>
<td>External force matrix</td>
</tr>
<tr>
<td>$R_{f}$</td>
<td>Reduction factor for the initial geometry</td>
</tr>
<tr>
<td>$R_{r}$</td>
<td>Reduction factor for the residual stress</td>
</tr>
<tr>
<td>$R_{e}$</td>
<td>Reduction factor for the shear stress</td>
</tr>
<tr>
<td>$P_{net50}$</td>
<td>Plate thickness</td>
</tr>
<tr>
<td>$T$</td>
<td>Nodal temperature</td>
</tr>
<tr>
<td>$T_{\infty}$</td>
<td>Surrounding temperature</td>
</tr>
<tr>
<td>$a$</td>
<td>Plate length</td>
</tr>
<tr>
<td>$b$</td>
<td>Plate breadth</td>
</tr>
<tr>
<td>$b_e$</td>
<td>Effective width</td>
</tr>
</tbody>
</table>
\( b_{\text{eff}-p} \)  Effective width for plate
\( b_{\text{eff}-s} \)  Effective width for stiffener
\( c \)  Surface heat flux radius
\( c_p \)  Specific heat coefficient
\( d_{\infty} \)  The long term corrosion wastage
\( d_n(t) \)  Mean corrosion wastage
\( f \)  Body force
\( h_f \)  Heat conduction coefficient
\( k \)  Buckling coefficient, index
\( m, n \)  Number of half sine waves on a bucked mode
\( q_{\text{con}} \)  The convection load
\( s \)  Stiffener span
\( t \)  Plate thickness
\( w_o \)  Initial deflection
\( w \)  Out-of-plane total deflection
\( x, y, z \)  Cartesian coordinates
\( \beta \)  Plate slenderness
\( \beta_{p} \)  Beta
\( \pi \)  Pi number
\( \varphi \)  Edge function
\( \phi_p \)  Perfect ultimate strength
\( \phi_u \)  Ultimate strength
\( \phi_r \)  Residual stress level
\( \eta \)  Tension extent parameter
\( \sigma_y \)  Yielding stress
\( \sigma_u \)  Ultimate strength
\( \sigma_c \)  Critical buckling load
\( \sigma_{\text{CR1}} \)  Critical buckling load
\( \sigma_{E1} \)  Euler buckling stress
\( \sigma_e \)  Maximum edge stress
\( \sigma_r \)  Residual stress
\( \sigma(0) \)  The welding residual stress at time zero
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{yu}$</td>
<td>Mean ultimate strength</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>Yielding strain</td>
</tr>
<tr>
<td>$\varepsilon_E$</td>
<td>Element strain</td>
</tr>
<tr>
<td>$\overline{\varepsilon}$</td>
<td>Relative strain</td>
</tr>
<tr>
<td>$\varepsilon_{th}$</td>
<td>Thermal strain</td>
</tr>
<tr>
<td>$\tau_c, \tau_t$</td>
<td>Coating life and transition time respectively</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Lag factor</td>
</tr>
<tr>
<td>$\tau_{SD,R}$</td>
<td>The time period when no residual stress reduction present</td>
</tr>
<tr>
<td>$\tau_{T,R}$</td>
<td>The time period at which no residual stress present</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Moving coordinate</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Material mass</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Expansion coefficient</td>
</tr>
<tr>
<td>$\delta_o$</td>
<td>Initial imperfection</td>
</tr>
</tbody>
</table>
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1 INTRODUCTION

Ships are built of plates, its associated stiffeners, girders, bulkheads and frames that form ship hull strength. The plates are supported by the stiffeners, which keep them in position to absorb the subjected loadings. The frames that prevent the stiffeners to buckle and support them are stiff and deeper than the stiffeners, which eliminate the overall grillage buckling. Ship structures are assembled of stiffened panels and an estimation of the maximum load carrying capacity or the ultimate strength of these elements is of high importance for the safety assessment and economical design of structures.

The structural strength assessment evolves ultimate strength. The ultimate strength assessment has become a criterion to assess the structural components. Essentially it implies the point where no additional external force absorption is allowed and in turn the structural strength declines, which is regarded as so-called structural instability point. The core reason of the characteristic of the unloading regime is geometrical shape that structures forms until they reach their ultimate stage accompanying plasticity.

From the manufacturing process to their scrape, the plates and their stiffeners are exposed to different external disturbance, imperfections that lead to their strength reduction and its failure namely the welding induced distortions and residual stresses arising from the manufacture, cyclic loadings due to the ship motions and the corrosive operating environment. In addition to those parameters that affect the panel strength, the failure modes of the stiffened panels add uncertainties due to its complexity in calculating the load carrying capacity. The formed plate shape due to the external forces may increase or decrease the load carrying capacity. The plate associated with the stiffener acts together to resist to the applied loads and in the case of the individual element instability of these combination leads to a dramatic load decline, which in turn affect the rest of the structural components, making up the entire ship hull girder. Therefore, these structural elements are to be designed in such a way that the individual instability is avoided. It can be concluded that the strength assessment of structural components is not a straightforward process to be achieved, which needs considering all the aspects of the structural uncertainty altogether.

1.1 Aim and scope

The method adopted in this thesis, deals with the material stress-strain modification within pre-set boundary conditions to capture the same response with welding induced stress and corrosion degradation and also to investigate the response of the different structural configurations accounting for the effect of welding induced residual stresses, time dependent shakedown and corrosion degradation on the structural strength. The logic behind this approach is to change the stress-strain developed within the analysed structure and to decrease the first yielding stress. An admissible stress strain curve is developed to capture the structural response accounting for residual stresses and corrosion degradation.
1.2 Motivation

The effect of welding induced distortions and residual stresses and corrosion wastage have been studied for plates and stiffened panels through the use of finite element method, which is commonly the most favourable to assess the structural strength. From the computational time point of view; it has been confined only to the local strength assessment. A fast finite element approach has been developed here. The point behind it is that each disturbance outlined here should have an implication on the structural material stress strain curve. Through the material stress strain modification, the intact plate in terms of the welding induced stresses and corrosion degradation is altered to find out the same response to that of the welding induced stresses and corrosion degradation. With this approach, the material stress strain curves are developed and readily incorporated into the ultimate strength assessment of structures implementing a progressive collapse analysis.

1.3 Organization of the thesis

The thesis is divided into seven other chapters that are organized as follows: Chapter 2 presents a state of the art of buckling, ultimate strength, welding induced residual stresses, shakedown, corrosion degradation modelling and the fast finite element approach; Chapter 3 deals with the uncertainties in the finite element analysis and paves the groundwork for Chapter 4; Chapter 4 is dedicated to the welding induced stresses for different type of structural joints and material stress-strain definitions; Chapter 5 is focused on the welding induced stresses, based on a moving source for a single plate and on the material stress-strain definition; Chapter 6 is dedicated to the welding sequence of a stiffened panel and the material stress-strain definition; Chapter 7 deals with the time-dependent shakedown and corrosion degradation and the material stress-strain definition and finally, Chapter 8 presents the conclusions of the present work and traces the future work.
2 STATE OF THE ART

2.1 Buckling

Instability is a condition wherein a compression member loses the ability to resist increasing loads due to plasticity and the geometrical shape formed and exhibits instead a decline in load-carrying capacity. It is a basic law of nature is that, whenever there is a choice between different paths, a physical phenomenon will follow the easiest path. Confronted with the choice of bending out or shortening, that point is called a bifurcation point where the structure has to decide to which direction it heads, the structure finds it easier to shorten when the applied load is small, when the load reaches its buckling load, the structure finds it easier which implies the stable zone to bend in order to satisfy the equilibrium otherwise it goes to the unstable zone considering the plate elements (see Figure 2-1).

![Figure 2-1: The stability phenomenon](image)

Buckling can be classified as a failure mode depending upon the phase that it occurs and upon the structure being analysed namely beams, column and plates etc. The buckling phenomena are considered as a failure mode for the beam and column elements since there is no lateral supports exist in relation to the plates to resist the increasing load. As for the plates, when the plate slenderness increases, the buckling is most likely present in the elastic range. Once the buckling has initiated, the structure can still resist the applied load which is considered stable post-buckling regime due to the in-plane behaviour developed by the curvature and as the structure is loaded, the material starts to yield to some extent until the axial rigidity becomes zero. However, as the plate slenderness decreases, the possible buckling occurrence is in the elastic-plastic or fully plastic range which is in compression is not possible. In this case, the buckling occurrence can be classified as a failure mode since in this range after the buckling occurrence; the capacity starts to reduce due to the axial rigidity reduction. The buckling occurrence can be also affected by the boundary conditions, structural configurations and imperfection and welding residual stresses and corrosion etc.

Stiffened or un-stiffened panels are widely used in shipbuilding industry as well as other fields of engineering. Due to the fact that ships largely suffers longitudinal forces which creates longitudinal bending stresses, the bending properties of the plates becomes a paramount property which depends greatly on its thickness as compared to its other dimensions. The strength assessment of the plate’s breaks down into two parts based on their thicknesses namely thin plate and thick plate theory, which can be further broken down into small deflection and large deflection theory comparing their deflection
to their thicknesses. The thin plate theory considers the problem as two dimensional while in thick plate theory three-dimensional solution is present due to the fact that the transverse shear distortions are comparable to the bending distortions.

The behaviour of the plates and stiffened plates has been studied substantially. It is worthwhile to point out that as the structure is loaded in compression; it passes through linear and plastic stages in terms of material non-linearity, and through small and large deformations in terms of geometrical non-linearity. These two aspects complete and correlate one another. It is important to point out that the structure may experience large deformation while it is still in the elastic range depending of its elasticity modulus which is the strength per strain. In case the experienced load exceeds the proportional limit developed by the large deformation, the material non-linearity comes into play and the behaviour against the applied load further changes.

![Figure 2-2: The structural strength history for idealized perfect structure under compression](image)

Figure 2-2 and 2.3 show different structural stress-strain curves accounting for the perfect and imperfect structure. For the perfect structure critical buckling stress is quite important and lead to the apparent elasticity reduction and the structure is still in the elastic range. As for the imperfect structure, due to the imperfect shape, residual stresses, structural configurations, boundary edge conditions and etc. it leads to early yielding and in turn ultimate load carrying capacity reduction. In the following the buckling and post-buckling phenomenon has been discussed.

When the plates are laterally loaded, it establishes the equilibrium through bending, shear, twisting or membrane forces. The strength assessment of the plates is similar to beam in terms of the assumptions taken. On the other hand, beams are regarded as one-dimensional elements whilst plates are generally regarded as two-dimensional elements. In this context, the applied load is distributed over two-dimensionally and therefore using the plate element for the same applied load leads to lighter structure. The following idealizations are made for the thin-walled small plate deflection:

The transverse strains are negligible and the lines perpendicular to the mid-plane remains perpendicular therefore the normal stress is zero which makes the problem to be solved two-dimensionally. The plate is initially flat and the material is homogenous which leads the uniform stress
distribution across the breath. The transverse displacements are small compared to its thickness.

Figure 2-3: The structural strength history for perfect and imperfect structure under compression

When the elements within the plate is bent, in addition to the in-plane forces which is equal to the applied external loads, the moments and shears which arise from the transverse bending come to act on the elements. To find out the total displacement, at first the equilibrium of these two forces, in-plane and bending forces, are to be statically equilibrated. Once the equilibrium is satisfied, through the bending moment-stress relation, the strain and ultimately the displacement can be related to the in-plane forces:

\[
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0
\] (1)

and

\[
\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0
\] (2)

Using the bending moment and stress relation leads to biaxial and uni-axial force–displacement equations respectively:

\[
D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}
\] (3)

\[
D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = N_y \frac{\partial^2 w}{\partial y^2}
\] (4)
where \( D = \frac{E t^3}{(12(1 - \nu^2))} \) is called the plate bending rigidity, \( w \) is the plate displacement considering both the bending and twisting effects and \( N_x, N_y, N_z \) are the constant in-plane forces per unit length. It is important to note that these forces are constant (small displacement) and arises only from the external forces and the plate is still in the elastic range. Another important note is that, the plate rigidity is larger than beam rigidity with similar width and depth due to the fact that the plate bending is treated in two dimensional planes which produce two curvatures forming anticlastic surface which gives more resistance and beams are free to deform laterally.

Considering the deflected surface is in the form of:

\[
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}
\]

(5)

where \( m \) and \( n \) is the number of the half-waves, \( a \) and \( b \) are the plate length and breadth respectively and \( A_{mn} \) is the maximum deflection in the middle of the plate:

\[
\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left[ m^4 \pi^4 \frac{1}{a^4} + 2 m^2 n^2 \pi^4 \frac{1}{a^2 b^2} + n^4 \pi^4 \frac{1}{b^4} - \frac{N_{cr}}{D} \frac{m^2 \pi^2}{a^2} \right] \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} = 0
\]

(6)

Here the solution turns into non-trivial solution which implies that at least one term of the equation is not equal to zero. Since \( A_{mn} \) is not zero:

\[
N_{cr} = \frac{D \pi^2}{b^2} \left( \frac{mb}{a} + \frac{n^2 a}{mb} \right)^2 = \frac{D \pi^2}{b^2} K_{mn}
\]

(7)

where \( K_{mn} \) is a buckling coefficient determined by a theoretical critical-load analysis. It is a function of plate geometry (aspect ratio) and boundary conditions. As the edge boundary conditions are strengthened, this value takes on larger value leads to higher buckling strength. \( a \) is the plate length and \( b \) is the plate breadth and \( D \) is the plate bending rigidity. As can be seen, increasing the plate breadth, the structural force absorption goes down and lead to less ultimate load carrying capacity due to the fact that the transverse strip elements which are supposed to support the longitudinal strip elements become more vulnerable to buckling.

Bryan (1891) calculated the elastic buckling stress for a long rectangular plate which was simply supported around its edges. The elastic stress was determined by the plate width-to-thickness ratio \( b/t \), by the restraint conditions along the longitudinal boundaries and by the elastic material properties. It is expressed as:

\[
\sigma_{cr} = k \frac{\pi^2 E}{12(1 - \nu^2) \left( \frac{b}{t} \right)^2}
\]

(8)

Bleich (1952) generalized the expression of the critical stress of a flat plate under uniform compressive stress in the inelastic range in the following manner:

\[
\sigma_{cr} = k \frac{\pi^2 E \eta}{12(1 - \nu^2) \left( \frac{b}{t} \right)^2}
\]

(9)

where \( \eta = \sqrt{E_t / E} \) and \( E_t \) is tangent modulus which is affected by the boundary edge conditions,
structural configurations and welding induced imperfections and stresses. This is the modification of Eqn (8) in order to calculate the stresses which are higher than proportional limit stresses.

