Flutter Models for the Preliminary Design of a Straight Bladed Vertical Axis Wind Turbine

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Abstract

The future of energy and the growing demand for wind power energy have led to more investigation in the field of vertical-axis wind turbines (VAWT). The work developed in this thesis is a contribution to the state-of-the-art of the aeroelastic analysis of VAWTs for the investigation of the onset of flutter in straight bladed vertical-axis wind turbines (SB-VAWT). This contribution consists in two reduced order models. The first model integrates the supporting structure of the blade as equivalent boundary conditions in terms of bending and torsion solicitation. The second model was based on the finite element method by assuming analytical dynamic rigidity matrices. The dynamics models were compared and verified by studying different structural configurations of a typical VAWT turbine. The necessity of experimental data of VAWT aeroelasticity was recognized given the lack of validation data in the literature. However, it was possible to study the behavior of the developed models in the estimation of the onset of flutter and recommendations for future work could be obtained. The development of a CFD tool to evaluate the lift and drag coefficients of a SB-VAWT airfoil in non-dynamic conditions was left out of the scope of this article for representing future work.

Keywords: aeroelasticity, flutter, vertical-axis wind turbine, straight blades

1 Introduction

The main purpose of this article is to present a contribution to the field of aeroelastic effects on SB-VAWTs, more specifically covering the occurrence of the flutter phenomenon. The contribution to the state-of-the-art consists in the development of two analytical methodologies to predict the onset of flutter in SB-VAWT. These tools can contribute greatly in the design of such devices, particularly in a preliminary or conceptual stage. A two dimensional classical airfoil flutter model was taken for analyzing a VAWT blade airfoil. This model has been successfully applied to wings and it is part of the present work to evaluate its validity on VAWT analysis. The first of the developed models was based on considering the supporting structures as boundary conditions to the beam representing the blade. This led to idealizing the supporting structures as equivalent elastic and inertial boundary conditions applied on the blade as a beam. The second approach was to model the SB-VAWT structure dynamics was based on the FEM method by deriving analytical dynamic elements to represent each of the structure beam spans dynamics. In trying to improve the flutter model a flow simulation was developed to evaluate the airfoil aerodynamic loads in flutter. However, due to time constraints only the steady-state simulations were performed on SB-VAWT airfoils. The discussion in this paper starts by introducing a classical flutter method applied to the special case of the SB-VAWT blade flow. Two blade dynamics models are then derived and implemented into the flutter model in trying to determine the SB-VAWT flutter speed of a test case and the flutter results from both methods are thus compared.

2 Classical Flutter Analysis Applied on SB-VAWT

The classical flutter analysis theory according to [2] was applied to a typical airfoil section of SB-VAWT illustrated in figure 1 with degrees of freedom of plunging $h$ and pitching $\theta$. We are only interested in determining the flutter boundary or
stability boundary. This corresponds to the flow conditions that produce simple harmonic motion at one of the oscillation modes. The speed at which this occurs is called the flutter speed $U_F$ which for the SB-VAWT case is the blade tip speed \[4\].

Following \[2\] closely we determine that the flutter analysis can be reduced to solving the system 4.59 of \[2\] where the airfoil natural frequencies of pitching $\omega_\theta$ and plunging $\omega_h$ are associated with the blade torsion and bending respectively. Hence, the blade dynamics will be analyzed in section 3 in order to determine these frequencies.

## 3 Reduced Models for the flutter Model Dynamics of Straight Bladed VAWT

Two methods of determining the natural frequencies of torsion and bending of straight blades of vertical axis wind turbines were developed. The natural frequencies of a SB-VAWT blade can be taken to represent its dynamics and be used as an input to the flutter model. The problem behind determining these frequencies is the influence of the supporting structure on the blade dynamics, as the former also takes part in the dynamic behavior of whole turbine structure. Three blade and support structural configurations were studied as represented in figure 2.

