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Medical Equipment Replacement: Economic Analysis under Deterministic and Uncertain Environments

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June 2012

Abstract

One of the most common capital budgeting decisions is the replacement of a plant or equipment. Replacement decisions have far reaching consequences on the ongoing life of a business, being particularly complex to model when it comes to medical equipment replacement. Health care institutions constantly deal with this problem. Finding the optimum timing to replace, regarding maintenance and operating costs, is of crucial importance to achieve an efficient budget management. The study considers a penalty measure, created with the objective to quantify the indirect costs incurred by the equipment downtime due to malfunction.

This document has developed a methodology to aid the decision-making process in Hospital de Santa Maria for medical equipment replacement. Two models are proposed to address this problem. A deterministic model, based on traditional investment evaluation tools and a model with uncertainty, based on the Real Options approach. A cost analysis is performed, not forgetting the penalty costs, as a measure to quantify the losses induced by equipment downtime.

The models were applied to the Linear Accelerators operating at the Radiotherapy Department. The results show that the optimum replacement timings obtained by the Real Options Model are, in average, longer than the ones obtained with the Deterministic Model. They also demonstrate that Penalty Costs play, in the replacement process, a more significant role than the Operating and Maintenance costs. The investigation on the effect of taxes revealed that these have small influence over the analysis results.

Keywords: Medical Equipment Replacement, Real Options, Discounted Cash Flow

1 Introduction

One of the most common capital budgeting decisions is the replacement of a plant or equipment (Lee & Petruzzi, 1984). Actually, all capital budgeting decisions can be viewed as replacement decisions. Expansion decisions are merely replacement decisions in which all cash flows from the old asset are zero (Guitman, 2003). Replacement decisions are ever-present in economic life, having far reaching consequences on the ongoing life of a business.

Equipment replacement is particularly relevant in the healthcare field. Medical equipment replacement is a particularly complex problem to model since it embraces a high number of considerations from many different natures. Some of these natures have subjective backgrounds. An increasing of costs associated with medical equipment, especially replacement costs, has been observed (Fennigkoh, 1992).

The concepts of capital investment planning are well established. However, they haven't found yet a widespread use within healthcare, especially when concerning the replacement planning of medical devices. Very few hospitals have any formal mechanisms for determining medical equipment replacement. This is also the reality in the Hospital de Santa Maria (Lisbon, Portugal), where this work has been developed. The replacement policy at this Hospital mainly consists on the usage of medical equipment until the end of its physical life. Such process is not based on any methodology or systematic procedure, constituting still a source of inefficiency in the Hospital management.

The objective of this work is to create a model to aid the equipment replacement decision-making in Hospital de Santa Maria (HSM). It will focus on the economical and financial aspects of the replacement problem, trying to identify the optimal replacement timing, minimizing the operation and maintenance costs. Therefore, the aim is to find the least-cost replacement interval, in order to achieve an equipment usage at the lowest possible cost.

2 Context

In order to achieve an efficient management of resources Healthcare Institutions need to elaborate strategic plans (Figueiredo, 2009). This work will use cost analysis as a replacement criteria. It is assumed that today replacements will affect the complete future chain of replacements. Future replacement decisions should not be independent from today actions. It is also assumed that the significance of present actions over future events becomes smaller as the time gap between them grows wider (Katz, 1998).

Hospital de Santa Maria (HSM) is the main assistance centre in the metropolitan area of Lisbon, providing direct services to over 350.000 inhabitants in the country's capital. The change of the juridical status for Public Enterprise in 2005 brought to the Hospital an opportunity to begin the modernization process. This change allowed for a new management model, with a special concern being given to measures with impact in the financial sustainability, whether by reducing costs or improving revenues. Action was taken in areas such as logistics, purchases, financial management, human resources and equipment and facilities.

The HSM has an annual budget of over 300 million euros. A significant part of the HSM expenses is attributable to operational and maintenance costs, associated with medical equipment. In what investment is concerned, medical equipment plays a significant role. Medical equipment roughly represents 30% of the Hospital annual investment, totalizing an amount of 13.3 million euros over the first three years as a Public Enterprise (Table 1).

Table 1. Equipment and Building Investments (Values in thousand of euros) (HSM, 2008).

Investment Program	2005	2006	2007	Total
Buildings/Facilities	3,468	3,611	6,239	13,318
Medical Equipment	3,304	4,124	5,900	13,328
Transportation Equip.	0	85	0	85
Administrative Equip.	230	311	828	1,369
Hardware/Software	2,373	2,542	2,706	7,621
Other Investments	433	265	0	698
Total	9,808	10,938	15,673	36,419

The biggest share of the investment in medical equipment was dedicated to the acquisition of a new Linear Accelerator for the Radiotherapy Department, with an approximate value of 2 million euros (HSM, 2008). Due to the huge financial effort necessary to replace it, and being one the most expensive items a hospital can purchase the Linear Accelerator was the selected equipment to analyze in this work.

The present replacement policy in HSM basically consists on the use of equipment until the end of its physical life. There is no specific methodology to determine when equipment should be replaced and when the replacement does happen, it does not happen in a previously planned way. This is a reactive process, which is triggered by an equipment malfunction, and only then, a replacement analyses is considered. Once a malfunction is detected, the origin department submits a repair order, to be analyzed by the department responsible for facilities management at HSM, the so called Serviço de Instalações e Equipamentos (SIE). SIE will then determine if it's worthwhile to repair the equipment, according to its record of failures. If a repair it's possible, the equipment is usually repaired. However, if the equipment is showing a high rate of failures or a major failure occurred, which will require an extensive and expensive repair, it will most likely be proposed for replacement. The proposition will then be submitted to the Administration Board, which has the final word in this process. In case of approval, a public examination will be launched for the acquisition of the equipment with the required features.

From the instant the equipment breaks down until the moment it is repaired or replaced a considerable period of time might pass. When a long stand-by period is predictable, due to a more extensive repair, and in order to comply with the commitment to provide an healthcare service of excellence, it is HSM policy to outsource the impaired services, paying to a third party institution to assure these services.

