Analysis of Laboratory Generated Sea Waves

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ABSTRACT: Spectral analysis and statistical analysis are two major methods in analyzing the characteristics of ocean waves. In the spectral analysis, both indirect and direct methods are adopted to obtain the wave spectra under 23 sea states generated in the laboratory. Comparing the obtained spectra with the standard one, it is drawn that these two methods almost give the same result and are consistent with the reality. Furthermore, some other useful laws are also summarized at the same time which mainly focus on the spatial variation and relationship with initial wave steepness for the significant wave height, wave period, spectral moment, steepness and spectral width parameter. In the statistical analysis, some relationships about skewness and kurtosis are researched and the exceedance distributions of wave height, crest and trough in the experiment are compared with Rayleigh distribution and GC approximation. Typical wave period distributions and joint distributions are also presented and compared with the theoretical models.

1 INTRODUCTIONS

1.1 General

Research on ocean waves is really meaningful considering that there are so many fields requiring wave information such as military, coast engineering, ship response and wave energy.

The available information on surface waves could be obtained with three methods: field experiments, laboratory experiments and numerical simulations. Laboratory method may be the most common way today. Observations and measurements in laboratory lead to theory being developed to explain it. Meanwhile, new phenomena predicted by theory lead to experiments being made to verify the prediction. Thus, discrepancies between theory and measurements stimulate further development of both.

In this work, wave series in 23 sea states generated by facility in deep water tank are analyzed. The main goal is to investigate and determine the wave parameters which are dependent on the wave steepness and the distance from the wave maker and to compare the experiment results with the existed theoretical distribution models as well.

2 BASIC THEORY OF WAVE ANALYSIS

2.1 Spectral Analysis of Surface Waves

The waves on the sea surface are not simple sinusoids. They usually exhibit irregular profiles randomly changing in space and time. In order to describe the characteristics of this random phenomenon clearly, the concept of stochastic process is introduced. The analysis of ocean waves is based on three important assumptions which are weakly stationary with zero mean, ergodic process and narrow-banded spectrum. As the development of digital computers, spectral analysis becomes more convenient and reliable than deterministic analysis. In the frequency domain, the spectral density function can be described in terms of its moments. The ordinary spectral moments are determined by the integral:

\[ m_n = \int_0^{\infty} \omega^n S(\omega) d\omega \]  \hspace{1cm} (1)

A measure which is frequently used in the analysis of wave data is the significant wave height which could define the level of severity of the given sea state. For a narrowband spectrum it is expected:

\[ H_s \approx 4.004 \sqrt{m_0} \]  \hspace{1cm} (2)

The carrier wave frequency and zero-up-crossing period are expressed as:

\[ \omega_{01} = \frac{m_1}{m_0} \]  \hspace{1cm} (3)

\[ T_z = 2\pi \sqrt{\frac{m_0}{m_2}} \]  \hspace{1cm} (4)

A very important measure of the distribution of the frequency components in the sea state is the spectral bandwidth parameter. One commonly used was proposed by Longuet-Higgins (1975):

\[ \nu = \left( \frac{m_0 m_2}{m_1^2} - 1 \right)^{1/2} \]  \hspace{1cm} (5)

Another one was introduced by Cartwright and Longuet-Higgins (1956):

\[ \varepsilon = \left( 1 - \frac{m_2^2}{m_0 m_4} \right)^{1/2} \]  \hspace{1cm} (6)

Steepness is also very useful for a random sea and defined as:

\[ S_p = \frac{2\pi^2 H_s}{g T_p^2} \]  \hspace{1cm} (7)
where $T_p$ is the peak wave period. There are several expressions used as standard forms of the frequency spectrum which can be regarded as having been derived empirically with some theoretical guidance. Pierson and Moskowitz (1964) used the field data and theoretical discoveries of Phillips (1958) to deduce a wind speed dependent spectrum. Later, the Pierson-Moskowitz spectrum was extended to including the fetch-limited seas. Hasselmann et al. (1973) published the famous JONSWAP spectrum which received almost instant recognition and became very well known in international literature. An alternative formulation of JONSWAP was proposed by Goda (1988):

$$S(\omega) = \alpha^* H^2 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega}{\omega_p}\right)^4\right]$$  \hspace{0.5cm} (8)

$$\alpha^* = \frac{0.0624}{\delta} \exp \left[-\frac{(\omega - \omega_p)^2}{2\sigma^2\omega_p^2}\right]$$  \hspace{0.5cm} (9)

$$\beta^* = 1.094 - 0.01915 \ln \gamma$$  \hspace{0.5cm} (10)

$$\delta = \exp \left[-\frac{(\omega - \omega_p)^2}{2\sigma^2\omega_p^2}\right]$$  \hspace{0.5cm} (11)

The method to estimate the wave spectrum proposed by Blackman and Tukey (1959) is based on the Wiener-Khintchine theorem. The Blackman-Tukey procedure, also known as the method of correlogram is an indirect method, because first it requires estimation of the autocorrelation function and then applies the Fourier transform to it. The procedure is described in Massel (1996) and will not be repeated here.

