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SENSITIVITY ANALYSIS OF THERMAL PERFORMANCE FACTORS OF BUILDINGS

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Lisbon, October 2012

EXTENDED ABSTRACT

1 - INTRODUCTION

Due to the large impact of burning fossil fuels on global warming, it is imperative to reduce energy consumption in all productive sectors. In Portugal, the energy consumption in the buildings sector represents an important share of final energy consumption, approximately 30% [1], and the tendency is to grow, given the improvement of living conditions of populations and the consequent increased demand for comfort in buildings. In order to face this trend, it has been published national and European legislation with the purpose of promoting the construction and rehabilitation of more efficient buildings with better thermal performance.

There are several factors involved in the thermal performance of a building. A greater knowledge of their importance on this performance can enhance the construction of more efficient buildings from the standpoint of energy. Therefore, this study will focus on the sensitivity analysis of some factors of thermal performance of buildings, using for this purpose the method of factorial design at two levels, both for the heating and the cooling seasons. It is intended to identify the factors with the greatest contribution in the response of the building and also to assess the possible interactions between them.

To perform this study it will be used a program of dynamic simulation of thermal performance of buildings and energy, the EnergyPlus [2], which, dealing with the heat transfer phenomena in detail, provides a deeper analysis of the effects of factors selected for this study.

In the heating season it will be applied a full factorial design at two levels, considering performance translated by the needs of useful energy for heating. In the cooling season it will be used a fractional factorial design at two levels, considering performance translated by the degree-hours of discomfort from overheating. It is also intended to deduce a mathematical equation that quickly allows the calculation of the useful energy needs for heating and the degree-hours of discomfort from overheating, without recourse to the dynamic simulation program.

2 - FACTORIAL DESIGN

There are several strategies for studying the effect of the input parameters (factors - independent variables) on the response (dependent variable) of a system. An effect is a change in response to a change of level of a factor. The simplest strategy to analyze the system sensitivity is varying the factors in turn. In a strategy where the factors are changed one-at-a-time, the effect on the response is calculated for each factor with the remaining maintained constant. The main inconvenient of this strategy is the impossibility of studying interactions between factors [3]. In fact, rarely factors act independently of each other. The influence of a factor on the response may depend on the level of other factors. Thus, interaction exists when the effect of a factor, set in a given level, depends on the level at which other factors are set. A better approach to evaluation of the effects, in cases involving several factors, is making an experiment in which all factors are changed simultaneously, according to specific plans, rather than being varied one-at-a-time. The main advantage of this strategy is that it allows the study of possible interaction among factors.

In a two-level factorial design, each factor may vary between a low level, x^-, and a high level, x^+. The full factorial design involves all possible combinations between levels of factors. If "k" is the number of factors, and 2 levels of variation are considered for each factor, the full factorial design is denoted by 2^k, with 2^k meaning the number of experiments (or simulations) needed for the study. The main drawback of a full design is that it quickly becomes prohibitive, due to the large number of experiments/simulations as the number of factors increases. To overcome this problem fractional factorial plans can be developed, which allow to study all factors but with a number of experiments/simulations reduced compared to the full design. These plans reduce costs, in terms of time and resources, but also decrease the obtained information about the system behavior. Therefore it is necessary that the combination of factors selected ensure a reliability of results compatible with the requirements of the problem. Considering only two levels for the factors, a fractional factorial design 2^{k-p} allows to analyze "k" factors only with 2^{k-p} experiments/simulations, where *p* defines the fraction of the design. For example, a plan $2^{K-1} = 2^k/2$ requires only half of the experiments, and a plan $2^{K-2} = 2^k / 4$ requires only one quarter of the experiments. In the general case, a fractional design 2^{k-p} requires $(1/2)^p$ experiments.

One of the objectives of a plan of experiments is the establishment of a mathematical equation that relates the response to the factors [4]. To this end, the development of the Taylor series function (multi-variable) is taken, which leads to a polynomial of degree higher or lower depending on the desired order of approximation:

$$y = a_0 + \sum a_i x_i + \sum a_{ij} x_i x_j + \ldots + \sum a_{ii} x_i^2 + a_{ij\ldots z} x_i x_j \ldots x_z \qquad (1)$$

in which:

y - response variable;

x_i - value assigned in each experiment/simulation by the researcher to factor i, between x_i^- and x_i^+, and is a known data;

a_0, a_i, a_{ij}, a_{ii} - coefficients of the mathematical model adopted - are not known and must be obtained from the results of experiments/simulations.

