Optimal Power Flow Including Wind Generation

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Abstract— Over the past years the inclusion of renewable energy sources in the Electrical Energy Systems (EES) has increased, with special emphasis to the intermittent renewable energy sources, such as wind, because they create a new challenge in the system operation optimization.

Thus the EES is gradually complex, requiring the development of tools to solve the Power Flow (PF), minimizing operating costs and maximizing the renewable energy advantages.

The purpose of this study is to develop software that performs the Optimal Power Flow (OPF) with inclusion of wind generation and applying several operational constraints, studying the impact in the system and in the operating costs.

Two different algorithms have been developed for solving the OPF (Newton and Interior Point Algorithm). They have been analyzed in order to pick up the one with best performance.

The software has been tested using several scenarios of generation and wind load distribution.

Index Terms— Optimal Power Flow, wind Energy, Newton, Interior Point, cost minimization, constrains.

1. INTRODUCTION

BEFORE the optimal power flow (OPF), the EES optimization had been performed by using the Economical Dispatch (ED), however the ED is a simplified and restricted method because it only respects the following equation:

\[ \text{Total Generation} = \text{Load} + \text{Losses} \]

where the reactive power influence is neglected, the voltages in all buses are considered as constants and the operational constrains that affect the system operation aren’t considered.

With the computational advances, the OPF was created. Using a large number of equations, the OPF solves the mentioned issues by adding to the ED the Power Flow (PF) equations and the operational constrains.

The OPF problem can be expressed as a classical mathematical formulation:

\[
\begin{align*}
\text{minimize } & F(\mathbf{x}, \mathbf{u}, \mathbf{p}) \\
\text{s.t. } & g(\mathbf{x}, \mathbf{u}, \mathbf{p}) = 0 \\
& h(\mathbf{x}, \mathbf{u}, \mathbf{p}) \leq 0
\end{align*}
\]

where \( F \) is the objective function, \( g \) represents the equality constrains and \( h \) the inequality constrains.

The arrays \( \mathbf{u}, \mathbf{x} \) and \( \mathbf{p} \) are respectively, the control, state and fixed variables.

The system operation limits may be from several types, such as:

- \( P_g \) Active power generated by thermal units
- \( Q_g \) Reactive power generated
- \( w_g \) Power generated by wind power generation units
- \( w_{aw} \) Rated power of wind power generation units
- \( C \) Thermal cost function
- \( C_w \) Wind power generation cost function
- \( C_p \) Penalty cost function for not using all available power from wind power generator
- \( C_r \) Required reserve cost function, relating to uncertainty of wind power
- \( k_p \) Penalty cost coefficient
- \( k_r \) Reserve cost coefficient

NOMENCLATURE

- \( n \) Number of generators
- \( n_g \) Number of thermal generation units
- \( n_w \) Number of wind power generation units
- \( n_p \) Number of buses
- \( l \) Number of inequality constrains
- \( F \) Objective Function
- \( \mathbf{g} \) Set of equality constrains
- \( \mathbf{h} \) Set of inequality constrains
- \( \mathbf{L} \) Lagrangean /Augmented cost function (without inequality constrains)
- \( \mathbf{L}^* \) Lagrangean /Augmented cost function (with inequality constrains)
- \( \mathbf{x} \) State variables
- \( \mathbf{u} \) Control variables
- \( \mathbf{p} \) Fixed variables
- \( \mathbf{\lambda} \) Lagrange multipliers referred to equality constrains
- \( \mathbf{\pi} \) Lagrange multipliers referred to inequality constrains
- \( \mathbf{\mu} \) Kuhn-Tucker coefficients
- \( \mathbf{J} \) Jacobian
- \( x_{\text{max}} \) Superior limit of variable \( x \)
- \( x_{\text{min}} \) Inferior limit of variable \( x \)
- \( s_{\text{max}} \) State variable (superior limit)
- \( s_{\text{min}} \) State variable (inferior limit)
- active and reactive generation power;
- voltage absolute value;
- flows on transmission lines and transformers;
- transformation ratio;

In this work will be used the first two limits listed above referring to the inequality constraints:

- Active and reactive generation limits:
  \[ p_{g_i}^{\text{min}} \leq p_{g_i} \leq p_{g_i}^{\text{max}} \quad i = 1, 2, \ldots, n_g \]  
  \[ q_{g_i}^{\text{min}} \leq q_{g_i} \leq q_{g_i}^{\text{max}} \quad i = 1, 2, \ldots, n_g \]  

- Voltage limits:
  \[ v_{i}^{\text{min}} \leq v_i \leq v_{i}^{\text{max}} \quad i = 1, 2, \ldots, n_b \]

The active and reactive power balances are given by (5) and (6) which gives the equality constrains.

\[ P_i - P_{g_i} + P_{c_i} = 0 \quad i = 1, 2, \ldots, n_b \]  
\[ Q_i - Q_{g_i} + Q_{c_i} = 0 \quad i = 1, 2, \ldots, n_b \]  

Where \( P_i \) and \( Q_i \) are the active and reactive injected power in bus \( i \):

\[ P_i = V_i \sum_{j=1}^{n_b} V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \]  
\[ Q_i = V_i \sum_{j=1}^{n_b} V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \]

The integration of wind generation in the OPF is an interesting challenge because this energy source has a different behavior compared to thermal generation. Its intermittency and unpredictability are to be taken into account, because it will influence the unit’s allocation. Therefore, to measure the wind power additional costs related to underestimation and overestimation, the addition of penalties will be considered.

The cost curves of wind power generators may be modeled differently from the thermal ones, and thus can cause difficulties in terms of OPF convergence.

II. POWER FLOW METHODS

A. Newton Method

The Newton method presents a fast convergence and a low computational requirements, although, is slightly difficult to deal with inequality constrains.

The problem formulation is the same presented in (1), but now there isn’t any distinction between state and control variables [1], [2].

To compute this method is necessary to create an expanded cost function based on objective function using lagrange multipliers:

\[ L([x]) = F([x]) - [\lambda]^T [g([x])] \]  

where \([x]\) represents the set of all variables and \([\lambda] = [\lambda_p, \lambda_q]\) the Lagrange multipliers.

Expanding the equation (9) using the (5) and (6) the Lagrangean is given by:

\[ L = F - \sum_{i=1}^{n_b} \lambda_p (P_{g_i} - P_{c_i} - P_i) - \sum_{i=1}^{n_b} \lambda_q (Q_{g_i} - Q_{c_i} - Q_i) \]

When the variables limits are inflicted is required the respective constraint activation. This can be done by using the Kuhn-Tucker approach.

The Kuhn-Tucker conditions are presented in (11).

\[ h(x) = \begin{cases} x - x_{\max} \leq 0 \\ x_{\min} - x \leq 0 \end{cases} \]  

Applying the conditions above in the Lagrangean the equation (12) is obtained.

\[ L^* = L + [\mu_{\max}]^T [h(x)]^+ + [\mu_{\min}]^T [h(x)]^- \]

where \(\mu_{\max}\) and \(\mu_{\min}\) are the upper and lower level Kuhn-Tucker coefficients.

Expanding the equation (12) we got the equation (13).

\[ L^* = L + \sum_{i=1}^{n_g} \mu_{p_{\max}} (p_{g_i} - p_{g_i}^{\text{max}}) + \sum_{i=1}^{n_g} \mu_{p_{\min}} (p_{g_i}^{\text{min}} - p_{g_i}) + \sum_{i=1}^{n_g} \mu_{q_{\max}} (q_{g_i} - q_{g_i}^{\text{max}}) + \sum_{i=1}^{n_g} \mu_{q_{\min}} (q_{g_i}^{\text{min}} - q_{g_i}) + \sum_{i=1}^{n_b} \mu_{v_{\max}} (v_{g_i} - v_{g_i}^{\text{max}}) + \sum_{i=1}^{n_b} \mu_{v_{\min}} (v_{g_i}^{\text{min}} - v_{g_i}) \]

The set of variables in this method is given by (14).

\[ [x] = \left[ P_G \; Q_G \; \lambda_p \; \lambda_q \; \theta \; V \; \mu_{p_{\max}} \; \mu_{p_{\min}} \; \mu_{q_{\max}} \; \mu_{q_{\min}} \; \mu_{v_{\max}} \; \mu_{v_{\min}} \right]^T \]