An alternative means named Cooley-Tukey method (1965) applies direct Fast Fourier Transform to the water surface displacements and is basically used lately as a result of saving a lot of computational time.

2.2 Statistical Analysis of Surface Waves

Since the propagation of wave systems can be represented by the sum of a large number of harmonic and statistically independent wave components, the wave surface displacement at one point will tend to normal law (Gaussian distribution) according to the central limit theorem as the number of harmonics increases infinitely.

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[\frac{-(x-\mu)^2}{2\sigma^2}\right], -\infty < x < +\infty$$  \hspace{0.5cm} (12)

In most cases, although the distribution of surface displacement is approximately Gaussian, a small asymmetry and different peakedness are observed. These deviations can be expressed in two parameters, coefficients of skewness $\gamma_3$ and kurtosis $\gamma_4$, which are higher order quantities and related to the nonlinearities in the wave basin.

Only when skewness and kurtosis are equal to zero at the same time, the random variable is normally distributed. For a narrow band unidirectional weakly nonlinear train up to second order, the coefficients of skewness and kurtosis are related to the steepness in Eq.(7) (Mori and Janssen, 2006)

$$\gamma_3 = \frac{3}{2} S_p$$  \hspace{0.5cm} (13)

$$\gamma_4 = 6 S_p^2$$  \hspace{0.5cm} (14)

In the linear approach proposed by Longuet-Higgins, the crest height and trough depth are both assumed to be Rayleigh distributed:

$$E_{\zeta^+}(z) = \exp \left(-\frac{z^2}{2\mu_1^2}\right)$$  \hspace{0.5cm} (15)

However, Rayleigh distribution could not give an exact prediction in ocean engineering design. Tayfun and Fedele (2007a) proposed the following exceedance distributions of crest and trough:

$$E_{\zeta^+}(z) = \exp \left[-\frac{1}{2\mu_1^2}\left(1+\sqrt{1+2\mu_1^2}z\right)^2\right]$$  \hspace{0.5cm} (16)

$$E_{\zeta^-}(z) = \exp \left[-\frac{1}{2}\left(1+\frac{1}{2}\mu_1^2\right)^2\right]$$  \hspace{0.5cm} (17)

If considering third-order nonlinearities, the preceding distribution could be modified further as the following forms:

$$E_{GC^+}(z) = E_{\zeta^+}(z) \left[1+\frac{\Lambda}{64}(z^2-4)\right]$$  \hspace{0.5cm} (18)

$$E_{GC^-}(z) = E_{\zeta^-}(z) \left[1+\frac{\Lambda}{64}(z^2-4)\right]$$  \hspace{0.5cm} (19)

where $\zeta^+$, $\zeta^-$ and $\mu_1$ are crest, trough and normalized wave steepness respectively.

$$\Lambda = \lambda_{40} + 2\lambda_{22} + \lambda_{04}$$  \hspace{0.5cm} (20)

$$\lambda_{mn} = \left\langle \eta^{m} \tilde{\eta}^{n} \right\rangle / \sigma^{(m+n)} + (-1)\frac{m}{2} (m-1)(n-1)$$  \hspace{0.5cm} (21)

The exceedance Rayleigh distribution for wave height is:

$$E_{\tilde{z}}(z) = \exp \left[-\frac{z^2}{8}\right], 0 \leq z < \infty$$  \hspace{0.5cm} (22)
However, it has been reported that Rayleigh exceedance distribution tends to overestimate the crest-to-trough heights of large waves by about 7%-8% (Tayfun and Fedele, 2007a). This discrepancy has been found to be mainly caused by the assumption in the Rayleigh theory of the wave height being double the crest height.