For a general use of equation (1), that is, for any factor irrespective of its nature (quantitative and qualitative), it is desirable to proceed with the following adimensionalization:

$$X = \frac{2 \times (x - \bar{x})}{x^{+} - x^{-}} \tag{2}$$

where $\bar{x} = (x^{+} + x^{-})/2$. It is easy to confirm that for $x = x^{+}$ then X = 1 and for $x = x^{-}$ then X = -1.

Each experimental point represents a response value. This response is modeled by a polynomial in the form of equation (1), whose unknowns are the coefficients to be determined. The plan of experiments/simulations provide a set of n equations (corresponding to the number of test/simulations) with p unknowns (the chosen model contains p coefficients to be determined), whose solution, that is, the determination of coefficients of the polynomial, may be found through the least-squares method if p <n.

3 - CASE STUDY AND SIMULATION

3.1 - CASE OF STUDY

The case study refers to a dwelling unit of a residential building, type *T2*, with double exposure (N-S), comprising two bedrooms, a lounge, a bathroom and a kitchen. It is assumed that the dwelling is located in the middle floor with sides - east to west - also bordered by similar air temperature, whence the exchange of heat is considered only through external walls. The dwelling has 120 m^2 (12 x 10m) of total area, and a ceiling height of 2.65 m, giving a total volume of 318 m^3. In Figure 1 it is possible to identify the different parts of the dwelling as well as the different walls.

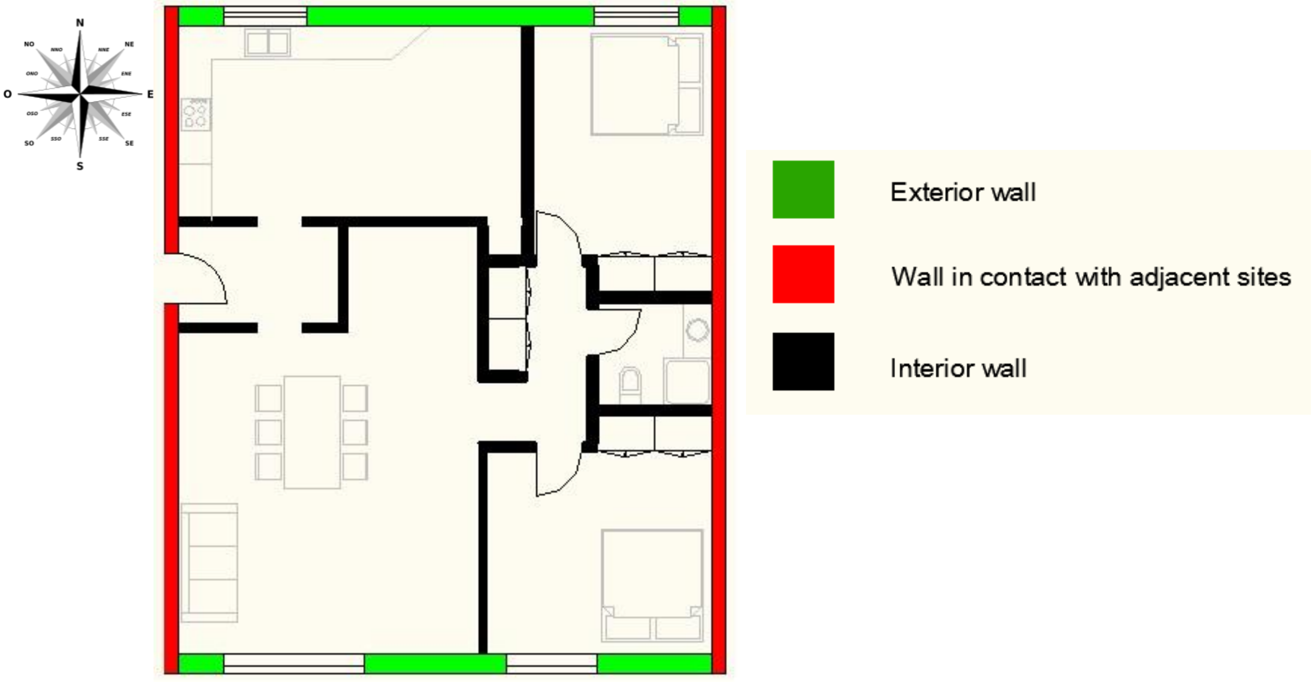

Figure 1 - Dwelling used as a study model

A sensitivity analysis was performed for the heating and cooling seasons, having been chosen specific factors and performance indicators to each season, as described below.