The optimization is made through an iterative process described by (13).

\[ [x^{k+1}] = [x^k] - [\nabla^2 L([x^k])]^{-1} \nabla L([x^k]) \]

where \([\nabla^2 L([x^k])] = \left[ \frac{d^2 L}{dx^2} \right] = [J]\) is the Jacobian of \(L\), \(\nabla L = \left[ \frac{dL}{dx} \right]\), and \(k\) the iteration number.
So, the difference between the old and the new value of x is given by (16).

\[
[\Delta x^{k+1}] = [x^{k+1}] - [x^k] = -[J]^{-1} \times \left[ \frac{\partial L}{\partial x} \right]
\]  

(16)

The matrix J, besides having very large dimension for large networks, is sparse because it has many null elements. This sparsity increases significantly with the number of violated constraints.

In this work, the OPF problem had been solved by the Newton approach using the following steps:

1. Initial PF conditions.
2. If necessary, order the admittance matrix to respect the nomenclature (generators in the first buses).
3. Compute \([J]\) and \(\frac{\partial L}{\partial x}\).
4. Compute \([\Delta x]\).
5. \([x^{k+1}] = [x^k] + [\Delta x^k].\)
6. Check if the new values are respecting the Kuhn-Tucker conditions. If so go to step 9.

7. Reconstruct \([J]\) and \(\frac{\partial L}{\partial x}\) adding the Kuhn-Tucker equations regarding the inflicted limits.
8. If there are fixed variables from previous iterations it is tested the feasibility of their re-entry in the Jacobian. If the respective Kuhn-Tucker coefficient is negative the variable is re-integrated. Go to step 4.

9. If \([\Delta x] \leq [\text{error}]\) the convergence is obtained. Else go to step 3.

In the eighth point, all variables previously fixed (already out of the Jacobian), including their Kuhn-Tucker coefficients, are reintegrated in the current conditions through an external function, being then calculated the auxiliary vectors \([\Delta x_{\text{aux}}]\) and \([x_{\text{aux}}]\). If a variable previously fixed no longer infringe the limits, is reintegrated in the Jacobian. This step is done to avoid performing all possible combinations of variables integration, since this would result in a high computational cost, resulting in a very slow process.

**B. Interior Point Algorithm**

The Interior Point Algorithm is a very fast OPF method that deals very well with inequality constraints.

It is called an Interior point the one where all variables are within their limits.

In this work is used the direct application of this method, which involves the transformation of inequality constrains in equality constrains in a nonlinear problem using slack variables.

The slack variables are defined as (17) and (18) [2]:

\[
s_i^{\text{max}} = h_i^{\text{max}}(u, x) = [x] - [x_{\text{max}}]
\]

(17)

\[
s_i^{\text{min}} = h_i^{\text{min}}(u, x) = [x_{\text{min}}] - [x]
\]

(18)

where \(s_i^{\text{min}}\) and \(s_i^{\text{max}}\) are the slack variables referred to the inferior and superior limits respectively.

To keep the nonnegativity of these variables is added a barrier function that is called “logarithmic barrier function” as presented in formulation (20) [2] [5].

\[
\begin{align*}
\min & \quad F([x], [u]) - \mu \sum_{i=1}^{l} [\ln(s_i^{\text{min}}) + \ln(s_i^{\text{max}})] \\
\text{s.t.} & \quad g([x], [u]) = 0 \\
& \quad [h_i^{\text{min}}([x], [u])] - [s_i^{\text{min}}] = 0 \\
& \quad [h_i^{\text{max}}([x], [u])] - [s_i^{\text{max}}] = 0
\end{align*}
\]

(19)

where \(\mu\) is called “barrier parameter”, a positive number that tends to zero as the algorithm converges to the optimum, and \(l\) the number um inequality constrains.

