Switching in Fiber Optic Systems

Daniel Filipe Ferreira dos Anjos
Departamento de Engenharia Electrotécnica e de Computadores
Instituto Superior Técnico
Av.Rovisco Pais 1, 1049-001 Lisboa, Portugal

ABSTRACT

This paper addresses one of the keys aspects of optical communications systems: optical switching. Pulse propagation in a linear regime and the effect of group velocity dispersion are analyzed. A numerical analysis method to simulate the propagation of several types of pulses along an optical fiber is developed.

Dispersion management, through the use non-linearity is addressed. The behavior of solitons, pulses with very specific characteristics, is studied when propagating along the fiber. An iterative model has been developed using the SSFM which allows get the signal output based on their initial conditions. With this model the fundamental, 2nd and 3rd order solitons are studied. Two other relevant points are studied: the interference generated by interaction between solitons and the mean soliton.

Finally, optical switching is discussed. The coupling equations are derived and the transmission coefficients are obtained, being, and a half-beat coupler is studied. Optical switching is analyzed both in the linear and the nonlinear regimes. The nonlinear coupling equations and a new method for calculation of the transmission coefficients between the two fibers is presented. The influence of intermodal dispersion on the pulse switching is considered in the linear and the nonlinear regime.

Index Terms- Optical switching, chirp, group velocity dispersion, soliton, SSFM, transmission coefficients, coupling, half-beat.

1-INTRODUCTION

One of the areas with the biggest technological developments is long distance communications. Nowadays, the most commonly used forms to communicate are mobile phones, internet and television.

A fiber optic network can be divided in three parts: transmitter, transmission cable and receiver, with need of amplification in long distance networks.

Nowadays almost every system is based on fiber optics. Hence the study of signal propagation across the fiber is a very important issue.

In 1984, Jonh Tyndall performed the first experiments related with light conduction through water thus proving it is possible guided propagation of optical signs. After several inventions related with optics over the years only in 1956 the fiber optic term was defined as a physical transmission medium in which information is transported in the form of light pulses.

A major advance to the use of optical transmission was the invention of the laser (Light Amplification by stimulated Emission of Radiation) in 1957, this device being a source of intense light that produces a monochromatic and coherent electromagnetic radiation.

In 1966, Kao and Hockman published a proposal to use fiber optics as the transmission medium for an optical signal from a laser, in cases when attenuation is less than 20 dB/km. Unfortunately the results of this study revealed that, largely due to impurities in the glass used, attenuations were in the orders of 1000 dB/km.

In 1970, with developments in the manufacture made by Corning Glass Works was made one single mode fiber with an attenuation of 16 dB/km working on a wavelength of 630 nm.

Over the years there have been several generations of fiber optics systems, the first one, in the 1980s, with single mode fibers that operated in the first window (800-900nm) with a bit rate around 45 MB/s and spacing between repeaters of about 10 km.

In 1987, appears the second generation, operating in the second window (1260-1360 nm) where the attenuation is below 1 dB/km and the dispersion is minimal. With this generation was obtained a bit rate around 1.7 GB/s with spacing between repeaters of 50 km. The first submarine fiber optic cable was installed in 1988 and named TAT-8 (Transatlantic Telecommunication Cable) operating in 1.3 μm with repeaters every 70 km.

The third fiber optics system generation appears in 1990, operating in the third window (1500-1600 nm) and reaching a bit rate of 10 Gb/s, this generation uses 3R’s repeaters making rescaling, reshaping and retiming of the signal, having the need making a transition from optical to electronic, being this system not totally optical. Because of this, only the appearing of optic amplifiers transformed the fiber optics system in truly photonic systems. The most important technology for optical amplification in this generation was the doped amplifier fiber, highlighting the EDFA’s (erbium doped fiber amplifiers) that allowed increased spacing between repeaters to 60-100 km.

Now, we are in the fourth generation using optical amplification to reduce the need for repeaters and wavelength-division multiplexing to increase data.
capacity, we can achieve bit rates in the order of terabits per second. In 1966 appeared the first submarine cable of fourth generation (TPC-5-Trans-Pacific Cable) operated in 1.55 μm reaching bit rates around 5.30 GB/s.