3.2 - HEATING SEASON

For the heating season (winter) it was chosen, as a performance indicator, the useful energy (N_u) that is required to ensure the minimum temperature of thermal comfort (20 °C) defined by the thermal regulation code. In turn, the performance factors chosen were: the heat transfer coefficient of the opaque surfaces (U_{par}), the heat transfer coefficient of the glazing (U_{env}), the relationship between the glazing area and wall total area facing south (A_{env} / A_{par}), the thermal inertia (I_t) and the air changes per hour (R_{ph}). The useful energy needs are calculated in dynamic regime per unit of floor area, using the computer simulation program *EnergyPlus*. Symbolically, the objective function is represented by:

$$N_u = f(U_{par}, U_{env}, \frac{A_{env}}{A_{par}}, I_t, R_{ph}) = \frac{N_H}{A_u} \text{,with } T_a \geq 20\ ^{\circ}\text{C} \qquad (3)$$

being T_a the inside air temperature (ºC), N_H the useful energy requirements for heating the unit dwelling (kWh), and A_u the useful surface area (m^2).

The requirements of useful energy for heating will be performed for the climatic region of Bragança, for presenting the most severe winter conditions and thereby better reveal the decisive factors for the thermal behavior of building in this season. The chosen factors for the sensitivity analysis in the heating season are presented in Table 1, together with the numerical values taken when placed in the "low" and "high" levels, respectively, designated by x^- and x^+.

Table 1 - Factors for the heating period (winter)

Factors		x^-	x^+	Units
x_1	Heat transfer coefficient of the opaque surfaces (U_{par})	1.02	0.48	W/m^2K
x_2	Heat transfer coefficient of glazing (U_{env})	2.79	1.64	W/m^2K
x_3	Relationship between glazing area and total area of façade in south (A_{env}/A_{par})	50	20	%
x_4	Thermal inertia (I_t)	Weak	Strong	Kg/m^2
x_5	Air changes per hour (R_{ph})	1.0	0.6	h^{-1}

The sensitivity analysis in the heating season is based on applying a full factorial design 2^5,which implies the realization of 32 experiments in this study, through the simulation program *EnergyPlus*, involving five factors and two levels of variation for each one.

3.3 - COOLING SEASON

For the cooling season (summer) it is considered the same performance factors of the heating season, except for the heat transfer coefficient of glazing, which was replaced by the solar transmittance of the glazing in south direction (τ_{sol}). As an indicator of the building thermal performance, it has been used the number of degree-hours of discomfort from overheating (ODH_{25}), defined as the sum of positive differences between the indoor air temperature and reference temperature of 25 °C for the period considered, that is:

$$ODH_{25} = f(U_{par}, \tau_{sol}, \frac{A_{env}}{A_{par}}, I_t, R_{ph}) = \sum (T_a - 25) \cdot \delta \qquad (4)$$

with $\delta = 1h \text{ if } T_a > 25^{\circ}C$

$\delta = 0 \text{ if } T_a \leq 25^{\circ}C$

The larger the value of this index, the lower the construction performance in regard to maintaining the comfort conditions in the summer.

The simulations for prediction ODH_{25} will be performed for the climatic region of Lisbon, for presenting summer conditions appropriate to the factors chosen for the study in this period. The chosen factors for the sensitivity analysis in the cooling season are presented in Table 2, together with the numerical values taken when placed in the "low" and "high" levels, respectively, designated by x^- and x^+.