2.2 Post buckling

After the buckling, due to the in-plane behaviour the buckled plate shows post-buckling strength. The in-plane forces developed becomes dependent on the location which leads to non-uniform stress distribution. The constraint part of the plate takes on the major applied load compared to the buckled part which in turn leads to the yielding of that part. To account for this effect, the implementation of the Pythagorean Theorem along with the Airy’s stress function, \( f(x, y) \) developed by Von Karman leads to:

\[
\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[ \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] - \sigma = E \varepsilon
\]  

(10)

\[
D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = t \left( \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right)
\]  

(11)

where \( F \) is the so-called stress function, \( E \) elasticity modulus and \( w \) is the vertical displacement, \( t \) is the plate thickness and \( D \) is the plate bending rigidity. With these two equations, the large displacement of the plates is covered in the elastic range considering the Hooke law.

However, the inherent plate out of flatness due to the welding induced residual stresses lead to render the bifurcation point for buckling impossible to define and the axial stiffness, the strength per strain, changes. The relevant expressions to cover the imperfect plate behaviour:

\[
\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[ \left( \frac{\partial^2 w_{0}}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_{0}}{\partial x^2} \frac{\partial^2 w_{0}}{\partial y^2} \right] - \sigma = E \varepsilon
\]  

(12)

where \( w_{0} \) is the initial vertical distortion, \( w \) is the total deflection which covers the imperfect plating in the elastic range. The general procedure follows that at first, the probable shape function is defined and substitutes the deformed shape function into the stress-strain definition and then by implementing the Galerkin method which adopts the differential equations, the total displacement can be found.

![Figure 2-5: Effective width of plates](image)

It is important to note that as the structure is loaded further the material non-linearity, that is, the
elasticity reduction come into play due to the yielding and it becomes difficult to find out the structural strength due to high non-linearity involved. Von Karman came up with the so-called effective width which has been discussed in the next chapter (see Figure 2-5).

2.3 Ultimate strength

The ultimate strength of structural components and systems is a real measure in strength assessment in a sense that the ultimate strength is the maximum carrying capacity that they can withstand. No additional load can carry beyond the ultimate strength (Paik et al. 2009). Achieving the ultimate strength implies that the resting capacity deteriorates due to plasticity, which appears since the axial rigidity is set to zero. When the structure is loaded with compressive forces, depending of the structural configuration in particular plate thickness, buckling and yielding, boundary conditions, applied load and etc. dominate the ultimate strength. Ultimate strength assessment involves a large amount of uncertainties and many factors may affect it. Garbatov et al. (2011) implemented a Monte Carlo simulation in order to find the most influential parameters on the ultimate strength. It has been found that plate slenderness and plate thickness have the most significant effect on the ultimate strength of the stiffened panels. In addition to that, residual stresses and different loading conditions, imperfection magnitude and material properties.

Figure 2-6: Ultimate strength assessment

Figure 2-6 shows the ultimate strength assessment of the structural components in compression. As told, the structure undergoes elastic and elastic-plastic stages when it is under compressive loads. Due to the shape changes as the structure is loaded, this adds to the geometrical non-linearity and in turn reduces the structural stiffness. Due to the presence of the buckling and material yielding, the structures lose its stiffness and the yielding propagates to some extent until the axial rigidity becomes zero and the ultimate strength is achieved, thereafter the structural strength reduces due to the presence of the plasticity and shape. Due to the many factors that may influence the structural stiffness, the elastic behaviour may decrease depending upon the welding induced distortions and stresses, the corrosion degradation and the structural joints and lead to the material early yielding. In this context, the post-collapse regime is highly affected by the stiffness lost until the ultimate strength is achieved. As for the thick plates, the plastic unloading line behaviour is smoother and load-shedding is not pronounced as much as the thin plates due to the fact that the thin plates lose its large stiffness before the ultimate strength is achieved. This holds also for the stiffened panels. In case the structural
instability of the individual elements is considered, this leads to the more pronounced load-shedding namely, the stiffener tripping. However, if the elemental instability is not considered like Beam-column approach, the post collapse regime is less pronounced and smoother. The point here is that the post-collapse regime is highly important in terms of the hull girder strength. They can still contribute to the ship strength although they have reached their ultimate strength. Therefore, the ships are designed in such a way that the stress values that individual structural element experience does not lead to this kind of failure.

Strength assessment has evolved from breaking strength to buckling strength and it has set to move onto ultimate strength. An allowable stress approach which uses a safety factor based on yielding stress of components have been used in class societies but it is now well recognized that the limit state approach is a better basis for the design and strength assessment of various types of structures than the traditional allowable working stress approach, because it is not possible to determine the true margin of structural safety as long as the limit states remain unknown (Paik and Thayamballi 2003).

Caldwell (1965) made the first attempt to calculate the ultimate hull girder strength. He idealised the cross-section composed of stiffened panels as that composed of panels with equivalent thickness. He implemented rigid plastic mechanism. He considered the presence of the buckling effect multiplying its yielding stress by a reduction factor.

Smith (1977) proposed a simplified method which is commonly called Smith’s method. Smith divided the hull cross-section into the elements which are composed of plating with its stiffener and performed progressive collapse analysis assuming that the cross-section remains plane as it is loaded. In this analysis, the elements fail one by one and each element behaves according to its structural stress-strain curve which is derived by applying axial load to each element accounting for the yielding and post-buckling effect before the ultimate state is reached.

An alternative method to perform progressive collapse analysis may be the ISUM, which was originally proposed by Ueda and Rashed (1996) to perform progressive collapse analysis on the transverse frame of a ship structure. Then, new elements have been developed to perform progressive collapse analysis of a hull girder under longitudinal bending. This could improve the accuracy of the method when it is applied to evaluate the strength of double-bottoms of ships. Many researchers have extended the existing ISUM plate element to consider combined uni-axial, bi-axial compression and lateral load.

Gordo et al. (1996) presented a method to estimate the ultimate moment based on a simplified approach to represent the behaviour of stiffened plate columns. The proposed method allows the prediction of the degradation of the strength due to corrosion and residual stresses. The method has been implemented in HULLCOL code.

Many researchers have applied the FEM to predict the ultimate strength of un-stiffened plates and stiffened plates where both geometric and material nonlinearities are taken into account. It may be said presently that it is fairly straightforward to use FEM for ultimate strength prediction of plates and stiffened plates. The FEM can also be a powerful method to perform progressive collapse analysis on
a hull girder. Generally speaking, the hull girder is too large to perform progressive collapse analysis by the ordinary FEM, and some simplified methods sometimes are required. However, it has become possible to perform the FEM analysis using ordinary elements applying a computer code (Yao and Nikolov 1991).

Paik et al. (1998) classified that, the panel collapse modes can be broken down into six types, namely:

- **Mode I**: Overall collapse after overall buckling.
- **Mode II**: Biaxial compressive-type collapse in plating between support members, i.e., without failure of support members.
- **Mode III**: Beam-column-type collapse of plate–stiffener combination, i.e., stiffener with associated plating.
- **Mode IV**: Buckling of stiffener web.
- **Mode V**: Flexural-torsional buckling or tripping of stiffener.
- **Mode VI**: Gross yielding.

Smith (1979) and Guedes Soares and Søreide (1983) categorized three main types of collapse, namely plate collapse, inter-frame flexural buckling and overall grillage collapse in addition to the tripping collapse of the stiffener which is considered the worst case for the stiffener panels due to the sharp unloading after the stiffener turns about the plate-stiffener junction and the collapse comes with the global plate buckling.

The response of a short stiffened panel, with a length approximately equal to the width of the plate between stiffeners, is dominated by the plate failure. In ships, the panels are generally much longer than stiffener spacing and therefore the possibility of having plate failure only exists in special cases, such as for example, a panel with high strength stiffeners and with relatively low strength nearly perfect plates. Under these conditions the plates show a very steep unloading characteristic and stiffeners are not able to accommodate the drop in load due to plate failure (Guedes Soares and Gordo 1997).

An overall column type of plate failure can occur in long uni-axially stiffened panels. In orthogonal stiffened panels the corresponding mode is the grillage collapse, which involves both longitudinal and transverse stiffeners. This collapse mode can be influenced by local buckling of the plate or of the stiffener and is generally not found in ships (Guedes Soares and Gordo 1997).

Some design studies led to the conclusion that the optimum design of a compressed stiffened plate would be obtained whenever the strength of the overall buckling mode equals the strength of the local buckling mode. However, such panels show an interaction between local and global modes that makes them imperfection sensitive, with a violent collapse (Tvergaard and Needleman 1975).

Inter-frame collapse, which is the most common failure mode in ships, attributed to the presence of heavy transverse girders. This is a typical case of interactive collapse in which the overall collapse of the beam-column is triggered by local buckling of the plate or stiffener (Reis and Roods 1977). It is
possible to have a failure towards the stiffener outstands which is called plate induced beam-column failure or towards the plate which is called stiffener induced beam – column failure (Soreide et al. 1978).

Tripping involves a rotation of the stiffener about one hinge developed at the junction of the plating and stiffener. This mode of panel failure is the most dangerous ones because it is always accompanied by a very quick shed of load carrying capacity of the column. Lateral – torsional instability may occur alone by twisting of the stiffener developing partial or full hinge on the intersection, or may be induced flexural plate induced buckling failure. In this case the stiffener will be subjected to a higher stress than the average column stress and the critical tripping stress could be easily reached (Guedes Soares and Gordo 1997).

The collapse modes given can be independent or interacted by the other collapse modes. This is heavily dependent upon structural configurations, boundary conditions and loading conditions. The buckling strength of plating between stiffeners will depend on the torsional rigidity of stiffeners as well as the dimensions of plating itself. Also, the tripping strength of a stiffener web will depend on the dimensions of plating as well as the stiffener (Paik et al. 1998).

There are several existing simple design formulations for the ultimate strength assessment of stiffened panels in the literature. The John-Ostenfeld approach is used to account for any plasticity effects since it considers the panel buckles in the elastic-plastic zone which is a general behaviour of stocky panels. This is an empirical approach used to account for plastic deformation in plates or struts that have a high elastic buckling stress and that develop a certain amount of plasticity before failure (Guedes Soares and Gordo 1997). According to this method, whenever the Euler buckling stress ($\sigma_e$) is higher than half the yield stress ($\sigma_y$), the critical buckling stress is given by $\sigma_{cr} = \left[1 - \left(\sigma_y / 4 \sigma_e\right)\right] \sigma_y$, assuming that the proportional limit is $0.5 \sigma_y$.

The Perry-Robertson approach is used to calculate the ultimate strength of either plate or stiffener induced stiffener panels. It is based on the assumption that the beam-column has an initial out-of-straightness, at mid-span. This formula assumes that the stiffener with associated plating will collapse as a beam-column when the maximum stress in the extreme fibre reaches the yield strength of the material. Two possible collapse modes are usually considered as follows:

- Plate induced failure
- Stiffener induced failure but without rotation of the stiffener about the stiffener and plating junction.

IACS (2010) provides an analytical method for calculating the load shortening curves for longitudinally stiffened plates used in Smith's method. For each increment in strain, the stress in a stiffened plate is taken as the lowest of the values determined, considering the following possible modes of failure:

- Elasto-plastic failure;
- Flexural buckling (Beam-Column) failure;
- Torsional buckling of stiffeners failure;
- Web local buckling of stiffeners failure.

Here only the flexural buckling failure has been covered herein:

\[
\sigma_{CR1} = \varphi \sigma_{C1} \left[ \frac{A_{S nett0} + 10^{-2} b_{eff-p} p_{net50}}{A_{S nett0} + 10^{-2} A_{p nett50}} \right]
\]

(13)

where \( \varphi \) is the edge function, \( \varphi = \begin{cases} 1, & \varepsilon < 1 \\ \varepsilon, & 0 < \varepsilon < 1 \\ -1, & \varepsilon > 1 \end{cases} \)

(14)

\[
\sigma_{C1} = \frac{\sigma_{E1}}{\varepsilon}, \sigma_{E1} \leq \frac{\sigma_y}{2} \varepsilon
\]

(17)

\[
\sigma_{C1} = \sigma_y \left[ 1 - \frac{\sigma_y}{4\sigma_{E1}} \right], \sigma_{E1} > \frac{\sigma_y}{2} \varepsilon
\]

(18)

where \( \sigma_{E1} \) is the Euler column buckling stress given by:

\[
\sigma_{E1} = \pi^2 E \frac{I_{E nett50}}{A_{E nett50} s^2} 10^{-4}
\]

(19)

\( E \) is the modulus of elasticity, \( A_{E nett50} \) is the stiffener area with effective attached plate width, \( I_{E nett50} \) is the moment of inertia of the stiffener with an effective width of attached plating given by:

\[
b_{eff-s} = \frac{s}{\beta_p}, \beta_p > 1
\]

(20)

\[
b_{eff-s} = s, \beta_p \leq 1
\]

(21)

where

\[
\beta_p = \frac{b}{t_{nett50} \sqrt{\varepsilon \sigma_y}}
\]

(22)

It is important also to accurately predict the effective width of plating in calculating the effective cross sectional area of a plate–stiffener combination. As the compressive loads increase, the effective width of the buckled plating should vary because it is a function of the applied compressive stresses. However, most simplified methods assume that the effective width of plating does not depend on the
applied compressive loads, and the ultimate effective width of plating is typically what is used for the effective width as a constant. If the effective width of plating were treated as a variable, one would find that the equation characterizing the ultimate limit condition for a stiffened panel will show a much higher degree of nonlinearity than it would otherwise (Paik et al. 1998). The effective width is dependent upon the applied axial compressive loading, the severity of the residual stress and the plate initial imperfection.

Von Karman et al. (1932) made the first attempt to the effective width concept. He derived an approximate formula for simply supported plates:

$$b_e = \left[ \frac{\pi}{\sqrt{3(1-v^2)}} \right] \frac{E}{\nu} t$$

where $$\sigma_e$$ is the maximum edge stress, $$t$$ is the plate thickness which in case of the simply supported plates as proposed by Ramberg et al. (1939) leads to:

$$\frac{b_e}{b} = \frac{\sigma_e}{\sigma_e}$$

where $$\sigma_e$$ is the buckling stress and $$b$$ is the plate breath.