As described, the replacement process in HSM is flawed, in a way that it's not based in any model or pre-determined criteria. As so, a previous planning of when to replace an equipment, in a proactive fashion, based on a definite methodology applied in systematic and standardized way to all Hospital inventory, is not in practice (Figueiredo, 2009). This is an inefficient process that can compromise the correct execution of the budget plan. The lack of a systematic planning of whether an item should be replaced introduces a large amount of uncertainty in the Hospital activities. As a consequence, unexpected expenses may appear, generated by equipments that should have been replaced already.

The delay of treatment due to equipment downtime, even for short periods, can have very serious implications in the quality of healthcare services provided. Not having an appropriate treatment in the correct timing can reduce the success rate of the treatment or reduce life expectancy after it. As well, a late diagnostic can compromise the treatment success, and so, downtime periods can ultimately result in human losses. Therefore, in addition to the economical advantages, an efficient replacement policy can actually save lives.

3 Literature Review

Literature proposing methodologies for optimal replacement planning can be found since the 1940's (Katz, 1998). Clapham (1957) proposes a way of calculating the average economic life in which the calculated economic life would then be used as reference for evaluating if the equipment should be replaced or not. Clapham assumes that all the costs associated with a specific equipment result from the acquisition and maintenance costs for each period. The sum of all the equipment related costs, until a certain age, divided by that same age will give us average cost per period specific for that age.

Eilon and King (1966) question about the validity of some of the assumptions used in the economic life studies made at the time. Specifically, the authors wondered if it was correct to use a single average function to describe the maintenance costs of a fleet, and if it was correct to consider in the same way costs occurring in distinct periods of time. In other words, a cost incurred in a more distant period of time shouldn't have the same weight in the decision process, as a cost incurred in a proximal future.

Both Clapham (1957) and Eilon & King (1966) disregard any kind of technological evolution and consider the linear rise of the maintenance costs with age a direct sign of deterioration. Christer & Goodbody (1980) started considering inflation rate as an issue in the formulation of replacement decision economic models. In this article the replacement process during a high inflation period is analyzed by introducing it in the calculation of the discount factor.

Christer & Scarf (1994) announced a methodology specifically aimed for medical equipment replacement. This new work appears as an evolution of Christer's previous 1980 and 1988 papers, and tries to encompass as many factors as technological evolution, equipment deterioration and inflation. It also tries to include subjective aspects, particularly important when it comes to medical equipment, such as, patients well being and safety. Kierulff (2007) elaborated a work where he points out some limitations of the traditional economic methods applied in replacement decisions, and proposes some approaches to overcome these limitations. One of these limitations is pointed to the net present value decision model, one of the most widely used model by companies, in a way that it discounts only the differences between cash flows and salvage values of the replace and do-not-replace alternatives, assuming that the risk and inflation factors associated with these values are the same. According to the author, this can lead to wrong decisions. In an attempt to overcome the problems shown the author proposes a model where each alternative cash flow and terminal value are made visible and where an appropriate risk and inflation discount factors are assigned to each one of them.

Under the multi-criteria methodology, a model to recommend and prioritize medical equipment replacement was introduced in the St. Luke's Medical Centre. The model, proposed by Fennigkoh (1992), contains a total of ten attributes addressing four primary replacement issues: *equipment service and support, equipment function, cost benefits and clinical efficacy* (Fennigkoh, 1992). To minimize the model's sensitivity to the subjective information a simple scoring system of "yes-no" (0,1) was used for each attribute (Fennigkoh, 1992). According to the author this can be an advantage regarding to conventional deterministic methods, like net present value or internal rate of return, when only incomplete or subjective information is available. Toporkov (2007) proposes the assessment of reliability, efficiency, and service life of sophisticated medical equipment in Russia Federation, as a response to the 80% of worn-out or obsolete equipment in use, at the time, in public health organizations (Toporkov, 2007). The author establishes the prognosis of the service life as an important parameter in the reliability and efficiency of technological equipment and proposes a method to make this prognosis. This method is based on mathematical principles such as stochastic principles resulting from equipment use or principles of risk conditions, related to the probability of failures and accidents.

The Real Options approach tries to apply financial options theory to evaluate real assets investments. A financial option is one type of derivative financial instrument that grants its owner rights over another financial asset, such as stocks or bonds (Baltazar, Opções Reais em Projectos de Engenharia, 2009). This asset is referred as the underlying asset and the value of the option depends on or is derived from it (Amram & Kulatilaka, Real Options - Managing Strategic Investment in an Uncertain World, 1998).

Lee & Petruzzi (1984) used formulas developed for the valuation of securities options to evaluate the timing option and to derive decision rules for optimal investment timing, providing, amongst others, an application example for plant and equipment replacement (Lee & Petruzzi, 1984). This paper considered the importance of timing for the decision maker's opportunity set. The authors establish a relation between the ability of the manager to choose when to undertake an investment and the ability of the holder of a securities option to choose when to exercise that option. Copeland & Antikarov (2001) proposed an approach to valuing Real Options that may be applied to problems where the underlying asset has no correlation with any market-traded asset. For this the authors use the so called *Marketed Asset Disclaimer (MAD)* assumption, which consists on the assumption that the present value of the project without options is the best unbiased estimator of the market value of the project. Considering the assumptions made by Copeland & Antikarov (2001) and Smith & Nau (1995), Brandão (2005) suggested an approach using decision tree analysis to solve Real Options valuation problems. The modulation of the problem was made with a binomial decision tree, rather than a binomial-lattice, in order to obtain a simpler, more appealing and intuitive solution, using commercial decision analyses software. The Monte Carlo Simulation allows to determine the standard deviation of a project returns which then provides an estimate of the project volatility. It is also assumed that the project returns have a normal distribution, and so, the project values have a log-normal distribution and can be modeled as a *GBM* with a constant volatility.

4 Methodology

Two models will be proposed in this work to evaluate replacement decision. The first model will be based on the discounted cash flows model proposed by Christer & Scarf (1994). The second model will be based on Real Options theory. In this model costs are assumed to evolve randomly, in an unpredictable way. Monte Carlo simulations will be used to determine the range of possible outcomes for cost evolution, and then Real Options valuation methods will be used to assess the replacement decision.

