Analysis of Laboratory Generated Sea Waves

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Abstract

Spectral analysis and statistical analysis are two major methods in analyzing the characteristics of ocean waves.

In the spectral analysis, both indirect Blackman-Tukey method and direct Cooley-Tukey method are adopted simultaneously in our work to obtain the wave spectra of wave series in 23 sea states generated in the laboratory. After comparing the obtained spectra with the standard one proposed by Goda, and significant wave heights and periods with the corresponding statistical results, it is drawn that these two methods almost give the same result. Furthermore, some other useful laws are also summarized at the same time which mainly focus on the spatial variation and relationship with initial wave steepness for the significant wave height, wave period, spectral moment, steepness and spectral width parameter.

In the statistical analysis, some relationships about skewness and kurtosis are researched and the exceedance distributions of wave height, crest and trough in the experiment are compared with Rayleigh distribution and GC approximation. Typical wave period distributions and joint distributions in three sea states which are characterized by the initial steepness are also presented and compared with the theoretical ones. At last, a few characteristics about the maximal wave height are simply considered in this thesis.

**Keywords:** surface displacement, wave crest and trough, significant wave height, wave period, peak frequency, wave basin, wave spectra, spectral moments, Rayleigh distribution, exceedance distribution, joint distribution of wave period and wave height, maximal wave height.
Resumo

O análise espectral e análise estatística são os dois métodos principais para a análise das características das ondas do mar.

Na análise espectral, tanto o método Blackman-Tukey indirecto e método directo Cooley-Tukey são adoptados simultaneamente no nosso trabalho para obter os espectros de onda da série em 23 estados do mar gerados em laboratório. Depois de comparar os espectros obtidos com o padrão proposto por Goda, e alturas e períodos com os resultados estatísticos correspondentes, ela é desenhada que estes dois métodos e quase dão o mesmo resultado. Além disso, algumas outras leis úteis são também resumidas, ao mesmo tempo que se centram principalmente sobre a variação espacial e relacionamento com declives na onda inicial para a altura de onda significativa, período da onda, momento espectral, declive e largura espectral parâmetro.

Na análise estatística, algumas relações sobre assimetria e curtose são pesquisadas e as distribuições de excedência de altura de onda, crista e calha no experimento são comparados com a distribuição Rayleigh e aproximação GC. Distribuições de onda típico período e distribuições conjuntas em três estados do mar, que são caracterizados pela inclinação inicial também são apresentados e comparados com os teóricos. Por fim, algumas características sobre a altura de onda máxima são simplesmente considerados nesta tese.

**Palavras-chave:** deslocamento da superfície, crista da onda e calha, altura de onda significativa, período de onda, freqüência de pico, onda de bacia, espectros de onda, momentos espectrais, distribuição de Rayleigh, distribuição de excedência, distribuição conjunta de período de onda e altura das ondas, altura de onda máxima.
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Chapter 1  

Introduction

1.1 Motivation

As soon as people ventured on the sea, waves became important to them. The causes of ocean waves are various and complicated. In the nature, ocean water is permanently subjected to the external forces, which dictate what types of waves can be induced in the ocean. Normally, the most obvious cause of surface waves is an action of wind. As early as 2300 years ago, the ancient Greeks were well aware of the interaction between the atmosphere and the sea surface. However, the development of learning about ocean waves was very slow. It was not until into the nineteenth century that more fundamental knowledge of what caused waves and how they behaved was accumulated.

Research on ocean waves is really meaningful considering that there are so many fields requiring wave information. During the World War 2, the requirement to predict wave characteristics for beach-landing operations generated a major research effort both in the USA and the UK. The pressure-operated mines which were laid on the sea bed and triggered by the pressure “signature” of a passing ship must not be triggered by waves.

In the field of coast engineering, waves not only damage breakwaters, but also move sediments. Taken over long periods of time and considerable stretches of coastline, long-shore drift can fill up harbors relatively quickly. The interaction of waves with beaches is fascinating but complicated.

The third important application of wave information is ship response to regular and irregular waves. The pitching and rolling of ships can be very uncomfortable for people. When the motions get severe, they also slow down the ship, cause cargo to shift, and the associated forces can do other damage. For warships, it makes it more difficult to operate weapon system accurately, or to operate aircraft.

Recently, oil companies pay more attention to the forces on the offshore structures which are
generated by the ocean waves. In traditional design techniques, the structure is first designed to withstand the most severe conditions which is likely to meet in 50 or 100 years. However, for much of the year, it has a significant level of wave activity, and the effects of fatigue must be allowed for. Thus, as well as an estimate of extreme wave conditions, the statistics of all waves throughout the year have to be specified. As the development of offshore oil and gas production in the world, there is a big push for the international companies to gain enough knowledge to ensure the safety of offshore structures in relation to environment forces.

The last field that will be pointed out is the wave power which is considerable and clean. Wave power generation will become a widely employed commercial technology in the future.

1.2 Wave Measurement

Normally, the available information on surface waves could be obtained with three methods: field experiments, laboratory experiments and numerical simulations. In wave experiment under field conditions, three groups of instruments are used: wave staffs, wave buoys and pressure gauges. Although field measurement could reflect the wave information directly, it is necessary to consider the instrument survivability, instrument deployment and retrieval. Moreover, in most cases it is unavailable to make actual measurements at the site of interest during major storms. Numerical simulations which include time domain simulation and frequency domain simulation are more and more concerned by marine engineers and oceanographers due to the interest in directional wave spectrum. With this method, a lot of money and time could be saved, but there are still some deviation between simulation and reality which need be further improved.

Laboratory method may be the most common way to obtain the wave information today. Observations and measurements in laboratory lead to theory being developed to explain it. Meanwhile, new phenomena predicted by theory lead to experiments being made to verify the prediction. Thus, discrepancies between theory and measurements stimulate further development of both. However, correct interpretation of measured results is indeed very difficult. On the one hand, failure to make correct measurements, or failure to measure the right parameters at the right spatial location will doom any attempted interpretation for the measurements. On the other hand, the reliability of laboratory results will depend on the engineer’s understanding of the physical process as well as the capabilities and limitations of laboratory test equipment and instrumentation.
When planning laboratory experiments or model studies, coastal engineers need to know at least four aspects: the measured physical parameters that could give a meaningful interpretation to the physical process, the right locations to take measurements, the suitable instruments for the experiments and the accuracy and reliability of the measurements.

According to different standard, laboratory measurements could be categorized into many different groups such as geometrical measurements, fluid property measurements, fluid motion measurements, force and transport measurements and other environmental measurements. The instruments used to obtain information on surface waves are termed wave gauges which can be grouped into resistance, capacitance and pressure types. Detailed review of laboratory method could be found in the book written by Hughes (1993).

1.3 Scope of the Present Thesis

The experiments are carried out in the Technical University of Berlin and the work has been performed within the project EXTREME SEAS (http://www.mar.ist.utl.pt/extremeseas/), “Design for Ship Safety in Extreme Seas”, which has been partially financed by the European Union through its 7th Framework program under contract SCP8-GA-2009-24175.

The job of this thesis is to analyze waves in 23 sea states generated by facility in deep water tank. The main goal is to investigate and determine the wave parameters which are dependent on the wave steepness and the distance from the wave maker and to compare the experiment results with the existed theoretical distributions as well.

In chapter 1, the meaning and motivation of ocean wave research are explained in a brief way from several perspectives. The methods to obtain wave information are also introduced and mainly focused on the laboratory experiment.

In chapter 2, the basic theories of wave analysis with spectral method and statistical method are introduced. In the first section of the theory, besides some concise introductions about sea state parameters, Blackman-Tukey method and Cooley-Tukey method are explained briefly. In the second section of the theory, the theoretical distribution models of surface displacement, wave crest, wave trough, wave period, etc. are described in few words.
In chapter 3, in the spectral analysis, the wave spectra are investigated with direct and indirect methods. The spatial variation of some wave parameters and their relationship with the initial wave steepness are also researched. In the statistical analysis, the observed distributions of wave surface displacement crest, trough and height and so on are compared with the theoretical models and meanwhile the dependence with nonlinearity is also considered. At last, some conclusions are summarized together and the future work is proposed.
Chapter 2  

Basic Theory of Wave Analysis

2.1  Spectral Analysis of Surface Waves

2.1.1  General

The waves on the sea surface are not simple sinusoids. They usually exhibit irregular profiles randomly changing in space and time. In order to describe the characteristics of this random phenomenon clearly, the concept of stochastic process is introduced. Furthermore, the probabilistic principles can be used to find the most favorable marginal and joint probabilistic laws applicable to the desired wave properties: periods, crests, heights, etc...

The stochastic approach spans three domains: time, frequency and probability. Usually, the derivation of the probability law requires several simplifying assumptions. The statistical properties of stochastic process are evaluated based on an ensemble, and hence they may or may not be the same as time progresses. The condition for a stationary stochastic process requires that the joint distribution be invariant irrespective of time, which is somewhat severe in practice. A more relaxed condition that is commonly acknowledged called weakly stationary with zero mean is taken in the analysis of ocean surface waves.

Another very important assumption for the analysis of ocean waves is ergodic process. It is developed by statisticians with the conditions under which the time average of a single record (sample mean) is equivalent to the ensemble average. It is taken for granted that the ergodic property holds for stochastic processes. Therefore, the wave surface displacement pertained to a single record can be used to instead of an ensemble of data.

Narrow-banded which implies that the amplitude and phase of the process vary slowly and randomly with time while the frequency retains a constant value is also assumed in the analysis of continuous-state and continuous-time stochastic processes. In other words, the distribution of wave energy in the frequency domain is concentrated around some characteristic value, the mode of the frequency.
Based on the assumptions mentioned above, the function of wave spectra can be deduced and some useful parameters could be determined. The weak stationarity, also known as the covariance stationarity, suggests that the mean of the process are constant independent of time and the autocovariance function of the process depends only on the time-lag for all time.

Let $x(t)$ represents the free surface elevation oscillating around the mean water level. Then, the weak stationarity will be expressed as:

$$E[x(t)] = \bar{x} = \text{const} \quad (2.1)$$

$$\text{var}[x(t)] = E[(x(t) - \bar{x})^2] = \sigma^2 = \text{const} \quad (2.2)$$

$$\text{cov}[x(t_1), x(t_2)] = E[(x(t_1) - \mu)(x(t_2) - \mu)] = B(t_1, t_2) = B(\tau) \quad (2.3)$$

where $E[ ]$ stands for the statistical expectation; $\tau = |t_2 - t_1|$ is the time lag; $B(\tau)$ is the autocovariance, the second order joint central moment of the process.

Similarly to $B(\tau)$, the second ordinary joint spectral moment, called the autocorrelation function, can be defined as:

$$\text{cor}[x(t_1), x(t_2)] = E[x(t_1)x(t_2)] = R(t_1, t_2) = R(\tau) \quad (2.4)$$

The definition in Eq.(2.4) implies that $R(\tau)$ is identical to $B(\tau)$ when the process has zero-mean, that is $\bar{x} = 0$.