Winter (1948) test results revealed that:

$$\beta \leq 1; \beta \leq 1.287$$

The effective width formulation is a way of expressing the diminishing of strength that a plate exhibits in the post-buckling regime. This weakening effect is expressed by a reduction of the width that effectively resists the compressive loads (Faulkner 1975)

$$\frac{b_e}{b} = 1 - \frac{2\eta}{(b - 2\eta)} \left( \frac{3.62\beta^2}{13.1 + 0.25\beta^4} \right)^2 : 0 < \beta < 1$$

$$\frac{b_e}{b} = \frac{2}{\beta} - \frac{1}{\beta^2} - \frac{2\eta}{(b - 2\eta)} \left( \frac{3.62\beta^2}{13.1 + 0.25\beta^4} \right)^2 : 1 < \beta \leq 2.69$$

$$\frac{b_e}{b} = \frac{2}{\beta} - \frac{1}{\beta^2} - \frac{2\eta}{(b - 2\eta)} : 2.69 < \beta$$

where $$\eta$$ is a parameter that defines the residual tension zone which is typically between 3 to 4.5 considering the shakedown effect. These equations accounts for some level of imperfection and residual stresses.

Guedes Soares and Gordo (1997) used the effective width formula derived by Faulkner and used the
following reduction factor to assess the ultimate strength of the stiffened panels implementing the Johnson-Ostenfeld approach:

\[ \frac{b_r}{b} = 2 - \frac{1}{\beta^2} \]  

(32)

And the reduction factors that accounts for the welding induced residual stresses, biaxial loading and shear stresses:

\[
R_r = \begin{cases} 
1 - \frac{2\eta}{b - 2\eta} \left( \frac{\beta}{2\beta - 1} \right) & \beta \geq 1 \\
1, \beta < 1
\end{cases}
\]

(33)

\[ R_y = 1 - \left( \frac{\sigma_y}{\sigma_{yw}} \right)^2, \sigma \leq 0.25\sigma_y \]  

(34)

\[ R_t = \left[ 1 - \left( \frac{\tau}{\tau_o} \right) \right]^{1/2} \]  

(35)

where

\[ \frac{E_t}{E} = \begin{cases} 
3.62\beta^2 & \text{for } 0 \leq \beta \leq 2.7 \\
13.1 + 0.25\beta^4 & \text{for } \beta \geq 2.7
\end{cases} \]  

(36)

where \( E_t \) is the tangent modulus and \( E \) is the material elasticity modulus and \( \beta \) is the plate slenderness and \( \sigma_{yw} \) is the mean ultimate strength.

2.4 Residual stress

Residual stresses may be induced in structural components during manufacture such as welding, forging, casting, age hardening, machining and other processing applications. It is important to distinguish the welding induced residual stresses from the service loading stresses that ship experience during their service life:

- Loading service stresses exist as a consequence of acting external forces, moments, pressure and/or internal pressure. Residual stresses exist as a consequence of inhomogeneous plastic deformation and/or as a consequence of constraints between the different constituents of a component and different material joints.
- Loading service stresses are not influenced by cyclic plastic deformation in stress-controlled loading. Residual stresses always relax, if the cyclic load exceeds certain threshold values
- Loading service stresses may be changed and finally disappear, if external loadings are changed and finally removed. Residual stresses may be changed and eventually disappear by thermal and/or mechanically induced relaxation.
- Structural response is in equilibrium with the external loading. In contrast, the residual compressive stresses are in equilibrium with the tensile residual stresses only.

The welding process to join the structural components inevitably leads to the welding induced residual...
stress development along with the accompanying distortions. It reduces the ultimate load carrying capacity and the linear behaviour of the structural components which causes the premature yielding. As to the post collapse regime, due to its influence, the post collapse regime becomes smoother and less load-shedding pronounced due to the plasticity propagation. It is important to point out that the welding residual stresses have different impact on the thin and thick plate provided that the same heat input and speed has been applied. Due to the thickness effect, it gives an extra constraining effect which leads to higher compressive stresses developed with respect to the thin plate and in turn more ultimate strength reduction. However, it has more impact on the linear behaviour reduction on the thin plates.

There are two distinct parameters which lead to the residual stress formation, namely the localized heat and time. The conductivity is performed basically through the metal electron movement. According to the heat conduction law proposed by the French mathematician, Fourier, and the implementation of the conservation of energy lead to a non-linear heat transfer equation (Teng and Chang 1998):

$$\frac{\partial}{\partial t} \left( K \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( K \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial T}{\partial z} \right) = \frac{\partial Q}{\partial t}$$

(37)

where $Q \left[ J/mm^3 \right]$ is the heat energy released or consumed per unit of volume, $k$ is the temperature dependent thermal conductivity, $\rho$ is the weak temperature-dependent material mass, $c_p$ is the temperature dependent specific heat coefficient which essentially is a measure for the temperature increase caused by the heat input and $T$ is the temperature and the heat loss due to convection has been given respectively as such:

$$q_{con} = h_f \left( T - T_\infty \right)$$

(38)

where $T$ is the body temperature, $T_\infty$ is the surrounding temperature, $h_f$ is the convection heat transfer, $\varepsilon$ is the emission coefficient and $\sigma$ is the Stefan – Boltzmann constant.

The combination of Eqns (37) to (39) lead to the general matrix form of:

$$[C] \{ e \} + [K] \{ T \} = \{ dE \}$$

(39)

where $C$ is the thermal stiffness, $T$ is the element nodal temperature and

$$\{ dE \} = \int_V Q_i [N] dV + \int_A h_f T_n [N] dA$$

(40)

where $N$ is the shape function. With these two equations the nodal temperature can be found and incorporated into the structural analysis to find out the welding induced strains and in turn the stresses. Figure 2-8 shows that, as the travelling heat source moves forward depending upon the plate location, it undergoes elastic-plastic compression and tension stages. Some local locations pass through compression and tension plastic strains. The total strain increment in the elastic and plastic region respectively can be expressed as ruling out the transformation-induced plasticity:

$$\{ dE \} = \{ dE^e \} + \{ dE^h \}$$

(41)
\[ \{d\varepsilon_2\} = \{d\varepsilon^e\} + \{d\varepsilon^p\} + \{d\varepsilon^h\} \]  
\[ \{d\varepsilon^h\} = \alpha T \]

where \( \{d\varepsilon^e\} \), \( \{d\varepsilon^p\} \), \( \{d\varepsilon^h\} \) is the elastic, plastic and thermal strain increment respectively and \( \alpha \) is the temperature dependent expansion coefficient.

Considering the virtual work and equilibrium equations:
\[ \int_v [B]^T \{\sigma\} dV = \{R\} \]

where
\[ \{R\} = \int_s \{N\}^T P dA + \int_v \{N\}^T \{f\} dV \]

where \( R \) is the external force matrix, \( B \) is strain-displacement matrix , \( N \) is the shape function, \( P \) is the surface force vector and \( f \) is the body force. The elastic and elastic-plastic stress-strain relations based on the von Misses yield criterion and the isotropic strain hardening rule can be written as:
\[ d\sigma_e = \left[D_e\right] [d\varepsilon] - \left[C^e\right] dT \]  
\[ d\sigma_{ep} = \left[D_{ep}\right] [d\varepsilon] - \left[C^{ep}\right] dT \]

where \( D_e \), \( D_{ep} \) is the elastic, elastic-plastic stiffness matrix respectively. \( C^e \) is the thermal stiffness.

Figure 2-7: The welding induced forces across the plate breadth

Figure 2-9 shows the welding induced stress development after the welding process has been completed. There is a rule of thumb that the place where cools down last, the tensile welding residual stresses are likely present. Figure 2-10 shows the welding induced stress equilibrium through the plate breadth where \( \sigma_{cr} \) and \( \sigma_T \) is the welding induced compressive and tensional stresses.
2.5 Shakedown

Ships experience cyclic loading, which results in the local plastic deformations. This gives to some degree the stress relaxation of the residual stresses due to the fact that the residual stresses reside in the rigid part of the structure. There are three kinds of shakedown in the literature, namely:

- Elastic shakedown: In the early cycles, the load values exceed the limit elastic limit and upon
the load release, the plastic strain remains along with the residual stress reduction, however after several cycles the load does not produce plastic strains and the structure behaves elastic.

- Plastic shakedown: The alternating strains are present which implies that the net plastic strain is zero due to the fact that the elastic-plastic strains are in a closed loop manner and it leads to low cycle fatigue.

- Ratcheting: For each cycle, there is plastic strain present and they are in an open loop which results in plastic collapse.

Figure 2-11 and 12 shows the interaction between the welding induced stresses and service stresses and the shakedown types respectively.

![Figure 2-11: Superposing of Residual and Service Stresses](image1)

The shakedown effect is to be included in the hull girder assessment and in addition to this; it has influence on the fatigue behaviour which is not covered herein.

![Figure 2-12: The shakedown types](image2)

2.6 Corrosion

Ships operate in corrosive environments and due to the interaction with ship structures; it leads to a gradual thickness reduction, which influences the ship durability. Several corrosion models have been developed to predict corrosion degradation in ship structures. Corrosion involves a big amount of uncertainty, which is based on many parameters. It is normally not straightforward to develop a corrosion wastage model solely based on theory, because corrosion is a function of many variables.
and uncertainties involved, such as the type of the corrosion protection system employed, type of cargo, temperature, humidity, etc. The corrosion models developed based on statistical analysis of operational data will usually be different according to the types of ships and cargoes or structural member locations and categories (Guedes Soares et al. 2008; Guedes Soares et al. 2009; Guedes Soares et al. 2011).

The corrosion rate is highly dependent upon corrosion protection effectiveness, component location and orientation, type of cargo, oxygen level, temperature and degree of movement etc. As the structural flexing increases, this gives rise to the coating breakdown and increases the corrosion rates. In this regard, as high tensile steel is more flexible than mild steel, the coatings breakdown may be earlier. During the ballasting and de-ballasting operation in order to keep the reasonable trim to sail, some ballast tanks may be half loaded and leads to higher rates of corrosion than fully loaded along with the exposure of the sunlight due to the high humidity. High stress concentration is another likely reason of the corrosion rate increase.

The corrosion wastage has a significant effect on the ultimate load carrying capacity reduction and stiffness due to the fact that the plate thickness is one of the most significant parameters on the ultimate strength. In addition, given that stiffness is the strength per strain, the corroded plates develops less strength with the same strain absorption with respect to the un-corroded plate. Due to the significant stiffness reduction, the post-collapse regime becomes more pronounced, which implies the strength contribution reduction to the hull girder.

It has been shown on many occasions that the non-linear corrosion wastage model is well accepted in representing realistic situations for steel plates in different areas of ship structures. A non-linear time dependent corrosion model developed by Guedes Soares and Garbatov (1999) has been adopted herein.

Guedes Soares and Garbatov (1999) proposed a corrosion wastage model which has been adopted herein (see Figure 2-13), where $d_x$ is the mean corrosion depth, $\tau_c$ is the coating life, which is equal to the time interval between the painting of the surface and the time when its effectiveness is lost and $\tau_t$ is the transition time under average conditions.
Paik et al. (2003) categorized the corrosion behaviour into three phases, namely durability of the coating, transition to corrosion and progress of corrosion. In this corrosion wastage model, the transition time is considered to be an exponentially distributed random variable. Once the corrosion has set to progress, the behaviour of the corrosion progress is either concave or convex curve depending upon the structure which is statically or dynamically loaded.

2.7 Stress strain definition approach

The structural components are subjected to the external disturbances which lead to their strength reduction. The manufacturing processes to join the structural component are accompanied with the welding induced distortions and residual stresses due to the corrosive environment the structures lose its material. To simulate their detrimental effect, the welding processes have been simulated or the randomly distributed corrosion application has been applied through finite element method. However, due to the computational cost, this has been confined to the local structural components. The objective here is to define the admissible material stress-strain curves that can be used to account for these detrimental factors on the ultimate strength assessment and to use it for the strength assessment of the ship hull girder.

The welding induced stresses and distortion reduces the structural linear behaviour and in turn reduces the ultimate strength of the structure which leads to the early yielding of the structure. To find out the same response, the material stress-strain slope modification or new data points have been incorporated on the other hand in order to find out the same response as to the corrosion, the linear behaviour has been completely changed against the compressive loads as shall be depicted in the next chapters. Due to the fact that in the finite element modelling, each element is weakened through the material stress-strain definition, this method becomes vulnerable to the boundary edge conditions owing to the structural rotations, the imperfection shapes and magnitude owing to geometrical non-linearity.

Faulkner (1977) suggested a model to account for residual stresses by implementing a modification in the elastic perfectly plastic stress-strain curve. Through this type of stress strain curve, the structure is subjected to premature yielding and in turn reduces both stiffness and strength.

To do that, at first, a reference strength curve is needed to match it to the response curve accounting for the material stress-strain definition. In the finite element method, in the first stage shell elements have been implemented to represent the structural behaviour and pre-stresses have been applied to simulate the welding induced residual stresses and to catch the same response, the material stress-strain curves have been developed. Similarly, in the second stage, a moving heat source has been introduced in a single and stiffened plate in order to assess the strength accounting for the residual stress and welding sequence and similarly the material stress-strain curves have been developed accounting for these effects. In the third stage, the time-dependent shakedown and corrosion have been accounted for and to catch the same response the material stress-strain curves have been developed for the intact single plate.
Figure 2-14: The stress development for any single node within the system

Figure 2-14 shows for any node within the analysed structure, the stress development and its constituents. Once the yielding stress point has been reached, the stress development ceases to be developed and according to the hardening rule the yielding surface follows the yielding curve which means the yielding is not expanded anymore.

Figure 2-15: The stress development for any single node within the system

Figure 2-15 shows the stress strain definition approach in which the material yielding stress is reduced and strain hardening introduced to capture the same response to the one showed in Figure 2-14. In this approach, when the new yielding point is reached, the stress development does not cease and carry on developing within the elastic-plastic stage till these nodes reach the original yielding stress.
3 ULTIMATE STRENGTH ASSESSMENT ACCOUNTING FOR THE EFFECT OF FINITE ELEMENT MODELLING

3.1 Introduction

In this chapter, the groundwork has been paved in terms of finite element modelling to overcome the finite element related uncertainties. Ships are built of plates, its associated stiffeners, girders, bulkheads and frames that form its strength at sea. The plates are supported by the stiffeners, which keep them in position to absorb the subjected loadings. The frames that prevent the stiffeners to buckle and support them are stiff and deeper than the stiffeners, which eliminate the overall grillage buckling. Ship structures are assembled of stiffened panels and an estimation of the maximum load carrying capacity or the ultimate strength of these elements is of high importance for the safety assessment and economical design of the structure.

The ultimate strength of structural components and systems is a real measure in strength assessment in a sense that the ultimate strength is the maximum carrying capacity that they can withstand. No additional load can carry beyond the ultimate strength (Paik et al. 2009). Achieving the ultimate strength implies that the resting capacity deteriorates due to plasticity, which appears since the axial rigidity is set to zero.

