Development of a computer based controller for PUMA 560 manipulator

André Miguel Ribeiro de Azevedo

Department of Mechanical Engineering - IDMEC, Instituto Superior Técnico, Technical University of Lisbon (TULisbon) Av. Rovisco Pais, 1049-001 Lisboa, Portugal; e-mail: andre.ribeiro.azevedo@ist.utl.pt

Abstract—The aim of this work is to integrate the Quanser Q8 acquisition card and the AMTI BP400600-1000 force plate with PUMA 560 manipulator. The software to be developed is based on Matlab/Simulink and xPC Target toolbox. This integration allows the identification of PUMA 560 dynamics parameters and the development of a controller based on it.

Firstly the importance of the knowledge of the dynamics parameters is presented along with the different estimation techniques. The estimation through identification is detailed and the advantages and disadvantages of each identification method are discussed.

The hardware used during the work and its main features are present as well as the work done to integrate the different devices.

The estimation of dynamic parameters procedure, which include the modeling and identification of the manipulator is presented. The algorithm to formulate the dynamic models is derived from the Newton-Euler equations. The identification technique derived uses the benefits of periodic excitation trajectories, allowing the calculation of joint velocities and accelerations from the measured response in an analytic way.

Estimation of dynamic parameters enables the implementation of a inverse dynamics control scheme. The performance of the implemented controller is evaluated.

Finally the results are checked, discussed and the conclusions drawn.

Index Terms—Robot Manipulator, PUMA 560, Identification, Force Plate, Dynamics Parameters, Inverse Dynamics Controller

I. INTRODUCTION

Mathematical models are required for various steps in the design, simulation or control design of mechatronic systems. There are mainly two ways to obtain these models, the theoretical modelling based on physical principles and design data, and the experimental modelling (or identification) which builds a model based on measured input and output variables. In many cases the basic model structure is known from theoretical modelling, however, some parameters are not known precisely or change with time. Hence, for obtaining precise mathematical models, generally, identification methods have to be applied. They can be divided into two categories according to the models and the type of sensors they use. In the classical and most widely used identification approach the parameters are estimated from motion data and actuator torques or forces, both measured by internal measurement devices. The dynamic model relating these inputs and outputs is called internal model.

An alternative approach to identify the inertial parameters makes use of the reaction or external model of the robot. This model relates the motion of the robot to the reaction forces and torques on its base plate and is, therefore, totally independent from internal torques such as joint friction torques. The robot motion is measured by means of joint encoders and the reaction forces and torques are measured by means of an external sensor: a force plate.

This two models can be combined into one identifiable minimal model. This model allows to combine the joint torque or force and reaction torque and forces measurements in on parameter estimation scheme. This combined model estimation will yield more accurate parameter estimates, and consequently better actuator torque predictions.

In the following section, the experimental procedure that allows the estimation of dynamic parameters is described and as result of the estimation, an inverse dynamics control scheme is implemented.

II. HARDWARE INTEGRATION

A. Interfacing the PUMA 560 Robot

To control the PUMA 560 arm using a PC, the TRC004 and TRC006 retrofit cards were removed. The Quanser Q8 acquisition card and its terminal board are installed, replacing the original boards. The original power amplifiers and current/torque controllers remain in the control architecture, as does the arm cable card. The hardware configuration is illustrated in figure 1.

B. Interfacing the force plate AMTI BP400600-1000

The force plate used is a BP400600-1000 manufactured by AMTI. A DigiAmp DSA-6 amplifier is provided with force plate that ensures the communication between the force plate and PC. The communication is based on the UDP protocol.
Numerical issues. The generated models were validated with the worst and most frequent computation errors related with Math toolbox from Matlab. The symbolic modelling avoids Simulink diagram, and to communicate with the Q8 board instantiations to be implemented in C code as S-functions. In addition rapid design of control algorithms and allows specific functions to be implemented in C code as S-functions. In addition, Simulink running under Windows XP. Simulink enables the hardware configuration is illustrated in figure 2.