From \[5\] it can be seen that the torsion and bending mode shape functions, $X_\theta (x)$ and $X_h (x)$ respectively, for a free vibrating beam are given by
\[ X_\theta(x) = A \sin(\alpha x) + B \cos(\alpha x), \]
\[ X_h(x) = C_1 \sin(\beta x) + C_2 \cos(\beta x) + C_3 \sinh(\beta x) + C_4 \cosh(\beta x), \]

with the natural frequencies of torsion \( \omega_\theta \) and bending \( \omega_h \) being given by

\[ \omega_\theta = \alpha \sqrt{\frac{GJ}{J_0}}, \]
\[ \omega_h = \beta^2 \sqrt{\frac{EI}{m}}. \]  

3.1 Modeling Blade Dynamics through Boundary Conditions Idealization of the Supporting Structure

The supporting structure of the blade was modeled for the torsion and bending solicitations according to the ideal elastic and inertial constraints as represented in figure 3.

The expressions for the equivalent rigidity constraints \( k_\theta \), \( k_{h,r} \), and \( k_{h,t} \) and inertial constraints \( I_{\theta} \), \( m_h \) and \( I_h \) of figure 3 were determined by assuming the supporting structure as a cantilever beam and performing a one-degree-of-freedom simplification of the support tip motion as described in [5].

We replace the torsional mode shape \( X_\theta \) from equation 1 into 3, and solve for \( \alpha \) that allows determining the blade natural frequency of torsion \( \omega_\theta \) as given by equation 2.

3.1.1 Blade Torsional Dynamics

Configuration 1 and 2 According to [2] the support equivalent boundary conditions to the blade as a beam at \( x = 0 \) and \( x = l \), the loads balance from the blade and support torques yields

\[ GJ X'_\theta(0) = + k_\theta X_\theta(0) - I_\theta \alpha^2 \frac{GJ}{J_0} X_\theta(0), \]
\[ GJ X'_\theta(l) = - k_\theta X_\theta(l) + I_\theta \alpha^2 \frac{GJ}{J_0} X_\theta(l). \]  

Figure 3: Supporting structure idealization through combined elastic and inertial constraints.
Configuration 3  For the third configuration of figure 2c with the support at \( x = l/2 \) a multi-span approach [3] was followed. Dividing the blade into two beam spans 1 and 2 for the leftmost and rightmost respectively, a torsion mode shape function is assumed for each of the spans in the form of equation 1 and coming as

\[
\begin{align*}
X_{\theta,1} (x) &= A_1 \sin(\alpha x) + B_1 \cos(\alpha x), \\
X_{\theta,2} (x) &= A_2 \sin(\alpha (x - l/2)) + B_2 \cos(\alpha (x - l/2)).
\end{align*}
\] (4)

Compatibility conditions must be introduced [3] between the two mode shape functions, translating the continuity in the blade mode shape \( X_\theta \), its rate \( X_\theta' \) and the beam torque balance across the support, that are given by

\[
\begin{align*}
X_{\theta,1} \left( \frac{l}{2}^- \right) &= X_{\theta,2} \left( \frac{l}{2}^+ \right), \\
X_{\theta,1}' \left( \frac{l}{2}^- , t \right) &= X_{\theta,2}' \left( \frac{l}{2}^+ , t \right), \\
GJX_{\theta,1}' \left( \frac{l}{2}^- \right) &= -k_\theta X_{\theta,1} \left( \frac{l}{2}^- \right) + I_\theta \alpha^2 \frac{GJ}{l_0} X_{\theta,1} \left( \frac{l}{2}^- \right) + GJX_{\theta,2}' \left( \frac{l}{2}^+ \right).
\end{align*}
\] (5)

Additional boundary conditions for the free ends at \( x = 0 \) and \( x = l \) yield

\[
\begin{align*}
X_{\theta,1}'(0) &= 0, \\
X_{\theta,2}'(l) &= 0.
\end{align*}
\] (6)

3.1.2 Blade Bending Dynamics

Configuration 1 and 2  According to [2] the support equivalent boundary conditions seen in figure 2 for the case of bending, the blade shear force and bending moment will balance with those from the support as given by

\[
\begin{align*}
l^3 X_h'''(0) &= - \frac{k_h l^3}{EI} X_h(0) + \frac{m_h}{ml^4} (\beta l)^4 X_h(0), \\
l^2 X_h''(0) &= + \frac{k_h l^2}{EI} X_h'(0) - \frac{I_h}{ml^2} (\beta l)^4 X_h'(0), \\
l^3 X_h'''(l) &= \frac{k_h l^3}{EI} X_h(l) - \frac{m_h}{ml^4} (\beta l)^4 X_h(l), \\
l^2 X_h''(l) &= - \frac{k_h l^2}{EI} X_h'(l) + \frac{I_h}{ml^2} (\beta l)^4 X_h'(l).
\end{align*}
\] (7)