Guedes Soares and Kmiecik (1995) found that edge conditions have no significant effect on ultimate strength for stocky plates with large deflections. They found that it increases the strength by 10% for practically perfect plate due to the restraining of the edges and it has been found that when initial imperfections are significant, the strength becomes insensitive to edge conditions. Due to that fact that strength reduction arising from the edge supports, when the ultimate strength of the ship is to be evaluated, the possible weakest structural element must be selected.

Guedes Soares and Kmiecik (1993) have shown that assessing the plate strength with a more accurate non-linear finite element code would lead to the same general type of results although with different numerical values.

Xu and Guedes Soares (2011) investigated the effect of geometry and boundary conditions on the strength behaviour of a short stiffened panel and it has been found that unconstrained longitudinal edges lead to strength reduction and as the structure is weakened, the dimensional influences on the ultimate strength become more evident.

Ultimate strength assessment involves a large amount of uncertainties and many factors may affect it. Garbatov et al. (2011) implemented a Monte Carlo simulation in order to find the most influential parameters on the ultimate strength. It has been found that plate slenderness and plate thickness have the most significant effect on the ultimate strength of the stiffened panels. In addition to that, residual stresses and different loading conditions, imperfection magnitude and material properties.
3.2 Finite element modelling

3.2.1 Structural description

Four structural components are modelled here. Models 1 to 3 are shown on Figure 3-1. Model 1 is a stiffened panel located between two neighbouring transverse frames, Model 2 is also a stiffened panel with a transverse frame in the middle and Model 3 is a plate bordered by two neighbouring transverse frames and two longitudinal stiffeners. Model 4, which can be seen in Figure 3-1, is composed of \(\frac{1}{2}+1+\frac{1}{2}\) plates and two transverse frames.

![Figure 3-1: Structural model 1 to 3 (left) and Structural Model 4 (right)](image)

The ultimate strength of the panels and plate are analysed based on the finite element method using commercial software ANSYS (2009). The software enables modelling of elastic plastic material properties and large deformations. Eight-node and four-node quadrilateral shell elements have been used to model the plates and stiffeners. The kinematic assumption is large displacement and rotation but small strain. The material modelling is assumed to be bilinear elastic-perfectly-plastic without hardening. Both a load-displacement control method and an automatic displacement control method are used in the solution scheme. The applied load is uni-axial compression.

![Figure 3-2: Model 1 (left) and Model 4 (right)](image)

The length of the stiffened panels are \(a = 400\) mm and breadth is \(b = 150\) mm respectively (see Figure 3-2). The web thickness is 3.6 mm and height is 25 mm and the plate thickness is 2.7 mm. Stiffened panel slenderness is \(\beta = 1.87\). The aspect ratio \(a/b\) of the plate is 2.66. The stiffener has a cross section of a standard type flat bar. The area of the stiffener, \(A_s\) is approximately 22 per cent of the
area of the plate, \( A_{pl} = ab \). The non-dimensional column slenderness of the stiffened panel calculated with a full plate width is \( \lambda = 0.69 \). The thickness, length, breadth of plates and the height of stiffeners is equal for the models 1 to 4.

The initial geometry imperfection of plates and stiffeners are generated by a pre-deformed surface. Faulkner (1975); Smith et al. (1988) have reported that the maximum imperfections in a plate can be assumed to be proportional to \( \beta^2 \). They suggested that the maximum initial deformation for an average imperfection can be calculated as

\[
W_{\text{max}} = 0.1 \beta^2 
\]

The maximum permissible camber tolerance, for a standard shape is usually assumed to be 0.2% of the length. The initial geometry surface imperfection is modelled as:

\[
w(x, y) = W_{\text{max}} \sin\left(\frac{\pi mx}{a}\right) \sin\left(\frac{\pi ny}{b}\right) 
\]

where \( a \) is the length of the panel and \( b \) is the breadth of the panel, \( x \) and \( y \) are the Cartesian coordinates of any location on the plate and \( m \) and \( n \) are number of half waves. The transverse frame for the Model 2 is not modelled by finite elements and its effect is accounted for by respective boundary conditions applied to the nodes associated with the connection between plate and stiffener. A buckling analysis with the prescribed surface of plates and stiffeners is performed before the incremental load-displacement analysis.

### 3.2.2 Element size effect

Element size of finite element models is a very important parameter, which highly influence the calculated load capacity of structures. To analyse this effect four different cases are studied here, related to different model lengths, breadths and number of half waves of imperfection shape (see Table 3.1 and 3.2).

Table 3.1: Structural dimension cases, Model 3

<table>
<thead>
<tr>
<th>Case</th>
<th>( a ), mm</th>
<th>( b ), mm</th>
<th>No of half waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>150</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>150</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>800</td>
<td>150</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>75</td>
<td>1</td>
</tr>
</tbody>
</table>

The element size of the web of stiffener has a big influence on the calculated ultimate strength of the models 1 and 2. As can be seen from Figure 3-3, the load carrying capacity of the stiffener panel, Model 2 achieves its minimum when the web has 5 elements and it is decreasing or increasing with the number of elements used to model the web. This point is so-called the inflection point which may
change according to structural configuration.

<table>
<thead>
<tr>
<th>Case</th>
<th>(a), mm</th>
<th>(b), mm</th>
<th>(h), mm</th>
<th>No of half waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>150</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>150</td>
<td>50</td>
<td>1</td>
</tr>
</tbody>
</table>

The trend of the calculated ultimate strength is stabilized once the web is modelled by 15 finite elements for Case 2, Model 2, which results in the lowest load carrying capacity. This can be interpreted that the use of coarse meshes results in overestimation of the load carrying capacity and for any specific configuration of analysed structure an appropriate finite element size should be identified by performing element size analysis. Figure 3-4 shows the respective element sizes when the Model 3 exhibits the minimum load carrying capacity according to the cases presented Table 3.1. In order to find out the most appropriate mesh size, first the dimensions and shape imperfection of the plate have been varied. It has been found that the mesh size is not affected by the plate failure mode in contrary to what has been observed in the case of stiffener tripping.

![Figure 3-3: Element size, Case 1, Model 2 (left) and Case 2, Model 2 (right)](image)

![Figure 3-4: The respective element sizes at minimum load carrying capacity, Model 3](image)

3.2.3 Element type effect

In addition to the element size effect analysis, element type effect analysis is performed here. The problem pointed here is finding the most suitable type of elements that may be used to model the
physical behaviour of analysed structure covering geometric and material non-linearity in any condition without leading to non-convergence problems. Shell 93 and Shell 181, which are eight-nodes and four-node quadrilateral element types are analysed here for finite element modelling of Model 1 and Model 2 and the resulting ultimate strength is compared to the closed-form solution stipulated by IACS (2010).

Shell 181 yields larger carrying capacity than Shell 93, which leads to different error in the estimation of ultimate strength. The first tangent modulus is the same for two cases and on the other hand the shell element 181 better captures the behaviour of the stiffened panels (see Figure 3-5). Shell 181 is used for Model 2 and Model 4 and Shell 93 is used for Model 1 and Model 3, which have been considered less volatile structures.

3.2.4 Structural cross section effect

In this section the cross-section effect has been investigated on the element size and on the ultimate strength. For this purpose, Model 2 has been selected and different element sizes and imperfection shapes have been applied. It has been found that the structural cross-section effect on the element size is insignificant. However, it has significant influence on the ultimate strength.

Figure 3-6 shows that the cross-section structural configuration has significant influence on the strength. Transverse configuration has more influence than longitudinal configuration on the ultimate
strength of this particular case. Garbatov et al. (2011) found that it does not always reflect in the strength reduction as increasing the longitudinal half wave number. They found that as the plate has 4 longitudinal half waves, it results in larger ultimate strength capacity than the one that has one longitudinal half wave.

3.2.5 Thickness effect

The thickness effect on the mesh size and the ultimate strength has been investigated and it has been found that there is no existing difference on the mesh size as either increasing or decreasing the plate and stiffener thicknesses, however it has a significant influence on the ultimate strength (see Figure 3-7). It can be concluded that the web height of this particular case is the most significant influence on mesh size.

![Figure 3-7: Thickness effect, Model 2](image)

3.3 Ultimate strength analysis

3.3.1 Model 1

Model 1 is made up of a stiffened panel between two neighbouring transverse frames (see Figure 3-8). The variety of the boundary conditions analysed is shown in Table 3.3. As can be seen in Figure 3-8, on the longitudinal edges, symmetry boundary conditions are applied since the maximum imperfection has been considered to occur there. The coupling conditions are employed on the longitudinal edges in order to keep the section plane. The transverse edges have also been exerted different rotation effect and appropriate translations given in Table 3.3 including coupling equations are implemented considering that the existing transverse frames are stiff enough to keep the sections plan.

Applying BCs 1 and 2, the torsion effect on the ultimate strength has been investigated. In these particular cases, there is no significant difference in the strength behaviour (see Figure 3-9), which can be explained with the fact that the transverse edges are stiff enough. The coupling effect on the ultimate strength has been also analysed for BCs 1 to 7 and in case where the loaded edges (transverse edges) are coupled in longitudinal direction, the post-collapse regime is better captured (see Figure 3-9). This is due to the fact that the longitudinal edges have been considered imperfect therefore edge effectiveness is low. The load carrying capacity is decreasing with less plasticity to make the collapse regime smoother. The coupling effect is also investigated for BCs 4 and 5. In case
when the longitudinal is coupled in transverse direction, this leads to a higher load carrying capacity, but sharper post-collapse regime (see Figure 3-9). The restrained transverse edge leads to higher load carrying capacity and also the stress strain curve behaviour changes to a plate failure observing BCs 3 and 4 (see Figure 3-9). Applying BCs 1 to 6 show that the transverse edge restraining in terms of rotations gives larger load carrying capacity. It can also be observed that the post collapse regime is more gradual due to the plasticity propagation (see Figure 3-9). As can be seen from Figure 3-10, when the plate slenderness increases, the structure becomes more sensitive to edge boundary changes. This kind of structural response, for an example, is characteristic of a structure subjected to uniformly distributed corrosion deterioration.

![Figure 3-8: Model 1](image)

<table>
<thead>
<tr>
<th>Lines</th>
<th>BC1</th>
<th>BC2</th>
<th>BC3</th>
<th>BC4</th>
<th>BC5</th>
<th>BC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-5: Uy, Uz, Rx</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L1-5: Uy, Uz</td>
<td>X</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>L1-5: Uy, Uz, Ry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>L1-5: C – Ux</td>
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<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L6-7: Uy, Uz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>L6-7: Uy, Uz, Rx</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>L6-7: Uy, Uz, Ry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2-4: Ux, Uy, Uz</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2-4: Ux, Uy, Uz, Rx</td>
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<td></td>
</tr>
<tr>
<td>L2-4: Ux, Uy, Uz, Ry</td>
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<td></td>
<td></td>
<td></td>
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<td>L3-8: Ux, Uy, Uz</td>
<td></td>
<td>x</td>
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<tr>
<td>L3-8: Ux, Uy, Uz, Rx</td>
<td></td>
<td>x</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>L1-2: Uy, Rx</td>
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<td></td>
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<td></td>
<td>x</td>
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<td>L5-4: C – Uy</td>
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<tr>
<td>L5-4: Rx</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* U-Displacement, R-Rotation, C-Coupling
3.3.2 Model 2

Model 2 is made up of a stiffened panel between two half bays, \( \frac{1}{2} + \frac{1}{2} \). The boundary conditions for the studied cases may be seen in Table 3.4. The longitudinal and transverse edges (see Figure 3-12) are subjected to symmetry boundary conditions, since the initial maximum imperfection has been generated in the longitudinal and transverse edges and also coupling conditions are employed in order to keep the section plane during loading.
The effect of four boundary conditions for the ultimate strength assessment is analysed here. BCs 1 and 2 show that using coupling conditions, on the longitudinal edges in transverse direction, give larger ultimate strength and because of the sufficient plasticity it makes the post-collapse regime smoother (see Figure 3-13). In addition to that, BCs 2 and 3 show that in the case when to the loaded transverse edges are not applied coupling conditions in longitudinal direction, it yields to a significant load carrying capacity reduction (see Figure 3-13). BCs 2 and 4 show that the longitudinal edge restraining leads to a larger load carrying capacity and also smoother post-collapse regime (see Figure 3-13). On the other hand, when the model 1 is observed closely (see Figure 3-9), the transverse restraining does not result in significant behaviour change in terms of ultimate point but has a similar effect on the post-collapse regime.
3.3.3 Model 4

Model 4 is made up of a stiffened panel between two half bays and one bay, $\frac{1}{2} + 1 + \frac{1}{2}$. The studied boundary conditions are presented in Table 3.5. The longitudinal and transverse edges subject to the symmetry boundary condition since the maximum imperfection has been considered in the longitudinal and transverse edges (see Figure 3-14). Coupling conditions are employed in order to keep the section plane during loading.

It has been expected that the Model 4 has larger carrying capacity than the Model 2 as can be seen from Figure 3-15 and 3-16. However, the Model 4 has larger ultimate strength but the Model 2 has larger post-collapse load carrying capacity. It has to be pointed out that the ultimate strength capacity is more important and therefore the Model 2 is more appropriate as a model to evaluate the ultimate strength of the ship structure.
Table 3.5: Boundary conditions, Model 4

<table>
<thead>
<tr>
<th>Line Number</th>
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<th>( \Phi )</th>
<th>( \Psi )</th>
<th>( \Theta )</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
</tr>
<tr>
<td>L1-3: Ry, C-Ux</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>L2-11: Rz</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L2-11: Rz, C-Ux</td>
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<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>L4-10: Uz</td>
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<td>x</td>
<td>x</td>
</tr>
<tr>
<td>L5-9: Uz</td>
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<tr>
<td>L8-6: Ux, Ry</td>
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<td>x</td>
</tr>
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<td>L7-16: Ux, Rz</td>
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</table>

* U-Displacement, R-Rotation, C-Coupling

Figure 3-15: Model 4, BCs 1 to 4 (left), Model 2 and 4 with thickness reduction, BC2 (right)

Figure 3-16: Model 2 and 4, No thickness reduction (left), with 1 mm thickness reduction (right)

3.3.4 Model 3

Model 3 is made up of a single plate bordered by two neighbouring transverse frames and longitudinal stiffeners as may be seen in Figure 3-17. The studied boundary conditions are presented in Table 3.6.
The longitudinal and transverse edges are all constrained in vertical direction. The longitudinal edges are not subjected to symmetry boundary condition. Since the plate has been considered bounded by the stiff transverse frames and longitudinal, coupling equations were not employed at the plate edges.

Boundary changes on the transverse edges do not change behaviour before the ultimate point and only gives a larger post-collapse loading capacity. As the loaded transverse edge is constrained against torsional rotation, this leads to the first yield occurrence (see Figure 3-17).