4.1 Deterministic Model

The deterministic model proposed consists on formulating the total expected cost of retaining the existing equipment for a further K years, replacing it with a new equipment, with a purchase cost Rn, retaining this for a period L and then replacing again with a new model, with an equivalent cost Rn. This process is represented in Figure 1. The criteria function would then be the total expected discounted costs annualized over a series of $(K+L)$ annuities, employing the Equivalent Annual Annuities method.

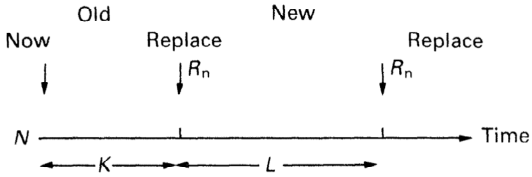

Figure 1. Replacement timeline (Christer & Scarf, 1994).

The total expected discounted cost of replacing a N years old existing equipment after K years, and again after further L years is denoted by $C(N;K,L)$ and is expressed by

$$C(N;K,L) = \sum_{i=1}^{K}\left(C_0(N+i) + p_0(N+i)\right)r^i + r^k\left\{R_n - S_0(N+k) + \sum_{i=1}^{L}(C_n(i) + p_n(i))r^i + (R_n - S_n(L))r^L\right\} \quad (1)$$

where:

N	Age of existing equipment in years;
K	Remaining life of existing equipment, expressed in years;
L	Economic life of the new replacement equipment in years;
$'o','n'$	suffix $'o'$ denotes old equipment, suffix $'n'$ denotes new equipment;
$C(i)$	total expected maintenance and operating costs per year for equipment in its i^{th} year of operation;
$p(i)$	penalty measure for equipment in its i^{th} year;
$S(i)$	Residual value of equipment i years old;
R_n	Purchase cost of new equipment;
r	Discount factor;
$C(N;K,L)$	Total expected discounted cost incurred over a $K+L$ period of time for equipment currently N years old.

The purpose is to minimize these costs relative to $(\mathrm{K}+\mathrm{L})$. This can be accomplished by converting the NPV of each (K,L) pair combination into an annuity. The annuity will be designated as Equivalent Annual Cost and the criteria function is equal to:

$$EAC(N;K,L) = C(N;K,L) * (A/P, i, K+L) \quad (2)$$

The objective is to find the (K,L) pair that minimizes $\mathrm{EAC}(\mathrm{N};\mathrm{K},\mathrm{L})$. The penalty cost associated with the equipment failure was modeled as a function of the item downtime. A cumulative measure of downtime periods was established. The cumulative downtime up-to-date $D(i)$ was modeled as folows:

$$D(i) = D(i-1) + \frac{\sum_{j=1}^{i} T(j)}{i} \quad (3)$$

where $D(i-1)$ represents the average of cumulative downtime up to month $(i-1)$, $T(j)$ is effective downtime of the item in month j (in hours). The cost associated to one single procedure outsourced C_{out} will simply be the difference between the cost paid to a third party institution to perform it $\mathrm{C}_{\mathrm{ext}}$ and the operational cost of performing that procedure in house $\mathrm{C}_{\mathrm{int}}$, as expressed by

$$C_{out} = C_{ext} - C_{int} \quad (4)$$

The penalty process can be obtained by multiplying the total downtime up to date by the cost incurred by one hour of downtime, as expressed by

$$p_n(i) = D_n(12i) \cdot C_{out} \quad (5)$$

where $\mathrm{p_n(i)}$ is the $\mathrm{n^{th}}$ penalty profile cost incurred in year i and $\mathrm{C_{out}}$ is the outsourcing cost of a service during one hour of equipment unavailability.

The cost of capital is composed by the costs of the various types of financing that a company or institution uses for its long-term capital investments (Cassimatis, 1988). The cost of capital will be the weighted average cost of each one of these sources of financing, and is designated as Weighted Average Cost of Capital (*WACC*) (Padrão, 2007). The weights to be used in this calculation should reflect realistically the proportions of the sources of funds the organization intends to use (Cassimatis, 1988). The *WACC* can be calculated using the following expression:

$$WACC = \frac{E}{A} \cdot i_e + \frac{D}{A} \cdot i_d(1 - T) \quad (6)$$

where:

i_e	Cost of equity;
i_d	Cost of debt;
E	Market value of the institution's equity;
D	Market value of the institution's debt;
$A = E + D$	Total liabilities and equity of the institution;
E/A	Percentage of financing that is equity;

The cost of debt i_d is the effective rate of interest the HSM has to pay on new debt issues (Cassimatis, 1988). The exponential equation to model the market value MV can be expressed by

$$MV(i) = R_n e^{-0.316i} \quad (7)$$

where $MV(i)$ is the market value of an equipment i years. Taxes will incur over earnings that occur when an asset is sold by a higher price than its book value. The cost associated with taxes will reduce the positive impact of selling the existing equipment, over cash flows. The influence of taxes in the residual value S can be mathematically expressed by the following expression

$$S(i) = MV(i) - [MV(i) - BV(i)]t \quad (8)$$

where $S(i)$ will be actual value received by the Hospital after taxes, considering that the sell is made in year i, $BV(i)$ is the equipment book value, and t is the tax rate incurred over profits at which the HSM is subjected.

4.2 Real Options Model

The purpose is to apply switching options concepts to the medical equipment replacement problem, based on the model proposed by Copeland & Antikarov (2001). With uncertainty and switching costs, the asset switch should only the take place when the value of expected costs is superior to the costs of the alternative in addition to the switching cost. The objective is to find the first switching point on the binomial tree. This point will provide the first time the replacement becomes advisable and will constitute a local optimum replacement point. The document assumed that total costs will follow a *Geometric Brownian Motion*. At each stage the cash flows can move up by u or down by d, according with the following expressions: $u = e^{\sigma}$ and $d = 1/u$ where σ represents the volatility of a specific type of costs. The Binomial Model requires an appropriate horizon of time that represents the maturity of the option. The Economic Life, can be determined by calculating the minimum Equivalent Uniform Annual Cost ($EUAC$) for the equipment's life cycle, as represented in Figure 2. The EUAC curve, which results from the combination of these two factors, will eventually meet an inflexion point that will correspond the equipment's Economic Life.