With the “new equality constrains”, new multipliers have to be added to the Lagrangean function (10):

\[
\begin{align*}
L = & \quad F - \mu \sum_{i=1}^{l} [\ln(s_i^{\text{min}}) + \ln(s_i^{\text{max}})] \\
& + [\lambda]^T [g([x], [u])] + [\pi]^T ([h([x], [u])] - [s])
\end{align*}
\]

(20)

where \([\pi] = [\pi^{\text{max}}, \pi^{\text{min}}]\) are the lagrange multipliers referred to the new equality constrains (variable limits).

Expanding the equation (21) we got the equation (22).

\[
\begin{align*}
L = & \quad -\mu \sum_{i=1}^{l} \lambda_i (P_{\text{gi}} - Q_{\text{ci}} - P_i) \\
& - \mu \sum_{i=1}^{l} \lambda_i (Q_{\text{gi}} - Q_{\text{ci}} - Q_i) \\
& - \mu \sum_{i=1}^{l} \lambda_i (s_i^{\text{min}}) \\
& + \sum_{i=1}^{l} \pi_i (g_i - P_i - P_{\text{max}} - s_i^{\text{max}}) \\
& + \sum_{i=1}^{l} \pi_i (g_i - Q_i - Q_{\text{max}} - s_i^{\text{max}}) \\
& + \sum_{i=1}^{l} \pi_i (Q_{\text{max}} - Q_{\text{min}} - s_i^{\text{max}}) \\
& + \sum_{i=1}^{l} \pi_i (V_{\text{max}} - V_{\text{min}} - s_i^{\text{max}}) \\
& + \sum_{i=1}^{l} \pi_i (V_{\text{max}} - V_{\text{min}} - s_i^{\text{max}})
\end{align*}
\]

(22)

Contrary to what occurs in the Newton’s approach, all restrictions remain in the matrix, ie the dimension is always constant with 2l more elements (new Lagrange multipliers).

The set of variables in this method is given by (22).

\[
[x] = [P, Q, \lambda, \lambda, \theta, V, S, \pi]^T
\]

(22)

where:

\[
P = \left[ s^\text{max}_p, s^\text{min}_p, s^\text{max}_q, s^\text{min}_q, s^\text{max}_v, s^\text{min}_v \right]
\]

(23)

\[
\pi = \left[ \pi^\text{max}_p, \pi^\text{min}_p, \pi^\text{max}_q, \pi^\text{min}_q, \pi^\text{max}_v, \pi^\text{min}_v \right]
\]

(24)
Choice of an Initial Point

The variables initialization is a critical point in this algorithm. Based on realized tests in this work is suggested the following technique [4]:

- Initialization of slack variables performed through the initial value of the inequality constraints multiplied by a corrective factor:

\[
[s^0] = (1 - \beta)[h^0] \tag{25}
\]

Choosing \( \beta \in [0.1, 0.3] \) good outcomes were obtained.

- Choosing a value of \( \mu^0 > 0 \), an efficient way to initialize the Lagrange multipliers’ is, given by the equation (26).

\[
[\pi^0] = \mu^0[S^0][e] \tag{26}
\]

where \([S^0]\) is a diagonal matrix with the slack variables initialization, and \([e]\) a vector with dimension \([1 \times 2]\) filled by ones.

- There isn’t any universal way to choose the initial point of \( \mu \), although, in a gross way is possible to define an interval \([0, 1 100]\).

Updating of the Variables

- The reduction of the barrier parameter, being \( k \) the iteration number is given by [4]:

\[
\mu^{k+1} = \sigma^k \times \frac{\rho^k}{2l} \tag{27}
\]

where \( \rho \) is called the complementarity gap, calculated using the equation (28), and \( \sigma \in [0,1] \) is called the centering parameter, a factor which represents a compromise between the algorithm feasibility and optimality, wish values close to of 0 (1) emphasis is given to optimality (feasibility). In present study, better performances were obtained with values of \( \sigma \) between 0.1 and 0.2. As the feasibility can be guaranteed in some iterations is practical to start with a higher value and go progressively reducing, for example, start at 0.2 and end at 0.1.

\[
\rho^k = [s][\pi^T] \tag{28}
\]

- For updating the variables is necessary to distinguish primal variables (control, state and slack variables) from the dual variables (Lagrange multipliers). In the first instance the updating is performed by equations (27) and (28) [4].