Table 2 – Factors for the cooling period (summer)

Factors		x^-	x^+	Units
x_1	Heat transfer coefficient of the opaque surfaces (U_{par})	1.02	0.48	W/m^2K
x_2	Solar transmittance of glazing in south (τ_{sol})	0.48	0.24	-
x_3	Relationship between glazing area and total area of facade in south (A_{env}/A_{par})	50	20	%
x_4	Thermal inertia (I_t)	Weak	Strong	Kg/m^2
x_5	Air changes per hour (R_{ph})	1.0	0.6	h^{-1}

The plan of experiments (simulation) selected to perform this evaluation was a fractional factorial design 2^{5-1}, with this choice being motivated by the interest in exemplifying the application of such a plan on a specific case. The use of this plan allows the study of five factors with only 16 experiments, which only need half of the effort required by the full factorial plan. However, there will be associations of effects directly related to the selected combinations of factors, that is, to a given level of design resolution, which leads to a loss of information of the system's behavior. In the case of the resolution plan selected in this study - named resolution plan V -, associations of effects occur between interactions of two factors and interactions of three factors. As the interactions of three factors are generally negligible, the degradation of information with this plan is not significant.

4 - RESULTS AND DISCUSSION

4.1 - HEATING SEASON

Running the 32 simulation corresponding to the 2^5 full factorial design, and proceeding to the determination of the coefficients (dimensionless) of the model equation (1), considering a first degree polynomial with interaction between factors, the values shown in Table 3 are obtained.

Table 3 - Values of the coefficients of the model equation for the 2^5 full factorial design

	Factor							
	I	X_1	X_2	$X_1 X_2$	X_3	$X_1 X_3$	$X_2 X_3$	$X_1X_2X_3$
Coef.	38.5691	-3.6795	-1.7052	-0.0198	7.6061	-0.3096	0.5023	-0.0107
	Factor							
	X_4	$X_1 X_4$	$X_2 X_4$	$X_1X_2X_4$	$X_3 X_4$	$X_1X_3X_4$	$X_2X_3X_4$	$X_1X_2X_3X_4$
Coef.	-0.6941	0.0147	0.0206	0.0087	0.4203	-0.0079	-0.0138	0.0002
	Factor							
	X_5	$X_1 X_5$	$X_2 X_5$	$X_1X_2 X_5$	$X_3 X_5$	$X_1X_3 X_5$	$X_2X_3 X_5$	$X_1X_2X_3X_5$
Coef.	-10.5874	0.0717	0.0375	0.0060	-0.4292	-0.0756	-0.0433	-0.0122
	Factor							
	$X_4 X_5$	$X_1X_4 X_5$	$X_2X_4 X_5$	$X_1X_2X_4X_5$	$X_3X_4X_5$	$X_1X_3X_4X_5$	$X_2X_3X_4X_5$	$X_1X_2 X_3X_4X_5$
Coef.	-0.0097	0.0060	0.0053	0.0041	-0.0538	-0.0031	-0.00004	0.0026

Considering only the effects with more influence on the response of the model - useful heating needs - the equation takes the form:

$$N_u = 38.57 - 3.68X_1 - 1.71X_2 + 7.61X_3 - 0.69X_4 - 10.59X_5 - 0.31X_1X_3 + 0.50X_2X_3 +$$

$$+ 0.42X_3X_4 - 0.43X_3X_5 \qquad (5)$$

In this equation it is still possible to observe a difference, in terms of weight in response, of the main and interaction effects. Also the main effects are not equally significant, with the differences among them being better observed through a graphical representation as shown in Figure 2.