Therefore, some more advanced theories have been presented in the present study. Tayfun and Fedele (2007a) presented the third-order approximations of wave heights based on the Gram-Charlier (GC) expansions:

$$ E_{sc}(z) = E_{c}(z) \left[ 1 + \frac{\Lambda}{1024} z^2 \left( z^2 - 16 \right) \right] $$

(23)

Longuet-Higgins (1975) proposed the joint distribution of wave heights and periods. And he further modified and improved it with some techniques to make the theoretical distribution agree more with the reality (L.H. 1983).

$$ f(R,T) = \frac{2L(v)R^2}{v\sqrt{\pi T^2}} \exp\left\{ -R^2 \left[ 1 + \frac{1}{v^2} \left( 1 - \frac{1}{T} \right)^2 \right] \right\} $$

(24)

$$ L(v) = \frac{2\sqrt{1+v^2}}{1+\sqrt{1+v^2}} $$

(25)

Furthermore, the wave period distributions could be given easily on the basis of this joint distribution.

$$ f(T) = \frac{L(v)}{2\nu T^2} \left[ 1 + \frac{1}{v^2} \left( 1 - \frac{1}{T} \right)^2 \right]^{-3/2} $$

(26)

3 EXPERIMENT RESULTS OF WAVE ANALYSIS

3.1 Facility and Data

In the experiment, the double-flap wave maker is installed at one of the short walls of the basin. The beach at the opposite side serves to absorb the incident wave energy. The size of the offshore basin is 152m×30m×5m as show in Fig. 1.

The water surface elevations have been measured by 6 gauges uniformly spaced along the wave basin at 10m interval. In the first test, the first gauge is 20m far away from the wave maker and the same experiments are carried out two times. Then the 6 wave gauges are moved simultaneously 10 meters away from the wave make. That is, the nearest gauge to the wave maker is 30m now and the same experiments are repeated as before. The duration of each recorded time series is near half an hour with sampling frequency 100Hz. The scale of this experiment is 1:50 and the following study study is based on a set of records obtained under 23 different model conditions. The spectrum used at the wave maker is of the JONSWAP type. The experiment satisfies the deep water condition for the reason that the maximal wave length, calculated from the linear dispersion...
relationship at the peak frequency, is less than 500m in all tests.

Considering that only 6 wave records are made at a time for each sea state, it is better to present the results with six gauges each time by the mean value in the same position. The respective mean results of the six gauges in the first position are denoted as Part A in the following analysis and the corresponding mean results in the second position are denoted as Part B. The initial steepness mentioned is the steepness measured at the first gauge in Part A and Part B respectively.

In order to make sure that the initial noise is cut off and that the waves analyzed from 6 gauges are the same, it is necessary to cut off the wave record at the right position by utilizing the wave group velocity. The errors such as present in Fig. 2 are also necessary to be eliminated by filtering the unnecessary frequency parts with MATLAB.

### 3.2 Spectral Analysis

#### 3.2.1 Spectra Comparison

Wave record in each gauge generates two wave spectra which are compared with the standard JONSWAP spectrum described in Eq.(8). For the purpose of simplicity, only the figures of Gauge4 are presented below in three typical sea states which are characterized by different initial steepness.

As it is shown in the 5, ‘G1’, ‘G4’ and ‘G6’ stands for three gauges respectively. ‘BT’, ‘CT’ and ‘Sta’ successively denote spectra obtained with Blackman-Tukey Method, Cooley-Tukey Method and the standard one proposed by Goda (1988).

According to the experiment, the two methods do not give two much difference except for the density of peak frequency. When the initial steepness is very small, the computed wave spectra are exactly the same as the standard one such as picture (a) of Fig. 3. When the initial steepness increases, the computed spectra will deviate from the standard one (see picture (c) of Fig. 3).

Observing picture (a) in Fig. 4, it is noted that in the sea state with small initial steepness the spectral slope behaves as $\mu^{-4}$ at the first interval above the peak frequency. Then a suddenly falling down appears. But the slope is still kept as before at the second interval. The third interval is very small and near to 4, the spectral slope is steepened to about $\mu^{-6}$ due to the viscous dissipation (Fedele et al., 2010) which plays a major role in energy dissipation when the wave steepness is not large enough in the wave basin.

When the initial steepness increases as present in picture (b) and (c) of Fig. 4, wave breaking begins to play a major role because the turning point of spectral slopes moves quickly to $\mu = 1$.

In Fig. 5, wave breaking becomes more and more serious along the wave basin because the part of spectral slope behaves as $\mu^{-3}$ above the peak frequency increases from G1 to G6.