The next technological move will be the arrival of the fifth generation of communication systems where the main problem is mitigate the signal dispersion, we have various ways to reach this goal: dispersion compensation, dispersion management and soliton systems, largely approached on this thesis and a solution with potential to be in the start of the fifth generation.

\[ L_D = \frac{\tau_0^2}{|\beta_2|}, \quad (2.6) \]

\[ \tau = \frac{t - \beta_1 z}{\tau_0}, \quad (2.7) \]

\[ \zeta = \frac{z}{L_D}, \quad (2.8) \]

Where \( L_D \) is dispersion length, \( \tau \) and \( \zeta \) are the normalization of time and space respectively. Then we have the following normalize equations

\[ \frac{\partial A}{\partial z} = i \frac{1}{2} \beta_2 A - \frac{\partial A}{\partial \tau} + \frac{1}{\tau_0} \frac{\partial A}{\partial \tau} = \frac{1}{\tau_0} \beta_4 \frac{\partial A}{\partial \tau}, \quad (2.9) \]

\[ \frac{\partial A}{\partial \tau} = \frac{1}{\tau_0} \frac{\partial A}{\partial \tau} = \frac{1}{\tau_0} \frac{\partial A}{\partial \tau}, \quad (2.10) \]

And also

\[ \frac{\partial^2 A}{\partial \tau^2} = \frac{1}{2} \frac{\partial^2 A}{\partial \tau^2} = \frac{1}{4} \frac{\partial^2 A}{\partial \tau^2} = \frac{1}{2} \frac{\partial^2 A}{\partial \tau^2}, \quad (2.11) \]

With that we can write a linear equation in a new form

\[ \frac{\partial A}{\partial \zeta} + \frac{1}{2} \beta_2 A + \frac{1}{4} \beta_4 A \frac{\partial A}{\partial \zeta} - \kappa \frac{\partial^3 A}{\partial \tau^3} = 0 \quad (2.13) \]

Where

\[ \beta_2 = |\beta_2| \text{sign}(\beta_2), \quad (2.14) \]

\[ \kappa = \frac{\beta_3}{6 |\beta_2| \tau_0}, \quad (2.15) \]

When we apply the Fourier transformation to (2.13) the equation is the following in the frequency domain

\[ \frac{\partial A}{\partial \xi} = i \left[ \frac{1}{2} \text{sign}(\beta_2) \xi^2 - \kappa \xi^3 \right] \tilde{A}(\xi, \eta), \quad (2.16) \]

With the normalization of the frequency

\[ \xi = \Omega \tau_0 = (\omega - \omega_0) \tau_0, \quad (2.17) \]

The equation (2.16) has the following solution

\[ \tilde{A}(\xi, \eta) = \tilde{A}(0, \eta) e^{i \left[ \frac{1}{2} \text{sign}(\beta_2) \xi^2 - \kappa \xi^3 \right]} \eta, \quad (2.18) \]
Thus for the numerical resolution of signal propagation we just need to follow this steps

\[
\tilde{A}(0, \xi) = FFT[A(0, \tau)] \tag{2.19}
\]

\[
\tilde{A}(\zeta, \xi) = \tilde{A}(0, \xi)e^{\frac{1}{2}m(\zeta - \zeta_0)^2} \tag{2.20}
\]

\[
A(\zeta, \tau) = IFFT[\tilde{A}(\zeta, \xi)] \tag{2.21}
\]

2.1-Numerical Simulation

2.1.1-Gaussian pulse

The general form of a super gaussian pulse is

\[
A(0, t) = e^{-\frac{t^2 + 4m^2}{2}} \tag{2.23}
\]

Where C is chirp parameter and m represents how fast the pulse reaches its maximum. When we have m=1 and C=0 we are in presence of a Gaussian pulse.

\[
A(0, t) = e^{-\frac{1}{2m^2}t^2} \tag{2.24}
\]

We obtain the following simulation

![Figure 2.3-Gaussian pulse at input and at output of fiber](image3.png)

![Figure 2.4- Gaussian pulse evolution in time and along the fiber (3D).](image4.png)

2.1.2-Super gaussian pulse

In order to study the effect of chirp parameter first we need to simulate super Gaussian pulse without chirp, that means m=3 and C=0

\[
A(0, t) = e^{-\frac{1}{2}t^2} \tag{2.25}
\]

For this simulation the value of \( \zeta \) was change to 2 instead of 10 to make easier to identify dispersion effect.