It is necessary to point out that according to the mathematical definitions, these statistical variables mentioned above should be calculated on the basis of the ensemble of the wave records, $X = \{x(t)_i\}, i = 1, \ldots, n$, given $n \to \infty$. However, for example, gathering data simultaneously at different points of space covered by the storm is a difficult and almost impossible task. Under the assumption of being weakly stationary and ergodic, it is possible to substitute the ensemble with a single record at a fixed point in space. The single sample in this case becomes representative for the wave population, irrespective of the wave directionality.

The autocorrelation function gives important information in the stochastic analysis for conversion from the time domain to the frequency domain. For convenience, it is assumed that the Fourier transform of a random process exists over the entire time domain and the truncated function is also
considered in the analysis. After that, it is possible to find the spectral density function $S(\omega)$ from the known autocorrelation function $R(\tau)$ by means of the Wiener-Khintchine theorem. The Wiener-Khintchine theorem represents $R(\tau)$ and $S(\omega)$ as a pair of Fourier transforms.

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau = \frac{1}{\pi} \int_{0}^{\infty} R(\tau) \cos \omega \tau d\tau$$  \hspace{1cm} (2.5)

$$R(\tau) = \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega = 2\int_{0}^{\infty} S(\omega) \cos \omega \tau d\omega$$  \hspace{1cm} (2.6)

Eq. (2.6) shows the most important property of $S(\omega)$ for $\tau = 0$.

$$R(0) = \int_{-\infty}^{\infty} S(\omega) d\omega$$  \hspace{1cm} (2.7)

According to the definition of $R(\tau)$ in Eq.(2.4), it results in the equation known as the Parseval theorem, i.e.

$$E[x(t)^2] = \int_{-\infty}^{\infty} S(\omega) d\omega$$  \hspace{1cm} (2.8)

The Parseval theorem specifies the relationship between the wave energy (the variance) of the process estimated in the time domain (left-handed side of the equation) and the wave energy in the frequency domain (right-handed side of the equation).

In order to make the above theorem clearly, we measure the elevation of the sea surface at the origin of the coordinate system (which can be done without loss of generality), then wave profile will become:

$$x(t) = \sum_{n} a_{n} \cos(\omega_{n} t + \theta_{n})$$  \hspace{1cm} (2.9)

Then

$$x^2(t) = \sum_{n} \sum_{m} a_{n} a_{m} \cos(\omega_{n} t + \theta_{n}) \cos(\omega_{m} t + \theta_{m})$$  \hspace{1cm} (2.10)

After some computation of trigonometric function, and taking a long-term average, it gives:

$$E[x(t)^2] = \sum_{n} \frac{1}{2} a_{n}^2$$  \hspace{1cm} (2.11)

Now with the help of Eq.(2.8), Eq.(2.11), it is much easier to understand that the area under the wave spectrum is equal to the wave energy. $S(\omega)$ is usually abbreviated to “the omnidirectional spectrum” or sometimes “the point spectrum”. It is also common practice to refer to it as “the wave energy spectrum”, but strictly speaking, the energy spectrum would be:
\[ E(\omega) = \rho g S(\omega) \]  \hspace{1cm} (2.12)

### 2.1.2 Sea State Parameters

In the early days of the modern approach, the parameters could only be determined directly from the time-histories which is now known as deterministic analysis. As the development of digital computers, spectral analysis by analogue method now becomes more convenient and more reliable. In the frequency domain, the spectral density function can be described in terms of its moments. The ordinary spectral moments are determined by the integral:

\[ m_n = \int_0^{+\infty} \omega^n S^*(\omega) d\omega \]  \hspace{1cm} (2.13)

In particular, the zeroth-order spectral moment \( m_0 \) (the area under the spectral density curve) represents the variance of the surface elevation process. It is also representative for the total energy of the waves as mentioned above. The second-order spectral moment represents the variance of the wave elevation velocity and the fourth-order spectral moment represents the variance of the wave elevation acceleration.

A measurement which is frequently used in the statistical analysis of wave data is the significant wave height. The significant wave height is a characteristic measurement that defines the level of severity of the given sea state. According to the statistical definition, it equals the average of one-third of the largest waves in a sample of wave heights, or the average of the largest waves having probability of exceedance 1/3. This wave height, \( H_{1/3} \), should be close to the average wave detected visually at sea.

The second definition of significant wave height, more frequently used in Gaussian sea, is based on the information from the wave spectrum.

\[ H_{m0} = 4.004\sqrt{m_0} \]  \hspace{1cm} (2.14)

For a narrowband spectrum it is expected that \( H_{m0} \approx H_{1/3} \). However, normally for the typical sea state, \( 0.9H_{m0} < H_{1/3} < H_{m0} \). Analysis of full-scale data shows that the spectral definition overestimates by approximately 5% the statistical formulation.

The peak period of the spectrum \( T_p \) corresponds to the frequency at the mode of the spectral
density, the peak frequency, \( \omega_p \):

\[
T_p = \frac{2\pi}{\omega_p}
\]  

(2.15)

Moreover, some characteristic wave frequencies and periods can be derived from the ordinary spectral moments. The mean spectral frequency (also could be defined as carrier wave frequency) and the associated mean spectral period are expressed as:

\[
\omega_{01} = \frac{m_1}{m_0}
\]  

(2.16)

\[
T_{01} = \frac{2\pi}{\omega_{01}} = 2\pi \frac{m_0}{m_1}
\]  

(2.17)

The average zero-up-crossing angular wave frequency and the associated average zero-up-crossing period are given as:

\[
\omega_z = \sqrt{\frac{m_2}{m_0}}
\]  

(2.18)

\[
T_z = \frac{2\pi}{\omega_z} = 2\pi \sqrt{\frac{m_0}{m_2}}
\]  

(2.19)

The crest period or the average period between wave crests is represented by:

\[
T_c = 2\pi \sqrt{\frac{m_2}{m_4}}
\]  

(2.20)

Another very important measure of the distribution of the frequency components in the sea state is the spectral bandwidth. Several measures of the width of the spectrum have been defined and used to validate the assumption of narrow spectrum. One of the parameters commonly used is proposed by Longuet-Higgins (1975), the spectral width parameter:

\[
\nu = \left( \frac{m_0 m_2}{m_1^2} - 1 \right)^{1/2}
\]  

(2.21)

which represents the normalized radius of gyration of the spectrum about the mean frequency \( \omega_{01} \).

For describing its derivation, it is convenient to think in mechanical terms. The moment of inertia about the axis \( \omega = 0 \) is \( m_2 \). The moment of inertia about the mean frequency \( \omega_{01} \) obtained in Eq.(2.16), that is, about the centre of gravity, is \( m_2 - \omega_{01}^2 m_0 \). The radius of gyration about the mean frequency is:
\[ \left( \frac{m_2 - \omega_0^2 m_0}{m_0} \right)^{1/2} = \left( \frac{m_2}{m_0} - \omega_0^2 \right)^{1/2} \] 

(2.22)

Normalizing Eq.(2.22) with the mean frequency \( \omega_0 \), Eq.(2.21) will be obtained. For a spectrum of narrow bandwidth, \( \nu \rightarrow 0 \). For example, for the Pierson-Moskowitz spectrum, \( \nu = 0.425 \); for JONSWAP spectrum with \( \gamma = 3.3 \), \( \sigma_a = 0.7 \), \( \sigma_b = 0.9 \), \( \nu = 0.390 \). The irregularity of the wave field is reflected in the large estimates of \( \nu \). Another parameter is the bandwidth parameter of Cartwright and Longuet-Higgins (1956):

\[ \epsilon = \left( 1 - \frac{m_2^2}{m_0 m_4} \right)^{1/2} \] 

(2.23)

Again, for a narrowband spectrum, it is expected that \( \epsilon \rightarrow 0 \). Since this parameter depends on the magnitude of the fourth-order spectral moment, \( m_4 \), instead of representing the energy distribution over the entire frequency range, it only represents the range of frequencies where the dominant energy exists. Because of this reason, it is rarely used recently.

The last very useful measure for a random sea is the steepness which is defined:

\[ S_p = \frac{2\pi^2 H_{m0}}{gT_p^2} \] 

(2.24)

It has no precise physical meaning. But by analogy with the case of the periodic wave, it can be considered as the steepness of the sea.

So far, there are several expressions used as standard forms of the frequency spectrum which can be regarded as having been derived empirically with some theoretical guidance. Probably the most popular spectrum among all proposed forms is that proposed by Pierson and Moskowitz (1964), who using the field data and theoretical discoveries of Phillips (1958), showed that:

\[ S(\omega) = \alpha g^2 \omega^{-5} \exp \left[ -B \left( \frac{g}{\omega U} \right)^4 \right] \] 

(2.25)

where \( \alpha = 0.0081 \), \( B = 0.74 \) and \( U \) is a wind speed at an elevation of 19.5m above the sea surface. The shape of the wave spectrum is controlled by a single parameter, wind speed \( U \). The spectrum of Eq.(2.25) was proposed for fully-developed sea, when phase speed is equal to wind speed. The experimental spectra given by Pierson and Moskowitz yields
Some mathematical problems arise when calculating the fourth order spectral moment using Eq. (2.26). This moment, which physically denotes the mean-squared acceleration measured at a Eulerian point, is unbounded.

Later, the JONSWAP spectrum extends the Pierson-Moskowitz spectrum to including fetch-limited seas. This spectrum is based on an extensive wave measurement program (Joint North Sea Wave Project) carried out in 1968 and 1969 in the North Sea. The JONSWAP spectrum, after publication in 1973, received almost instant recognition and became very well known in international literature. The resulting spectral model is taken as the following form (Hasselmann et al., 1973):

\[
S(\omega) = \alpha g^2 \omega^{-5} \exp \left[ -\frac{5}{4} \left( \frac{\omega}{\omega_p} \right)^4 \right] \gamma^{\delta} \tag{2.27}
\]

where \( \delta = \exp \left[ -\frac{(\omega - \omega_p)^2}{2\sigma_0^2 \omega_p^2} \right] \).

Spectrum (2.27) contains five parameters which should be known: \( \alpha \), the Phillips’s constant, limited by the value 0.0081 in the case of the Pierson-Moskowitz spectrum for fully-developed seas; \( \omega_p \), the spectral peak frequency; \( \sigma_0 = \sigma_a = 0.07 \) for \( \omega < \omega_p \) and \( \sigma_0 = \sigma_b = 0.09 \) for \( \omega > \omega_p \); \( \gamma \) the peak enhancement parameter that describes the degree of peakedness of the spectrum.

The term \( \gamma^{\delta} \) is the peak enhancement factor, added to the Pierson-Moskowitz formulation in order to represent a narrower and more peaked spectrum which is typical for growing seas. Also, \( \gamma \) is a random Gaussian variable with mean 3.3 and variance 0.62, such that \( \gamma = 3.3 \) represents the mean JONSWAP spectrum and \( \gamma = 1 \) represents the Pierson-Moskowitz spectrum. The width of the enhancement at the peak region is given by the values of \( \sigma_0 \). The values of \( \alpha \) and \( \gamma \) depend on the stage of development of the sea.