We replace the bending mode shape \( X_h \) from equation 1 into 7, and solve for \( \beta \) that allows determining the blade natural frequency of bending \( \omega_h \) as given by equation 2.
Configuration 3  The third configuration was analyzed with the multi-span method [3]. Mode shape functions were assumed for each of the spans 1 and 2 in the form of 1 and as given by

\[
X_{h,1}(x) = C_1 \sin (\beta x) + C_2 \cos (\beta x) + C_3 \sinh (\beta x) + C_4 \cosh (\beta x), \\
X_{h,2}(x) = D_1 \sin (\beta (x - l/2)) + D_2 \cos (\beta (x - l/2)) + D_3 \sinh (\beta (x - l/2)) + D_4 \cosh (\beta (x - l/2)).
\]  

(8)

Compatibility conditions at \( x = l/2 \) were introduced in order to set the continuity of the blade plunging mode shape \( X_h(x) \) and its rate \( X'_h(x) \) as well as the shear force and bending moment across the support location. Those conditions are given by equations 9. The free end conditions at \( x = 0 \) and \( x = l \) are translated by equations 10.

\[
X_{h,1} \left( \frac{l}{2} \right) = X_{h,2} \left( \frac{l}{2} \right), \\
X'_{h,1} \left( \frac{l}{2} \right) = X'_{h,2} \left( \frac{l}{2} \right), \\
l^3 X''_{h,1} \left( \frac{l}{2} \right) = \varepsilon_l X_{h,2} \left( \frac{l}{2} \right) + l^3 X''_{h,2} \left( \frac{l}{2} \right), \\
l^2 X'''_{h,1} \left( \frac{l}{2} \right) = -\varepsilon_r X_{h,2} \left( \frac{l}{2} \right) + l^2 X'''_{h,2} \left( \frac{l}{2} \right).
\]  

(9)

3.2 Modeling Blade Dynamics through Analytic Finite Elements

The second approach followed to model the blade dynamics was based on the finite element method. Individual beam spans of the SB-VAWT structure were assumed as analytical elements. These elements were derived for the bending and the torsion solicitation with degrees of freedom as represented in figure 4 and 5, respectively.

![Figure 4: Analytic finite element of beam bending.](image)

![Figure 5: Analytic finite element of beam torsion.](image)

In free vibration conditions the elements motion is assumed to be given by the mode shape functions from equation 1. Provided with that, a relation between the element loads \( \{Q\} \) and degrees of freedom \( \{q\} \) from figures 4 and 5 can be found according to [7] for dynamic conditions, being given by
\[
\{Q\} = [K] \{q\}, \tag{11}
\]

For the case of bending the matrix \([K]\) takes the form \([K] = [K_h (\beta)]\) and is called the **bending dynamic rigidity matrix** since it exposes the relation between the bending degrees of freedom and the loads of the finite element at its nodes. The derived analytical element is suitable to represent the bending dynamics of a beam of length \(l\) and constant flexural rigidity \(EI\). For the case of torsion \([K]\) takes the form \([K] = [K_\theta (\alpha)]\) and is called the **torsion dynamic rigidity matrix** that is suitable to represent the torsional dynamics of a beam or shaft of length \(l\) and constant torsional rigidity \(GJ\).

### 3.2.1 Blade Dynamics

In order to determine the blade natural frequency of bending, the derived elements must be assembled. For the bending solicitation of the blade, the SB-VAWT structure was modeled with only bending elements and the degrees of freedom of the structure were assumed as illustrated in figures 6a and 6b for the second and third configurations of figure 2.