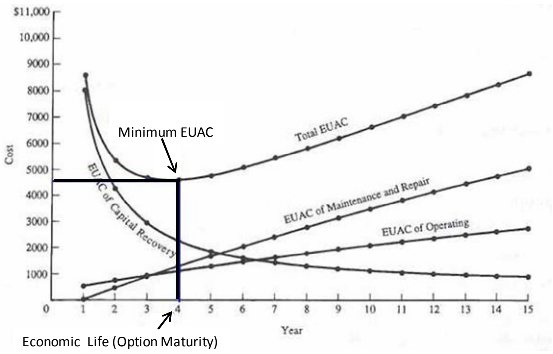

Figure 2. Graphical determination of the Economic Life (Option Maturity) (Oliveira, 2011).

Once determined the challenger's economic life, the time horizon can be settled for the Real Options analysis. The analysis with switching option consists on elaborating two binary trees to calculate the present cost value of each alternative. Considering an initial cash flow CF, Figure 3. describes the range of possible future cost flows , according with the different types of costs volatilities. At each step, the cash flows can go up (u) or down (d), as described by

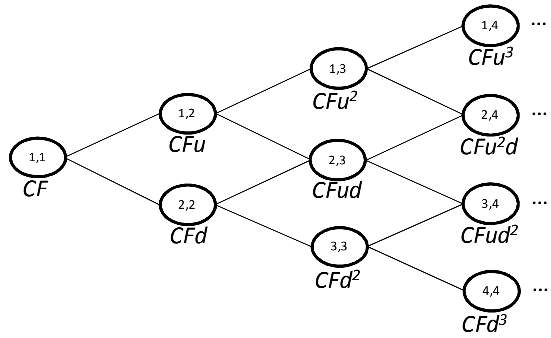

Figure 3. Binomial Cash Flow's tree.

It is now possible to discount the expected cash flows along the event tree of each alternative to determine the corresponding present value of cost:

$$PV_{i,j} = \frac{[PVd_{i,j+1}p + PVd_{i+1,j+1}(1 - p)]}{(1 + WACC)} + FCF_{i,j} \quad (9)$$

where $PVd_{i,j}$ represents the present value of costs associated with the defender equipment at specific stage i, j and $FCF_{i,j}$ corresponds to free cash flow from operations at each stage of the fold back process. This free cash flow is composed by operation and maintenance costs plus the penalty costs incurred in stage i, j. This process is represented in Figure 4.

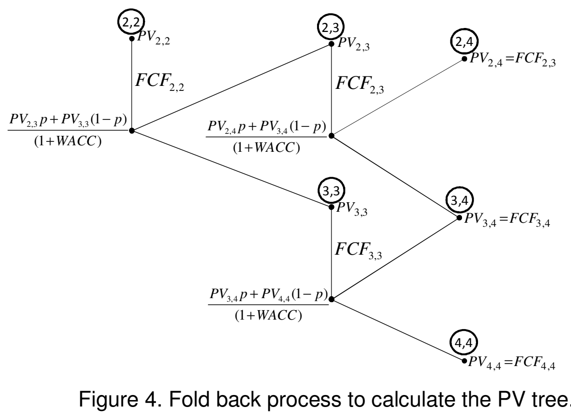

Figure 4. Fold back process to calculate the PV tree.

Two distinct methods will be used to compute the present value of the equipment costs. The defender's present cost will be calculated using the risk neutral approach while the challenger's present cost will be constructed from a replicating portfolio. It will be constituted of m units of the underlying asset, constituted by the defender's cost flows, and B units of the risk-free bonds, which represent an adjustment measure to discount the defender's cash flows:

$$m = \frac{PVc_{i,j+1} - PVc_{i+1,j+1}}{PVd_{i,j+1} - PVd_{i+1,j+1}} \quad (10)$$

$$B_{i,j} = \frac{PVc_{i,j+1} - mPVd_{i+1,j+1}}{1 + r_f} \quad (11)$$

At each stage, the present value of costs for the challenger equipment will be given by

$$PVc_{i,j} = mPVd_{i+1,j+1} + B + FCFc_{i,j} \quad (12)$$

The fold back process starts at the end nodes of the decision tree, where the following condition applies:

$$if\ PVd > PVc + Rn \Rightarrow Switch\ to\ New\ Equipment \quad (13)$$

And so, the end-of-period states of the tree are represented by $S = MIN(PVd, PVc + Rn)$. The present value of cost with flexibility is calculated at each node to determine whether is preferable to remain in the present state or to switch to the challenger alternative. At each node, the decision will be given by the following expression:

$$PV^f d_{i,j} = \frac{PV^f d_{i,j+1} p + PV^f d_{i+1,j+1}(1-p)}{(1 + WACC)} + FCFd_{i,j} \quad (14)$$

The flexibility to switch at any given time during the analyses can be obtained by $S_i = MIN(PV^f d_i, PVc_i + Rn)$.

5 Application and Results

The two models were applied in the Hospital de Santa Maria equipment replacement context, to the two Linear Accelerators models in operation.

5.1 Deterministic Model

A compilation of monthly downtime periods for each Linear Accelerator is presented in the following table:

Table 2. Effective monthly downtime periods (hours).

Equipment	Year 2009												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	TOTAL
CLINAC (VARIAN)	12.5	0.5	40.5	28.0	19.0	35.1	12.2	9.7	18.0	49.0	0.0	0.0	224.5
Mevatron (Siemens)	16.1	3.9	55.4	42.8	14.6	40.4	33.6	11.0	16.3	3.0	0.0	0.0	237.1
ONCOR (SIEMENS)	69.0	25.0	5.2	29.7	28.7	16.2	15.2	2.5	21.4	38.9	77.2	24.8	353.8

Making use of expression (3) it is possible to calculate cumulative downtime (in hours) over the year 2009, as graphically represented in Figure 5. The data was used to extrapolate the evolution of downtime over the complete equipment economic life.