The alternative formulation of JONSWAP as a function of the significant wave height and peak frequency was proposed by Goda (1988):
\[ S(\omega) = \alpha^* H_s^2 \frac{\omega^5}{\omega_p^3} \exp \left[ -\frac{5}{4} \left( \frac{\omega}{\omega_p} \right)^4 \right] \gamma^\delta \]  

(2.28)

where

\[ \alpha^* = \frac{0.0624}{0.23 + 0.0336\gamma - \frac{0.185}{1.9 + \gamma}} \beta^* \]

\[ \beta^* = 1.094 - 0.01915 \ln \gamma. \]

The standard formulations in Eq.(2.27) and Eq.(2.28) can be fitted by the measured data. For example, the values of the significant wave height, peak period and peak factor used in Eq.(2.28) can be found with the best fit minimizing the sum of squared differences between the smoothed measured spectrum and the JONSWAP standard formula in Eq.(2.28). As another option to fit the measured data with the standard JONSWAP formula is to use the Günter method which is given in detail in Tucker and Pitt (2001). The logic of this method is clear and simple while the values of parameters computed tends to deviate from the normal ones.

### 2.1.3 Blackman-Tukey Method

The method proposed by Blackman and Tukey (1959) for the estimation of the wave spectrum is based on the Wiener-Khintchine theorem. The Blackman-Tukey procedure, also known as the method of correlogram is an indirect method, since first it requires estimation of the autocorrelation function and then applies the Fourier transform to it. The procedure is described in Massel (1996) and it will be repeated briefly below.

**a) Transformation of variables**

Let the surface elevation process is \( x(t) \). The total duration of wave recordings is \( T = N\Delta t \), where \( N \) is the number of the sampled data and \( \Delta t [\text{s}] \) is the constant interval of data sampling. A new time series \( \tilde{x}_i \), with zero-mean and unit standard deviation could be obtained:

\[ \tilde{x}_i = x_i - \bar{x} \sigma \]  

(2.29)

**b) Estimation of the autocorrelation function**

The autocorrelation function, \( R(\tau) \), is directly obtained from the time series, by computing a set
of average products among the sample data values. One of the possible estimators for $R(\tau)$ is expressed as:

$$R(r\Delta t) = \frac{1}{N} - r \sum_{n=1}^{N-r} x_n x_{n+r}, r = 0, 1, 2, \ldots, m$$  \hspace{1cm} (2.30)

where $r$ stands for the lag number; $m$ is the maximum lag number ($m \ll N$). Selection of the $m$ value, which provides the optimum estimate for the autocorrelation function, is also very important. The finite value of $m$ implies that the surface elevations at large time $t > m\Delta t$ are uncorrelated.

c) **Suppression of the spectral energy leakage**

A window should be used for the autocorrelation function to suppress the spectrum energy leakage. The window aims at tapering the autocorrelation function, so as to eliminate the discontinuity at the end of $R(\tau)$. There are numerous such windows in use. A typical window is the cosine Hanning window:

$$u_h(r\Delta t) = \begin{cases} \frac{1}{2} \left(1 + \cos \frac{\pi r}{m}\right), & \text{for } r = 0, 1, 2, \ldots, m \\ 0, & \text{for } r > m \end{cases} \hspace{1cm} (2.31)$$

Applying the filter in Eq.(2.31) to the autocorrelation function, the latter is recalculated as:

$$\tilde{R}(r\Delta t) = R(r\Delta t) u_h(r\Delta t) \hspace{1cm} (2.32)$$

d) **Calculation of the frequency spectral density**

The spectral density is calculated by numerical integration from the Wiener-Khintchine equations in Eq.(2.5):

$$S(\omega_k) = \frac{\Delta t}{\pi} \left\{ \tilde{R}(0) + 2 \sum_{r=1}^{m-1} \tilde{R}(r\Delta t) \cos \left( \frac{\pi r k}{m} \right) + \tilde{R}(m\Delta t) \cos \left( k \pi \right) \right\}$$  \hspace{1cm} (2.33)

where $\Delta t = dt$ in the Wiener-Khintchine integral. The frequency $\omega [\text{rad/s}]$ corresponding to each time-lag is expressed as:

$$\omega_k = k\Delta \omega = \frac{k\pi}{m\Delta t}, k = 0, 1, 2, \ldots, m$$  \hspace{1cm} (2.34)

The fundamental increment $\Delta t$, also named Nyquist sampling interval, is the maximum interval required to properly describe the data $x(t)$. According to the Nyquist frequency, the maximum
useful frequency in the spectrum is given by:

\[ \omega_{\text{max}} = \omega_{N} = \frac{\pi}{\Delta t} \]  

(2.35)

The spectral estimate in Eq.(2.33) describes the mean square value \( \left[ \tilde{x}(t) \right]^{2} \) in terms of frequency components laying inside the frequency band \( \{ \omega - (B_{e}/2), \omega + (B_{e}/2) \} \), where \( B_{e} = 2\pi/(m\Delta t) \) [rad/s] is the obtained frequency resolution for the chosen sampling interval \( \Delta t \). Eq.(2.33) gives \( m/2 \) independent estimates of the spectrum for the reason that the estimates separated by frequency increments smaller than \( 2\omega_{e}/m \) are correlated. For a given bandwidth \( B_{e} \), the required maximum lag number \( m \) can be determined as:

\[ m = \frac{2\pi}{B_{e} \Delta t} \]  

(2.36)

e) **Smoothing of the spectral estimate**

The spectral estimation in Eq.(2.33) exhibits large variance, so it becomes necessary to smooth it in order to reduce the statistical variability. This can be done by simple block-averaging, moving-average or by means of weighted averaging. The simple moving average method is expressed by:

\[ \tilde{S}(\omega_{k}) = \frac{1}{(2p+1)} \sum_{j=-p}^{p} S(\omega_{k+j}) \]  

(2.37)

where \( \tilde{S}(\omega_{k}) \) is the smoothed spectrum resulting from averaging of \( S(\omega_{k}) \) over \( 2p+1 \) neighboring values.

In order to further increase the estimate accuracy, some techniques should be used and the maximal lag number needs to be selected carefully in the procedure. The method taken in this thesis for the maximum lag is to find the envelope of the autocorrelation function and then to choose the lag at the position where the envelope reaches its first minimum as shown in Fig. 2.1.
2.1.4 Cooley-Tukey Method

An alternative method, known as Cooley-Tukey method applies direct Fast Fourier Transform to the water surface displacements and is basically used lately as a result of saving a lot of computational time. In the following part, some critical procedures in this method are briefly explained.

a) Data preprocessing
As mentioned in step 1 of Blackman-Tukey method, a mean value from the digital data needs to be subtracted. Trend removing and filtering will be processed if they are necessary.

b) Window function selection
In signal processing, there are two bothering problems: the first one is that the signal can only be measured for a limited time which means nothing could be known about the behavior outside the measurement interval; the second one is that Fast Fourier Transform is base on the fact that the signal is periodic which is not true in reality. When the FFT assumes the signal repeats, it will assume discontinuities that are not really there. Therefore, the spectral leakage will happen. In order to obtain the periodic signal, the procedure is to multiply the signal within the measurement time by some function that smoothly reduces the signal to zero at the end point. One of the simple and useful window functions is Hanning window which should be applied first before FFT. It is expressed in the following formula:
\[ \hat{x}_i = \tilde{x}_i \cdot w_i \]  

(2.38)

The Hanning window is defined as follows:

\[
w_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi \cdot (i-1)}{N} \right) \right]; i = 1...N \]

(2.39)

c) Fast Fourier Transform

The fast Fourier Transform is a DFT with special algorithm developed by Tukey and Cooley (1965) which reduces the number of computations from something on the order of \( N^2 \) to \( N \log(N) \). Although many variants of the FFT algorithm (Oran, 1974) exist, but the critical parts of this algorithm do not change.

- **Bit reversed indexing**
  It is a good choice to reverse the bits of binary representation of \( i \) to get the corresponding array index for the input “time domain” data at the initial stage of the program. Therefore, the output result will be in the normal order.

- **Nested loops instead of recursive decimation in time**
  Three nested loops should be included in the program. Pass Loop: the outer loop, an \( N = 2^p \) point transform will perform \( P \) "passes", indexed by \( l = 1, 2, ..., p; \) Block Loop: the inner loop, each pass \( l \) has \( 2^{p-l} \) sub-blocks whose size is \( 2^l \), indexed by \( m = 1, 2, ..., 2^{p-l}; \) Butterfly Loop: the middle loop, each sub-block operation will perform \( 2^{l-1} \) butterflies, indexed by \( n = 1, 2, ..., 2^{l-1}; \)

- **Twiddle factor tricks**
  In order to avoid the time-consuming work of computing sinusoidal function and cosine function for every butterfly. In the program, it is convenient to set the step size of twiddle factor as \( W_{2^l} \). Therefore, the twiddle factors used in each pass should be \( W_{2^{l-n}} \), where the term \( W_N = \exp \left( \frac{2\pi j}{N} \right) \).
Because the input data $x_i$ are always real, after fast Fourier transform, the output array $X_i$ obeys the following relationship:

$$X_{N-i+2} = X_i^*, i = 2, 3, ..., N/2$$

(2.40)

where * denotes complex conjugation. Since $N$ is also even, $X_1$ and $X_{N/2+1}$ are both real. Thus, only the first $N/2 + 1$ distinct real and complex numbers which should be expressed in the form of square of absolute value would be useful in the spectrum analysis. These correspond to the frequencies:

$$\omega_i = (i-1)\Delta\omega, i = 1, 2, ..., N/2 + 1$$

(2.41)

where the width of a frequency bin is given by

$$\Delta\omega = \frac{2\pi}{N\Delta t}$$

(2.42)

The apparent loss of information is explained by the fact that the output consists of complex numbers while the input is real.

d) Averaging and overlap

If we just compute one estimate of a spectrum with the first three steps described so far, it will be found that the result is rather variant. It does not help to increase the length $N$ of the FFT and that only reduces the width of frequency bin without impairing the variance.

The practical remedy is to take the average of $k$ estimates and hence reduce the standard deviation of the averaged result by a factor of $1/\sqrt{k}$. However, the precondition is that the properties of the signal must remain stationary during the average. Thus, it is necessary to select a suitable value for the length $N$ of the FFT. In conjunction with the use of window functions, this method of averaging several spectra is known as “Welch’s Overlapped Segmented Average”.