C. Experimental setup

The implemented software architecture is based on Matlab and Simulink running under Windows XP. Simulink enables rapid design of control algorithms and allows specific functions to be implemented in C code as S-functions. In addition, xPC Target 5.1 is used for real-time execution of the compiled C-code generated by the Real Time Workshop from the Simulink diagram, and to communicate with the Q8 board and force plate. The experimental setup is illustrated in figure 3.

III. Modeling

The dynamic models were formulated using the Symbolic Math toolbox from Matlab. The symbolic modelling avoids the worst and most frequent computation errors related with numerical issues. The generated models were validated with SymMechanics toolbox from Matlab.

A. External Model

The joint force and torque due to the movement of its own link can be expressed by simply treating the link as a load and applying the following equation:

\[
\begin{bmatrix}
F_{i,i} \\
M_{i,i}
\end{bmatrix} = \begin{bmatrix}
\ddot{p}_i^T - g \
\omega_i^T \Omega_i^T \
0 \\omega_i^T + \omega_i \omega_i^T
\end{bmatrix} \begin{bmatrix}
m_i \\
m_i c_i \\
I_i
\end{bmatrix}
\]

more compactly,

\[ W_{i,i} = A_i \dot{\phi}_i \]

where \( W_{i,i} \) is the wrench (vector of forces and torques) at joint \( i \) due to movement of link \( i \) alone. \( A_i \) is a dynamic matrix that describes the motion of link \( i \) and \( \dot{\phi}_i \) is the vector of unknown link inertial parameters.

The total wrench \( W_i \) at joint \( i \) is the sum of the wrenches \( W_{i,j} \) for all links \( j \) distal to joint \( i \):

\[ W_i = \sum_{j=1}^{n} W_{i,j} \]

Each wrench \( W_{i,j} \) at joint \( i \) is determined by transmitting the distal wrench \( W_{j,j} \) across intermediate joints. This is a function of the geometry of the linkage only. The forces and torques at neighboring joints are related by

\[
\begin{bmatrix}
F_{i,i+1} \\
M_{i,i+1}
\end{bmatrix} = \begin{bmatrix}
R_{i+1}^T \\
R_{i+1}^T \Theta_{i+1} \\
0
\end{bmatrix} \begin{bmatrix}
F_{i+1,i+1} \\
M_{i+1,i+1}
\end{bmatrix}
\]

more compactly,

\[ W_{i,i+1} = T_{i+1}^i W_{i+1,i+1} \]

where \( R_{i+1}^T \) is the rotation matrix rotating the link \( i+1 \) coordinate system to the link \( i \) coordinate system, \( \Theta_{i+1} \) a vector from the origin of the link \( i \) coordinate system to the link \( i+1 \) coordinate system and \( T_{i+1}^i \) a wrench transmission matrix.

To obtain the forces and torques at the \( i^{th} \) joint due to the movement of the \( j^{th} \) link, these matrices can be cascaded:

\[ W_{i,j} = T_{i+1}^i T_{i+2}^i \cdots T_{j-1}^i W_{j,j} = U_{i,j} \dot{\phi}_j \]

where \( U_{i,j} = T_{i+1}^i T_{i+2}^i \cdots T_{j-1}^i A_j \) and \( U_{ii} = A_i \). A simple matrix expression for a serial kinematic chain be derived from equations 3 and 6:

\[
\begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_n
\end{bmatrix} = \begin{bmatrix}
U_{11} & U_{12} & \cdots & U_{1n} \\
0 & U_{22} & \cdots & U_{2n} \\
0 & 0 & \ddots & \vdots \\
0 & 0 & 0 & U_{nn}
\end{bmatrix} \begin{bmatrix}
\phi_1 \\
\phi_2 \\
\vdots \\
\phi_n
\end{bmatrix}
\]
\[ W_1 = T_b^1 W_b = \begin{bmatrix} U_{11} & U_{12} & \cdots & U_{1n} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \]  