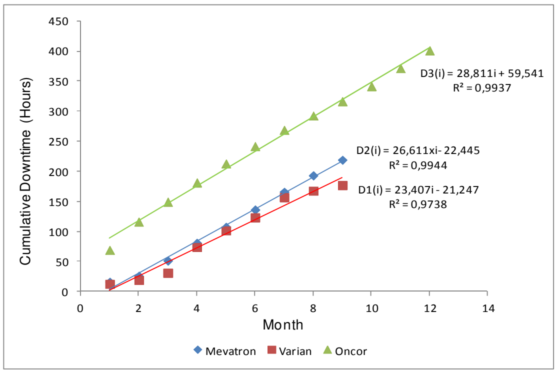

Figure 5. Downtime profiles for the three Linear Accelerators (2009).

Considering the operational costs of the Radiotherapy Department it is possible to determine a global operational cost of 1,599,207.00€, mainly attributable to the activity of the three Linear Accelerators in this service. Assuming a similar usage between these equipments, an average operational cost of 533,069.00€ can be attributed to each one of them a total of 34,641 treatments have been performed in the HSM Radiotherapy Department during the year of 2009. This represents a unitary cost per treatment of 46.17€. According with the data obtained from the HSM accounting department, the external cost C_{out} is 102.00€. Applying expression (4) it is possible to establish the effective outsourcing cost C_{out} for the HSM in 55.83€ per procedure. The average number of 3.59 procedures per hour of effective labor. This represents a cost of 200.56€ per hour of equipment unavailability.

The data obtained from the financial report HSM (2008) allowed to determine the *WACC* for HSM. The capital structure gives an E/A equal to 48% and a D/A equal to 52%. Due to the lack of market relative information in this report, it was decided to use the financial data from José de Mello Saúde. In this way it was possible to use actual market data to calculate the *WACC* parameters, which was found to be a more adequate solution. The 2009 year data was considered more appropriate to describe the present economic environment and therefore selected for the WACC calculation. This way the cost of debt can be settled in $\mathrm{i_d} = 5.22\%$ and the cost of equity can be settled in $\mathrm{i_e} = 8.98\%$. Computing the values one obtain

$$WACC = 0.48 \cdot 0.0898 + 0.52 \cdot 0.0522(1 - 0.25) = 0.0635 \quad (15)$$

The Residual Value $\mathrm{S(i)}$ is obtained after considered the tax effect over the Market Value $\mathrm{MV(i)}$ obtained by the Hospital as revenue, if a replacement is performed in year i. Calculations were made considering different combinations of penalty profiles for both defender and challenger alternatives. The results for the optimum replacement time (K, L), as well as the corresponding equivalent annual cost $EAC(N; K, L)$ values are presented in Table 3. for the analysis without taxes:

Table 3. Replacement analysis without taxes.

EAC(K,L) (K,L)		Defender Penalty Profile: Oncor 1	Oncor 2	Oncor 3	Philips 1	Philips 2	Philips 3
Challenger Penalty Profile Philips	1	1,707,445 € (3 , 9)	1,669,106 € (1 , 10)	1,676,321 € (1 , 10)	1,467,361 € (6 , 5)	1,492,056 € (4 , 7)	1,509,539 € (4 , 7)
	2	1,689,851 € (3 , 9)	1,705,864 € (2 , 9)	1,717,896 € (1 , 10)	1,484,002 € (6 , 5)	1,506,873 € (5 , 6)	1,528,162 € (4 , 7)
	3	1,720,821 € (5 , 8)	1,743,686 € (2 , 9)	1,822,171 € (3 , 8)	1,497,264 € (6 , 5)	1,521,599 € (6 , 5)	1,547,733 € (5 , 5)

K - Number of remaining years in operation (Defender)

L - Recommended age for replacement of the Challenger

The results in Table 3. for the Oncor as a Defender exhibit average *EAC* values of 1,717,018€. The Philips replacement study exhibits an average *EAC* value of a 1,506,065€. The average *EAC* of the Philips, as a Defender, is 12.29% lower than the average *EAC* obtained for the equipment Oncor. The reason for a lower average EAC value for the Philips equipment is that the Oncor has been in operation for 5 years. Consequently, it will present a higher Penalty Cost that will determine a higher *EAC*. Regarding the replacement periods results, the Oncor has average retaining periods for the Defender (K) of 2.33 years. The replacement periods for the Challenger (L) have an average value of 9.11 years.

The Philips replacement study results exhibit average K values of 5.11, and L values of 5.78 years. The Oncor results present an average replacement time 54.35% lower for the Defender and 57.69% higher for the Challenger, than the Philips results. The lower average replacement value for the Oncor (as a Defender) is explained by the fact that it is in operation for 5 years when compared with the 1 year operation for the Philips. Therefore the Oncor is expected to have an earlier replacement recommendation.

The combination of a low penalty costs for the Defender and high penalty costs for the Challenger result in a retaining of the low cost equipment for a longer period. The 57.69% difference between the replacement timing of the Philips as a Challenger, in both studies, can demonstrate that the economic life of equipment it is not independent on the predecessor equipment. Considering now the tax effect in the analysis, the following results are obtained:

Table 4. Replacement analysis considering taxes.

EAC(K,L) (K,L)		Defender Penalty Profile: Oncor 1	Oncor 2	Oncor 3	Philips 1	Philips 2	Philips 3
Challenger Penalty Profile Philips	1	1,718,894 € (3 , 9)	1,684,203 € (1 , 10)	1,694,336 € (1 , 10)	1,491,868 € (6 , 7)	1,513,296 € (5 , 7)	1,534,760 € (4 , 8)
	2	1,701,309 € (3 , 9)	1,775,690 € (3 , 9)	1,735,151 € (2 , 10)	1,503,550 € (7 , 6)	1,527,486 € (6 , 6)	1,551,572 € (5 , 7)
	3	1,729,287 € (5 , 8)	1,756,354 € (3 , 9)	1,835,967 € (3 , 8)	1,515,764 € (7 , 5)	1,543,432 € (6 , 6)	1,570,943 € (6 , 6)

K - Number of remaining years in operation (Defender)

L - Recommended age for replacement of the Challenger

Analyzing the results of Table 4., it is possible to observe that the Oncor exhibit average *EAC* values of 1,736,799€. The Philips results exhibit average *EAC* values of 1,528,074€, therefore 12.02% lower than the average value obtained for the Oncor replacement study. The effect of taxes over the replacement periods results originates, for the Oncor study, average values for the Defender (K) of 2.67 years, and average replacement periods for the Challenger (L) of 9.11 years. The Philips replacement study results show K values of 5.78 years, and average L values of 6.44 years. The Oncor results present an average replacement time 53.85% lower for the Defender and 41.38% higher for the Challenger, than the Philips results.