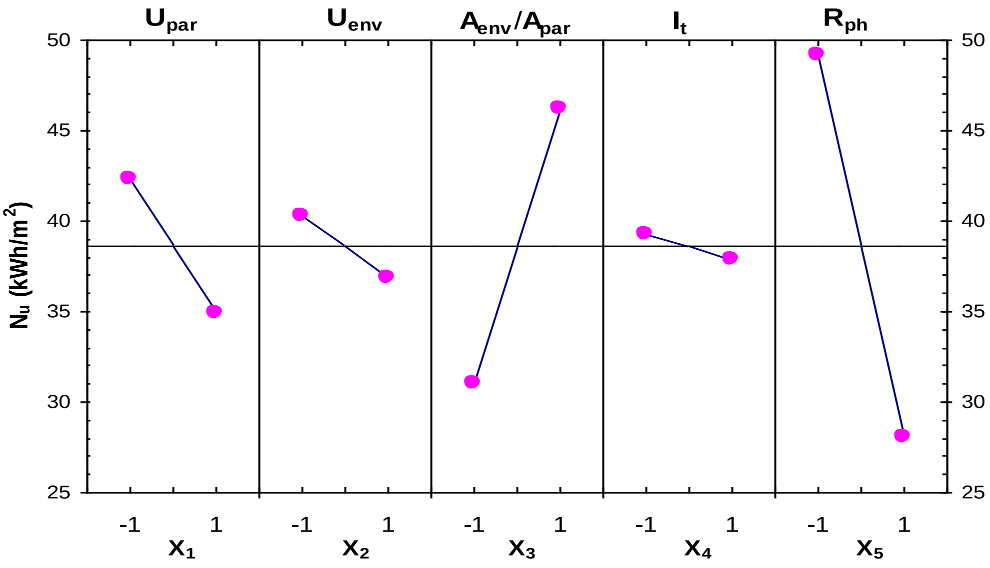

Figure 2 - Effects of the main factors in the heating season

It appears that in the case of heat transfer coefficients, thermal inertia and ventilation rate, a positive variation of the level - transition from low to high level - induces a negative change in the response, which is desirable in view of the objective of energy savings that should always be behind a project. In contrast, the decrease in glazing area when compared to the total façade area (in south direction) produces a positive variation in the response, which is undesirable because it leads to increased energy needs. By observation of Figure 2 we can see that the negative impact on the energy needs of the reduction of the south wall glazed area - resulting in the decrease of direct solar gain - is not compensated by the increased insulating characteristics of the facade – opaque and glazed parts. The factors of lower and higher impact on energy needs are, respectively, the thermal inertia and the ventilation rate, producing both a lowering of the energy needs when passing from low level - weak thermal inertia and high ventilation rate - to high level – strong thermal inertia and low ventilation rate.

4.2 - COOLING SEASON

Running the 16 simulations corresponding to the 2_V^{5-1} fractional design, the determination of the coefficients of the model equation (linear polynomial with interactions) for the cooling season provides the values listed in Table 4.

Table 4 - Values of the coefficients of the model equation for the 2_V^{5-1} fractional factorial design

	Factor							
	I	X_1	X_2	X_1 X_2	X_3	X_1 X_3	X_2 X_3	X_4 X_5
Coef.	4300.06	242.56	-1932.69	-154.87	-2274.28	-181.69	1049.85	83.98
	Factor							
	X_4	X_1 X_4	X_2 X_4	X_3X_5	X_3 X_4	X_2X_5	X_1X_5	X_5
Coef.	-74.51	128.50	-76.43	-529.37	-101.10	-480.80	147.74	1250.60

Considering only the most representative effects, the model equation is:

$$ODH_{25} = 4300.1 - 1932.7X_2 - 2274.3X_3 + 1250.6X_5 + 1049.9X_2X_3 \quad (6)$$

Figure 3 shows in graphical form the effect of the major factors in the amount of the degree-hours of discomfort from overheating, which is the response of the model for the summer season.