![Figure 2.5- Super Gaussian pulse at input and at output of fiber.](image5.png)

![Figure 2.6- Super Gaussian pulse evolution in time and along the fiber (3D).](image6.png)

2.1.3- Chirp effect

Now we can introduce chirp parameter in input pulse, so with C=2 we have

![Figure 2.7- Super Gaussian pulse with C=2 at input and at output of fiber.](image7.png)
On other hand with a negative chirp (C=-2) we get

In case of negative chirp, the broadening effects are exacerbated, this makes it impossible to use this solution. For a positive chirp there is a compensation of dispersive effects at the beginning of propagation, and then from a certain point aggravated these effects, it makes this solution only beneficial for short distance fibers.

The enlargement of the pulses due to the dispersion has the effect of limiting the bitrate allowed on the fiber, since it is conditioned by intersymbol interference. This enlargement depends on several factors: the spectral width of the semiconductor laser, the initial width of the pulses and the dispersion, mainly group velocity dispersion.

Enlargement of the pulses being defined by

\[
\left( \frac{\sigma}{\sigma_0} \right)^2 = \left( 1 + C \frac{\beta_2 L}{2 \sigma_0^2} \right)^2 + \left( \frac{\beta_2 L}{2 \sigma_0^2} \right)^2
\]  

(2.26)

Is then possible to show progress on enlargement of pulses for different values of chirp.

3-PULSE PROPAGATION ON NONLINEAR REGIME

When a wave propagates in time, unchanged and collision immune is considered a soliton, what happens is a balance between the dispersive effects, which lead to a decrease of amplitude and pulse width extension, with the nonlinear effects, responsible for the compression of the width and increasing the amplitude.

In fiber optics this commitment is reached between the group velocity dispersion (DVG) and Self-Phase Modulation (AMF), in the region of anomalous dispersion and in the absence of losses, they work together to keep unchanged the shape of the pulse.

3.1-Propagation Equation on Nonlinear Regime

The equation that governs the propagation in this regime is

\[
\frac{\partial A}{\partial z} + \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial t^3} + \frac{1}{\sigma_0} \frac{\partial A}{\partial t} = - \frac{\sigma}{2} A + i \gamma |A|^2 A
\]  

(3.1)

This being the nonlinear Schrödinger equations (NLS) which describes the propagation of an optical pulse on the effect of the losses, the group velocity dispersion and the nonlinearity of the fiber.

To our case we don’t have losses and the higher-order dispersion is neglected. So the equation is

\[
\frac{\partial A}{\partial z} + \beta_2 \frac{\partial^2 A}{\partial t^2} + i \frac{\beta_3}{2} \frac{\partial^3 A}{\partial t^3} = i \gamma |A|^2 A
\]  

(3.2)

When we changed variables as shown in [3], we get

\[
i \frac{\partial U}{\partial \zeta} + \frac{sgn(\beta_2)}{2} \frac{\partial^2 U}{\partial \zeta^2} + N^2 |U|^2 U = 0
\]  

(3.3)
3.2-Self-Phase Modulation

Pulses which propagate in an optical fiber is subject to nonlinear effects, ie, the response varies with the system input signal.

The nonlinear phase generated by the Kerr optical effect is given by

\[ \phi_{NL} = \gamma P_{in}(t) L_{eff} \]  \hspace{1cm} (3.4)

is called self-phase modulation.

The frequency shift caused by SPM is given by

\[ \delta \omega(t) = -\frac{d\phi_{NL}}{dt} = -\gamma L_{eff} \frac{dP_{in}}{dt} \] \hspace{1cm} (3.5)

So there is a redshift in front of the pulse

\[ \frac{dP_{in}}{dt} > 0 \Rightarrow \delta \omega(t) < 0 \]  \hspace{1cm} (3.6)

And a shift to the blue in the tail of the pulse

\[ \frac{dP_{in}}{dt} < 0 \Rightarrow \delta \omega(t) > 0 \]  \hspace{1cm} (3.7)

3.3-Split Step Fourier Method

For the simulation in Matlab of nonlinear regime will be used the iterative method SSFM. This method follows these steps:

\[ u_0(\tau) = \text{sech}(\tau) \] \hspace{1cm} (3.9)

Getting the following results in the simulation:

![Figure 3.2-Fundamental soliton evolution in time and along the fiber (3D).](image)

Is possible to verify that the pulse keeps its shape when it propagates along the fiber, does not occur any change in the amplitude and pulse width. Is possible then verify that the GVD is totally compensated by SPM.