The investigation on the effect of taxes over the model *EAC* results indicates that the tax effect is more relevant on the Philips replacement study than on the Oncor one. This result is according with expectations, since the Philips, as a Defender, is still on its depreciation period. The tax effect is only felt during this period and therefore it will become more relevant for the Philips replacement. Notice that both replacement studies experience the influence of taxes in the replacement of the Challenger.

The analysis of the tax effect over the replacement periods indicates that this is more relevant in the Philips replacement study, generating changes in the replacement periods of all the Penalty Profile combinations considered in the analysis. Again these results are explained by the fact the tax effect is felt in the Defender and Challenger replacements for the Philips replacement study, as long as it is only felt in the Challenger replacement in the Oncor replacement study.

The results highlight the sensitivity of the model as even for small changes in the *EAC* values produce significant changes in the replacement periods. This signifies that small changes in acquisition or maintenance contracts can become very relevant for the recommended service life of the equipment.

5.2 Real Options Model

The first step for the Real Options analysis is to establish an appropriate time horizon for the option exercise. This is accomplished by calculating the challenger's Economic Life, as graphically represented in Figure 6. establishing an 11 years Economic Life for Philips Elekta.

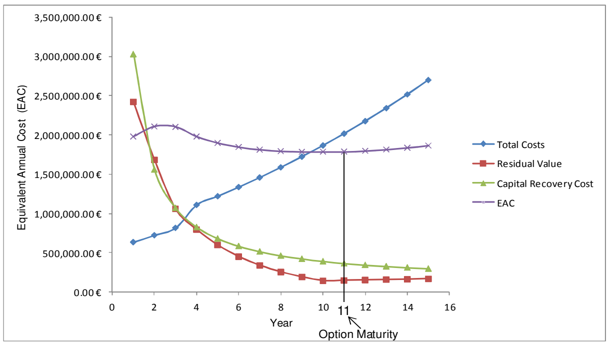

Figure 6. Graphical determination of the option maturity.

Once determined the time horizon (maturity) for the analysis, a Monte Carlo simulation is performed to evaluate a stochastic evolution of future costs. The Monte Carlo simulation parameters are determined according with the historical data available from the HSM Radiotherapy Department. The uncertainty lies over three types of costs: Maintenance, Operating and Penalty Costs. The remaining cost sources have a pre-established or an estimated nature. So, it was considered that there was no uncertainty about their future evolution.

Costs volatility can be obtained by measuring the standard deviation of the costs growth rate over time. Considering the monthly Operating Costs data available for the year 2009, it was possible to determine the average growth rate of -0.17% and a corresponding standard deviation of 7.65%, obtained through a data fit. Using these inputs from the Monte Carlo simulation was employed to determine the standard deviation that will characterize the stochastic growth rate for the Operating Costs. The simulation is conducted for 10,000 iterations. Each one generates, according to an adequate distribution, a random growth rate value.

The binomial model can assume a Normal or a log-normal distribution for the data. Since the operating costs can exhibit positive and negative variations every month, negative growth rates can be obtained. Therefore, the Normal distribution was used to run the simulation. From the simulation output presented in Figure 7., it is possible to establish a growth rate standard deviation of 7.52% per month, for the Operating Costs. The extracted standard deviation will allow to determine the up (u) and down (d) movements, used in the binomial tree construction.

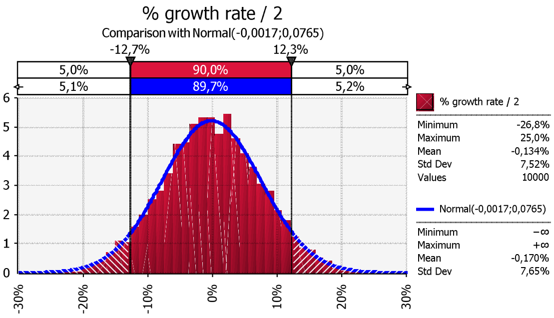

Figure 7. Simulation results for the Monthly Operating Costs. Standard Deviation = 7.52%.

It was considered that a satisfactory approach to describe maintenance costs volatility would be to calculate the standard deviation of the available data, regarding the update in the maintenance contracts for the three linear accelerators in operation in 2009. Therefore the maintenance costs standard deviation value used in the Cash Flows Tree construction was 0.95%.

Once determined the implicit volatilities for both maintenance and operating costs it was necessary to combine these volatilities to build a single Cash Flows Tree. In order to determine the volatility associated with the Penalty Cost, a measure of the cumulative downtime growth rates is made. Table 5. shows the average growth rates of the effective cumulative downtime periods, and corresponding standard deviations, for each equipment.

Table 5. Historical effective cumulative downtime and corresponding growth rates

Equipment	Average Growth Rate	Standard Deviation
Profile 1 - Varian	56.64%	17.80%
Profile 2 - Mevatron	54.12%	17.70%
Profile 3 - Oncor	16.54%	6.06%

Since there are three data sets (one per equipment), the Real Options analysis was made considering each one of this profiles for the Penalty Cost Tree construction. Since a cumulative measure of downtime is being considered negative growth rates cannot be obtained. Therefore a log-Normal distribution represents a better fit for the Penalty Cost data, being used to run the simulations. Considering the parameters obtained from the Penalty Costs historical data, it was possible to run the Monte Carlo simulations and determine the stochastic volatilities. The monthly standard deviation values σ_m obtained for the penalty costs and respective conversion to annual values are presented in Table 6.

Table 6. Conversion of simulated monthly standard deviations into annual values.

Standard deviation (σ)	σ_m	$\sigma_m\sqrt{T}$	σ_{annual}
Profile 1 - Varian	18.2%	$18.2\sqrt{12}$	63.05%
Profile 2 - Mevatron	17.5%	$17.5\sqrt{12}$	60.62%
Profile 3 - Oncor	6.18%	$6.18\sqrt{12}$	21.41%

Once obtained the standard deviation for all types of costs, it is possible to apply the Real Options model that begins by constructing the Cash Flows Trees for each type of cost. Analogously to the deterministic model, the Real Options analysis will be made considering the combination of all the penalty profiles for Defender and Challenger. In the replacement analysis Oncor and Philips acted alternatively as defenders and Philips acted as the challenger in both studies.