![Figure 3-17: Model 3 (left), Model 3, BCs 1 to 3 (right)](image)

**Table 3.6: Boundary conditions, Model 3**

<table>
<thead>
<tr>
<th>Line Number</th>
<th>BC1</th>
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<th>BC3</th>
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</tr>
<tr>
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</tr>
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<td>x</td>
<td></td>
</tr>
<tr>
<td>L2-3: Uy, Uz</td>
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<td>x</td>
<td></td>
</tr>
<tr>
<td>L3-4: Ux, Uy, Uz</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>L4-1: Uy, Uz</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

* U-Displacement, R-Rotation

**3.4 Chapter conclusions**

In this chapter the boundary condition effect has been investigated on the ultimate strength of the structural elements. It has been found that in the case when the boundary edges are not well constrained, this leads to a sudden and smaller carrying capacity and to non-convergence problems. The post-collapse regime is highly affected by the edge boundary conditions. The restraining boundaries give a larger post-collapse loading capacity. The boundary conditions may change the failure mode too. As the plate slenderness is increased, its sensitivity to boundary changes increases. The structure cross-section configuration has significant influence on the ultimate strength. Model 2 and Model 4 are similarly affected by the boundary condition changes with respect to the ultimate strength point. Model 2 has larger post-collapse loading capacity than the Model 4. Although the Model 1 is stiffer, it has similar ultimate strength point however it has a larger post collapse load carrying capacity than the Model 3. As the thickness is decreased, Model 2 and Model 4 behave closer in terms of ultimate point and strength history. Model 2 is a more appropriate finite element model in order to calculate the ultimate strength of ship structural components.
4 ULTIMATE STRENGTH ASSESSMENT OF A STIFFENED PANEL
ACCOUNTING FOR THE EFFECT OF RESIDUAL STRESS

4.1 Introduction

Residual stresses may be induced in structural components during manufacture such as welding, forging, casting, age hardening, machining and other processing applications. Due to their negative effect on structural strength, a wide range of studies has been performed in order to evaluate what the reasons are leading to their development. Residual stresses, which arise from non-uniform induced heat and plastic strain development, differ from the service loading stresses that the structure is subjected during its service life. Structural response is in equilibrium with the external loading. In contrast, the residual compressive stresses are in equilibrium with the tensile residual stresses only.

Guedes Soares and Kmiecik (1993) have found that the residual stresses tend to be more important for thicker plates and are likely to contribute to geometrical nonlinearity due to the presence of buckling and plastic propagation. They also found that when the compressive forces are predominant, the transverse residual stresses effect is very small until the ultimate load is reached, but they influence the post-collapse regime.

Ueda and Yao (1991) showed that, both welding residual stresses and initial geometrical imperfections reduce the compressive buckling and ultimate strength of plates, and this reduction achieves its maximum when the plate slenderness is about 1.8. Welding residual stresses reduce the compressive buckling and ultimate strength of stiffened panels when local buckling takes place, but increase them when overall buckling occurs.

Ueda et al. (1997) concluded that, welding residual stresses reduce the buckling strength remarkably, but have a little effect on the ultimate strength when the plate is thin. On the other hand, when the plate is thick, welding residual stresses reduce the ultimate strength remarkably if there is an initial geometrical imperfection accompanied by local bending stresses.

Grondin et al. (1999) applied three levels of residual stresses and showed that, at plate slenderness, $\beta$ larger than 1.7 the residual stresses in the plate decrease the strength roughly in direct proportion to the magnitude of the compressive residual stresses in the plate. However, when yielding occurs before buckling, the effect of residual stresses is diminished. When failure of stiffened plates is due to overall Euler buckling, the effect of the residual stresses in the plate is less pronounced.

4.2 Residual stress modelling

The residual stress reduces the strength of the structural components and moreover it reduces the structural stiffness. Some authors decided, for a better approximation, to divide the ultimate strength in various terms, accounting for initial geometrical imperfection and residual stresses as:

$$\phi_0 = \phi_p R_e R_s$$  \hspace{1cm} (50)

where $\phi_p$ is the perfect ultimate strength, $R_e$ is the reduction factor due the initial geometry
imperfection and $R_r$ is the reduction factor due to the residual stresses. Dwight and Moxham (1969) related the level of compressive residual stress to the width of the tensile block:

$$
\phi_t = \frac{\sigma_y}{\sigma_t} = \frac{2\eta t}{b - 2\eta t}
$$

where $\eta t$ is the width of the reduction factor area and the reduction factor due to residual stresses may be calculated as:

$$
R_r = (1 - \phi_t)
$$

Faulkner (1975) developed an expression for ultimate strength fitting it to data of ultimate plate strength leading to:

$$
\varphi_p = \frac{a_1}{\beta} - \frac{a_2}{\beta^2}, \quad \beta \geq 1.0
$$

where the constants $a_1$ and $a_2$ are given $a_1 = 2.0$ and $a_2 = 1.0$ for simple supports and $a_1 = 2.5$ and $a_2 = 1.56$ for clamped supports. This equation accounts implicitly for average levels of initial deflection and it can be complemented with others that dealt explicitly with the effect of residual stresses.

Guedes Soares (1988) has extended that formulation by deriving a strength assessment expression for the compressive strength of plate elements under uni-axial load, which deals explicitly with initial defects as:

$$
\varphi_z = (\varphi_p B_p) (R_y B_y) (R_z B_z)
$$

where $\varphi_p$ is given by Eqn (53), $B_p$, $B_y$, and $B_z$ are model uncertainty factors and $R_y$ and $R_z$ are strength reduction factors which are due to the presence of weld induced residual stresses and initial distortions respectively. These expressions are:

$$
B_p = 1.08
$$

$$
R_y = 1 - \frac{\Delta \varphi_p}{1.08 \varphi_p}
$$

$$
B_y = 1.07
$$

$$
R_z = 1 - (0.626 - 0.121\lambda) \frac{\delta_z}{t}
$$

$$
B_{z0} = 0.76 + 0.01\theta + 0.24 \frac{\delta_z}{t} + 0.1\lambda
$$

where $\delta_z$ is the initial imperfection and $t$ is the strength reduction due to the residual stress is defined as:

$$
\Delta \varphi_p = \phi_t \frac{E_t}{E}
$$

where the tangent modulus of elasticity, $E_t/E$ ratio accounts for the development of plasticity. This modulus can be approximated by the expression was used by Guedes Soares and Faulkner (1987):

$$
\frac{E_t}{E} = \frac{\lambda - 1}{1.5}, \quad \text{for } 1 \leq \lambda \leq 2.5
$$
Expressions resulting from the same type of approach have been derived for plates subjected to transverse were derived by Guedes Soares and Gordo (1996) and to biaxial loading by Guedes Soares and Gordo (1996).

The approaches presented here and accounting for residual stresses can be directly used for design and they estimate only the ultimate strength of steel structure subjected to compressive load. The second group of approaches, dealing with the residual stress modelling for the ultimate strength assessment, are those by using a modified stress strain elasto-plastic curve or by using direct prescribed pre-stresses, simulating the residual stresses. These methods employ the finite element method and estimate not only the ultimate strength but also the pre and post collapse regime behaviour.

Faulkner (1977) suggested a model to account for residual stresses by implementing a modification in the elastic perfectly plastic stress-strain curve. Through this type of stress strain curve, the structure is subjected to premature yielding and in turn reduces both stiffness and strength. The structural yielding stress point is reduced allowing the structure to absorb more strain and reduces the strength per strain. This implies that the structural linear behaviour is reduced. The residual stress may also be modelled as direct pre-stresses induced to the finite element model before the start of non-linear ultimate strength analysis. The results achieved for the ultimate strength, when direct prescribed pre-stresses are used to model the residual stresses, are twice smaller if the residual stresses are modelled by equivalent temperature (Paik 2009).

Figure 4-1: Modified stress-strain relationships

Figure 4-1 shows four types of admissible stress strain curve definitions, which have been implemented in this thesis. In the present study pre-stresses induced to the finite element model and modified elastic perfectly plastic stress-strain curve are used for residual stress modelling in the finite element analysis. To find out the shape of the modified stress-strain curve an iterative procedure is created here (see Figure 4-3). In this iterative procedure, the material stress-strain curve is modified according to the best fitted results of pre and post collapse estimated by directly prescribed residual stresses. The entire procedure is divided into two stages. In the first one, for three levels of the residual stresses defined as slight, average and severe pre-stresses are induced into the finite
element model. The defined residual stresses are applied over the structure according to predefined residual stress distribution. Then the structural normalized strength against strain is estimated. In the second stage, the iterative procedure is implemented to identify the best modification to the material stress-strain curve used for finite element calculations of ultimate strength in order to find out the best fit the structural response to that of the first stage, which has been based on the direct pre-stress modelling of residual stresses. However, the pre and post collapse regime assessment, based on modified stress-strain curve, is influenced by the structural configurations, boundary conditions of edges and the level of the residual stresses. The first step in the modification of the stress-strain curve is to define the stress level of proportional limit in order to capture the first yielding point. Once the first yielding limit has been defined, the next point in the stress-strain curve has to be identified by changing the slope angle as can be seen on Figure 4-1, up or adding a new point as can be seen in Figure 4-1, down. The iterations are ended when the two structural responses from both methods have been found almost identical.

4.3 Finite element modelling

Three structural models are analysed here. Models 1 to 3 are shown in Figure 4-2, where Model 1 represents a stiffened panel located between two neighbouring transverse frames; Model 2 is also stiffened panel with a transverse frame in the middle. Model 3 is composed by $\frac{1}{2}+1+\frac{1}{2}$ plates and two transverse frames. The ultimate strength of the panels is analysed based on finite element method using commercial software ANSYS (2009). The software enables modelling of elastic plastic material properties and large deformations. Four-node quadrilateral shell elements have been used to model the plates and stiffeners.

![Figure 4-2: Model structural configurations](image)

Three distinguish stiffened models are studied here as can be see them from Figure 4-2. The length of the stiffened panels of the Model 1 is $a = 400$ mm and the breadth is $b = 150$ mm respectively. The web thickness is 3.6 mm and the height is 25 mm and the plate thickness is 2.7 mm. Plate slenderness is $\beta = 1.87$. The aspect ratio, $a/b$ of the plate is 2.66. The stiffener has a cross section
of a standard type flat bar. The area of the stiffener, $A_s$, is approximately 22 per cent of the area of the plate, $A_p = ab$. The non-dimensional column slenderness of the stiffened panel calculated with a full plate width is $\lambda = 0.69$.

The thickness, length, breadth of plates and the height of stiffeners are equal for all finite element models. The initial geometry imperfection of plates and stiffeners are generated by a pre deformed surface. Faulkner (1975); Smith et al. (1988) have reported that the maximum imperfections in a plate can be assumed to be proportional to $\beta^2$. They suggested that the maximum initial deformation for an average imperfection can be calculated as

$$w_{max} = 0.1\beta^2$$  \hspace{1cm} (63)

The maximum camber tolerance, for a standard shape is usually assumed to be 0.2% of the length of the length. The initial geometry surface imperfection is modelled as proposed by Smith et al. (1988):

$$w(x, y) = w_{max} \sin\left(\frac{mx}{a}\right)\sin\left(\frac{ny}{b}\right)$$  \hspace{1cm} (64)

![Interactive procedure for identifying the modified stress strain descriptors (PRS-pre stress, SSM-modified stress-strain)](image)

where $a$ is the length of the panel and $b$ is the breadth of the panel, $x$ and $y$ are the Cartesian coordinates of any location on the plate and $m$ and $n$ are number of half waves. The transverse frame for Model 2 and Model 3 is not modelled by finite elements and its effect is accounted for by the respective boundary conditions applied to the nodes associated with the connection between plate and stiffener. Several studies have been performed in order to find the appropriate finite element type. Four-node quadrilateral shell 181 element has been chosen to be used in order to model the plate and stiffener for all models. The kinematic assumption of the finite element analysis calculation is large displacement and rotation, but small strain. The material stress strain model, when the residual
stresses are not accounted for, is assumed to be bilinear elastic-perfectly-plastic without hardening with the yield stress of $\sigma_y = 235$ MPa, the elastic modulus is $E = 2.1 \times 10^{11}$ Pa and the Poisson coefficient is $\nu = 0.3$. Geometric and material non-linearity has been taken into account. Stress stiffening effect, in order to avoid sharp post-collapse regime, has been accounted for. The applied load is uni-axial compression.

4.4 Ultimate strength assessment

4.4.1 Model 1

4.4.1.1 Directly prescribed residual stress

Finite element Model 1, presenting a stiffened panel, is located between two neighbouring transverse frames as can be seen from Figure 4-2 and Figure 4-4. The applied boundary conditions are shown in Table 4.1, the symmetry boundary conditions are applied on the longitudinal edges, since the maximum displacement is considered to occur there. Coupling conditions are employed on the longitudinal edges in order to keep the structural net section plane.

Three levels of the compressive part of the applied residual stress distribution are considered as proposed by Smith et al. (1988): slight, average and severe as defined by Eqn (65) and only the longitudinal residual stresses are investigated.

$$\frac{\sigma_r}{\sigma_y} = \begin{cases} 
-0.05, & \text{slight} \\
-0.15, & \text{average} \\
-0.3, & \text{severe}
\end{cases} \quad (65)$$

Figure 4-4: Model 1

Table 4.1: Boundary conditions, Model 1

<table>
<thead>
<tr>
<th>Lines</th>
<th>BCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-5: Uy, Uz</td>
<td>0</td>
</tr>
<tr>
<td>L1-5: C- Ux</td>
<td>√</td>
</tr>
<tr>
<td>L6-7: Uy, Uz</td>
<td>0</td>
</tr>
<tr>
<td>L6-7: C – Ux</td>
<td>√</td>
</tr>
<tr>
<td>L2-4: Ux, Uy, Uz</td>
<td>0</td>
</tr>
<tr>
<td>L3-8: Ux,Uy,Uz</td>
<td>0</td>
</tr>
<tr>
<td>L1-2: Uy, Rx</td>
<td>0</td>
</tr>
<tr>
<td>L5-4: Rx</td>
<td>0</td>
</tr>
</tbody>
</table>

* U-Displacement, R-Rotation, C-Coupling
The compressive and tensile parts of the residual stresses are equilibrated, leading to a specific residual stress distribution, as can be seen in Figure 4-5.

The residual stresses reduce the first yielding point of the structural response and due to the plasticity it results in large strain absorption, but less strength (see Figure 4-6). However, the strain absorption after the first yielding point may be affected by the boundary conditions and may lead to a less strain absorption. The strength reduction, resulted from the initial geometry imperfection and residual stresses, tends to be a linear function of $b/t$ ratio as can be seen from Figure 4-7.