\[
x_p^{k+1} = x_p^k + \alpha_p^k \Delta x_p \tag{29}
\]

\[
x_d^{k+1} = x_d^k + \alpha_d^k \Delta x_p \tag{30}
\]

where \( \alpha_p^k \) and \( \alpha_d^k \) are the parameters that define the step dimension realized in each iteration and computed by the following expressions:

\[
\alpha_p^k = \tau \min_{\Delta x_i < 0} \frac{x_i}{|\Delta x_i|} \tag{31}
\]

\[
\alpha_d^k = \tau \min_{\Delta x_i < 0} \frac{x_i}{|\Delta x_i|} \tag{32}
\]

where \( \tau = 0.9999 \).

As there is a dependence between the primal and dual variables it is not feasible to use the equations above, so it is necessary to define a common parameter \( \alpha_{pd} \), given by:

\[
\alpha_{pd} = \min(\alpha_p, \alpha_d) \tag{33}
\]

Finally, the variables update is performed by equation (34).

\[
x^{k+1} = x^k + \alpha_{pd}^k \Delta x \tag{34}
\]

In this work, the OPF problem had been solved by Interior Point Algorithm using de following steps:

1. Initial PF conditions.
2. If necessary, order the admittance matrix to respect the nomenclature (generators in the first buses).
3. Compute \([J]\) and \(\frac{\partial q}{\partial x}\).
4. Compute \([\Delta x] = -[J]^{-1} \times [\frac{\partial q}{\partial x}]\).
5. \([x^{k+1}] = [x^k] + \alpha_{pd}[\Delta x^k]\)

6. If \([\Delta x] \leq [error]\) the convergence is obtained. Else go to step 3.

As we can see, the first four points in both methods are identical.

C. Wind Power Integration

Wind energy due to its limited predictability and variability is considered problematic for the power systems operation. Due to the possibility of wind unavailability it is required the existence of sufficient reserve capacity, and in case of any quick changes of speed, lack of wind energy will be quickly compensated by conventional thermal units, otherwise it may cause:

- Great voltage variation;
- Great frequency variation;
- System crash.

then, is essential to have a good wind forecast.

The impact of wind power in the PF becomes a gradually more important question with the crescent integration of this
technology in power systems. Some of the key factors in this analysis are the operation costs, reservation costs and emissions.  

In this analysis, only the changes in objective functions and/or constraints to be made in the two OPF models referred in the previous subchapters will be presented.  

In the OPF, the satisfaction of the expected wind power must be taken into account, so it is required to have reserves in case of this power be less than the scheduled. The opposite, i.e., overestimation of wind power also has to be taken into account because it can lead to unnecessary allocation of generators. So there will be introduced cost functions for the underestimation and overestimation of the power generated by each wind generator. 

This model consists in adding to the cost functions of conventional thermal and wind generators the two functions mentioned in the previous paragraph [3]:

\[
F(P_t, w_g) = \sum_{i=1}^{n_g} C_i(P_{gi}) + \sum_{i=1}^{n_w} C_w(w_i) + \sum_{i=1}^{n_w} C_p(w_{iav} - w_i) + \sum_{i=1}^{n_w} C_r(w_i - w_{iav}) \tag{35}
\]

Where:

- \( n_g \) – Number of thermal generation units
- \( n_w \) – Number of wind power generation units
- \( P_{gi} \) – Active power generated by thermal unit i
- \( w_{gi} \) – Power generated by wind power generation unit i
- \( w_{iav} \) – Rated power of wind power generation unit i
- \( C_i \) – Thermal cost function
- \( C_w \) – Wind power generation cost function
- \( C_p \) – Penalty cost function for not using all available power from wind power generator
- \( C_r \) – Required reserve cost function, relating to uncertainty of wind power

Usually the cost functions of thermal generators are modeled as quadratics (36), on the other hand, in the case of wind generators will be assumed a linear behavior (37).

\[
C_i(p_i) = a_i + b_i P_{gi} + c_i P_{gi}^2 \tag{36}
\]

\[
C_w = d_i w_{gi} \tag{37}
\]

The cost functions related to the underestimation and overestimation are given by (38) and (39) respectively [3].

\[
C_p(w_{iav} - w_i) = k_p(w_{iav} - w_{gi}) \tag{38}
\]

\[
C_r(w_{iav} - w_i) = k_r(w_{iav} - w_{iav}) \tag{39}
\]

where \( k_p \) and \( k_r \) are the penalty (underestimation) and reserve (overestimation) cost coefficients respectively. These two coefficients are added in the set of variables of the two methods studied in this work.