The next step is to build the Present Value Tree without flexibility. The Risk-Neutral rate of return used to construct the replicating portfolio and to calculate the Risk-Neutral probability, has to be similar to the return rate of a traded financial product considered to be a safe investment, hence without risk. Considering these requirements it was decided to take as a reference the Treasury Obligations issued by the Portuguese Government with a coupon rate of 4.95% with a maturity of 11 years, in October 2023.

The Real Options analysis ends by presenting a Binomial Tree with flexibility (Figure 8), highlighting the nodes where the replacement is advisable. These can occur when the Present Value of the expected future cost of Defender is higher than the equivalent Present Value of Challenger. The last one includes the cost of acquiring the Challenger equipment in that same year minus the Residual Value obtained from the sale of the Defender. Figure 8. permits to observe that the first replacement will occur in the fourth year.

Year 0	1	2	3	4	5	6	7	8	9	10	11
9,031,361	12,756,786	18,065,104	25,291,409	34,144,466	44,616,806	58,544,818	76,107,898	95,719,484	111,686,025	110,478,590	69,845,899
	5,856,321	8,066,883	11,189,722	15,468,235	20,880,669	26,412,275	31,971,863	37,783,277	41,965,535	40,317,827	26,161,041
		3,848,918	5,148,895	6,947,297	9,397,480	12,565,589	16,108,035	18,360,518	19,332,063	18,084,160	12,507,372
			2,558,734	3,312,501	4,311,922	5,627,223	7,301,642	9,239,704	10,889,623	10,391,103	7,881,802
				1,715,300	2,144,949	2,675,187	3,314,874	4,043,295	4,753,171	5,112,674	4,244,454
					1,152,914	1,391,224	1,654,891	1,922,676	2,138,358	2,166,001	1,690,598
						770,125	893,758	1,006,072	1,075,809	1,039,569	768,678
							505,468	558,923	583,801	547,365	389,756
								320,953	331,039	305,179	212,520
									192,126	175,575	120,822
										102,648	70,215
											41,256

Defender Present Value Tree

First Switch Year 4

Figure 8. PV tree with flexibility, highlighting the nodes where a switch can occur (Penalty Profile combination (1,1)).

The analysis is performed for the all the Penalty Profile combinations for both equipments. The results of the Real Options analysis are presented in Table 7. They show the year where the first replacement incurred and the *EAC* obtained. Penalty profile combinations (3,1) and (3,2), for the Oncor (defender), and combinations (3,1), (3,2) and (3,3), for Philips (defender), originated never replacement decisions. In this case, the option to switch is never exercised during the option maturity, and so, it is considered that the replacement is made for obsolescence reasons.

Table 7. EAC of the first replacement and year of first replacement.

		Defender Penalty Profile					
EAC		Oncor			Philips		
1st Switch		1	2	3	1	2	3
Challenger Penalty Profile Philips	1	9,302,044 €	11,135,000 €	2,337,448 €	9,015,740 €	10,038,283 €	1,564,124 €
		4	4	11	11	11	11
	2	5,170,518 €	10,190,001 €	2,337,448 €	14,004,214 €	8,130,217 €	1,564,124 €
		4	4	11	10	11	11
	3	6,679,318 €	6,820,918 €	2,000,139 €	5,357,573 €	8,512,226 €	1,564,124 €
		3	3	11	6	6	11

Table 7. exhibits, for Oncor (Defender), an average *EAC* value of 6,219,204€. The Philips (Defender) has an average *EAC* value of 6,638,959€, which corresponds to a value 6.75% higher than the value obtained for the Oncor replacement study. For the year of the first replacement, the Oncor shows an average result of 6.11 years. The Philips results exhibit average replacement values of 9.78 years. The results for Oncor exhibit a first replacement average value 37.5% lower than the average value obtained for the Philips.

The Real Options Model results highlights that, as expected, low uncertainty, characterized by low volatility values, conduce to later replacements. In fact, low volatility scenarios can actually conduce to never-replace decisions. It is also possible to realize that volatility is the more relevant parameter in the analyses outcomes, as similar volatilities produce similar results even if the starting cash flows are different.

5.3 Results Comparison and Sensitivity Analysis

Comparing the results obtained for the two models in analysis, the considerable higher *EAC* values obtained for the Real Options Model are explained by the shorter time horizon of the analysis. The Deterministic Model exhibits an overall time horizon K+L of 2.67+9.11 = 11.78 years for the Oncor Replacement study and 5.78+6.44 = 12.22 years for the Philips one. This represents a 92.79% higher time horizon for the Deterministic Oncor Replacement study, and a 24.95% higher time horizon for the Deterministic Philips Replacement study, when compared with the time horizon values obtained for the Real Options Model.

Since the number of annuities to distribute the costs is smaller for the Real Options Model it will originate higher equivalent annuities. The replacement time results exhibit a 56.30% lower value for the Defender replacement time in the Deterministic Oncor Replacement study and a 40.89% lower value for the Defender replacement time for the Deterministic Philips replacement study. The higher replacement periods exhibited by the Real Options model are explained by the fact that some Penalty Profile combinations originate late (11 years) or never replace recommendations, elevating the average replacement recommendation.

These elevated values are explained by the fact that low volatility values for the Defender and high volatility values for the Challenger tend to delay the replacement, as the uncertainty about the Challenger future costs increases. This way it is preferable to maintain the low uncertainty equipment for longer.

5.3.1 Deterministic Model

Considering a 30% increase in the *WACC*, to 8.25%, the results indicate that an increase in the *WACC* produce a reduction in the *EAC* values, being the reduction more relevant in the Philips Replacement study. The decrease in the *EAC* values in expected and is justified by the *EAC* calculation formula. Since the discount rate is higher, the discounted cash flows will be lower, and so the corresponding *EAC* value. The results obtained for the variation in the replacement periods indicate that even small variations in the *EAC* values will generate changes in the replacement periods. It is also possible to observe that an increase in *WACC* produces similar changes in both replacement studies.