For the larger plate slenderness values, the residual stress becomes less influential on the ultimate strength (see Figure 4-7). In addition to that, as the structural capacity is reduced, through increasing the number of the longitudinally induced half-waves of the initial geometry imperfection, the residual stresses become almost insignificant as can be seen from Figure 4-7, right.
4.4.1.2 Modified material stress strain curve accounting for residual stress

When structures are subjected to a tensile loading, their structural stress-strain responses follow the material stress-strain response. This means that the transition from elastic to plastic behaviour is smooth and there is no disturbance. When the structure is subjected to compressive loading the transition from elastic to plastic behaviour is with disturbance due to the presence of buckling and depends upon the structural configurations and boundary conditions, which lead to a non-uniform plastic propagation. It is strongly dependent on the plate thickness, which gives more strength and therefore more plastic propagation may be achieved.

As can be seen from Figure 4-8, modelling the residual stresses by the use of a modified stress strain curve, the stress strain response match very well the stress strain response calculated by the use of directly prescribed residual stresses, except for the post collapse regime. By increasing the severity of the residual stress, the first yielding stress is reduced as can be seen from Figure 4-9.

Once the first yielding point is achieved, the next point has been found according to the structural response which is highly affected by the boundary conditions of the edges. As the first yielding stress point is reduced, the structure absorbs more strains. However, if the structure is not well constrained then this may lead to less strain absorption. As for Model 1, the first cut of $\sigma - \varepsilon$ curve leads to more strain absorption. The second cut of $\sigma - \varepsilon$ curve makes the structure to absorb less strain. Therefore, the modified stress-strain curve used for modelling residual stresses has to be defined for any particular structural configuration and applied boundary conditions at the edges. The modified material stress-strain data points have been shown in Table 4.2 where $\varepsilon_y$ is the material yielding strain.
4.4.2 Model 2

4.4.2.1 Directly prescribed residual stress

Model 2 is defined as a stiffened panel between two half bays, $\frac{1}{2}+\frac{1}{2}$. The boundary condition may be seen in Table 4.3. The longitudinal and transverse edges (see Figure 4-10) are subjected to the
symmetry boundary conditions, since the initial maximum imperfection has been generated in the longitudinal and transverse edges and also the coupling conditions are employed in order to keep the section plane during loading.

![Figure 4-10: Model 2](image)

**Table 4.3: Boundary conditions, Model 2**

<table>
<thead>
<tr>
<th>Lines</th>
<th>BCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3-5: Ry, C-Ux</td>
<td>0</td>
</tr>
<tr>
<td>L4-12: Rz, C-Ux</td>
<td>√</td>
</tr>
<tr>
<td>L2-6: Uz</td>
<td>0</td>
</tr>
<tr>
<td>L10-11: Uz, C-Uy</td>
<td>√</td>
</tr>
<tr>
<td>L1-7: Ux, Ry</td>
<td>0</td>
</tr>
<tr>
<td>L8-9: Ux, Rz</td>
<td>0</td>
</tr>
<tr>
<td>L1-3: Rx, Uy</td>
<td>0</td>
</tr>
<tr>
<td>L5-7: Rx, C-Uy</td>
<td>√</td>
</tr>
</tbody>
</table>

*U-Displacement, R-Rotation, C-Coupling

As for Model 2, longitudinal residual stresses have been taken into account and three levels of residual stress have been applied, where the magnitude of compressive stresses is defined as Eqn (65) where for severe residual stresses are considered as -0.2.

![Figure 4-11: Model 2, Residual stress distribution (left) and geometry shape imperfections, Model 2 (right)](image)

Three different initial geometry imperfection shapes have been analysed in order to find the residual stress effect on the ultimate strength. The shapes of initial geometry imperfections are shown in Figure 4-11.
The strain absorption is also affected by the geometry non-linearity. Due to the premature yielding, more strain is absorbed and geometry non-linearity increases and turn reduces the linear part of structural behaviour (see Figure 4-12 and Figure 4-13).

As can be seen from Figure 4-13, right, as the structural complexity increases, which depends on the shape on the initial geometry imperfection, leads to strength reduction in the ultimate strength. This also can be interpreted in the way that as the geometry non-linearity increases; the residual stress reduction on ultimate strength becomes more significant.

4.4.2.2 Modified stress strain curve accounting for residual stress

Shape 3 has been selected as a reference to identify the modified stress strain descriptors. As opposed to Model 1, after firstly the welding residual stresses reduce the structural first yielding point and strength (see Figure 4-14). The structure may have less strain by increasing the material strain.
since the boundary edges are imperfect which leads to less plasticity propagation.

The modified stress-strain curve is influenced by the boundary conditions at the edges, which in turn increases the edge rotations and reduces the stiffness and structural strain. Therefore, for the modified severe stress-strain, the second cut has been defined closer to the first cutting point. The modified stress-strain descriptors have been shown in Table 4.4 where $\varepsilon_y$ is the material yielding strain. As for Model 2, the average and severe levels of residual stresses have been considered in identifying the descriptors of the modified stress-strain curve. The post-collapse regime is highly affected by the plasticity propagation, which in turn is affected by the boundary conditions of edges. The post-collapse regime of structural response accounting for the presence of residual stresses is less smoother than the one developed based on the modified stress-strain curve (see Figure 4-14).

![Figure 4-14: Stress strain response, Model 2 (SSM- modified stress-strain, RS-residual stresses, WRS-welding residual stresses) (left), Material stress strain relationship, Model 2 (right)](image)

Table 4.4: Model 2, Modified stress –strain descriptors

<table>
<thead>
<tr>
<th>Severe Level</th>
<th>Average Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pts</td>
<td>Stress (MPa)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>2</td>
<td>95</td>
</tr>
<tr>
<td>3</td>
<td>235</td>
</tr>
<tr>
<td>4</td>
<td>235</td>
</tr>
</tbody>
</table>
4.4.3 Model 3

Model 3 represents a stiffened panel between two halves and one bays, $\frac{1}{2} + 1 + \frac{1}{2}$. The applied boundary conditions are presented in Table 4.5. The longitudinal and transverse edges are subjected to the symmetry boundary conditions since the maximum imperfection has been considered in the longitudinal and transverse edges (see Figure 4-15). The coupling effect is employed in order to keep the section plane during loading.

The effect of the residual stresses on the ultimate strength reduction for the Model 3 is less significant in a comparison to Model 2 since the structural capacity of the Model 3 is larger than the one of Model 2 (see Figure 4-16). There is no significant behaviour change between Model 2 and 3. Model 2 is better representative of the stiffened panel since the Model 3 always present more optimistic loading carrying capacity results.

![Figure 4-15: Model 3](image)

Table 4.5: Boundary conditions, Model 3

<table>
<thead>
<tr>
<th>Line Number</th>
<th>BCs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-3: Ry, C-Ux</td>
<td>✓</td>
</tr>
<tr>
<td>L2-11: Rz, C-Ux</td>
<td>✓</td>
</tr>
<tr>
<td>L4-10: Uz</td>
<td>0</td>
</tr>
<tr>
<td>L12-13: Uz, C-Uy</td>
<td>✓</td>
</tr>
<tr>
<td>L14-15: Uz, C-Uy</td>
<td>✓</td>
</tr>
<tr>
<td>L5-9: Uz</td>
<td>0</td>
</tr>
<tr>
<td>L8-6: Ux, Ry</td>
<td>0</td>
</tr>
<tr>
<td>L7-16: Ux, Rz</td>
<td>0</td>
</tr>
<tr>
<td>L1-8: Rx, Uy</td>
<td>0</td>
</tr>
<tr>
<td>L3-6: Rx, C-Uy</td>
<td>0</td>
</tr>
</tbody>
</table>

* U-Displacement, R-Rotation, C-Coupling
The residual stress effect on the ultimate strength has been investigated using different structural finite element models in this chapter. It has been found that the residual stresses decrease the first yielding point of the structure response in turn the ultimate strength. As the structural capacity increases, the effect of residual stresses on the ultimate strength decreases. Initial geometry imperfection increases the strength reduction along with the residual stresses.

Modified stress-strain curve has been found as a good and fast method to introduce residual stress effect. It has been found that the modified stress-strain curve descriptors are influenced by the structural configurations and boundary conditions at the edges.
5 ULTIMATE STRENGTH ASSESSMENT OF A THIN PLATE ACCOUNTING FOR THE RESIDUAL STRESSES BASED ON A MOVING HEAT SOURCE

5.1 Introduction

The welding induced distortions and residual stresses, which lead to the structural strength reduction due to the non-uniform heat application, is a major concern in steel construction. The applied welding heat results in a non-uniform temperature distribution and a non-uniform cooling, which in turn inevitably leads to welding residual stress and induced distortions. Several types of heat and mechanical treatment are applied to reduce the welding induced distortion and stresses and to enhance steel constructions.

In the current industrial practice, welding processes are developed based on trial and error experiments, incorporating with engineers’ knowledge and experience of previous similar designs (Gery et al. 2005). On the other hand, finite element method is a very powerful and fast tool to analyse the welding induced residual stresses and distortions.

There are several types of heat source models developed in the literature. Pavelic et al. (1969) proposed the disc model, which is considered to be associated with the Gaussian distribution. Friedman (1975); Krutz and Segerlind (1978) suggested an alternative method to the Pavelic disc model, which is expressed with the moving coordinate system along with the heat source that has been adopted herein.

Goldak et al. (1984) proposed a double ellipsoidal heat source model to simulate deep or shallow penetrated welding processes.

Long et al. (2009) used the Goldak model to find out the welding induced distortions and residual stresses for a single plate. They found out that, the plate thickness and heat source speed have a significant influence on the welding induced stresses and distortions.

Biswa et al. (2006) used the disc model in order to simulate a line-heating process for a single plate changing the plate thicknesses. They found that as the plate thickness is increased, the out-of plate deflection becomes smaller for the same heat and speed input. Gannon et al. (2010) used the disc model to simulate the welding influence on the ultimate strength for a stiffened panel. They concluded that the welding residual stresses may decrease the ultimate load carrying capacity by 16.5 %.

5.2 Modified stress strain definition

The detailed procedure has been given in the previous chapter. Here the material stress-strain curves have been established according to the moving heat source.

The entire procedure is divided into two stages. In the first one, the moving heat is applied to the plate accounting for the thickness, heat speed and level applied over the structure defining the residual stress distribution resulting from the welding process. Then the structural normalized strength against strain is estimated as can be seen in Figure 5-1. In the second stage, the iterative procedure is
implemented to identify the best modification to the material stress-strain curve used for finite element analysis of the ultimate strength to find out the best fit the structural response to that of the first stage, which has been based on the travelling heat source (see Figure 5-3). However, the pre and post-collapse regime assessment, based on the modified stress-strain curve, is influenced by the structural configurations, boundary conditions of the plate edges and the level of the residual stresses. The first step in the modification of the stress-strain curve is to define the proportional stress to capture the first yielding point. Once the first yielding has been defined, the next point in the stress-strain curve definition is to define the changing of the slope. The iterations are concluded when the two structural responses have been found almost identical ones.

![Diagram](image)

Figure 5-1: Strength assessment procedure accounting for the welding residual stress and distortion.

### 5.3 Thermal structural analysis

The structural model analysed here is shown on Figure 5-2. The ultimate strength of the plate is analysed based on finite element method using a commercial software ANSYS (2009). The software enables modelling of elastic plastic material properties and large deformations. The kinematic assumption of the finite element analysis calculation is large displacement and rotation, but small strain. The material yielding stress, $\sigma_y = 350$ MPa, the elastic modulus is $E = 2.05 \times 10^{11}$ Pa and the Poisson coefficient is $\nu = 0.297$.

Geometric and material non-linearity has been taken into account. The stress stiffening effect, to avoid sharp post-collapse regime in the ultimate strength assessment, has been accounted for. The applied load is uniaxial compression. Eight-node quadrilateral solid elements have been used to model the plate. Solid 70 and Solid 185 has been used in respectively thermal and structural analysis. The boundary conditions of ultimate strength assessment are shown in Figure 5-2.
The material properties have been taken from Gery et al. (2005) for the thermal analysis. The circular moving heat source proposed by Friedman (1975); Krutz and Segerlind (1978) has been adopted to simulate the welding heat source (see Figure 5-4). The mathematical heat source model is expressed as:

\[ q(z, \xi) = \frac{3Q}{\pi c^2} e^{-\frac{(x^2+\xi^2)}{c^2}} \]  

(66)

where \( Q \) is energy input rate and \( c \) is the characteristic radius of surface flux distribution, \( x \) is the transverse distance and \( \xi \) is the moving longitudinal location which is expressed as:

\[ \xi = z + v(\tau - t) \]  

(67)

where \( v \) is the welding speed and \( t \) is time and \( \tau \) is a lag factor needed to define the source at time \( 0 \).

First, the thermal analysis is performed by the moving heat source accounting for the plate thickness, heat input and heat level. The output temperature distribution is used as a load input in the subsequent structural analysis to find out the welding residual stress distribution induced by the
thermal process. In the next phase, the residual stress distribution is imposed over the plate along to find out the ultimate load carrying capacity of the plate (see Figure 5-1).

5.4 Ultimate strength assessment

5.4.1 Thermal structural analysis

5.4.1.1 Thickness effect

Firstly, the thermal analysis is performed by changing the plate thicknesses to investigate the thickness effect on the plate distortion and on the welding residual stress distribution. In the second phase, the ultimate strength assessment is conducted for each case by keeping the heat input and heat speed the same. The applied heat input is 7500 W and the heat speed is 3 cm/s.

For the thermal and structural analyses, the appropriate boundary conditions have been applied to achieve a symmetrical distortion and also to avoid rigid body motions (see Figure 5-5). In order to release the system in transverse direction, the opposite side is set to be free in a transverse direction. As for the thermal analysis, the top surface of the plate is considered to be insulated during the welding; the remaining part of the plate has been subjected to the convection and radiation loads. The temperature distribution of mid-plate layer is shown on Figure 5-6. The welding residual stress equilibrium is locally and globally satisfied as can be seen in Table 5.1. The local equilibrium is satisfied through the plate thickness. All the plate thicknesses, investigated herein, have been subjected to the equilibrium criteria and similar results have been found for each thickness. As the plate thickness is increasing, the area occupied by the welding induced tensile stress becomes smaller and to comply with the equilibrium, the compressive stresses start to propagate. The longitudinal residual stress distribution is shown and schematic representation of the residual stress equilibrium is given on Figure 5-7.
Figure 5-5: Welding residual stress assessment BC (left) and plate layers (right)

Figure 5-6: Temperature distribution, mid-plate

Figure 5-7: Longitudinal residual stress distribution (left), Residual stress equilibrium (right)
There is no significant displacement change through the plate layers as can be seen from Figure 5-8. As the plate thickness is increasing, assuming the same heat input and speed, the vertical displacement becomes smaller as can be observed from Figure 5-8. Figure 5-9, left, shows the longitudinal stress distribution through the plate breath accounting for each layer that the plate has. As can be seen, through the thickness, the welding compressive stresses start to distribute in order to satisfy the cross-sectional equilibrium with the tensile stresses. Figure 5-9, right, shows the thickness effect on the welding residual stress development for the third layer. As the plate thickness is increasing, the area occupied by the tensile stresses gets smaller and the compressive stresses start to propagate which leads to more reduction in ultimate load carrying capacity.