3.4.2-Second-order soliton

With N=2 we have a second-order soliton

\[ u_0(\tau) = 2\text{sech}(\tau) \] \hspace{1cm} (3.10)

![Figure 3.3- Second-order soliton evolution in time and along the fiber (3D).](image)

It is possible to observe that characteristic of the pulse varies along the propagation through the fiber, nevertheless periodically the impulse returns to its original shape. In this case the soliton period is \( \frac{\pi}{2} \).

3.4.3- Interaction between solitons

Is important to study the interaction between solitons, since a pulse do not propagate alone through the fiber. To analyze the interaction between solitons in the same channel should be considered as input signal

\[ U(0, t) = N\text{sech}(t) \]  \hspace{1cm} (3.8)

Where N is the soliton order, then for N=1 we have the fundamental soliton
\[ u_0(x) = \text{sech}(x - q_0) + r \text{sech}[r(x + q_0)] e^{i\theta} \] (3.11)

Where \( q_0 \) is half of the normalize temporal separation, \( r \) is the relation between solitons amplitudes and \( \theta \) is the difference between solitons phases. Will be analyzed different values for these parameters

**3.4.3.1- First case: \( \theta=0, r=1 \)**

**Figure 3.4-Interaction between solitons (first case 3D view)**

In this case the two solitons are in phase and have the same amplitude, it can be concluded that periodically solitons overlap causing amplitude peaks. This is not a good solution for telecommunications due to possible misinterpretation of the signal in areas of overlap.

**3.4.3.2- second case: \( \theta=\frac{\pi}{4}, r=1 \)**

**Figure 3.6-Interaction between solitons (second case 3D view)**

Considering a phase shift of \( \theta = \frac{\pi}{4} \) and signals with the same amplitude. At start occurs an approximation of signals, but then the distance between the solitons increases as they propagate through the fiber. The solutions with phase shift do not present themselves as viable in a real system since the distance increases can lead to increased intersymbol interference.

**Figure 3.7- Interaction between solitons (second case top view)**

**3.4.3.3- Third case: \( \theta=0, r=1.1 \)**

**Figure 3.8- Interaction between solitons (third case 3D view)**

In this example there is no phase shift, but the signals have different amplitudes, is possible to verify
that the solitons do not converge or move away indefinitely. In theory this is the best solution to implement in a real system.

4- OPTICAL SWITCHING

4.1- Coupling between Two Identical Fibers

Will be analyzed the coupling between two cores identical and parallel, of radius \( a \) in a monomodal linear regime.

According to the theory of scalar coupling the total longitudinal field is given by

\[
E(x, y, z, t) = B_1(x, t)F_1(x, y) + B_2(x, t)F_2(x, y)
\]  

(4.1)

According [7], it is also important emphasize the relationship between fields in the fiber at position \( z \) and intensity of the input signal.

\[
\begin{bmatrix}
\tilde{B}_1(z, \omega) \\
\tilde{B}_2(z, \omega)
\end{bmatrix} = 
\begin{bmatrix}
\cos[C(\omega)z] & i\sin[C(\omega)z] \\
i\sin[C(\omega)z] & \cos[C(\omega)z]
\end{bmatrix}
\begin{bmatrix}
\tilde{B}_1(0, \omega) \\
\tilde{B}_2(0, \omega)
\end{bmatrix}
\]  

(4.2)

In this way considering a coupler length \( L_C \) and a input signal at fiber \( \tilde{B}_1(0, \omega) = 0 \), it is possible establish the following transmission coefficients

\[
t(L_C, \omega) = \left| \frac{\tilde{B}_1(L_C, \omega)}{\tilde{B}_1(0, \omega)} \right|^2 = \cos^2[C(\omega)L_C]
\]  

(4.3)

\[
t_a(L_C, \omega) = \left| \frac{\tilde{B}_1(L_C, \omega)}{\tilde{B}_1(0, \omega)} \right|^2 = \sin^2[C(\omega)L_C]
\]  

(4.4)

It is possible then simulate transmission coefficients for a half-beat coupler.