D. Newton Vs Interior Point

Below there is a comparative table about the four methods presented in this section where scores are assigned (1-5) to the most important features in the algorithms performance, as well as the major critical points of them. [1] [2] [4]

<table>
<thead>
<tr>
<th></th>
<th>Newton</th>
<th>Interior Point</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Convergence</strong></td>
<td>★★★</td>
<td>★★★</td>
</tr>
<tr>
<td><strong>Accuracy</strong></td>
<td>★★★★★★</td>
<td>★★★</td>
</tr>
<tr>
<td><strong>Velocity</strong></td>
<td>★★★</td>
<td>★★★★★★</td>
</tr>
<tr>
<td><strong>Complexity</strong></td>
<td>★★★★★</td>
<td>★★</td>
</tr>
</tbody>
</table>

**Notes**

- The high number of violated limits increases the complexity of the problem and the matrix sparsity, which may lead to divergence.
- The unique and great disadvantage of this method is the strong dependence on initial conditions, especially the startup barrier parameter.

To test the performance of both algorithms, tests were performed using two different grids:

- 12 bus, 3 generators;
- IEEE 57 bus, with 7 generators;

Different demand and wind power distribution scenarios were tested, with and without wind powered generators.

With all tests performed in the study and development of these two algorithms, it was concluded that generally the IP proved to be better for smaller grids, although, that superiority becomes less evident with larger grids.

It is important to note that using the model proposed for wind generators (linear cost function and penalty functions associated to the wind power forecast), the convergence of IP method is much more difficult, being necessary to test different parameters of barrier and / or increase the value of the variable \( \beta \) until it finds a combination that leads to the method success. This is due to the fact that this method works best with convex objective functions, such as quadratic functions proposed for thermal generators and also because the wind associated penalty functions may obstruct the barrier function.

Another factor that may hinder the convergence in both methods occurs when there is an over- or underestimation of the wind, causing the coefficients \( k_p \) and / or \( k_r \) to take nonzero values. When one of these two situations occurs, it may cause a fluctuation around the result, and the tolerance of convergence will not be reached.

With respect to the failure situations resolution, while the interior point method is performed by changing the initial parameters \( \beta \) and \( \mu_0 \), in the Newton's method it is only possible to change the initial conditions (the closer they are
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For all that was seen in this section, and counting that will be used the 12 bus grid (Figure 1) in the next chapter, “Simulation & Computational Results”, the chosen method will be the Interior Point Algorithm.

III. SIMULATION & COMPUTATIONAL RESULTS

In this chapter the developed software will be used in simulations performed in daily time intervals of 15 minutes, ie, for each day, the program will run from 00am to 11:45pm from 15 to 15 minutes, making a total of 96 runs.

The data relating to expected demand, real demand, real and expected wind power, were provided by REN and properly adapted to the grid under study. The information provided is from year 2011, having been selected four months of this year to study the subject of this work: January, April, July and October, each corresponding to a different season.

The following cases will be studied [5]:

1. Only thermal units.
2. Unit 1 and 2 – Thermal.
   Unit 3 – Wind power generator with reduced cost (15€/MW) and estimation penalties.
3. Unit 1 and 2 – Thermal.
   Unit 3 – Wind power generator with competitive cost (25€/MW) and estimation penalties.
4. Unit 1 and 2 – Thermal.
   Unit 3 – Wind power generator with reduced cost (15€/MW) and without estimation penalties.
5. Unit 1 and 2 – Thermal.
   Unit 3 – Wind power generator with competitive cost (25€/MW) and without estimation penalties.