Table 5.1: Residual stress equilibrium, 4 mm plate thickness

<table>
<thead>
<tr>
<th>Stresses</th>
<th>Local-mid (Pa)</th>
<th>Global (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>$\approx 3.00$</td>
<td>$\approx -0.2500$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$\approx 6.91$</td>
<td>$\approx 0.0048$</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>$\approx -0.30$</td>
<td>$\approx 0.0000$</td>
</tr>
</tbody>
</table>

Figure 5-8: Vertical displacement, 4 mm thickness (left), for various plate thicknesses (right)

Figure 5-9: Longitudinal stress distribution through plate breath (left), through plate breath, the third layer (right)

Figure 5-10 shows the longitudinal stresses across the plate length. The stresses get larger as the
thickness increases. Due to the shape of deformation (see Figure 5-11), the longitudinal ends rises resulting in structural load carrying capacity increases and in addition to that, the introduced welding residual stresses lead to more distortions, which consequently larges the load carrying capacity (see Figure 5-12). Since the net section centroid of deformed section is shifted into a location where the cross-sectional area moves, the inertia moment grows resulting in a bigger stiffness, as can be seen from Figure 5-12, for $B/T$ ratios less than 50 and more than 100 this effect negligible.
5.4.1.2 Heat input and speed effect

The heat input and speed influence is investigated on the welding induced distortions, residual stresses and the ultimate strength by varying the heat input and speed by 20%. For this purpose, the reference plate thickness of 6 mm has been selected.

The speed and heat has reverse effect on the welding induced distortions (see Figure 5-13). This holds also for the welding induced stresses (see Figure 5-14). Figure 5-15 shows the speed and heat effect on the strength reduction. The speed and heat input has been decreased and increased with respect to the reference heat and speed input which are 7500 W and 3 cm/s respectively.

![Figure 5-13: Vertical displacement, 6 mm thickness, speed effect (left), heat input (right)](image1)

![Figure 5-14: Longitudinal stress distribution through plate breath, heat input effect (left), Speed effect (right), third layer](image2)
Increasing the speed and decreasing the heat input leads to more strength reduction and this is opposite when the speed and heat input vary respectively. This is due to the fact that as the speed is decreasing or the heat input increasing, the area occupied by the tensile stresses becomes larger and gives more strength against the applied compressive forces. The heat speed effect has a larger impact on the ultimate strength reduction as can be seen on Figure 5-15.

5.4.2 Modified stress strain curve accounting for the residual stress

The modified material stress-strain curve is developed according to the thickness variations. For 4 and 5 mm plate thicknesses, the strain hardening has been introduced to capture the response to the welding residual stress and the shape effect. As for 3 mm plate thickness, the stress-strain curve has been weakened through a slope modification.

The descriptors for the modified material stress-strain curve have been shown in Table 5.2 where $\sigma_y$ is the yield stress, which for the present study is 350 MPa, $\varepsilon_y = \sigma_y / E$ denotes the material yielding strain and $E = 2.05 \times 10^{11}$ Pa is the material elasticity modulus.

<table>
<thead>
<tr>
<th>Table 5.2: Modified stress –strain descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pts</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
As the plate thickness is increasing, the strain hardening is decreased. As for 6 mm plate thickness, the adopted method here is not capable to capture the strength history until the ultimate load carrying capacity. The introduced strain hardening contributes to the ultimate and post-collapse strength increase (see Figure 5-16).

5.5 Chapter conclusions

The residual stress effect on the ultimate strength has been investigated through a moving heat source by changing the heat input, heat speed and plate thickness. It has been found that the plate thickness is the most influential parameter that affects the vertical displacement due to a moving heat source. As the plate thickness is increasing, the compressive residual stresses start to take over the area occupied by the tensile residual stresses, which leads to more strength reduction. The shape effect of deformed plate results in larger ultimate strength in certain $B/T$ ratios. The speed is more significant to the heat distribution in relation to the residual stress formation.

The developed modified stress-strain curve has been found as a good and fast approach to introduce both residual stress and the shape effect that arises during the welding process for a non-linear finite element analysis of ultimate strength.
6 ULTIMATE STRENGTH ASSESSMENT OF A STIFFENED PLATE
ACCOUNTING FOR THE RESIDUAL STRESSES AND WELDING SEQUENCE

6.1 Introduction
The welding induced distortions and residual stresses, which lead to the structural strength reduction due to the non-uniform heat application, is a major concern in steel construction. The welded area is subject to a sharp heating relative to the surrounding area and the heat is propagates depending on the material thermal expansion coefficient and the current temperature. Since the surrounding area is colder and restrains the heat expansion, this causes elastic thermal stresses. Due to the fact that the yielding stress of the structural material is time-dependent, it starts to be lowered and when the elastic stresses exceed the proportional limit, it causes local plastic deformations. Residual stress will not be created provided that the structure is homogeneous and no-temperature gradient is developed through uniform heating and cooling process. On the other hand, even though the favourable temperature gradient is achieved, if the structure is non-homogeneous, this leads to the residual stress development due to the different expansion coefficient values.

Gannon et al. (2010) implemented the Gaussian heat source model for stiffened panels to investigate the effect of the welding sequence on the residual stresses and distortions. They showed that, the sequence of the welding may lead to larger welding compressive stresses and it may lead to larger out-of plate deflections, which decreases the plate ultimate strength.

Deng et al. (2006) used a surface heat source with the Gaussian heat distribution to investigate the welding induced distortions for a stiffened panel. They found that the temperature gradient through thickness is a main factor that governs the angular (vertical) distortions in a fillet welding.

6.2 Modified stress strain definition
The procedure has been detailed in the previous chapter. Here the material stress-strain curve has been defined based on the welding sequence.

6.3 Thermal structural analysis
The structural model analysed here is shown on Figure 6-1. The ultimate strength of the stiffened plate is analysed based on finite element method using a commercial software ANSYS (2009). The software enables modelling of elastic plastic material properties and large deformations. The kinematic assumption of the finite element analysis calculation is large displacement and rotation, but small strain. The material yielding stress, \( \sigma_y = 350 \text{ MPa} \), the elastic modulus is \( E = 2.05 \times 10^{11} \text{ Pa} \) and the Poison coefficient is \( \nu = 0.297. \)
Figure 6-1: Finite element model with a mesh pattern (left) and ultimate strength assessment BC (right)

Geometric and material non-linearity has been taken into account. The stress stiffening effect, to avoid sharp post-collapse regime in the ultimate strength assessment, has been accounted for. The applied load is uni-axial compression. Eight-node quadrilateral solid elements have been used to model the plate. Solid 70 and Solid 185 has been used in respectively thermal and structural analysis. The boundary conditions of ultimate strength assessment are shown in Figure 6-1. The material properties have been taken from Gery et al. (2005) for the thermal analysis. The circular moving heat source proposed by Friedman (1975); Krutz and Segerlind (1978) has been adopted to simulate the welding heat source along with the volumetric heat source (see Figure 6-2). The mathematical surface heat source model is expressed as:

\[
q(z, \xi) = \frac{3Q}{\pi c^2} e^{-\frac{(x^2 + \xi^2)}{c^2}}
\]

where \(Q\) is energy input rate and \(c\) is the characteristic radius of surface flux distribution, \(x\) is the transverse distance and \(\xi\) is the moving longitudinal location which is expressed as:

\[
\xi = z + v(\tau - t)
\]

where \(v\) is the welding speed and \(t\) is time and \(\tau\) is a lag factor needed to define the source at time 0.

Figure 6-2: Moving heat source

The total heat input is divided into as shown in Figure 6-2 surface heat flux and volumetric heat flux 40% and 60% respectively. First, the thermal analysis is performed by the moving heat source accounting for the welding sequence. The output temperature distribution is used as a load input in the
subsequent structural analysis to find out the welding residual stress distribution induced by the thermal process. In the next phase, the residual stress distribution is imposed over the stiffened plate along to find out the ultimate load carrying capacity of the stiffened plate. In the thermal analysis, the element birth and death method has been implemented in order to simulate the welding droplets. In this method, physically all the elements are modelled however they are not included into the calculation when they are deactivated by a severe multiplier factor through this method.

6.4 Ultimate strength assessment

6.4.1 Thermal structural analysis

6.4.2 Welding sequence effect

Firstly, the thermal analysis is performed by changing welding sequence to investigate the welding sequence effect on the plate and stiffener distortion and on the welding residual stress distribution. In the second phase, the ultimate strength assessment is conducted for each case by keeping the heat input and heat speed the same. The applied heat input is 5500 W and the heat speed is 5 cm/s.

Figure 6-3: Welding residual stress assessment BC (left) and welding sequence (right)

The analysed welding sequences have been demonstrated in Figure 6-3. All the welding sequences start at the same time. As for the thermal and structural analyses, the appropriate boundary conditions have been applied to achieve a symmetrical distortion and also to avoid rigid body motions (see Figure 6-3). In order to release the system in transverse direction, the opposite side is set to be free in a transverse direction. As for the thermal analysis, the top surface of the welding is considered to be insulated during the welding; the remaining parts of the plate and stiffener have been subjected to the convection and radiation loads. The temperature distribution of mid-plate layer is shown in Figure 6-4.

The welding residual stress equilibrium is locally and globally satisfied as can be seen in Table 6.1. The local equilibrium is satisfied through the cross-section of the stiffened panel. As has been shown in Table 6.1, the local and global welding induced stresses in all direction have been satisfied. All the welding sequences, investigated herein, have been subjected to the equilibrium criteria and similar results have been found in each case. The equilibrium is satisfied with the cross-sectional combination of the plate and stiffener. The longitudinal residual stress distribution for each case is shown on Figure 6-5 and Figure 6-6 and schematic representation of the residual stress equilibrium is given on Figure 6-7. As can be seen, the stiffener is gravitating toward where the welding passes. As for the welding
sequence 2, the stiffener turns about the mid-plate which gives the least ultimate strength (see Figure 6-13).

Figure 6-4: Temperature distribution, mid-plate

Figure 6-5: Longitudinal residual stress distribution for welding sequence 1 (left), welding sequence 2 (right).

Figure 6-6: Longitudinal residual stress distribution for welding sequence 3

Figure 6-7: Residual stress equilibrium
There is no significant change on the plate vertical displacement through the plate thickness (see Figure 6-8). The stiffener starts to rotate about the mid-plane when the welding sequence 2 is being conducted which give the least ultimate load carrying capacity, as for the rest; they are gravitating toward where the plate is fixed in the transverse direction (see Figure 6-8).

Figure 6-8: Vertical displacement, for the welding sequence 1 (left), the stiffener lateral displacement (right)

Figure 6-9: The vertical displacement of the plate (left), The vertical displacement of the mid-plate accounting for the welding sequence, the third layer (right)

Table 6.1: Residual stress equilibrium, welding sequence 1

<table>
<thead>
<tr>
<th>Stresses</th>
<th>Local-mid (Pa)</th>
<th>Global (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x$</td>
<td>$\approx 0.01$</td>
<td>$\approx -0.0073$</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>$\approx 0.00$</td>
<td>$\approx -0.0001$</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>$\approx 0.00$</td>
<td>$\approx 0.0000$</td>
</tr>
</tbody>
</table>

There is no significant change on the plate vertical displacement accounting for the welding sequence;
however the plate edge vertical displacement is somewhat influenced by the welding sequence (see Figure 6-9). Since the energy is swept towards the mid-plate in the sequence 2, there is more vertical displacement present.

The welding induced compressive stresses get smaller through the thickness to meet the equilibrium. There is no significant difference between the welding sequence 1 and 2 but there is a significant difference between welding sequence 3 and the others (see Figure 6-10).

![Figure 6-10: Longitudinal stress distribution through plate breath, the welding sequence 1 (left), the welding sequence, the third layer (right)](image1)

![Figure 6-11: Longitudinal stress distribution through plate length (left), through stiffener height (right)](image2)

The longitudinal stress distribution is quite influenced by the welding sequence. The welding sequence 2 and 3 is more symmetrical in contrast to the welding sequence 1. The equilibrium is not satisfied through single entities, it is satisfied as a combination of plate and stiffener (see Figure 6-11). Figure 6-12 shows the ultimate strength assessment of the stiffened panel accounting for the welding sequence. As can be seen, the welding sequence 2 yields the least the ultimate load carrying
capacity. Figure 6-13 shows the ultimate strength reduction.

![Graph](image)

**Figure 6-12:** Ultimate strength assessment accounting for the welding sequences

![Bar chart](image)

**Figure 6-13:** Ultimate strength variation accounting for the welding sequence

### 6.4.3 Modified stress strain curve accounting for the residual stress

The material stress-strain relation has been established accounting for the best welding sequence in terms of the ultimate load carrying capacity reduction. For this purpose, the welding sequence 1 has been selected. The descriptors for the modified material stress-strain curve have been shown in Table 6.2 where $\sigma_y$ is the yield stress, which for the present study is 350 MPa, $\varepsilon_y = \sigma_y / E$ denotes the material yielding strain and $E = 2.05 \times 10^{11}$ Pa is the material elasticity modulus. As can be seen in Figure 6-14, the response of the structure that has been modelled with the stress-strain modification is almost the same to that of the response of the welding induced residual stresses.
The residual stress effect on the ultimate strength has been investigated through a moving heat source by changing the welding sequence. It has been found that the welding sequence is the most influential parameter that affects the lateral displacement of the stiffener due to a moving heat source which leads to more ultimate load carrying capacity. The vertical displacement of the plate edges are quite influenced by the welding sequence and govern the buckling shape either up or down depending on the welding sequence. The welding residual stress equilibrium is satisfied through cross-section of the stiffened plate. The welding sequence 1 for this particular case has been found the best sequence in terms of the ultimate load carrying capacity and the plate and stiffener distortions. It has been found that the extent of the tensile zone of the stiffener is almost the same to that of the plate.