4.2- Optical Switching in linear regime

In our case, only one of the fibers has input signal

\[
A_1(0, \tau) = e^{-\frac{\tau^2}{2}}, \quad A_2(0, \tau) = 0
\]  

(4.5)

The starting point for numerical simulation is the coupling equations between two optical fibers.

\[
\begin{aligned}
\frac{\partial A_1}{\partial \tau} + \delta \frac{\partial A_2}{\partial \tau} + \frac{1}{2} [\operatorname{sgn}(\beta_2) \frac{\partial A_1}{\partial \omega} - \mu \frac{\partial A_2}{\partial \omega}] &= ikA_2 \\
\frac{\partial A_2}{\partial \tau} + \delta \frac{\partial A_1}{\partial \tau} + \frac{1}{2} [\operatorname{sgn}(\beta_1) \frac{\partial A_2}{\partial \omega} + \mu \frac{\partial A_1}{\partial \omega}] &= ikA_1
\end{aligned}
\]  

(4.6)

After some manipulation of equations, according to [6], we obtain a set of steps that allow the simulation of the switching in linear regime.

**Step 1**

\[
\tilde{A}_1(0, \zeta) = \text{FFT}[A_1(0, \tau)]
\]  

(4.7)

**Step 2**

\[
\tilde{A}_1(\zeta, \xi) = e^{i\frac{\theta}{2}\operatorname{sgn}(\beta_2^2\xi^2)}\cos[\theta(\zeta, \xi)]\tilde{A}_1(0, \zeta)
\]  

(4.8)

\[
\tilde{A}_2(\zeta, \xi) = e^{i\frac{\theta}{2}\operatorname{sgn}(\beta_1^2\zeta^2)}\sin[\theta(\zeta, \xi)]\tilde{A}_1(0, \zeta)
\]  

(4.9)

**Step 3**

\[
A_1(\zeta, \tau) = \operatorname{IFFT}[\tilde{A}_1(\zeta, \xi)]
\]  

(4.10)

\[
A_2(\zeta, \tau) = \operatorname{IFFT}[\tilde{A}_2(\zeta, \xi)]
\]  

(4.11)

Obtaining the following results.
The most important conclusion to be drawn is that when the fiber 1 has a null signal, the signal on fiber 2 is maximum, and vice versa. This transient behavior can be explained due to the influence of the electromagnetic field of the signal in a fiber has in other fiber and also due the principle of energy conservation.

4.3- Optical Switching in Nonlinear Regime

In the nonlinear regime, the propagation is governed by the coupling nonlinear equation which takes account Kerr nonlinear effect.

\[ i \left( \frac{\partial u_1}{\partial \zeta} + \delta \frac{\partial u_1}{\partial \tau} \right) + \frac{1}{2} \left( \frac{\partial^2 u_1}{\partial \tau^2} - \mu \frac{\partial^2 u_2}{\partial \tau^2} \right) + |u_1|^2 u_1 + ku_2 = 0 \]  
(4.12)

\[ i \left( \frac{\partial u_2}{\partial \zeta} + \delta \frac{\partial u_2}{\partial \tau} \right) + \frac{1}{2} \left( \frac{\partial^2 u_2}{\partial \tau^2} - \mu \frac{\partial^2 u_1}{\partial \tau^2} \right) + |u_2|^2 u_2 + ku_1 = 0 \]  
(4.13)

Using the normalized amplitude

\[ u_n(\zeta, \tau) = N \frac{\Delta_n(\zeta, \tau)}{\sqrt{P_0}} \]  
(4.14)

And normalize coupling coefficient

\[ k = C_{DP} \]  
(4.15)

In the above equations is not considered intermodal dispersion, if we take this into account, we have the following equations of propagation

\[ i \left( \frac{\partial u_1}{\partial \zeta} + \delta \frac{\partial u_1}{\partial \tau} \right) + \frac{1}{2} \left( \frac{\partial^2 u_1}{\partial \tau^2} - \mu \frac{\partial^2 u_2}{\partial \tau^2} \right) + |u_1|^4 u_1 + ku_2 = 0 \]  
(4.16)

\[ i \left( \frac{\partial u_2}{\partial \zeta} + \delta \frac{\partial u_2}{\partial \tau} \right) + \frac{1}{2} \left( \frac{\partial^2 u_2}{\partial \tau^2} - \mu \frac{\partial^2 u_1}{\partial \tau^2} \right) + |u_2|^4 u_2 + ku_1 = 0 \]  
(4.17)