If the original data is split up into several non-overlapping segments of length $N$ and each segment is processed by a FFT with a window function, some information may be ignored in the analysis due to the fact that the window function is typically very small or zero near its boundaries. It is clearly not very good in some situations such as the maximal possible wave height is to be extracted from it. This situation could be improved by letting the segments overlap. Normally, different windows need different overlapped segments. For Hanning window which is relatively
wide in the time domain, 50% is a commonly recommended. For narrower window such as flat-top window, a higher overlap up to 84% may be appropriate. On the other hand, if the overlap becomes too big, the spectrum estimates from subsequent stretches become strongly correlated, even if the signal is random. What is worse, it will waste computational efforts.

Sometimes, the confident interval would be used in the estimate. Just keep one thing in mind, when the overlapping is zero, confident interval will be needed and 95% is the recommend value in the engineering project.

e) Scaling the result

Now it is time to normalize the results of the FFT. Based on what has been mentioned above and the use of window function, a new normalized parameter is needed to instead of $N$:

$$B_2 = \sum_{i=1}^{N} w_i^2$$

The power spectral density could be obtained in the following form:

$$S(\omega) = \frac{1}{kB_2\omega_{max}} \sum_{j=1}^{k} |X_y|^2, i = 1, 2, ..., N/2 + 1$$

The factor $k$ comes from the fact that $k$ segments have been summed up together during the whole process.

f) Smoothing of spectral estimation and Parameters Selection

If a smoothed spectral is needed, the same procedure in step 6 of Blackman-Tukey Method could be taken and will not be repeated here again.
2.2 Statistical Analysis of Surface Waves

2.2.1 General

Besides the spectral analysis of the ocean surface waves, the statistical distribution of waves are also very useful in the wave research, especially in ocean engineering. For example, the statistics of extreme waves plays a very important role in the determination of the design wave height for offshore and coastal structures. In the statistical analysis, wave parameters such as wave height and wave period are considered as random variables with statistical characteristics.

Normally, the statistics of ocean waves will be considered in a short-term and long-term scale. In the following parts of this chapter, the short-term statistics based on the Gaussian processes will be discussed for the distribution of surface displacement. The theoretical distributions of wave height, crest, trough and wave period are also briefly introduced.

Firstly, it is necessary to explain the thorough procedure for finding the zero-up-crossing points, wave maxima, wave minima and some practical problems in application.

The points of zero-up crossing of the wave profile are determined by the following criteria:

\[
\begin{align*}
\tilde{x}_i, \tilde{x}_{i+1} &< 0 \\
\tilde{x}_i, \tilde{x}_{i+1} &> 0
\end{align*}
\]  
\begin{equation}
(2.45)
\end{equation}

where \( \tilde{x}_i \) denotes the \( i \)-th data point of the surface elevation obtained from Eq.(2.29). Then, the time of the zero-up-crossing is found by linear interpolation between the sampling time of \( \tilde{x}_i \) and \( \tilde{x}_{i+1} \). The time difference between one zero-up-crossing point and the successive one in the same direction gives the zero-up-crossing wave period. The global maximum surface elevation is defined as the wave crest between two consecutive zero-up-crossing points where

\[
\begin{align*}
\tilde{x}_{i-1} < \tilde{x}_i \\
\tilde{x}_i > \tilde{x}_{i+1}
\end{align*}
\]  
\begin{equation}
(2.46)
\end{equation}

It is suggested that the time and the magnitude of the maximum point should be estimated by fitting three consecutive points \( \tilde{x}_{i-1}, \tilde{x}_i \), and \( \tilde{x}_{i+1} \) with a parabolic equation to avoid the possibility of underestimating the true maximum value among those discrete sampling points. The parabolic fitting is given by:
\[ \tilde{x}_{\text{max}} = D - \frac{B^2}{4A} \]  
\[ t_{\text{max}} = t_i - \frac{\Delta t B}{2A} \]

where \( \Delta t \) is the time sampling interval and

\[ A = \frac{1}{2(\Delta t)^2} \left( x_{i-1} - 2\tilde{x}_i + \tilde{x}_{i+1} \right) \]  
\[ B = \frac{1}{2\Delta t} (x_{i+1} - x_{i-1}) \]  
\[ D = \tilde{x}_i \]

The global minimum wave profile between two consecutive zero-up-crossing points is defined as the wave trough which is determined in the same procedure as wave crest. And the only difference is the requirement:

\[ \begin{cases} 
\tilde{x}_{i-1} > \tilde{x}_i \\
\tilde{x}_i < \tilde{x}_{i+1}
\end{cases} \]

The wave height is defined by the difference between the crest and wave trough in a given wave cycle.

Sometimes due to the very small time interval and the problem of equipment, the recorded wave elevation may fluctuate around the mean water level. It satisfies Eq.(2.45) but is not the desired point. As a result, the obtained probability of small wave height and wave period will be enlarged. Therefore, it is necessary to make full use of some techniques to solve this problem such as filtering wave data at the beginning or adding more conditions in Eq.(2.45).

For example (see Fig. 2.2), the conditions described in Eq.(2.45) will result in some errors in the statistical analysis. Fortunately, some functions in MATLAB could be used to filter the undesired high frequency and low frequency parts in the wave record. However, as shown in Fig. 2.3, although the error record could be eliminated, some other errors will be induced inevitably due to the missed information such as the wave height.
2.2.2 Surface Displacement

Since the propagation of wave systems can be represented by the sum of a large number of harmonic and statistically independent wave components, the wave surface displacement at one point will tend to normal law (Gaussian distribution) according to the central limit theorem as the number of harmonics increases infinitely.

The Gaussian distribution could be defined by two parameters, the mean value $\mu$, and the variance $\sigma^2$. Generally speaking, it always holds for the deep water waves, and the probability density function takes the following form:
The mean and variance are equal to the first-order and second-order cumulants respectively, or the corresponding central moments if the mean is zero.

In most cases, although the distribution of surface displacement is approximately Gaussian, a small asymmetry and different peakedness are observed. These deviations from the Gaussian distribution can be expressed in two parameters, coefficients of skewness $\gamma_3$ and kurtosis $\gamma_4$, which are higher order quantities and related to the nonlinearities in the wave basin.

$$\gamma_3 = \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{\kappa_3}{\sigma^3}$$

$$\gamma_4 = \frac{\kappa_4}{\kappa_2^2} = \frac{\kappa_4}{\sigma^4}$$

These Eq.(2.54) and Eq.(2.55) also represent the normalized third-order and fourth-order cumulants respectively. Only when coefficients of skewness and kurtosis are both equal to zero at the same time, the random variable is normally distributed.

The coefficient of skewness measures the steepness in the wave basin or the vertical asymmetry in the wave profile provoked by second-order effects and is exemplified by the sharp crests and rounded troughs of gravity waves. The positive coefficient of skewness will mean a bigger probability of occurrence of large crests and that the empirical distribution will be skewed to the right with respect to the mode of the Gaussian distribution, such that the mode of the distribution is located at a value smaller than the mean. On the other hand, although coefficient of kurtosis is more commonly defined as the fourth cumulants divided by the square of the second cumulants, it is equal to the fourth order central moment divided by the square of the probability distribution minus 3. The positive coefficient of kurtosis reflects the total increase of crest-to-trough wave height, due to third order nonlinear interactions.

For a narrow band unidirectional weakly nonlinear train up to second order, the coefficients of skewness and kurtosis are related to the steepness $S_p$ in Eq.(2.24) (Mori and Janssen, 2006)
\[ \gamma_3 = \frac{3}{2} S_p \]  
\[ \gamma_4 = 6 S_p^2 \]  

2.2.3 Wave Crest and Trough

In the linear approach of Longuet-Higgins, the crest height and trough height are both assumed to be Rayleigh distributed:

\[ f_r(z) = z \exp \left( -\frac{1}{2} z^2 \right) \]  

and the exceedance probability will be:

\[ E_r(z) = \exp \left( -\frac{1}{2} z^2 \right) \]  

where the normalized crest or trough is \( z = \eta/\sigma \).

However, second order nonlinear wave is characterized by higher, sharper crests and shallower, more rounded troughs. Therefore, Rayleigh distribution could not give an exact prediction in ocean engineering design. Tayfun and Fedele (2007a) proposed the following exceedance distribution of crest and trough:

\[
E_\zeta^+ (z) = \Pr \{ \zeta^+ > z \} = \exp \left[ -\frac{1}{2} \mu_0^+ \left(-1 + \sqrt{1+2 \mu_1 z}\right)^2 \right]
\]

\[
E_\zeta^- (z) = \Pr \{ \zeta^- > z \} = \exp \left[ -\frac{1}{2} \mu_0^- \left(1 + \frac{1}{2} \mu_1 z\right)^2 \right]
\]

where \( \zeta^+, \zeta^- \) stands for the second order crest and trough amplitudes, respectively, and \( \mu_0 \) represents a dimensionless measure of wave steepness.

\[ \mu_1 = \frac{\sigma \omega_p^2}{g} \]  

If considering third-order nonlinearities, the preceding distribution could be modified further as the following forms:

\[ E_{GC}^+ (z) = E_\zeta^+ \left[ 1 + \frac{\Lambda}{64} z^2 (z^2 - 4) \right] \]
\[ E_{GC}^{-}(z) = E_{z'}^{-}(z) \left[ 1 + \frac{\Lambda}{64} z^2(z^2 - 4) \right] \]  
\[ (2.64) \]

where

\[ \Lambda = \lambda_{40} + 2\lambda_{22} + \lambda_{04} \]  
\[ (2.65) \]

\[ \lambda_{mm} = \left\langle \eta^m \hat{\eta}^n \right\rangle / \sigma^{(m+n)} ; \text{ for } m + n = 3 \]  
\[ (2.66) \]

\[ \lambda_{mm} = \left\langle \eta^m \hat{\eta}^n \right\rangle / \sigma^{(m+n)} + (-1)^{m/2} (m - 1)(n - 1) ; \text{ for } m + n = 4 \]  
\[ (2.67) \]

\( \hat{\eta} \) is the Hilbert transform. Detailed procedure about the above cumulants was explained by Tayfun and Lo (1990).

### 2.2.4 Wave Height

The classical linear theory, besides the assumption of the surface elevation being a large sum of independent harmonic waves, assumes a narrowband spectrum. Thus, this theory leads to the well-known Rayleigh distribution for the prediction of wave heights. In this case, the wave heights can be represented as the double linear wave amplitudes and have Rayleigh distribution of the following form:

\[ f_{h'}(h) = \frac{2h}{H_{rms}^2} \exp \left( -\frac{h^2}{H_{rms}^2} \right), 0 \leq h < \infty \]  
\[ (2.68) \]

where \( H_{rms}^2 = 8\sigma^2 \) is the mean-square value of the sample of wave heights.