The thermal cost function coefficients are shown in Table 2:

<table>
<thead>
<tr>
<th>Generator</th>
<th>$a$ [€/h]</th>
<th>$b$ [€/MWh]</th>
<th>$c$ [€/MW$^2$h]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>750</td>
<td>20</td>
<td>0.055</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>12</td>
<td>0.0425</td>
</tr>
<tr>
<td>3</td>
<td>1600</td>
<td>10</td>
<td>0.060</td>
</tr>
</tbody>
</table>

So there is one case where there are only thermal generators, and four where are both thermal and wind generators. In cases 2 and 4 the respective wind power generation cost is low compared to the thermal cost, in turn, in cases 3 and 5 is higher, so the remaining generators have competitive costs in relation to the wind power cost.

The cases 4 and 5 were introduced here to measure the significance of the underestimation and overestimation penalties of wind energy.

Primarily it is performed an analysis of different scenarios for the distribution of wind energy. To this purpose the simulation will be done for one selected day, applying the cases described above.

The generated power plots have their values in [pu] and operating costs are presented in [k€].

The cases 4 and 5 do not have graphical representation since they are very similar to the analogous cases 2 and 3.

Scenario 1

In the first situation analyzed here there is a significant production of wind generator (Gen. 3) during the off-peak, opposing with the peak period when the same generation is very low.

Looking at Figure 1.b and 1.c, as in the off-peak period the predicted wind energy is high, consumption is achieved with a thermal generator and a wind farm, so the most expensive generator (generator 1) can be disconnected.

![Figure 1 – Topology of the twelve bus grid [1]](image)
Figure 1 – Active and reactive power evolution: (a) case 1; (b) case 2; (c) case 3.

Figure 2 – Temporal evolution of total operating cost: (a) case 1; (b) case 2; (c) case 3.

From Figure 2 analysis it is verified that the inclusion of a wind generator reduces the operation cost at off-peak period, however outside those points, the same fact increases the wind generation total cost.

Then, it is important to analyze the average daily cost in the three situations:
- Case 1: 14,375 k€;
- Case 2: 13,537 k€;
- Case 3: 14,361 k€;

As we can verify, the difference between the average cost of cases 1 (no wind generator) and 3 (wind generator at a cost of 20€/MWh) is minor. It turns out that in simulations with identical wind scenarios, on several occasions the average cost of production is higher when there is a wind generator with the referred cost (Case 3).
In Figure 3 is analyzed the difference between the expected power, available and useful for the wind generator (generator 3). In Figure 3 (a), corresponding to the situation where we have the generator with lower cost, the entire available power is used, which does not occur in the situation where the cost is higher (Figure 3 (b)), because in the off-peak period is no longer economically viable use all available wind power.

**Scenario 2**

In the second case, as opposed to what is observed in the first one, there is more wind power available at peak periods than in the off-peak. This scenario, considering the wind power cheaper than thermal, is ideal for the ESS, because it is possible to disable the more expensive if the consumer satisfaction is guaranteed, such as in this example where the generator 1 is shut down between 16h and 21h.

Here, when there is wind power generation, the costs are lower compared to the case 1 (three thermal generators) in the whole range of values.

It is worth noting that when there is a reduced wind generation cost and when the more expensive generator is off, it occurs a significant cost reduction.

**Scenario 3**

After presenting situations where there are high levels of wind generation or on off-peak or on peak, it is now considered a situation where throughout the day, this same production is predominant in relation to the other generators, and nearly stable.

Here the most expensive generator is deactivated during periods when the consumption is higher, noting that in the off-peak period, approximately 80% of generation is handed to the wind generator (Figure 4).

Regarding operating costs, it can be seen again that comparing the cases 1 and 2 the cost difference is negligible. However, in the remaining period, there are higher costs in the first situation. This is due to the fact that the thermal curves are modeled as quadratics, which causes from a certain point, this technology be less economical than wind (linear costs).

Finally in Figure 5, the change in the wind generator production when the wind estimation penalties are not applied (cases 4 and 5) is graphically analyzed.

As shown in Figure 5, the inexistence of the penalty function related with non-use of the entire wind power, leads to a search for an operating point where it utilization is inferior. This fact leads, in most situations, to a lower operation cost when there aren’t penalties associated with the wind.
Final Results

In total 11,808 executions of the program developed were carried out, corresponding to the 123 days simulated.