For the blade torsion case illustrated in figure 6c, elements 1 and 3 of the second configuration, or double supported blade, are elements of bending whereas the blade was represented by the torsional element 2. For the third structural configuration of figure 6d, or the single supported blade, elements 1 and 2 are of torsion whereas the support was modeled with a bending element. The assumed degrees of freedom for both cases are a compromise solution between the complexity of the model and its accuracy. For each of the torsion and bending analysis, the individual element matrices must be assembled into a global system rigidity matrix \([K_G]\) of degrees of freedom \(\{q_G\}\). This procedure follows the one described in [6] for the common FEM method. Having determined the global dynamic rigidity matrix of the system, the free vibration motion will be characterized by

\[
[K_G] \{q_G\} = 0 \iff \det ([K_G]) = 0 \tag{12}
\]

The roots \(\alpha\) for the blade torsion case and \(\beta\) for the blade bending case of equation 12 must then be determined. These can thus be used to compute the blade natural frequencies \(\omega_h\) and \(\omega_\theta\) from equation 2.
3.3 Verification

In order to verify the accuracy of the developed models, a verification analysis was conducted through the use of a numerical FEM analysis with the help of Ansys software. The second and the third configurations from figure 2 were studied. The test structures were solid and entirely made of aluminum, assumed as a linear isotropic material with a Young's modulus $E = 70$ GPa, density $\rho = 2700$ kg/m$^3$ and Poisson's ratio $\nu = 0.33$. A circular cross section of radius $r = 0.02$ m was chosen for the supporting structures, while a NACA0015 airfoil profile of chord $c = 0.15$ m at $0^\circ$ angle of incidence was taken for the solid blades. Blade total length was set as $l = 1$ m, with the effect of the support length $l_s$ being studied. A model analysis was conducted with numerical FEM in Ansys-APDL with beam188 elements [1], suitable for modeling moderately thick beams. It was observed that of the studied configurations, no mode shape would present pure blade torsion, and therefore these results will only be used for the blade bending. Modeling pure blade torsion of SB-VAWT in a numerical FEM analysis for validation purposes was thus left as future work.

3.3.1 Blade Bending Dynamics

The parameter $(\beta l)$ represents the non-dimensional blade bending natural frequency $\omega_b$ and therefore it was plotted in figure 7 against the support to blade length ratio $l_s/l$ for the single and double supported blade and for the boundary conditions and the analytic DFEM method.

![Figure 7: Results comparison for $(\beta l)$ as a function of $l_s/l$.](image)

The results difference between methods and blade configurations is clear. The results for the bending frequency of the double supported blade, or second configuration, suggest the analytic DFEM method is capable of predicting the natural frequency of an idealized structure, namely of a SB-VAWT. However, results for the single supported blade in figure 7b indicate the solution accuracy of this must depend on the assumed problem conditions. These must meet the method requirements for accuracy and therefore the range of conditions the analytic DFEM method can be applied on must be investigated. The boundary conditions method has generally presented greater error in predicting the blade dynamics. This must largely be because of the assumptions made in idealizing the supports. These were assumed as simple cantilever beams which neglects the influence of the blade oscillations on the supports and thereby not considering the oscillations interaction of the complete structure. The observed convergence trend of the boundary conditions method and the analytic DFEM show that for a range of conditions both methods must be equivalent. In fact, assuming the supporting structures as cantilever beams for the boundary conditions method could not be valid for the cases where the blade takes higher importance in the structure dynamics, as it is the case of a much stiffer and dense blade than the supports. This agrees with the convergence observed for $l_s/l \gg 1$ as in that case it will be the supports dominating the structure dynamics.
3.3.2 Blade Torsional Dynamics

The parameter \( \alpha_l \) represents the non-dimensional blade torsion natural frequency \( \omega_\theta \) and therefore it was plotted in figure 8 against the support to blade length ratio \( l_s/l \) for the single and double supported blade and for the boundary conditions and the analytic DFEM method.

![Graphs](a) Double supported blade. (b) Single supported blade.

Figure 8: \( \alpha_l \) as a function of \( l_s/l \) for different structural configurations. The approximate solution refers to the natural frequency of the support as a bending cantilever beam.