It is necessary to normalize Rayleigh distribution when comparing with the data and other distribution models. Different normalization factor will result in different normalized form. Most commonly, the normalization is done by dividing the standard deviation of the surface elevation, \( \sigma \). Then the Rayleigh distribution takes the following form:

\[ f_{z}(z) = \frac{z}{4} \exp \left( -\frac{z^2}{8} \right), 0 \leq z < \infty \]  
\[ (2.69) \]

where \( z = h/\sigma \). The interest in the probability is that a certain threshold value will be exceeded. The probability of exceedance associated with the Rayleigh probability density function in Eq.(2.69) has the following form:
\[ E_r(z) = \Pr(H > \sigma z) = \exp\left(-\frac{z^2}{8}\right), 0 \leq z < \infty \] (2.70)

However, it has been reported that Rayleigh exceedance distribution tends to overestimate the crest-to-trough heights of large waves by about 7%-8% (Tayfun and Fedele, 2007a). This discrepancy has been found to be mainly caused by the assumption in the Rayleigh theory of the wave height being double the crest height.

Therefore, some more advanced theories have been presented in the present study. Tayfun and Fedele (2007a) presented the third-order approximations of wave heights based on the Gram-Charlier (GC) expansions:

\[
R_{f_{2413}} = \left[1 + \frac{\Lambda}{1024} (z^4 - 32z^2 + 128)\right] \\
E_{E_{2211}} = \left[1 + \frac{\Lambda}{1024} z^2 \left(z^2 - 16\right)\right]
\] (2.71) (2.72)

The wave height here is defined as double the wave envelope. Actually, it differs appreciably from the crest-to-trough definition height because it ignores the variation of the wave envelope over the time interval between a wave crest and the following trough in a typical zero-up-crossing cycle. That variation is \(O(\nu)\) to the leading order, and can be rather significant for relatively broad-band ocean waves. However, if \(\nu \to 0\), then the crest-to-trough wave height is approximately equal to the double wave envelope. Further assuming that \(\lambda_{04} \to 3\lambda_{22} \to \lambda_{40}\) as in the case of long-crested waves, Eq.(2.71) and Eq.(2.72) becomes:

\[
R_{f_{2404}} = \left[1 + \frac{\lambda_{40}}{384} (z^4 - 32z^2 + 128)\right] \\
E_{E_{2240}} = \left[1 + \frac{\lambda_{40}}{384} z^2 \left(z^2 - 16\right)\right]
\] (2.73) (2.74)

Eq.(2.73) and Eq.(2.74) are identical to formulae given by Mori and Janssen (2006), which are referred to as a modified Edgeworth-Rayleigh (MER) distribution. Clearly, MER is a special case for GC. Later we will show that these two exceedance distributions are almost the same in the experiment for the reason that \(\Lambda_{app} \approx \Lambda = 8\lambda_{40} / 3\).
2.2.5 Joint Distribution

In practice, wave height and wave period may respectively have different distribution types. And the theory of a multivariate distribution of correlated random variables with different marginal distribution is still unavailable to solve the practical problems. An alternative method that could circumvent this difficulty is to use the multivariate normal distribution to describe the joint probability distribution of correlated random variables.

Longuet-Higgins (1975) proposed the joint distributions of wave heights and periods, he further modified and improved it with some techniques to make the theoretical distribution agree more with the reality (Longuet-Higgins, 1983). His theory is based on several assumptions. Firstly, the amplitudes of waves in deep water are small which allows for linearized superposition. Secondly, the wave spectrum is assumed to have energy with independent amplitudes and phases for the different frequencies. Furthermore, the narrow-band hypothesis will be necessary in order to satisfy the condition that the envelop is a slowly varying function of time, thus the maxima and minima of wave elevation will lie nearly on the envelope function.

Besides Eq. (2.9), another representation of sea surface profile in complex quantity is:

\[
Re \exp \left( \sum_n a_n \exp \left( i \left( \omega_n t + \theta_n \right) \right) \right)
\]

(2.75)

With the help of the mean frequency \( \omega_{\theta 1} \) in Eq. (2.16), the new derivation frequency could be obtained:

\[
\omega'_n = \omega_n - \omega_{\theta 1}
\]

(2.76)

Therefore, Eq. (2.75) could be expressed in a new form:

\[
x(t) = \text{Re} \left\{ \sum_n a_n \exp \left( i \left( \omega'_n t + \theta_n \right) \right) \exp \left( i \omega_{\theta 1} t \right) \right\}
\]

(2.77)

If the complex envelope is defined as:

\[
\rho e^{i\phi} = \sum_n a_n \exp \left( i \left( \omega'_n t + \theta_n \right) \right)
\]

(2.78)

then

\[
x(t) = \text{Re} \left\{ \rho e^{i\phi} \exp \left( i \omega_{\theta 1} t \right) \right\} = \text{Re} \left\{ \rho \exp \left( i \left( \phi + \omega_{\theta 1} t \right) \right) \right\} = \text{Re} \left\{ \rho \exp \left( i \chi \right) \right\}
\]

(2.79)

Eq. (2.79) is a carrier with fixed frequency modulated by a complex wave envelope of amplitude
\( \rho \) and phase \( \phi \) which are time-varying random variables. The detailed approach was deduced by Longuet-Higgins (1975). Finally, the joint density distribution of the envelope amplitude, phase, and their time derivatives will be

\[
f(\rho, \phi, \rho', \phi') = \frac{\rho^2}{(2\pi)^2 \mu_0 \mu_2} \exp\left(-\frac{\rho^2}{2\mu_0}\right) \exp\left(-\frac{\rho'^2 + \rho'^2 \phi'^2}{2\mu_2}\right)
\]

(2.80)

Integrating with respect to \( \rho \) over \((-\infty, \infty)\) and with respect to \( \phi \) over \((0, 2\pi)\), the resulting density will be:

\[
f(\rho, \phi) = \frac{\rho^2}{\sqrt{2\pi\mu_2^2 \mu_0}} \exp\left(-\frac{\rho^2}{2\mu_0}\right) \exp\left(-\frac{\rho'^2 \phi'^2}{2\mu_2}\right)
\]

(2.81)

The definition of the envelope in Eq.(2.78) indicates that \( \rho \) and \( \phi \) will vary slowly if the spectral energy is concentrated in a small band of frequencies near \( \omega_{01} \), where \( \omega' \) is small. As a result, the wave crests of \( x(t) \) will lie almost on the envelope which leads to a wave height is twice the amplitude.

\[
h = 2\rho
\]

(2.82)

The rate of change of the total phase of wave profile

\[
\dot{\chi} = \dot{\phi} + \omega_{01}
\]

(2.83)

will be almost equal to \( \omega_{01} \) for the reason that \( \dot{\phi} \) is small and varies little over a wave period. The wave period can be approximated by:

\[
\tau = \frac{2\pi}{\dot{\chi}} = \frac{2\pi}{\dot{\phi} + \omega_{01}}
\]

(2.84)

The wave height and period are normalized by

\[
R = \frac{h}{\sqrt{8m_0}} = \frac{\rho}{\sqrt{2m_0}}
\]

(2.85)

\[
T = \frac{\tau}{T_{01}} = \frac{m_{11}}{m_1 + m_0 \phi}
\]

(2.86)

One thing should be noted is that the parameter used to normalize wave height in Eq.(2.85) is not identical with the one used in Eq.(2.69) for the reason that the joint distribution of experiment data will be not very good if the normalized wave height is too large comparing with the normalized period. Applying Jacobian transformation, the resulting density is:
\[ f(R,T) = \frac{2R^2}{\nu \sqrt{\pi T^2}} \exp\left\{ -R^2 \left[ 1 + \frac{1}{\nu^2} \left( 1 - \frac{1}{T} \right)^2 \right] \right\} \]  

(2.87)

where the spectral width parameter \( \nu \) is given in Eq.(2.21). Since the negative wave period is not physically sensible, the normalization factor \( L(\nu) \) is introduced. Thus,

\[ f(R,T) = \frac{2L(\nu) R^2}{\nu \sqrt{\pi T^2}} \exp\left\{ -R^2 \left[ 1 + \frac{1}{\nu^2} \left( 1 - \frac{1}{T} \right)^2 \right] \right\} \]  

(2.88)

where

\[ L(\nu) = \frac{2\sqrt{1+\nu^2}}{1+\sqrt{1+\nu^2}} \]  

(2.89)

The position of mode, or maximum value of Eq.(2.88) is found from the condition that \( \frac{\partial f}{\partial R} \) and \( \frac{\partial f}{\partial T} \) both vanish. Hence we could find:

\[ \begin{align*}
R &= \frac{1}{\sqrt{1+\nu^2}} \\
T &= \frac{1}{1+\nu^2}
\end{align*} \]  

(2.90)

and the value of \( f(R,T) \) at this point is:

\[ f_{\text{max}} = \frac{2L(\nu)(1+\nu^2)}{e \nu \sqrt{\pi}} \]  

(2.91)

From Fig. 2.4 to Fig. 2.6, typical three-dimensional joint distributions and the corresponding two-dimensional contour lines are present. When the spectral parameter width is very small, the figure is approximately symmetric with respect to the vertical line \( T=1 \). As it increases, this symmetry is broken and the ranges of normalized period and wave height will become much wider.

Furthermore, the wave period distributions could be given easily on the basis of joint distribution. The probability density function of period \( T \), regardless of wave height \( R \), is found by integrating \( f(R,T) \) with respect to \( R \) over \( (0, \infty) \), to give:

\[ f(T) = \frac{L(\nu)}{2\nu T^2} \left[ 1 + \frac{1}{\nu^2} \left( 1 - \frac{1}{T} \right)^2 \right]^{-3/2} \]  

(2.92)
Fig. 2.4 three-dimensional and two-dimensional joint distributions in theory for $\nu=0.1$

Fig. 2.5 three-dimensional and two-dimensional joint distributions in theory for $\nu=0.2$

Fig. 2.6 three-dimensional and two-dimensional joint distributions in theory for $\nu=0.3$
Chapter 3  

Experiment Results of Wave Analysis

3.1 Facility and Data

Wave tanks are usually characterized as long, narrow enclosures with some kind wave-maker at one end. In the experiment, the double-flap wave maker is installed at one of the short walls of the basin. The beach at the opposite side serves to absorb the incident wave energy. The size of the offshore basin in the Technical University of Berlin is 152m×30m×5m as show in Fig. 3.1.

![Fig. 3.1 Dimensions of offshore basin](image)

As shown in Fig. 3.2, the water surface elevations have been measured by 6 gauges uniformly spaced along the wave basin at 20m interval. In the first test, the first gauge is 20m far away from the wave maker and the same experiment is done two times. Then the 6 wave gauges are moved simultaneously 10 meters away from the wave make. That is, the nearest gauge to the wave maker is 30m now and the same experiment is repeated as before. The duration of each recorded time series is nearly half an hour with sampling frequency 100Hz. The scale of this experiment is 1:50
and the following study is based on a set of records obtained under 23 different model conditions. The spectrum used at the wave maker is of the JONSWAP type with the same peakedness parameter $\gamma = 3$. And the peak periods and significant wave heights are present in full scale in Table 3.1 where the steepnesses are from the wave maker.

The experiments describe deep water waves propagating on the full-scaled constant depth $h = 250m$. It satisfies the deep water condition $h/L_p > 0.5$ for the reason that the maximal wave length, calculated from the linear dispersion relationship at the peak frequency, is less than 500m in all tests.