(8)

where \( T_b^1 \) is a wrench transmission matrix from base to joint 1. Reorganizing the previous equation, the external dynamic model can be expressed as:

\[ W_b = (T_b^1)^{-1} U_1 \phi \]  

(9)

more compactly,

\[ \tau_e = Y_e \phi_e \]  

(10)

B. Internal Model

The equation 7 is linear in the unknown parameters, but left side is composed of a full force-torque vector at each joint. Since only the torque about joint axis can usually be measured, each joint wrench must be projected onto the joint rotation axis, reducing the equation 7 to:

\[ \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} T_{1,1}^{11} U_{11} & T_{1,1}^{12} U_{12} & \cdots & T_{1,1}^{1n} U_{1n} \\ 0 & T_{2,2}^{11} U_{22} & \cdots & T_{2,2}^{1n} U_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{n,n}^{1n} U_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \]  

(11)

more compactly,

where \( T_{i,j}^{p} \) represent the projection of the joint wrench \( i \) onto the joint \( j \) rotation axis. The previous equation represents the internal dynamic model.

\[ \tau_i = Y_i \phi_i \]  

(12)

C. Combined Model

The previous internal and external model can be combined into one identifiable minimal model. This model allows to combine joint torque or force and reaction forces measurements in one parameter estimation scheme.

Since the previous models are linear in the desired unknown parameters, the derivation of a model that combine both models is very simple and can be obtained by:

\[ \begin{bmatrix} W_b \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} T_{1,1}^{b1} U_{11} & T_{1,1}^{b2} U_{12} & \cdots & T_{1,1}^{b1} U_{1n} \\ T_{1,1}^{11} U_{11} & T_{1,1}^{12} U_{12} & \cdots & T_{1,1}^{1n} U_{1n} \\ 0 & T_{2,2}^{11} U_{22} & \cdots & T_{2,2}^{1n} U_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{n,n}^{1n} U_{nn} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \]  

(13)

Besides inertial parameters, parameters that model joint friction have to be considered in the internal model. It is common to model friction in robot joints by means of viscous and Coulomb friction, however in this work only the viscous term is considered, yielding the following friction model for joint \( i \):

\[ \tau_{fi} = f_v \dot{q}_i \]  

(14)

where \( f_v \) represent the viscous friction parameter. The modelling of the friction always yields parameters that have to be added to the set of robot inertial model parameters. This parameters however do not influence the external model. Considering that vector \( \phi_{fi} \) contains the friction parameter of joint \( i \), the combined model can be expressed as

\[ \begin{bmatrix} W_b \\ \tau_1 \\ \tau_2 \\ \vdots \\ \tau_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \dot{q}_1 & 0 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{bmatrix} \]  

(15)

with,

\[ \chi = \begin{bmatrix} T_{1,1}^{b1} U_{11} & T_{1,1}^{b2} U_{12} & \cdots & T_{1,1}^{b1} U_{1n} \\ T_{1,1}^{11} U_{11} & T_{1,1}^{12} U_{12} & \cdots & T_{1,1}^{1n} U_{1n} \\ 0 & T_{2,2}^{11} U_{22} & \cdots & T_{2,2}^{1n} U_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T_{n,n}^{1n} U_{nn} \end{bmatrix} \]  

more compactly,

\[ \tau_c = Y_c \phi_c \]  

(16)

The equation 15 represents the combined dynamic model.

IV. IDENTIFICATION

The dynamic model of a robot is linear in dynamic parameters. After a model reduction, the dynamic model can be written as a minimal set of linear equations

\[ \tau = Y(q, \dot{q}, \ddot{q}) \phi \]  

(17)

Robot identification deals with the problem of estimating the model parameters \( \phi \) from the measurements during a robot excitation experiment.

A. Trajectory generation

The generation of an optimal robot excitation trajectory involves nonlinear optimization with motion constraints.