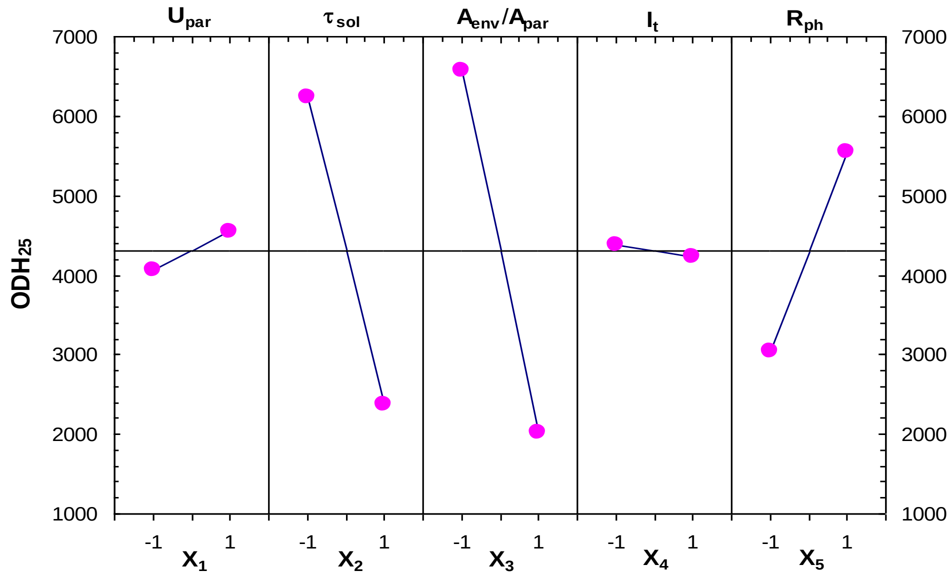

Figure 3 - Main effects of factors on the cooling season

Although it is not very important, it is possible to see that the decrease in the heat transfer coefficient that was advantageous in the heating season, in the summer leads to a larger number of cases of overheating. In fact, if the solar gain is high, the heat accumulated inside the building finds a greater resistance to dissipate to the outside when the thermal insulation of the building increases. The result of this fact is longer periods of overheating and a larger deviation, in such periods, of the indoor air temperature from the comfort limit temperature.

The major factors that impact the value of the response are the solar transmittance of glazing and the relationship between the glazing and total area of the south facade. To a positive change in the level of these factors (a positive change in the level implies a negative variation of the actual values of these factors) corresponds a negative response, that is, a lower value of the degree-hours of discomfort from overheating, which is the desired behavior in the cooling season. The increased thermal inertia is again beneficial from the standpoint of energy saving and comfort, while its impact is considerably lower than the rest of other factors. Finally, in relation to the ventilation rate, an increase of the level also yields a positive variation of the response. In real terms this means that the reduction in ventilation increases the number of degree-hours of discomfort from overheating and therefore has a negative effect on comfort.

As opposed to the heating season, in the cooling season the existence of interaction is evident, but with different amounts in response. Figure 4 shows the highest interaction that is the one occurring between the solar transmittance of south glazing (X_2) and the relationship between the glazing and the total area of the south facade (X_3), referred as interaction X_2X_3.

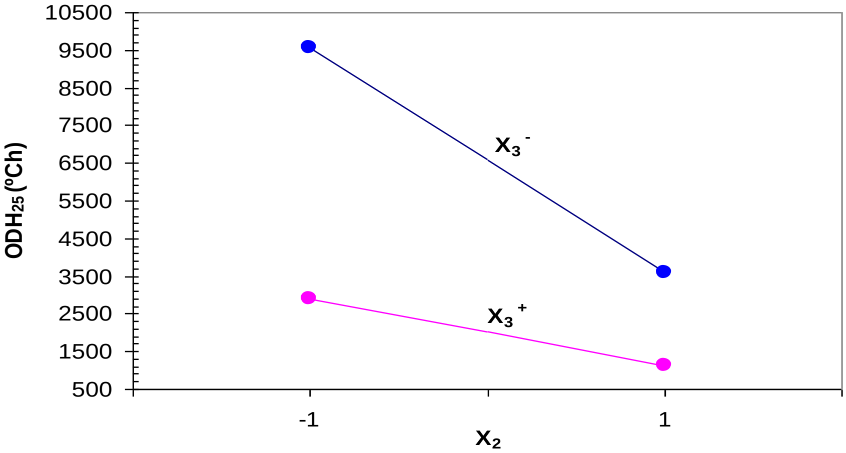

Figure 4 - Interaction X_2X_3 - Solar transmittance of south glazing / Relationship between glazing and total area of the south facade (cooling season)

Figure 4 shows that the effect of X_2 is higher when X_3 is at the low level and lower when X_3 is at the high level.

5 - CONCLUSIONS

For the heating season, the study showed that reducing the heat transfer coefficients of the facade components - glazing and the opaque areas - and the air change rate, on one hand, and increasing thermal inertia, on the other hand, have a benefit effect in terms of thermal comfort and energy saving, since they contribute to the reduction of the useful energy needs for heating. Of these, the factors with higher and lower impact are the air change rate and the construction thermal inertia, respectively. On the contrary, decreasing the relationship between the glazing and total area of the south facade, which implies the decrease of the direct solar gain, causes an increase of the energy needs for heating.