<table>
<thead>
<tr>
<th>Sea state</th>
<th>Hs(m)</th>
<th>Tp(s)</th>
<th>Spectrum</th>
<th>Steepness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.25</td>
<td>15.66</td>
<td>J3</td>
<td>0.05126</td>
</tr>
<tr>
<td>2</td>
<td>7.50</td>
<td>15.66</td>
<td>J3</td>
<td>0.06151</td>
</tr>
<tr>
<td>3</td>
<td>7.50</td>
<td>14.53</td>
<td>J3</td>
<td>0.07140</td>
</tr>
<tr>
<td>4</td>
<td>10.00</td>
<td>15.66</td>
<td>J3</td>
<td>0.08202</td>
</tr>
<tr>
<td>5</td>
<td>15.00</td>
<td>17.89</td>
<td>J3</td>
<td>0.09421</td>
</tr>
<tr>
<td>6</td>
<td>6.25</td>
<td>11.18</td>
<td>J3</td>
<td>0.10052</td>
</tr>
<tr>
<td>7</td>
<td>15.00</td>
<td>16.77</td>
<td>J3</td>
<td>0.10718</td>
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<tr>
<td>8</td>
<td>3.75</td>
<td>7.83</td>
<td>J3</td>
<td>0.12302</td>
</tr>
<tr>
<td>9</td>
<td>11.25</td>
<td>13.41</td>
<td>J3</td>
<td>0.12568</td>
</tr>
<tr>
<td>10</td>
<td>10.00</td>
<td>12.30</td>
<td>J3</td>
<td>0.13294</td>
</tr>
<tr>
<td>11</td>
<td>8.75</td>
<td>11.18</td>
<td>J3</td>
<td>0.14073</td>
</tr>
<tr>
<td>12</td>
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<td>J3</td>
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</tr>
<tr>
<td>13</td>
<td>7.50</td>
<td>10.06</td>
<td>J3</td>
<td>0.14890</td>
</tr>
<tr>
<td>14</td>
<td>13.80</td>
<td>13.41</td>
<td>J3</td>
<td>0.15417</td>
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<td>11.18</td>
<td>J3</td>
<td>0.16084</td>
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<tr>
<td>16</td>
<td>5.00</td>
<td>7.83</td>
<td>J3</td>
<td>0.16403</td>
</tr>
<tr>
<td>17</td>
<td>15.00</td>
<td>13.41</td>
<td>J3</td>
<td>0.16757</td>
</tr>
<tr>
<td>18</td>
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<td>J3</td>
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<td>11.18</td>
<td>J3</td>
<td>0.18094</td>
</tr>
<tr>
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<td>12.30</td>
<td>J3</td>
<td>0.18279</td>
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<tr>
<td>21</td>
<td>7.50</td>
<td>8.94</td>
<td>J3</td>
<td>0.18842</td>
</tr>
<tr>
<td>22</td>
<td>15.00</td>
<td>12.30</td>
<td>J3</td>
<td>0.19941</td>
</tr>
<tr>
<td>23</td>
<td>6.25</td>
<td>7.83</td>
<td>J3</td>
<td>0.20504</td>
</tr>
</tbody>
</table>
Fig. 3.3 Time series cut from gauge 1

Fig. 3.4 Time series cut from gauge 3

Fig. 3.5 Time series cut from gauge 5
Considering that only 6 wave records are made at a time for each sea state, it is better to present the results with six gauges each time by the mean value in the same position. The respective mean results of the six gauges in the first position are denoted as Part A in the following analysis and the corresponding mean results in the second position are denoted as Part B. The initial steepness mentioned is the steepness measured at the first gauge in Part A and Part B respectively.

In order to make sure that the initial noise is cut off and that the waves analyzed from 6 gauges are the same, it is necessary to cut off the wave record at the right position by utilizing the wave group velocity as done in Fig. 3.3–Fig. 3.5.
3.2 **Spectral Analysis**

3.2.1 **Spectral Comparison**

With the help of Blackman-Tukey method and Cooley-Tukey method mentioned in chapter 2, the wave record in each gauge generates two wave spectra which are compared with the standard JONSWAP spectrum described in Eq.(2.28) where significant wave height and peak period are the expected values from the former two methods. For the purpose of simplicity, only the figures of Gauge1, Gauge4 and Gauge6 are presented below in three typical sea states which are characterized by different initial steepness. These three gauges could reflect the spatial variation of the wave spectra very well, because Gauge1 is the nearest point to the wave maker which shows what happens at the beginning; Gauge4 is in the middle of the wave basin which shows what appears at the intermediate position; Gauge6 is the longest distance from the wave maker which shows what could be present at the end of the wave basin.

As it is shown in the Fig. 3.6, ‘G1’, ‘G4’ and ‘G6’ stands for three gauges respectively. ‘BT’, ‘CT’ and ‘Sta’ successively denote spectra obtained with Blackman-Tukey Method, Cooley-Tukey Method and the standard one proposed by Goda (1988). According to the experiment, it is noted that the two methods do not give two much difference except for the density of peak frequency. When the initial steepness is very small, the computed wave spectra are exactly the same as the standard one as shown in Fig. 3.6.

![Wave spectra with initial steepness s₀=0.0514](image)

**Fig. 3.6 Wave spectra with initial steepness s₀=0.0514**

When the initial steepness increases, the computed spectra will deviate from the standard one. With reference to Fig. 3.7, in the sea state with intermediate initial steepness, the computed spectra
are not as narrow as the theoretical one in the vicinity of the peak frequency. In other words, the standard wave spectrum underestimates the density of wave frequencies close to the peak frequency. It is also noted that the high frequency part decreases much more quickly than that of the standard one. These phenomena are more obvious in the sea state with large initial steepness as show in Fig. 3.8.

In order to see the variation of high frequency part clearly, it is necessary to normalize the wave spectra and present the figures in logarithmic scale. Now the focus is only on the normalized high frequency part, that is, $1 < \mu < 4$.

$$\mu = \frac{\omega}{\omega_p}$$  \hspace{1cm} (3.1)

In Fig. 3.9, a very interesting phenomenon in the sea state with small initial steepness is that the slope of wave spectrum behaves as $\mu^{-4}$ at the interval (1, 2.55). And then a sharp drop appears. But the slope is still kept as before at the interval (2.8, 3.5). After this section, the spectral slope is
steepened to about $\mu^{-6}$ due to the viscous dissipation (Fedele et al., 2010). It plays a major role when the initial wave steepness is not large enough in the wave basin. In other word, when the initial steepness is very small, it is the viscous effect that dissipates the high frequency wave energy.

When the initial steepness increases to 0.0879 as present in Fig. 3.10, wave breaking begins to dissipate wave energy with the viscous effect. Moreover, since the part of $\mu^{-5}$ increases along the wave basin, it means more and more energy is dissipated in the propagation.

As the initial steepness continues to increase, the turning point of spectral slopes moves quickly to $\mu = 1$. This is confirmed explicitly if comparing Fig. 3.11 and Fig. 3.12 together. In Fig. 3.12, since the initial steepness is really high, the spectral slopes tend to behave as $\mu^{-5}$ above the peak frequency. In other words, the wave breaking is really severe and dissipates the most energy in the high frequency part.
3.2.2 Significant Wave Height

The change of the significant wave heights (Hs) along the wave basin could be catalogued into three groups and attributed to two reasons. The major factor that affects the evolution trend is the initial wave steepness. Normally, if the initial steepness is smaller than one specific value, the significant wave height will increase along the wave basin while if it is larger than another specific value, the significant wave height will decrease. Between these two values, the general trend will maintain level. The critical values of initial steepness found in the experiment are listed in Table 3.2. Initial steepness used below for classification is the mean value of the first steepness in part A and part B.

<table>
<thead>
<tr>
<th>Group</th>
<th>Initial Steepness</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0514</td>
<td>0.1086</td>
</tr>
<tr>
<td>2</td>
<td>0.1148</td>
<td>0.1680</td>
</tr>
<tr>
<td>3</td>
<td>0.1702</td>
<td>0.2100</td>
</tr>
</tbody>
</table>

Fig. 3.11 Normalized wave spectra with initial steepness $s_0=0.1152$

Fig. 3.12 Normalized wave spectra with initial steepness $s_0=0.2084$
The second factor which has an influence on the significant wave height is the initial amplitudes of significant wave height. This effect is very obvious in group 2. Generally speaking, the large amplitudes are apt to generate an increased trend for the significant wave heights while the small ones are inclined to give a decreased trend.

In group 1 (Fig. 3.13) where the steepness is very small, significant wave heights increase along the wave basin. Since there is no input energy after the wave generator, the only reason that could explain this kind of wave evolution in the propagation is the reflection in the wave basin and the superposition among the waves. It increases the wave amplitude without changing its frequency.

![Fig. 3.13 Spatial variation of significant wave height in group 1](image)

Note: “Sta” represents the significant wave heights obtained by statistical method, “BT”, “CT” stands for the significant wave heights gotten from Blackman-Tukey method and Cooley-Tukey method by utilizing Eq.(2.14) respectively.

The most complicated case is the transition steepness in group 2. Wave breaking and wave reflection work together and reach a balance. Thus the general trend of significant wave height keeps level along the wave basin although with a little fluctuation at times which is shown in Fig. 3.14. That is, the significant wave heights decrease or increase a little after the wave maker and reach the minimal or maximal value around the middle of wave basin. However, they could go back to the initial significant wave height again. Moreover, sometimes the amplitudes of initial significant wave height could play an important role in group 2. Small amplitudes could result in a little descended trend while large amplitudes will lead to some ascended trend.
Finally, as present in Fig. 3.15 significant wave heights in group 3 decrease along the wave basin due to serious wave breaking in the sea state with large initial wave steepness. As far as we know, 0.10–0.12 is the critical value for wave breaking. Therefore, it is to be expected that most energy will be lost as a result of wave breaking and thus leads to a descended trend.

Another conclusion should be mentioned is that according to the empirical results the laboratory significant wave heights got from statistical method are not always smaller than those obtained from spectral methods.
3.2.3 Wave Period

Based on the Eq.(2.19), we could get the zero-up-crossing wave periods by spectral method. In order to improve the accuracy, the mean value of zero-up-crossing wave periods obtained by two spectral methods is used below and denoted by “SpAv” in the following figures. For the purpose of better comparing the spectral method with statistical method, periods computed by statistical method are also presented in the same figures where “StDn”, “StUp” represent zero-down-crossing period and zero-up-crossing period respectively.

![Spatial variation of zero-crossing wave period with small initial steepness](image)

The experiments results reveal some interesting spatial variation about zero-crossing wave periods which could be categorized into two groups. When the initial steepness is less than 0.0936, the wave periods almost keep constant along the wave basin as shown in Fig. 3.16. On the other hand, when it is larger than 0.1086, the wave periods will increase along the wave basin which are proved in Fig. 3.17 and Fig. 3.18. Normally, larger initial steepness will lead to larger increase, but the largest one is not more than one second which is plausibly due to the physical dimensions of the wave basin.