Where the coupling coefficients of the first and second order are, respectively

\[ \delta = \frac{C_{1D}}{\tau_0} \]  
(4.18)

And

\[ \mu = \frac{C_{2D}}{\tau_0} \]  
(4.19)

4.4- Soliton Switching with Different Wavelengths

Defining the transmission coefficient

\[ T = \frac{1}{Q} \int_{-\infty}^{+\infty} |u_1(\zeta, \tau)|^2 d\tau \]  
(4.20)

Where

\[ Q = \int_{-\infty}^{+\infty} (|u_1(\zeta, \tau)|^2 + |u_1(\zeta, \tau)|^2) d\tau \]  
(4.21)

Represents total energy.

Figure 4.5- Transmissivity as function of normalized input potency for different wavelengths.
4.5 - Influence of Intermodal Dispersion in Switching Soliton

To study the effects of intermodal dispersion in solitons switching in WDM systems we analyzed behavior to an input

\[ u_1(0, \tau) = \sqrt{\text{psech}(\sqrt{\pi r})} \sum_{k=1}^{3} e^{-i\xi_k \tau} \]  \hspace{1cm} (4.22)

With \( u_2(0, \tau) = 0 \) and for \( \xi_1 = \xi(\lambda = 1.52 \mu m) \), \( \xi_2 = \xi(\lambda = 1.55 \mu m) \), \( \xi_3 = \xi(\lambda = 1.58 \mu m) \). Obtaining, at the output of fiber 2, the results shown in the next sections.

4.5.1 - Linear Regime

We will analyze the response with and without dispersion intermodal

| Figure 4.6 - |u2| in linear regime without intermodal dispersion |
| Figure 4.7 - |u2| in linear regime with intermodal dispersion |

Is observable in Figure 4.6, under a linear regime without dispersion, the existence of a perfect signal switching. On the other hand in figure 4.7, is noticeable the effect of intermodal dispersion mainly in frequency adjacent to the central.

4.5.1 - Nonlinear Regime

| Figure 4.8 - |u2| in nonlinear regime without intermodal dispersion |
| Figure 4.9 - |u2| in nonlinear regime with intermodal dispersion |

Is easily observable the difference in the results with and without intermodal dispersion in nonlinear regime. Because there are so obvious change in this regime, becomes obvious that one can not disregard the intermodal dispersion in the nonlinear propagation equations.
5- CONCLUSIONS

The main conclusion to be drawn on the propagation in the linear regime has to do with the influence of intersymbol interference on pulse propagation, this being a consequence of the enlargement of the pulses and the dispersion phenomenon.

The influence of the chirp parameter and how it can compensate the dispersion was verified. At the pulse generation the influence of a positive chirp produces a smaller enlargement, until a certain point. However at the end of the fiber the best solution is a pulse without chirp.

A method that allows solving propagation equations on a nonlinear regime was presented. In the absence of losses, the fundamental soliton shows the ideal characteristics for an optical telecommunications system, there is a total commitment between self-phase modulation and the group velocity dispersion. The behavior of second and third order solitons is not suitable for optical communication.

The interaction between solitons, with the phase and amplitude relation was studied. The best solution was the one which used different amplitudes. Although the distance between pulses is random, its variation is limited.

From this analysis it is possible to conclude that the use of solitons has tremendous advantages, but some limitations that should be taken into account when of its use in optical communications.

Finally pulse switching between two identical optical fibers was addressed. We analyzed the effect of intermodal dispersion in soliton switching at different wavelengths. The main conclusion is that it is not possible to disregard the intermodal dispersion in the equations that define the coupling dynamic.

The soliton propagation in the central frequency in a half-beat coupler can be analyzed despising the intermodal dispersion.

A model without considering IMD cannot be used in analysis of soliton switching in WDM systems, since there is a variation coefficient $k$ as the frequency channel changes. A Model with IMD is very important in the analysis of a system that operates with multiplexers and desmultiplexers.

REFERENCES