In regard to interactions between factors, they have little relevance in the heating season. An interaction effect exists when the effect of a factor depends on the level of another factor. In the heating season, the interactions of factors have a small interference with the results of the sensitivity analysis.

Relatively to the weight that the factors have on the energy needs in the study case, it is shown in Figure 5 the percentage variation of the model response (useful energy needs N_u) when each factor is varied between the "average" level (0) and the "high" one (+1) with the remaining factors kept at their "average" level (0).

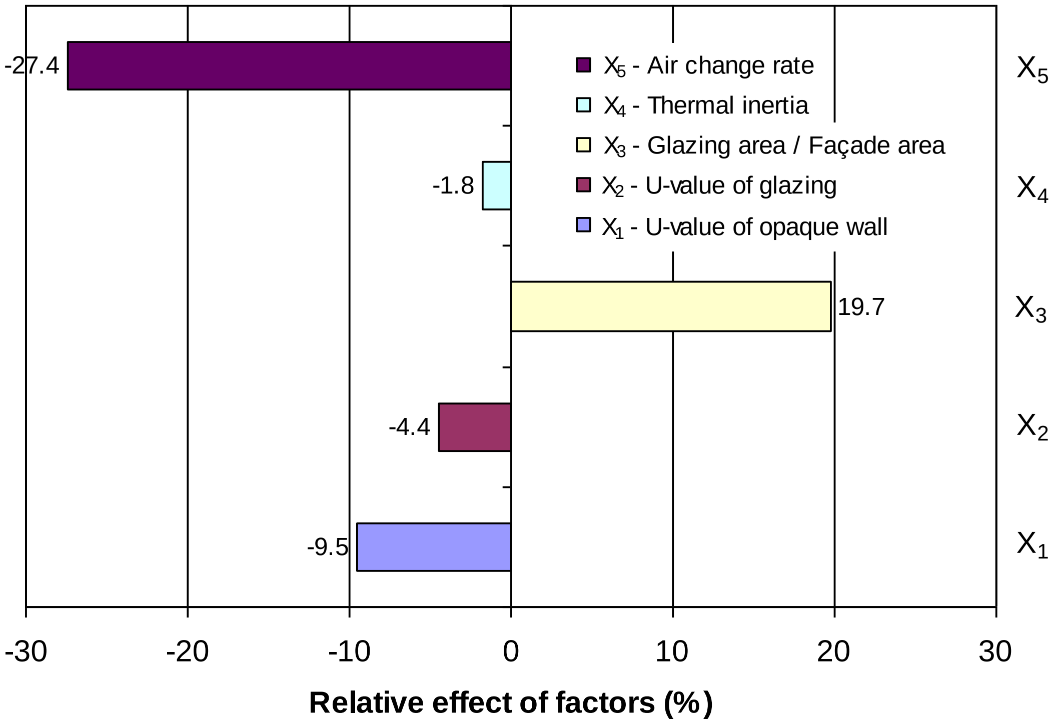

Figure 5 - Percentage change in energy needs

It is visible in Figure 5 the great weight of the air change rate and the relationship between the glazing area and total area of façade on the energy needs for heating, with the positive variation of these factors producing opposite effects on the response, contributing in the first case to the reduction of the energy needs and in the second case to its increase.

In regard to the cooling season, the study showed that reducing the heat transfer coefficient of the opaque areas, contrarily to the heating season, is harmful because it leads to a discomfort by overheating of the indoor air. Similarly, lowering the air change rate is also prejudicial in the cooling season, but with a much higher importance than the heat transfer coefficient. On the contrary, the decrease of the glazing areas in the south, accompanied by the decrease of the solar transmittance of glazing, is beneficial in terms of summer comfort. Also the thermal inertia of the building has a positive influence on thermal comfort, but without the impact of the two previous factors.

Regarding the interactions between factors, they are much stronger in the cooling season than in the heating one, being relevant the interaction between the transmittance of glazing of the south facade (X_2) and the relationship between glazing and total area of the same facade. It is observed that the effect X_2 is greater when X_3 is at the low level and smaller when X_3 is at the high level.