It is also could be noted that zero-up-crossing periods and zero-down-crossing periods in statistical analysis almost give the same result in all sea states. This could provide some facts for the argument that analysis from zero-up-crossing methods and zero-down-crossing methods normally give the same conclusion except for some special cases. Moreover, it seems that zero-up crossing period computed with statistical method is larger than that obtained with spectral method.
Another very useful wave period in spectral analysis is the mean spectral period as expressed in Eq.\((2.17)\). Considering that the spatial variation of zero crossing wave periods is almost linear, it is feasible to measure the change of mean spectral periods during the propagation along the tank by the ratio between the value measured in the longest distance to the value measured closest to the wave maker, proposed by Cherneva and Guedes Soares (2011). This ratio is denoted by index \(r\) to differ from the relevant parameters.

Since the reciprocal value of the mean spectral period which is known as the carrier frequency has more physical meaning, the relative carrier frequency as a function of initial steepness for all sea
states is preferred and plotted in Fig. 3.19. It is certain that the relative frequency is equal or small than 1. The series with higher nonlinearity characterized by large initial steepness are modulated more due to the reduction of the spectrum tail in its high frequency part as found previously in section 3.2.1.

![Fig. 3.19 Relationship between relative carrier frequency and initial steepness](image)

### 3.2.4 Zero Spectral Moment and Steepness

Since we have analyzed the spatial variation of significant wave heights, it is not difficult to predict the propagation law of the zero spectral moments along the wave basin for the reason that they have a direct relationship as listed in Eq.(2.14). All the conclusions made in section 3.2.2 still hold for the zero spectral moment. In the following three figures, “m0” denotes the mean zero spectral moment obtained by “BT” and “CT” spectral methods. And “steep” stands for the wave steepness of each gauge. The two independent variables have the same dependent variable which is the distance from the wave maker. But their values are presented in different vertical axes. The left one is for zero spectral moment while the right one is for wave steepness.

It is pronounced that the change of wave steepness is almost in direct proportion to the change of zero spectral moment and also depends on the initial steepness. It is obviously correct in that when the wave energy is large, that is, large zero spectral moment, it definitely will generate large wave height and thus results in large steepness at that position. Therefore, the spatial variation of steepness will be the same as that of significant wave height.
Fig. 3.20 Spatial variation of $m_0$ and steepness with small initial steepness

Fig. 3.21 Spatial variation of $m_0$ and steepness with intermediate initial steepness

Fig. 3.22 Spatial variation of $m_0$ and steepness with large initial steepness
For the sake of observing the relationship between the change of wave energy and the initial steepness clearly, the ratio of the sixth zero spectral moment in the wave basin to the first one could be used again as a parameter as mentioned by Cherneva and Guedes Soares (2011). One thing should be kept in mind is that only for large and small initial steepness, Fig. 3.23 could reflect the change of wave energy well while for the intermediate initial steepness, this method does not always work due to the irregular variation as shown in Fig. 3.21.

As shown in the experiment, the relative zero spectral moment is larger than 1.0 in the case of small initial steepness. In other words, the wave energy is increased nearly 10% plausibly because of the reflection in the wave basin. When initial steepness is larger than 0.1, the increase speed is not as fast as before which means that the speed of energy dissipation is much faster. As initial steepness continues to increase, wave breaking plays a more and more important role in the dissipation of energy and thus leads to a further decrease in relative zero spectral moment. The largest decrease is nearly 15% and the second critical change point of initial steepness in the experiment is near to 0.15 where the total wave energy begins to decrease. It is demonstrated in the value of relative zero spectral moment less than 1.0.

![Fig. 3.23 Relationship between relative zero spectral moment and initial steepness](image)

**3.2.5 Spectral Width Parameter**

One of the most important parameters in spectral analysis is the spectral width parameter described in Eq.(2.21) and Eq.(2.23). According to the experiment, a very interesting phenomenon is that
spectral width parameter has the same critical value as wave period for initial steepness. That is, if the initial steepness is less than 0.0936, spectral width is almost constant along the wave basin as shown in Fig. 3.24. However, if it is larger than 0.1086, spectral width will decrease which is proved in Fig. 3.25 and Fig. 3.26. The unchanged spectral width means that the reflected waves almost have the same frequency as before and only increase the amplitudes of incident waves. Furthermore, the decreased wave spectral width parameter is just another demonstration of wave dissipation in the sea states with intermediate and large initial steepness.

Fig. 3.24 Spatial variation of spectral width parameter with small initial steepness

Fig. 3.25 Spatial variation of spectral width parameter with intermediate initial steepness
Fig. 3.26 Spatial variation of spectral width parameter with large initial steepness

Considering that the spatial variation of the spectral width is also almost linear along the wave basin, it is correct to use the relative spectral width parameter (Cherneva and Guedes Soares, 2011) to identify the relationship between change of spectral width and initial steepness. Since the two spectral width parameters have the same change rule, that is, $\varepsilon$ is approximately two times of $\nu$ for all sea states, only relative $\nu_r$ is presented in Fig. 3.27.

Fig. 3.27 Relationship between relative spectral width and initial steepness

Apparently, 0.1 could work as a threshold value again to identify the change trend. When initial steepness is less than 0.1, the relative spectral width extends to one. But after that turning point, the relative wave spectral width decreases quickly and reaches its minimal value in the position
where the initial wave steepness is near 0.15. And then, the relative wave spectral width parameter increases but still smaller than 1.0. Whatever, it could be noted that the averaged decrease is nearly 8% in most cases. The last thing that needs to be emphasized is that all the wave spectral widths $V$ are in the range from 0.2 to 0.3 in the experiment which could be considered as narrow band spectra. It is very strange that the maximal decrease is present in the intermediate sea state rather than the most serious one.
3.3 **Statistical Analysis**

3.3.1 **Skewness and Kurtosis**

Skewness and kurtosis are two important parameters in statistics analysis. As mentioned above, only when the kurtosis and skewness are both close to zero, wave surface displacements could be considered as Gaussian as presented in (A) of Fig. 3.28. More often than not, the most common distribution appeared in the experiment is presented in (B) of Fig. 3.28 where the peak value is located smaller than the mean value zero because of the positive skewness and it is sharper than (A) due to the large positive kurtosis.

![Fig. 3.28 Distribution of normalized surface displacement in the experiment](image)

In the laboratory experiment, only in the case of very small initial steepness, the coefficient of kurtosis is negative as presented in the first picture of Fig. 3.29 and Fig. 3.30. When the initial steepness is intermediate or large, it is positive and increases to a maximal value at about 60m from the wave maker. It is constant in the later propagation. The same spatial variation was presented by Petrova et al. (2008). At the place near to wave maker, the third order nonlinearity effect is not very strong because its value is very small. What is more, the largest kurtosis appears in the sea state with intermediate initial steepness rather than the largest one. That is also what happens to the decrease of spectral width parameter discussed in section 3.2.5. The coefficient of skewness is almost positive everywhere and its spatial variation is very small along the wave basin. It means that the second order nonlinear effect is constant along the wave basin. Moreover, larger coefficient of skewness is related with larger initial steepness, that is, the severer sea state.
To further describe the above conclusions clearly, it is feasible and necessary to study the coefficients of kurtosis and skewness of the outmost gauge as a function of the initial steepness as presented in Fig. 3.31. Apparently, the relationship between the coefficient of skewness and initial steepness is linear, that is, large initial steepness leads to large skewness. As for the coefficient of
kurtosis, the largest value is in the interval from 0.1 to 0.15. When the initial steepness is small, the coefficient of kurtosis is small, while the initial steepness is larger than 0.15, it almost extends to a constant with a little decreased trend.

If we research the coefficients of skewness, kurtosis as a function of the wave steepness for each gauge, almost the same conclusions could be drawn. In Fig. 3.32, although the theoretical Eq.(2.56) overestimates the coefficient of skewness, the linear relationship still holds and the fitted lines almost have the same slope with the theoretical one. One thing should be kept in mind is that part of the decrease of coefficient of skewness is due to filtering wave series. In Fig. 3.33, Eq.(2.57) does not predict the coefficient of kurtosis well for the reason that our fitted curve decreases for the larger wave steepness which coincides with the conclusion obtained in Fig. 3.31.

Fig. 3.32 Relationship between coefficient of skewness and wave steepness

Fig. 3.33 Relationship between coefficient of kurtosis and wave steepness
The coefficients of skewness and kurtosis also have some relationship with each other as presented in Fig. 3.34. The data are fitted with two standard mathematical models. The first one is proposed by the author in the following form:

\[ y = a \exp(bx + cx) + d \]  \hspace{1cm} (3.2)

And the second one is a five order polynomial function which could be used directly from MATLAB function “polyfit”. Apparently, our mathematical model presents more merits since it could keep its shape in both cases and begin to decrease for high coefficient of skewness which plausibly is true in reality. However, based on a set of wave series, Cherneva et al. (2006) also proposed another formula.

![Fig. 3.34 Relationship between coefficient of skewness and kurtosis](image)

### 3.3.2 EDF of Wave Crest Trough and Height

The same exceedance distribution will be obtained by Eq.(2.72) and Eq.(2.74) in the experiment because \( \Lambda_{app} \) is approximately equal to \( \Lambda \) as proved in Fig. 3.35 although the slope of the fitted line is a little larger than the theoretical one. As a result, only Eq.(2.72) will be used to analyze the exceedance distribution of wave height. By the way, the reason why the probability density functions are not compared with the experiment data is that only little useful information could be seen there.

In order to see these exceedance distributions clearly, it is better to arrange the exceedance distributions of wave crest, trough and height together. Since there are so many sea states, only
three typical ones with small, intermediate and large initial steepnesses are considered and their spatial variation of coefficients of skewness and kurtosis could be found in Fig. 3.29 and Fig. 3.30. For economy of space and ease of interpretation, only measurements at Gauge 1, 3, 4 and 6 will be compared where spatial variation of coefficients of skewness and kurtosis could be described well.

Note: “GC_H”, “GC_C” and “GC_T” denote exceedance distribution of wave height, crest and trough for GC approximations. “Ray_H,C&T” denote the Rayleigh exceedance distributions of wave height, crest and trough.

Normally, kurtosis plays a much more important role in the exceedance distribution of wave crest, trough and height than skewness, especially for those larger waves in the severer sea state.

Fig. 3.35 Relationship between $\Lambda$ and $\Lambda_{app}$

Fig. 3.36 EDF of wave crest, trough and height for G1 and G3 with small initial steepness
In Fig. 3.36 and Fig. 3.37, when the initial steepness is very small, skewness and kurtosis will not be very large anywhere in the wave basin. Thus leads to that the GC distribution is almost consistent with the Rayleigh distribution except for a little difference in the middle of the wave basin where both of them overestimate the larger wave heights. The theoretical modals of wave crest and trough agree with the experiment data reasonably well and they almost overlap with each other satisfying the linear wave theory.