The variations in the results for the analysis with taxes produced by a 30% increase in the *WACC*, are similar to the ones obtained for the results without taxes. It is reasonable to say that if an increase in the *WACC* is observed, an increase in the optimal replacement periods is expected, with higher prevalence in the defender replacement time (K). Considering now an opposite economical evolution, with a reduction in *WACC* to 4.44%, an opposite effect will be observed for the *EAC* values. A 30% decrease in *WACC* originates a reduction in the replacement periods. However this reduction is not very significant. As expected a 30% decrease in the *WACC* induces an opposite effect over the analysis results when compared to the one created by a 30% increase in *WACC*. When the *WACC* decreases, the replacement periods K and L tend to decrease. These results indicate that economic scenarios characterized by low interest rates and low risk, accelerates the replacement. However this effect is not very significant, indicating that the *WACC* parameter has a little influence in the model. Analyzing the results of the *WACC* variation it is also possible to observe that positive variations in the *WACC* have higher impact, over the Model results, than negative ones.

Analyzing the results obtained for the sensitivity analysis over the 30% variation in the Penalty Cost parameter it is possible to conclude that, as expected, a variation in penalty originates a considerable change in results, both for the *EAC* values and for the replacement periods K, L. These results demonstrate the high importance of this parameter in the Deterministic Model. It is also possible to observe that, contrary to the *WACC* variation, negative Penalty Cost variations produce a higher impact in the results. A decrease in the Penalty Cost profile will originate a higher variation both in the *EAC* values and replacement periods. This reduction is particularly relevant for the retaining periods for the Defender. Consequently, in the replacement decision context low penalty profiles equipments shall have higher retaining periods.

The model is considerable more sensitive to Penalty Costs variations, rather than changes in the *WACC*. As so, for the replacement process in HSM the analysis of the equipment failures profile shall be more relevant than the analysis of the Operating and Maintenance Costs. As the penalty costs arise from the downtime periods, it becomes of extreme importance for HSM to minimize such periods.

5.3.2 Real Options Model

The results of the sensitivity analyses of the 30 % variation in volatility over the Real Options model highlighted that the model is more sensitive to negatives variations in this parameter. A decrease in volatility produces a slightly bigger impact in the timing of first replacement. An increase in volatilities results on a decrease of the time until the first replacement and a decrease of volatilities extends the retaining period of the equipment. The *EAC* results obtained indicate that higher volatilities generate higher values to the project options. This can be explained by the fact that higher volatilities generate higher possible final payoffs, elevating the reduction of costs obtained if the option to replace is exercised. High volatilities are a result of economic uncertainty, and therefore, the reduction in replacement periods is expected, as the option owners will try to dispose of the risky asset sooner, in order to avoid costly future outcomes. Highly uncertainty periods will tend to precipitate the replacement. In an opposite way the equipment should be retained for longer for predictable future evolutions of economic conditions.

6 Conclusions

The objective of developing a methodology to address the medical equipment replacement process at HSM has been accomplished. A deterministic and a stochastic model were constructed, with the purpose to identify the optimum replacement timing for selected medical devices, in a cost-effective way, potentiating a reduction of costs in HSM. The models proposed managed to accurately identify optimum replacement timings for the two linear accelerator models in service in the Radiotherapy Department. Another goal of the dissertation was to create a measure to value the subjective aspects regarding medical activity, such as patient safety or quality of medical service provided. This goal was accomplished by creating a penalty measure as a function of the costs incurred with equipment downtime due to malfunction.

From the analysis performed, considering different combinations of Penalty Profiles, the Deterministic Model managed to establish an average replacement horizon for the Siemens Oncor (defender) of 2.33 years. Assuming that it will be replaced by a Philips Elekta like equipment, it was possible to determine an average replacement time for this substitute (Challenger), within the next 9.11 years.

The effect of taxes over the analysis was investigated, determining a change in results to 2.67 and to 9.11 years, respectively. This allows to conclude that taxes have little influence over the results, especially when the depreciation period of the existing equipment already expired. The replacement analysis for the Philips Elekta (Defender) allowed to obtain an average replacement value of 5.11 years. Assuming a like-by-like replacement for this equipment, a 5.78 years average value was obtained for the replacement of this new item (Challenger). The tax effect induces a change in values to 5.78 years and 6.44 years, respectively.

It has been demonstrated that the Penalty Cost parameter is more relevant for the deterministic analysis than the Operating and Maintenance costs evolution. This indicates that under the HSM replacement context, a privileged importance should be given to the analysis of the equipment's failure profile when compared with the analysis of the associated Operating and Maintenance costs.

In the Real Options model, a more complete range historical data would increase the accuracy of the model results. Even though, it was possible to obtain reliable values for potential costs of equipment replacement, highlighting the relevance of studying the volatility as a tool for the Real Options analysis. The model determined an average replacement timing for the Siemens Oncor of 6.11 years and 9.78 years for the Philips Elekta. It is so possible to conclude that the Real Options model exhibits longer replacement periods than the Deterministic model, highlighting the influence of volatility over the results. High volatility values for the defender and low volatilities for the challenger equipment originate faster replacements. In opposition when the defender exhibits low volatility values and the challenger exhibits high volatilities, the replacement periods tend to increase.

The Real Options value also revealed a higher influence of Penalty Costs volatility over the final outcomes when compared with the volatility of Operating and Maintenance costs. This indicates that uncertainty about equipment failures and consequent downtime shall be more relevant in the replacement process in HSM, than the uncertainty about the evolution of operating costs or the value of future maintenance contracts. The sensitivity analysis performed indicates that the Deterministic model is more sensitive to positive variations in the *WACC* and negative variations for the Penalty Costs. The Real Options model revealed to be more sensitive to negative variations in the volatility parameter.

It would be advisable to implement, in the HSM, a database to keep an accurate historic record of equipment failures and related costs. The existence of extensive data would allow for a higher number of analysis, with more parameters, in a more accurate and reliable fashion. This decision would constitute a contribution to promote future works of this nature at HSM. In conclusion, despite of the data limitations, this work made an useful contribution for the creation of an cost-effective and methodological management of medical equipment in HSM.

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