Table 1 shows the average costs calculated for each month in the five cases studied in this work, as well as the average costs of the set of the same months.

As shown through the case 1 (without wind generation), January is the month with the highest average cost because, being a winter month the consumption is higher.

Table 1 – Average costs (per month and totals) [€] [5]

<table>
<thead>
<tr>
<th>Case</th>
<th>January</th>
<th>April</th>
<th>July</th>
<th>October</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.9301</td>
<td>12.6034</td>
<td>131059</td>
<td>12.7180</td>
<td>13.3395</td>
</tr>
<tr>
<td>2</td>
<td>13.4074</td>
<td>11.2133</td>
<td>11.4461</td>
<td>11.3403</td>
<td>11.8518</td>
</tr>
<tr>
<td>4</td>
<td>13.3978</td>
<td>11.2109</td>
<td>11.4411</td>
<td>11.3370</td>
<td>11.9445</td>
</tr>
</tbody>
</table>

As shown through the case 1 (without wind generation), January is the month with the highest average cost because, being a winter month the consumption is higher.

As was seen in Figure 5.a, the overestimation penalty implies a greater utilization of available wind power because there are associated costs when it is not entirely used. However such penalty influence isn’t very significant as can be seen analyzing the difference between the total cost among the cases 2 and 4 and the cases 3 and 5:

- \( \text{Cost}_{\text{av}}^{\text{case 2}} - \text{Cost}_{\text{av}}^{\text{case 4}} = 11851.8 - 11846.9 = 4.9\€; \)
- \( \text{Cost}_{\text{av}}^{\text{case 3}} - \text{Cost}_{\text{av}}^{\text{case 5}} = 12984.6 - 12944.5 = 40.1\€; \)

As can be seen, the cost difference is greater when we have the wind with the highest price.

Finally it is important to check the average deviations of useful wind power in relation to the expected. Since this is a wide range of results, this study is important to the extent that provides a reference in relation to the necessary reserve power. The upper deviation means that the generator wind power is higher than expected, taking the lower deviation the opposite meaning.

Table 2 – Average deviations of useful wind power in relation to the expected in \( \text{pu} \) [5]

<table>
<thead>
<tr>
<th>Case</th>
<th>Upper deviation</th>
<th>Lower deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0416</td>
<td>0.0269</td>
</tr>
<tr>
<td>3</td>
<td>0.0333</td>
<td>0.0195</td>
</tr>
<tr>
<td>4</td>
<td>0.0419</td>
<td>0.0254</td>
</tr>
<tr>
<td>5</td>
<td>0.0381</td>
<td>0.0418</td>
</tr>
</tbody>
</table>

In cases 2 to 4 it is verified that the upper deviation is larger than the lower. This is due to the fact that in approximately 80% of days (calculated from data provided by REN) the available wind power is higher than expected and also because of the presence of the penalties that cover the cases 2 and 3 . In case 5, the same isn’t verified, as there is any underestimation penalty and the wind generation has competitive costs with the thermal units, the production level goes down, inverting the upper deviation superiority.

IV. CONCLUSIONS

The main goals of this study were to develop a computational tool to solve the Optimal Power Flow with inclusion of wind power, and studying the effects of such production against different scenarios of demand and wind power availability. Both objectives were reached with the development of two programs that solve the problem using each one a different algorithm (Newton Method and Interior Point Algorithm).
With the analysis done in this study it was concluded that, depending on the wind energy cost and of having or not associated penalties, it is not always beneficial to have this type of generation, specifically in periods of off-peak where the consumption is relatively low. However in peak situations, where consumption is much higher, the fact of having wind generators may become quite advantageous because in this case there is a reduction or even elimination of generators with higher costs.

It was also estimated the deviation of the wind useful energy in relation to the expected, providing useful information regarding the reserve power to take account.

It is important to say that the study carried out in these last chapters was based on the cost functions assigned to the generators. For a more efficient and complete analyses it would be necessary to study further combinations and using different cost functions, based on data as close as possible to reality.

REFERENCES