With regard to the weight that the factors mentioned have on the degree-hours of discomfort from overheating, is shown in Figure 6 the percentage variation of the model response (degree-hours of discomfort ODH_{25}) when each factor is varied from the "average" level (0) to the " high "(+1) one with the remaining factors kept at their "average" level (0).

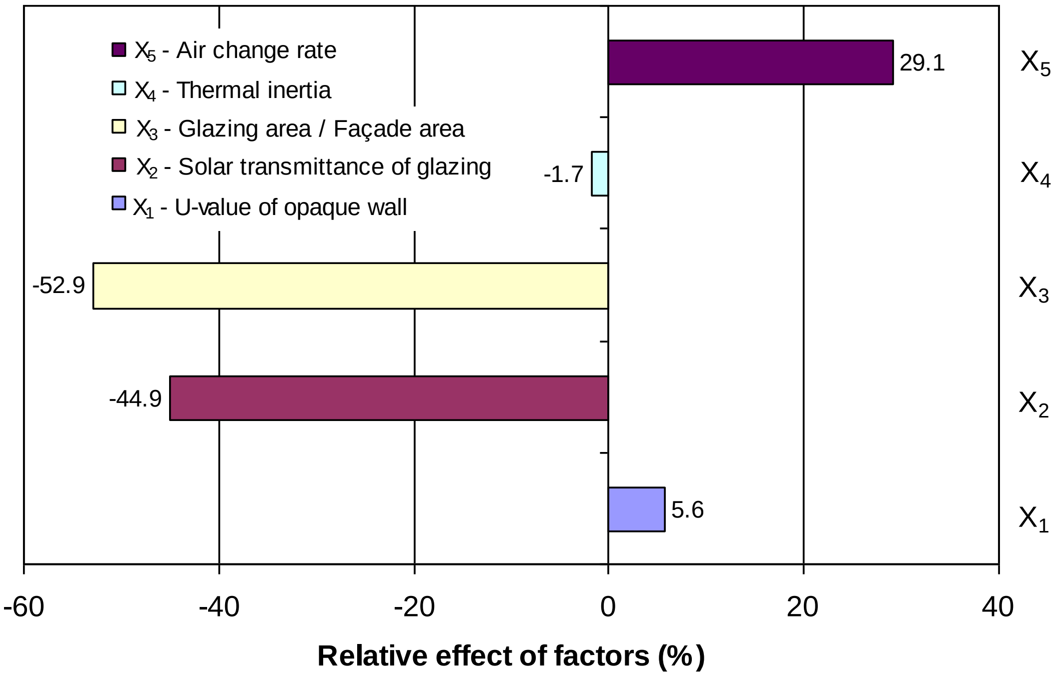

Figure 6 - Percentage change of degree-hours of discomfort from overheating (ODH_{25})

From Figure 6 it appears that the air change rate, the relationship between glazing and total area of façade and the solar transmittance of glazing have a weight on the degree-hours of discomfort from overheating incomparably superior to the other factors, so it is on these factors that the design should be focused on if the thermal performance is determined by the heating season.

In summary, it can be seen from Figures 5 and 6, except for the thermal inertia, that factors that are common to the heating and cooling seasons show opposite effects in the two seasons - what is advantageous for a season is unfavorable for the other: in the heating season is favorable to reduce the heat transfer coefficient of the opaque areas and the ventilation rate and increase the relationship between the glazing and total area of the south façade; on the contrary, in the cooling season, the variation of these factors in the opposite direction is favorable: it reduces the number of degree-hours of discomfort. This behavior of the factors, which have a different tendency in the heating and cooling seasons, paves the way for an optimization study if the objective is to maximize the global building thermal performance, that is, involving the whole year, in terms of the winter and summer seasons.

REFERENCES

[1] “Energy Efficiency in Buildings”, Directorate General for Energy – Economy Ministry, February 2002 in Portuguese.

[2] Lawrence Berkeley National Laboratory - EnergyPlus Engineering Document: The Reference to EnergyPlus Calculations, 2010.

[3] Hamby, D.M. 1994. "A Review of Techniques for Parameter Sensitivity Analysis of Environmental Models”, Environmental Monitoring and Assessment. Vol. 32, no. 2, pp. 135-154.

[4] Fisher, R.A., The Design of Experiments. New York: Hafner; 1935.