The developed modified stress-strain curve has been found as a good and fast approach to introduce residual stress that arises during the welding process for a non-linear finite element analysis of ultimate strength.
7 ULTIMATE STRENGTH ASSESSMENT OF A PLATE ACCOUNTING FOR THE EFFECT OF SHAKEDOWN AND CORROSION DEGRADATION

7.1 Introduction

Welding induced distortions and residual stresses lead to a structural strength reduction due to a non-uniform heat distribution and are a major concern in steel construction. Ship structures are subjected to cyclic loadings and because of that, the accumulated residual stresses are reduced through local plastic strains (shakedown) because residual stresses only reside in the rigid part of the structure. In addition to the shakedown effect, the structure is vulnerable to a corrosion degradation, which leads to a plate thickness reduction and in turn, the ultimate load carrying capacity is reduced.

Gannon et al. (2010) investigated the effect of shakedown on the reduction of welding induced stresses and on the improvement of ultimate strength. They concluded that the shake down might increase the ultimate load carrying capacity by 6% and it decreases the tensile and compressive residual stresses as much as 40%, when the applied load causes 50% yielding stress on the structure.

Paik et al. (2005) found that the longitudinal residual stresses were reduced by 36% and 21% in tension and compression respectively when the loading is applying in three cycles, which produces on the extreme fibre stresses equal to 88% of the yield stress.

Ships operate in corrosive environments and due to the interaction with ship structures; it leads to a gradual thickness reduction, which influences the ship durability. Several corrosion models have been developed to predict corrosion degradation in ship structures. Corrosion involves a big amount of uncertainty, which is based on many parameters. It is normally not straightforward to develop a corrosion wastage model solely based on theory, because corrosion is a function of many variables and uncertainties involved, such as the type of the corrosion protection system employed, type of cargo, temperature, humidity, etc. The corrosion models developed based on statistical analysis of operational data will usually be different according to the types of ships and cargoes or structural member locations and categories (Guedes Soares et al. 2008; Guedes Soares et al. 2009; Guedes Soares et al. 2011).

It has been shown on many occasions that the non-linear corrosion wastage model is well accepted in representing realistic situations for steel plates in different areas of ship structures. A non-linear time dependent corrosion model developed by Guedes Soares and Garbatov (1999) has been adopted herein.

7.2 Modified material stress strain definition

The procedure has been detailed in previous chapter 3. Here the material stress-strain curve has been defined based on the shakedown and corrosion. Figure 7-1 shows the several admissible material stress-strain definitions.
7.3 Thermal structural analysis

The structural model analysed here is shown in Figure 7-2. The ultimate strength of the plate is analysed based on finite element method using commercial software ANSYS (2009). The software enables modelling of elastic plastic material properties and large deformations. The kinematic assumption of finite element analysis calculations is large displacement and rotation, but small strain. The material yielding stress, $\sigma_y = 350$ MPa, the elastic modulus is $E = 2.05 \times 10^{11}$ Pa and the Poison coefficient is $\nu = 0.297$ and the plate thickness is 6mm.

First, the thermal analysis is performed. The output temperature distribution is used as a load input in the subsequent structural analysis to find out the welding residual stress distribution induced by the thermal process. In the next phase, the residual stress distribution is imposed over the plate along to find out the ultimate load carrying capacity of the plate accounting for the shakedown level. A formula is developed to account for the welding induced stress reduction as a function of time, which is expressed as:

$$
\sigma(t) = \sigma(0) \exp \left( -\frac{t - \tau_{SD,R}}{\tau_{T,R}} \right)
$$

where $t$ is time, $\tau_{T,R}$ is the specified time at which the welding residual stresses drastically degrease, $\sigma(0) = \sigma_{max}$ is the welding residual stress at time zero which is equal to the material yielding stress in the tensile region. It is important to note that the reduction of the compressive and tensile residual stresses have been considered equal and $\tau_{SD,R}$ represent the time up to where no welding residual stress reduction is present.

The detailed explanation regarding the heat input values and the mathematical model used have been given in chapter 3 (see Figure 7-3).
As for the corrosion wastage model, the one developed by Guedes Soares and Garbatov (1999) has been adopted here:

\[
d_c(t) = \begin{cases} 
  d_c \left[ \exp \left( -\frac{t-t_r}{\tau_c} \right) \right] & t > \tau_c \\
  0, & t \leq \tau_c 
\end{cases}
\]

(71)

\[
\text{StDev}[d(t)] = a\ln(t) - b
\]

(72)

where \( d_c(t) \) is the mean corrosion depth, \( \tau_c \) is the coating life, which is equal to the time interval between the painting of the surface and the time when its effectiveness is lost and \( \tau_r \) is the transition time under average conditions and \( d_\infty \) is the long-term corrosion wastage which is assumed here as 1.85 mm and \( a \) and \( b \) are the coefficient defined as 0.384 and 0.710 for the standard deviation of the corrosion degradation in ballast tanks (Garbatov et al. 2007). Schematic representation of the effect of shakedown and corrosion deterioration is shown in Figure 7-4.

The descriptors of the time dependent functions of shakedown and corrosion degradation are given in Table 7.1. The shakedown effect and the corrosion initiation take place simultaneously in order to find out the lowest load carrying capacity and their interaction. The random corrosion surface of the plate is
generated through the mean value and standard deviation given in Table 7.1 and the rough surface is created after the welding induced distortion has been formed through the welding process. The corrosion has been applied as seen in over the top and bottom layers. The middle layer stays intact. Figure 7-5 shows the corroded plate surface at the 9th year.

Figure 7-4: Residual stress and corrosion depth as a function of time (left) and plate (right)

Figure 7-5: Corroded plate surface at 9th year

<table>
<thead>
<tr>
<th></th>
<th>1st year</th>
<th>2nd year</th>
<th>3rd year</th>
<th>6th year</th>
<th>9th year</th>
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<tr>
<td>Mean value</td>
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<td>N/A</td>
<td>0.04</td>
<td>0.14</td>
<td>0.23</td>
</tr>
<tr>
<td>St Dev.</td>
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<td>N/A</td>
<td>0.71</td>
<td>1.24</td>
<td>1.46</td>
</tr>
<tr>
<td>Residual stress</td>
<td>%100</td>
<td>%100</td>
<td>%57</td>
<td>%10</td>
<td>0</td>
</tr>
<tr>
<td>Coating life</td>
<td>2 years</td>
<td>2 years</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full RS</td>
<td>2 years</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.4 Ultimate strength assessment

7.4.1 Shakedown and corrosion effect

Firstly, a thermal analysis is performed to find out welding residual stresses at time 0. In the second phase, the welding residual stresses are multiplied by the factor derived from the formula given herein accounting for the reduction of residual stresses as a function of time simulating the shakedown effect.
In the third phase, the ultimate strength assessment is estimated over the application of compressive uni-axial load and the welding residual stresses at the respective time. The applied heat input is 7500 W and the heat speed is 3 cm/s.

For the thermal and structural analyses, the appropriate boundary conditions have been applied to achieve a symmetrical distortion and to avoid rigid body motions (see Figure 7-6). In order to release the displacement in the transverse direction, the opposite side is set to be displacement free. As for the thermal analysis, the top surface of the plate is considered to be insulated during the welding and the remaining part of the plate has been subjected to the convection and radiation loads.

Figure 7-6: Welding residual stress assessment BC

Figure 7-7 shows the applied cyclic loadings on the plate that have been used to analyse the shakedown effect on the tensile and compressive stresses along with the longitudinal stress distribution. Figure 7-8 shows the welding residual stress distribution through the plate thickness before the cyclic loadings have been applied and the cyclic loading effect on the welding residual stress. As can be seen, the compressive stresses get larger through the plate thickness to satisfy the equilibrium. The cyclic loading-1 leads to 40% and 22.2% reduction on the tensile and compressive stresses respectively. The cyclic loading-2 results in 63.1% and 27.2% reduction in the tensile and compressive stresses respectively. However, it has been considered that the reduction is proportional for the tensile and compressive stress herein.

Figure 7-7: Longitudinal residual stress distribution (left), Applied cyclic loadings (right)
Through the input data given in Table 7.1, first, the shakedown effect has been investigated. In the shakedown analysis, two types of boundary condition have been implemented (see Figure 7-2). Figure 7-9 shows the stress strain of the plate accounting for both the shakedown effect and boundary change at 1, 2, 3, 6 and 9 year.

Figure 7-8: Longitudinal stress distribution (left), Residual stress loading effect, third layer (right)

Figure 7-9: Normalized stress-strain, shakedown effect, BC-1 (left), BC-2 (right)

Figure 7-10: Ultimate strength, only shakedown
It has been found that as the structure is strengthened through constraining the plate edges, the ultimate strength increases due to the shakedown effect. Figure 7-10 shows that the strength increases as the residual stress goes down. It has been considered that for the first and second year, there is no change on the welding residual stress due to the structural component settlement and because of that, there is no structural load carrying capacity increase.

The time dependent corrosion degradation has been assumed after the welding process has been made. As given in Table 7.1, the structure has full residual stress within a 2 year period and after then it contains both the partial residual stress and corrosion degradation and in the 9th year, only contains the corrosion degradation. Figure 7-11 and Figure 7-12 show the ultimate stress distribution accounting for both the time dependent shakedown and corrosion effects in the respective time.

Figure 7-11: Longitudinal stress distribution of the ultimate state, shakedown and corrosion deterioration effects, at 3rd year (left), at 6th year (right)

Figure 7-12: Longitudinal stress distribution of the ultimate state at 9th year, shakedown and corrosion deterioration effects
As can be seen, the corrosion degradation changes the stress distribution and in turn, the failure location is changing. It has been found that as the time goes by, the structural stress strain changes in terms of the ultimate strength dramatically and its behaviour against the compressive load as can be seen in Figure 7-13.

### 7.4.2 Modified stress-strain curve accounting for the time dependent residual stresses and corrosion degradation

The material stress-strain relation has been established accounting for the time dependent shakedown and corrosion effects. The aim here is to find out the best material stress-strain definition for the intact plate to represent the structural behaviour accounting for the time dependent corrosion and shakedown. It has been found that the residual stresses changes the ultimate load carrying capacity and to some degree the linear behaviour on the other hand, as the time dependent corrosion thickness reduction is introduced. Because of that, the ultimate strength capacity declines significantly and the behaviour of the structure subjected to compressive load changes.

![Figure 7-14: Strength assessment survey](image.png)
The descriptors for the modified material stress-strain curve of the first two years of the plate life have been shown in Table 7.2 where $\sigma_y$ is the yield stress, which for the present study is 350 MPa, $\varepsilon_y = \sigma_y / E$ denotes the material yielding strain and $E = 2.05 \times 10^{11}$ Pa is the material elastic modulus.

Figure 7-14 shows a procedure to find out the structural strength of the component in question. Figure 7-15 show the material stress strain behaviour as a function of time and the mean corrosion depth. It is important to point out that the first two years have not been incorporated due to the material stress-strain definition change. Once the corrosion becomes active, the elastic-perfectly plastic material has been implemented with different tangent modulus depending on the corrosion degradation. In contrast, the welding residual stress does change the linear behaviour to some degree.

As can be seen in Figure 7-16, the response of the structure that has been modelled with the stress-strain modified curve is almost the same as for the one accounting for the effect of the shakedown and corrosion. It has to be noted that the roughness of the surface of the plate changes towards compressive loads. This implies that with the same strain absorption, in the corroded plate, there is less stress development present in relation to the un-corroded plate, which can be expressed as directional tangent elasticity. It also shows the material stress-strain definition defined for the shakedown and corrosion effect.

Table 7.2: Modified stress –strain descriptors

<table>
<thead>
<tr>
<th>Points</th>
<th>Stress (MPa)</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
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<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.94 $\sigma_y$</td>
<td>0.94 $\varepsilon_y$</td>
</tr>
<tr>
<td>2</td>
<td>1.00 $\sigma_y$</td>
<td>1.50 $\varepsilon_y$</td>
</tr>
<tr>
<td>3</td>
<td>1.00 $\sigma_y$</td>
<td>5.85 $\varepsilon_y$</td>
</tr>
</tbody>
</table>

Figure 7-15: Mean corrosion depth, Material yielding stress (left), Material yielding strain (right)
7.5 Chapter conclusions

The time dependent shakedown and corrosion effects on the ultimate strength have been investigated. It has been found that the boundary edge conditions affect the ultimate load carrying capacity through the shakedown effect. The corrosion degradation has significant influence on the ultimate strength leading to different failure locations. The new developed stress-strain curves have been found as a good and fast solution to introduce the time dependent shakedown and corrosion behaviour in a non-linear finite element analysis of ultimate strength.
8 FINAL CONCLUSIONS AND FUTURE WORK

The work just presented here dealt with ultimate strength assessment of plate and stiffened panels. The residual stress effect, modelled as initially induced pre-stresses, on the ultimate strength of steel plates and stiffened panels has been investigated using different structural finite element models. It has been found that the residual stresses decrease the first yielding point of the structure response in turn the ultimate strength. As the structural capacity increases, the effect of residual stresses on the ultimate strength decreases. Initial geometry imperfection increases the strength reduction along with the residual stresses. The residual stress effect on the ultimate strength has been investigated through also a moving heat source by changing the heat input, heat speed and plate thickness. It has been found that the plate thickness is the most influential parameter that affects the vertical displacement due to a moving heat source. As the plate thickness is increasing, the compressive residual stresses start to take over the area occupied by the tensile residual stresses, which leads to more strength reduction. The shape effect of deformed plate results in larger ultimate strength in certain $B/T$ ratio. The speed is more significant to the heat distribution in relation to the residual stress formation. The welding sequence effect on the ultimate strength for a stiffened panel has been also investigated through a moving heat. It has been found that the welding sequence is the most influential parameter that affects the lateral displacement of the stiffener due to a moving heat source leading to more ultimate load carrying capacity. The vertical displacement of the plate edges is quite influenced by the welding sequence and governs the buckling shape either up or down depending on the welding sequence. The welding residual stress equilibrium is satisfied through a cross-section of the stiffened plate. The welding sequence 1 has been found as the best welding sequence in terms of the ultimate load carrying capacity and the plate and stiffener distortions. It has been found that the extent of the tensile zone of the stiffener is almost the same as to that of the plate. The time dependent shakedown and corrosion effects on the ultimate strength have also been investigated. It has been found that the boundary edge conditions affect the ultimate load carrying capacity through the shakedown effect. The corrosion degradation has significant influence on the ultimate strength leading to different failure locations.

The developed admissible stress-strain curve has been found as a good and fast method to introduce residual stress and corrosion degradation effect. It has been found that the modified stress-strain curve descriptors are influenced by the structural configurations and boundary conditions at the edges.

The present work is not concerned with the material stress strain curve definition under tension, accounting for different imperfection, residual stresses and structural degradation, and it will be a good step further to accomplish the full definition of the stress stain material curve for a non-linear finite element analysis.
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REFERENCES