The excitation trajectory for each joint is a finite sum of harmonic sine and cosine functions, i.e., a finite Fourier series. The angular position \( q_i \), velocity \( \dot{q}_i \) and acceleration \( \ddot{q}_i \) trajectories for joint \( i \) are written as
\[ q_i(t) = q_i + \sum_{l=1}^{N_i} \alpha_{i,l} \sin(\omega_f t) - \frac{b_{i,l}}{\omega_f} \cos(\omega_f t) \]
\[ \dot{q}_i(t) = \sum_{l=1}^{N_i} \alpha_{i,l} \cos(\omega_f t) + b_{i,l} \sin(\omega_f t) \]
\[ \ddot{q}_i(t) = \sum_{l=1}^{N_i} -\alpha_{i,l} \omega_f \sin(\omega_f t) + b_{i,l} \omega_f \cos(\omega_f t) \]

with \( \omega_f \) the fundamental pulsation of the Fourier series. This Fourier series specifies a periodic function with period \( T_f = \frac{2\pi}{\omega_f} \). The fundamental pulsation is common for all joints, in order to preserve the periodicity of the overall robot excitation. Each Fourier series contains \( 2 \times N_i + 1 \) parameters, that constitute the degree of freedom for the optimization problem: \( a_{i,l} \) and \( b_{i,l} \), for \( l = 1 \) to \( N_i \), which are the amplitudes of the cosine and sine functions, and \( q_i \), which is the offset on position trajectory. The offset determines the robot configuration around which excitation will occur. The parameters for all joints are grouped into vector \( \delta \).

The robot excitation trajectory is optimized according the criteria \( - \log(\det(Y^T Y)) \). The optimization problem is formulated as:
\[
\delta = \text{arg min}_\delta \; - \log(\det(Y^T Y))
\]
subject to
\[
\begin{align*}
-2.5089 &\leq \theta_1 \leq 2.5089 [\text{rad}] \\
-1.6760 &\leq \theta_2 \leq 1.6760 [\text{rad}] \\
-2.1049 &\leq \theta_3 \leq 2.1049 [\text{rad}] \\
-1.4312 &\leq \dot{\theta}_1 \leq 1.4312 [\text{rad s}^{-1}] \\
-0.9425 &\leq \dot{\theta}_2 \leq 0.9425 [\text{rad s}^{-1}] \\
-2.1293 &\leq \dot{\theta}_3 \leq 2.1293 [\text{rad s}^{-1}] \\
-10 &\leq \ddot{\theta}_1 \leq 10 [\text{rad s}^{-2}] \\
-5 &\leq \ddot{\theta}_2 \leq 5 [\text{rad s}^{-2}] \\
-15 &\leq \ddot{\theta}_3 \leq 15 [\text{rad s}^{-2}]
\end{align*}
\]

The excitation trajectories are five-term Fourier series, yielding \( 11 \) parameters for each joint. The fundamental frequency of the trajectories is \( 0.2 \)Hz. The sampling rate for the simulation is \( 1 \)kHz. The length of the data sequence is \( 5000 \) data samples, i.e., one period of the trajectory. The figure 4, 5 and 6 show the optimized and the real trajectories for internal, external and combined models respectively. The cost function corresponding to optimized and real trajectories are presented in tables I and II.

The classical least-squares method is a well-known method to solve an overdetermined set of linear equations.
The LS estimator supposes that the measurements are corrupted with Gaussian white noise and that the standard deviation is equal for all channels. In practice, this condition is not satisfied which leads to a bias on parameter estimates. However, due to its simplicity, is often used. As the dynamic model is linear in dynamic parameters the least square solution is

$$\phi_{LS} = (Y^TY)^{-1}Y^T \tau$$  \hspace{1cm} (20)

The internal, external and combined parameters estimates are presented in tables III, IV and V respectively.