In Fig. 3.38 and Fig. 3.39, the coefficients of skewness and kurtosis are both positive and very large in the sea state with intermediate initial steepness. As a result, GC approximations for wave crest and height largely deviate from the Rayleigh distribution. What is more, GC approximation could predict the experiment data better in most cases comparing with the Rayleigh distribution.
The overestimation is due to wave breaking in the offshore basin and filtering wave at the beginning.

Fig. 3.39 EDF of wave crest, trough and height for G4 and G6 with intermediate initial steepness

Fig. 3.40 EDF of wave crest, trough and height for G1 and G3 with large initial steepness

Fig. 3.41 EDF of wave crest, trough and height for G4 and G6 with large initial steepness
For the case of large initial steepness as shown in Fig. 3.40 and Fig. 3.41, almost the same rules could be obtained as the case of intermediate initial steepness except for the very large wave height and crest. Considering that the more serious sea state generates more wave breaking, it is no wonder that exceedance probability will drop in larger wave part in Gauge3 and Gauge4 because of the very large kurtosis. Although GC approximation overestimates the wave crest, it is still much better than Rayleigh distribution. Cherneva, Tayfun and Guedes Soares (2009) also pointed out that GC approximation could not fit the empirical data well sometimes. Last, although kurtosis is negative at the gauge near to the wave maker, GC approximation could predict the exceedance distribution reasonably well.

Finally, based on what we have discussed above, it is strongly supported that GC approximations could predict wave crest and height much better than Rayleigh distribution in most cases.

3.3.3 Wave Period

In section 3.2.3, the spatial variation of specified wave periods has been researched with spectral method for different sea states. Now we will consider the distributions of wave periods in a statistical view. The theoretical distribution in Eq.(2.92) is only related to the wave spectral width, and section 3.2.5 has proved that the variation of wave spectral width is not large enough to apparently affect the theoretical distribution.

From Fig. 3.42 to Fig. 3.44, periods from three represented sea states are chosen where “ν” and “S” stand for the spectral width parameter and the wave steepness for each gauge respectively. Obviously, the theoretical wave period distribution changes slowly because the spectral width parameter does not decrease quickly as the sea state changes. However, the distribution of observed wave period changes rapidly and the shape becomes sharper and narrower as the wave steepness increases. Therefore, the wave period distribution should be related not only with the wave spectral width parameter but also with the initial wave steepness. It seems that the wave periods are asymptotically uniformly distributed around the carrier wave period when the sea state is mild as described in Fig. 3.42. As the sea state becomes severer, the periods will concentrate on the peak wave period which is a little larger than the carrier wave period as presented in Fig. 3.43 and Fig. 3.44. Moreover, the wave period will increase along the wave basin as we discussed in spectral analysis.
It seems that the theoretical distribution agrees with the experiment data well in the sea state with intermediate steepness. But as the sea state becomes severer, the theoretical distribution underestimates part of the periods. And it will be much better if the wave periods are normalized by peak period rather than the mean wave period for the reason that the mode of the observed wave period will move backward and conform with the theoretical maximum value.
3.3.4 Joint Distribution

From Fig. 3.45 to Fig. 3.50, the theoretical joint distribution and the experiment data are compared together. In the same manner as wave period distribution, only figures of three typical sea states are presented below. The symbols “Pomax” and “Pmax” successively denote observed maximal probability and theoretical maximal probability. The other symbols are still the same as before. The black asterisk represents the location of the observed maximal probability which may be not only one sometimes. In order to describe the comparison clearly, the probabilities are normalized by the maximal probability respectively when plotting the theoretical contour lines and drawing the experiment data with different colors.

Apparently, the mean spectral width parameter does not decrease too much and thus the theoretical joint distribution almost keeps the same shape for all sea states for the reason that spectral width parameter is the only variable in Eq.(2.88). However, the observed distribution changes greatly as the wave steepness increases.

For the mild sea state, this theoretical joint distribution does not predict the experiment result well although the maximal value seems good. The observed shape is more or less like a triangle and it is obvious that the large wave period and large wave height are not enough to fit the theoretical distribution. Sharpe (1990) once obtained the same result without explaining the reasons. If we review the deducing procedure, Longuet-Higgins once assumed the wave frequencies mainly concentrated on the carrier wave frequency and thus leaded to the bad prediction in the sea state with small initial steepness considering that Fig. 3.42 presents an approximately flat distribution for the wave periods rather than a sharp one.

As the sea state becomes serious, the theoretical joint distribution agrees with the observed data reasonably well. Therefore, the theoretical joint distribution should be not only related to the wave spectral width parameter but also connected with the initial wave steepness. The theoretical joint distribution is also based on the linear superposition which may be not suitable in the severe sea state due to the strong nonlinearity and thus maybe explain the deviation from the observed results.
Fig. 3.45 Joint distribution with small initial wave steepness 1

Fig. 3.46 Joint distribution with small wave steepness 2

Fig. 3.47 Joint distribution with intermediate initial wave steepness 1
Fig. 3.48 Joint distribution with intermediate initial wave steepness 2

Fig. 3.49 Joint distribution with large initial wave steepness 1

Fig. 3.50 Joint distribution for large initial wave steepness 2
If we check the location of the observed maximal probability from Fig. 3.47 to Fig. 3.50, it is very interesting that the observed positions stay in the bottom right corner of the theoretical location in most cases. It means that the parameter for normalizing wave periods is too small while the parameter for normalizing wave heights is too large. Therefore, the peak wave periods may be a better choice as we have proposed in section 3.3.3. Maybe another smaller parameter should be chosen to replace the mean-square value of wave height. But the shape of the theoretical distribution will also be changed at the same time. Apparently, this topic could be further continued but is out of the scope of this thesis.

### 3.3.5 Maximal Wave Height

The prediction of location of the maximal wave height in the wave basin will be really useful if we do some research on the freak waves. But according to the experiment, it is found that the spatial variation of maximal wave height is rather complicated and irregular. Even for the same sea state, different experiments normally present different variations. Only in several sea states such as presented in Fig. 3.51, a general trend could be obtained from the two tests. Therefore, statistical distribution will be much better for describing the spatial variation of maximal wave height.

![Fig. 3.51 Spatial variation of maximal wave height](image)

In Fig. 3.52, the relationship between the maximal wave height and the corresponding period is researched. “ZeroUp” and “ZeroDn” represent the experiment data obtained from the analysis of zero-up-crossing wave points and zero-down-crossing points respectively. Apparently, the fitted lines are nearly overlapped with each other. That is, analysis of zero-up-crossing points or
zero-down-crossing points does not have too much difference. Moreover, as maximal wave height increases, the related wave period will increase. What calls for special attentions is that there seems to be two functions for the maximal wave height and period. One is linear and the other needs to be further researched to check if it is true.

![Fig. 3.52 Relationship between maximal wave height and the corresponding wave period](image1)

![Fig. 3.53 Relationship between initial steepness and maximal wave steepness](image2)

The last very interesting relationship for the maximal wave height in this thesis is shown in Fig. 3.53 where “Data1” and “Data2” are only different in “L” used for the computation of vertical coordinates. L is obtained from wave period T by the linear dispersion relationship. For “Data1”, T is the period related with the maximal wave height. And for “Data2”, T represents the peak wave period. “S0” stands for the initial wave steepness and the black dash line represents the Stockes
limit. It is obvious that the maximal wave steepness will increase as the initial steepness increases and reach a maximal value when the initial steepness is around 0.17. After that, it will begin to fall down due to the serious wave breaking. This change is consistent with the result obtained by Cherneva and Guedes Soares (2011). One interesting phenomenon is that the red solid line almost presents the same change rule as the blue dot dash line. In other words, the period corresponding to maximal wave height is directly proportional to the peak wave period. The blue dot dash line has more realistic meaning in application because if it could be obtained from a wave basin, it will be convenient for the researcher to design the wave experiment to obtain the desired maximal wave height.
3.4 Conclusion

In the work 552 laboratory wave series have been analyzed. The spectral and statistical methods are used in this study. Several wave parameters like significant wave height, zero spectral moment, wave steepness of each gauge, carrier wave frequency, spectral width parameter, zero-up-crossing period, skewness and kurtosis are researched. The main focus is on the spatial variation and their relationship with the initial wave steepness. The spatial variation is determined by the sea state and could be categorized into three groups by the value of initial steepness. According to the experiment, the relationship between the spatial variation and the value of initial steepness are summarized together in the Table 3.3.

Table 3.3 Values of initial steepness for spatial variation

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“I.S.” stands for the mean value of initial steepnesses in Part A and Part B in the same sea state. “I”, “C” and “D” represent “increased”, “constant” and “decreased” trends respectively.

As it was discussed before, significant wave height, zero spectral moment and steepness almost present the same spatial variation in that large wave energy is characterized by large wave height which definitely will result in large steepness. The carrier wave frequency and spectral width parameters illustrate the same spatial variation due to the energy dissipation mainly caused by wave breaking. Considering the direct relationship between zero-up-crossing period and mean period, it is easy to understand that zero-up-crossing wave period must have the same critical value of initial steepness as carrier wave frequency which is the reciprocal of mean period. The coefficient of skewness almost keeps constant along the wave basin and its value increases as the initial steepness increases. Kurtosis is negative and nearly constant only in two sea states with small initial steepness. In most cases, it is small near to the wave maker and increases to the maximal value approximately in the middle of the wave basin which will be kept constant in the later propagation.

When comparing the experiment data with the theoretical distribution models, the useful information are summarized into three groups denoted by small, intermediate and large initial wave steepness. Wave spectra calculated with Blackman-Tukey method and Cooley-Tukey method are almost the same. Compared with the standard JONSWAP spectrum, only in the sea state with small initial wave steepness, they coincide. For the large initial wave steepness, the discrepancy could be apparently observed. It is identified that the high frequency part is tailed from $\mu^{-4}$ to $\mu^{-5}$ as a result of wave breaking as the initial steepness increases. The observed distributions of wave crest, trough and height are compared with Rayleigh distribution and GC approximation. As a result, in most sea states GC model fits the empirical data better than Rayleigh distribution. Wave periods are also compared with the theoretical distribution which is the marginal distribution of joint distribution proposed by Longuet-Higgins. The experiment data fit the theoretical distribution well in the sea state with intermediate steepness. Based on our results it is suggested to normalize the wave periods with peak period considering that most energy is concentrated on the peak frequency. The same conclusion could be obtained for the joint distribution. The observed location of the maximal probability is always in the bottom left corner of the theoretical one. It is shown that as the initial wave steepness increases, the maximal wave height increases too.
The goal of the thesis has been achieved successfully, but there are still some problems which need future work. If we could have some wave data from other tanks, the main conclusion drawn here could be checked to confirm if they are the same. The results from this experiment also could be used to compare with that from numerical simulation in the future.
Reference