The disparity between the two methods is evident for both configurations. The behavior demonstrated by the analytic DFEM model in figure 8a for \( l_s/l < 0.35 \) can be explained by the prediction of distinct mode shapes from those predicted for \( l_s/l > 0.35 \) which raises the importance of correctly predicting the mode shape solution. In the case of the single supported blade, the solution of the boundary conditions method was found to be independent of the problem physics. For the example shown, both blade dynamics models match exactly for \( l_s/l < 0.3 \), with \( \alpha_l = \pi \). However, for values higher than that the analytic DFEM method solution moves away from the solution of the boundary conditions method, predicting a decreasing blade torsional frequency with increasing support length of the support \( l_s \). To understand the behavior shown in figure 8 the natural frequency of the support as a bending cantilever beam was plotted. Figure 8 shows that both methods predict the blade torsional frequency to converge for the support bending frequency for larger values of \( l_s \).

4 SB-VAWT Flutter Speed Determination

In order to validate the introduced flutter model, this was compared with experimental data from reference [8] for the case of wing flutter. The goal of this analysis is to obtain reliable results from available experimental results in order to gain confidence on the predictions of the occurrence of blade flutter. The wing flutter experimental model from [8] made use of several cantilever wings with an airfoil chord of 30.48 cm, 4.2% relative thickness and span 205.74 cm. The wings parameters were determined in [8] from testing and calculation and included into the flutter model. The error between the predicted flutter speed \( U_{F,pred} \) and the observed \( U_{F,exp} \) comes calculated with,

\[
\epsilon [\%] = 100 \left( \frac{U_{F,pred} - U_{F,exp}}{U_{F,exp}} \right),
\]
The theoretical results correlate well with the experimental values. According to studies performed in [8] it is expected that the flutter speed $U_F$ reaches a minimum value at a frequency ratio near unity which agrees with figure 9a.

### 4.1 Flutter boundary results for a SB-VAWT

The blade dynamics models were included into the flutter model for the SB-VAWT test cases of section 3. The flutter speed results from the boundary conditions and the analytic DFEM methods were compared for the single and double supported blades and are plotted in figure 10a against the support length $l_s$. In order to better understand the results the frequency ratio $\sigma$ was plotted for the same range of $l_s$. That range was chosen to avoid the behavior demonstrated by the analytic DFEM method in predicting the blade natural frequency of torsion as shown in figure 8a.
The results obtained indicate the importance of correctly predicting the natural frequencies of the system, given the strong influence of the frequency ratio $\sigma$ parameter. Similarly to the wing case the flutter speed reaches a minimum value near $\sigma = 1$. The boundary conditions method and the analytic DFEM predict this value for different values of $l_s$, which could be interpreted as a range of support lengths for which the flutter speed would come as minimal, thus increasing the likelihood of the flutter phenomenon at lower speeds. This result indicates that with additional investigation on improved dynamics models the developed methodology can effectively be used for the preliminary design of SB-VAWTs. For the single supported blade the results difference arise from the constant solution found by the boundary conditions method for the torsional natural frequency of the blade. For longer supports the boundary conditions method predicts the frequency of the system will be mostly dominated by the frequency of torsion and thus the convergence trend shown. This demonstrates that on SB-VAWT flutter analysis the flutter model and the dynamics model must be evaluated together, raising once more the need for validation data from future work.

5 Conclusions

Given past work for SB-VAWT aeroelasticity analysis was nonexistent, the modeling of analytical methods to determine the blade dynamics proposed in this thesis is was a novel contribution to the state-of-the-art. The boundary conditions method and the analytic DFEM method were derived and the effect of the blade support on the blade dynamics was investigated, through its length and structural arrangements. The two methods presented considerable differences, mainly due to the assumptions made. After validating the flutter model for the case of wings, the dynamics models were then implemented into the flutter model. Due to the lack of validation data for SB-VAWT, both dynamics methods could only be compared against each other in predicting the flutter speed. However it was possible to understand the flutter speed behavior with the increasing support length and points of minimum flutter speed could be determined, indicating a design point that should be avoided. This demonstrated the capability of the developed tool to be used on the parametric design of SB-VAWTs although further investigation on modeling the blade dynamics should be made. In conclusion, a methodology for the evaluation of flutter conditions was derived that although requiring additional investigation demonstrated to be suitable to assess the optimization design of SB-VAWTs.

References