**Table III**  
**INTERNAL MODEL PARAMETERS ESTIMATES**

<table>
<thead>
<tr>
<th>#</th>
<th>Internal Model ((\phi_i))</th>
<th>Physical Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(I_{xz_3})</td>
<td></td>
<td>0.1567</td>
</tr>
<tr>
<td>2</td>
<td>(I_{yy_3})</td>
<td></td>
<td>-0.0323</td>
</tr>
<tr>
<td>4</td>
<td>(I_{xy_3} + a_3m_3c_{y_3})</td>
<td></td>
<td>-0.1481</td>
</tr>
<tr>
<td>5</td>
<td>(m_3c_{y_3} - a_3m_3)</td>
<td></td>
<td>-0.0170</td>
</tr>
<tr>
<td>7</td>
<td>(I_{yz_3} - a_3^2m_3)</td>
<td></td>
<td>0.2339</td>
</tr>
<tr>
<td>8</td>
<td>(I_{yz_2})</td>
<td></td>
<td>0.0072</td>
</tr>
<tr>
<td>9</td>
<td>(I_{yy_2} + a_2^2m_3)</td>
<td></td>
<td>0.5039</td>
</tr>
<tr>
<td>10</td>
<td>(I_{xx_3} + I_{yy_3} + I_{yz_3} - a_2^2m_2 - a_2^2m_3 - a_3^2m_3c_{y_3})</td>
<td></td>
<td>2.1246</td>
</tr>
<tr>
<td>11</td>
<td>(m_3c_{y_3} - a_3m_3)</td>
<td></td>
<td>-0.5532</td>
</tr>
<tr>
<td>12</td>
<td>(I_{xx_2} - (a_2^2 + d_{xy}^2 + 2a_3) m_3 - a_2 (m_3c_{y_3} + m_3c_{y_3}))</td>
<td></td>
<td>-0.3193</td>
</tr>
<tr>
<td>13</td>
<td>(a_2(m_2 + m_3) + m_2c_{x_2})</td>
<td></td>
<td>2.1131</td>
</tr>
<tr>
<td>14</td>
<td>(f_{c_{y_3}})</td>
<td></td>
<td>10.1970</td>
</tr>
<tr>
<td>15</td>
<td>(f_{c_{y_1}})</td>
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<td>(I_{c_{y_3}})</td>
<td></td>
<td>20.5084</td>
</tr>
<tr>
<td>17</td>
<td>(I_{xx_3} + I_{yy_3} + I_{yz_3} - a_3^2m_3 + 2d_{xx}m_3c_{y_3})</td>
<td></td>
<td>1.9853</td>
</tr>
<tr>
<td>18</td>
<td>(I_{xx_2} - a_2^2(m_2 + m_3))</td>
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<td>3.5732</td>
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</table>

**Table IV**  
**EXTERNAL MODEL PARAMETERS ESTIMATES**

<table>
<thead>
<tr>
<th>#</th>
<th>External Model ((\phi_e))</th>
<th>Physical Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(m_3c_{x_3} - a_3m_1)</td>
<td></td>
<td>-0.1364</td>
</tr>
<tr>
<td>2</td>
<td>(m_3c_{x_3})</td>
<td></td>
<td>0.5043</td>
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<tr>
<td>4</td>
<td>(I_{yy_3} + a_3m_3c_{y_3})</td>
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</tr>
<tr>
<td>5</td>
<td>(I_{xy_3} + a_3m_3c_{y_3})</td>
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<td>0.0188</td>
</tr>
<tr>
<td>6</td>
<td>(m_2c_{x_2} - a_2m_1)</td>
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<td>-27.9918</td>
</tr>
<tr>
<td>7</td>
<td>(m_2c_{x_2})</td>
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<td>-0.0632</td>
</tr>
<tr>
<td>8</td>
<td>(I_{xx_3} - a_3^2m_3)</td>
<td></td>
<td>0.7745</td>
</tr>
<tr>
<td>9</td>
<td>(I_{yy_2})</td>
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<td>-0.1293</td>
</tr>
<tr>
<td>10</td>
<td>(I_{yy_3} - a_2^2m_3)</td>
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<tr>
<td>11</td>
<td>(m_3c_{x_2})</td>
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</tr>
<tr>
<td>12</td>
<td>(I_{yy_2})</td>
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<td>-0.0573</td>
</tr>
<tr>
<td>13</td>
<td>(d_3m_3 + m_1c_{x_2} + m_2c_{x_2} + m_4c_{y_4})</td>
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</tr>
<tr>
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<td>(I_{xx_2} + a_2m_2c_{x_1})</td>
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</tr>
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<td>(m_1 + m_2 + m_3)</td>
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</tr>
<tr>
<td>16</td>
<td>(I_{xx_2} - a_2m_2^2)</td>
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<td>-12.3473</td>
</tr>
<tr>
<td>17</td>
<td>(I_{yy_1})</td>
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<td>18</td>
<td>(I_{yy_2})</td>
<td></td>
<td>0.2769</td>
</tr>
<tr>
<td>19</td>
<td>(I_{yy_1} + I_{yy_2} + I_{xx_3} + a_2^2m_1 + d_{xx}m_3 + 2d_{xx}m_3c_{y_3})</td>
<td></td>
<td>14.1570</td>
</tr>
</tbody>
</table>

**Table V**  
**COMBINED MODEL PARAMETERS ESTIMATES**

In order to validate the estimated parameters a new set of data were measured during a trajectory different from the one optimized. The match of predicted and measured torques and reaction forces determines the quality of the estimation. In figure 7, 8 and 9 are presented the validation of internal, external and combined dynamic parameters estimates respectively.

The least mean square error between the predict and measured torque and reaction are presented in tables VI, VII and VIII.
Figure 7. Internal model parameters validation

Figure 8. External model parameters validation

Figure 9. Combined model parameters validation

V. CONTROL

Good estimates for internal model parameters were obtained, as determined from the match of predicted torques to measured torques. This allows the implementation of an inverse dynamics control scheme based on internal model as shown in figure 10.

The desired and real trajectories of the robot are shown in figure 11. The error between desired and real trajectory is shown in figure 12. Once again the results shown in figure 11 and 12 prove that the estimates of dynamic parameters of the internal model are good.

As the results of estimates of dynamic parameters of combined model are not satisfactory it was impossible to implement a inverse dynamics controller based on this model.

VI. CONCLUSIONS

The integration of the hardware was necessary because the previous acquisition cards are based in bus ISA. This bus was replaced by bus PCI in computers. The integration of Quanser Q8 acquisition card enables the total connectivity between all devices needed for the identification of dynamic parameters.

The results shown prove that estimates of dynamic parameters of internal model are good as determined from the

<table>
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<tr>
<th>MSE</th>
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<th>$\tau_2$ [N m$^2$]</th>
<th>$\tau_3$ [N m$^2$]</th>
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<th>$M_y$ [N m$^2$]</th>
<th>$M_z$ [N m$^2$]</th>
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<td>44.8051</td>
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<table>
<thead>
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<th>$\theta_2$ [rad]</th>
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<table>
<thead>
<tr>
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<th>$\tau_2$ [N m$^2$]</th>
<th>$\tau_3$ [N m$^2$]</th>
</tr>
</thead>
<tbody>
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<td>1355.7726</td>
<td>79.4018</td>
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<table>
<thead>
<tr>
<th>MSE</th>
<th>$\sigma_1$ [N m$^2$]</th>
<th>$\sigma_2$ [N m$^2$]</th>
<th>$\sigma_3$ [N m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>92.2090</td>
<td>314.0775</td>
<td>104.0090</td>
</tr>
</tbody>
</table>
match of predicted torques to measured torques and the match of desired trajectory to real trajectory. The biggest tracking errors were observed at highest velocities, meaning that the friction model considered is insufficient. In order to improve the results a complex friction model has to be considered. The results for estimates of dynamic parameters of combined model are not satisfactory so it was impossible to implement an inverse dynamics controller based on this model. This unsatisfactory results can result from the induced moment around \( z \) by tightening the screws and induced moment around \( x \) by the robot cable. The match of predicted reaction forces and measured reaction forces determines the good estimates of dynamic parameters of external model. Both data measured for identification and validation of the parameters of this dynamic model are corrupted by the same induced moments referenced above.

Figure 11. Inverse dynamics controller validation

Figure 12. Error between real and desired trajectory

Referências