#### 3.2.2 Significant Wave Height

The change of the significant wave heights along the wave basin could be catalogued into three groups and attributed to two reasons. The major factor that affects the evolution trend is the initial wave steepness. The second one is the initial amplitude of significant wave height which is very obvious in the intermediate sea state. Normally large amplitudes are apt to generate an increased trend for the significant wave heights while small ones are inclined to give a decreased trend.

In group 1 such as picture (a) of Fig. 6 where the steepness is very small, significant wave heights increase along the wave basin. Since there is no input energy after the wave generator, the only reason that could explain this kind of wave evolution in the propagation is the reflection in the wave basin and the superposition among the waves. It increases the wave amplitude without changing its frequency.

The most complicated case is the transition steepness in group 2 ((b) of Fig. 6). Normally, the general trend keeps level along the wave basin although with a little fluctuation at times determined by the initial wave amplitudes.

In group 3 (c) of Fig. 6 with large initial steepness, significant wave heights decrease along the wave basin due to the serious wave breaking. As far as we know, 0.10-0.12 is the critical value for wave breaking. Therefore, it is to be expected that most energy will be lost as a result of wave breaking and thus leads to a descended trend.

#### 3.2.3 Wave Period

The mean values of zero-up crossing periods obtained from spectral methods are used below and denoted by “SpAv” in Fig. 7. For the purpose of better comparing the spectral method with statistical method, periods computed by statistical method are also presented in the same figures where “StDn”, “StUp” represent zero-down crossing period and zero-up crossing period respectively.

When the initial steepness is less than 0.0936, the wave periods almost keep constant along the wave basin as shown in (picture (a) of Fig. 7). If it is larger than 0.1086, the wave periods will increase along the wave basin which is proved in the rest two pictures.

It is also could be noted that zero-up-crossing periods and zero-down-crossing periods in statistical analysis almost give the same result in all sea states which provide some facts for the argument that analysis from zero-up-crossing methods and zero-down-crossing methods normally give the same conclusion except for some special cases.

Another very useful wave period in spectral analysis is the mean spectral period. Considering that the
Spatial variation of zero crossing wave period is almost linear, it is feasible to measure the changes of mean spectral periods during the propagation along the tank by the ratio between the value measured in the longest distance to the value measured closest to the wave maker, proposed by Cherneva and Guedes Soares (2011). This ratio is denoted by index (r) to differ from the relevant parameters.

Since the reciprocal of mean spectral period known as the carrier frequency has more physical meaning, the relative carrier frequency as a function of initial steepness for all sea states is preferred and plotted in (picture (a) of Fig. 8). The series with higher nonlinearity characterized by large initial steepness are modulated more due to the reduction of the spectrum tail in its high frequency part as found previously in section 3.2.1.

3.2.4 Zero Spectral Moment and Steepness
Since the spatial variation of significant wave heights has been analyzed, it is not difficult to predict the propagation law of the zero spectral moments for the reason that they have a direct relationship. All the conclusions made in section 3.2.2 still hold for the zero spectral moment.

It is pronounced in Fig. 9 that the change of wave steepness is almost in direct proportion to the change of zero spectral moment and also depends on the initial steepness. It is obviously correct in that when the wave energy is large, that is, large zero spectral moment, it definitely will generate large wave heights and thus result in large steepness at that position. Therefore, the spatial variation of steepness will be the same as that of significant wave height.

The same procedure is done as before in picture (b) of Fig. 8). The relative zero spectral moment is larger than 1.0 in the case of small initial steepness. In other words, the wave energy is increased nearly 10% plausibly because of the reflection in the wave basin. When initial steepness is larger than 0.1, the increase speed is not as fast as before. As initial steepness continues to increases, wave breaking plays a more and more important role in the energy dissipation and thus leads to a decrease in relative zero spectral moment. The largest decrease is nearly 15% and the second critical change point of initial steepness in the experiment is near to 0.15.

3.2.5 Spectral Width Parameter
Spectral width parameter has the same critical value as wave period for initial steepness. If the initial steepness is less than 0.0936, spectral width is almost constant along the wave basin as shown in picture (a) of Fig. 10. However, if it is larger than 0.1086, spectral width will decrease which is proved in picture (b) of Fig. 10. The unchanged spectral width means that the reflected waves almost have the same frequency as incident waves and only modulate their amplitudes. Furthermore, the decreased wave spectral width parameter is just another demonstration of wave dissipation in the sea states with intermediate and large initial steepness.

Since the two spectral width parameters have the same change rule, that is, ε is approximately two times of ν for all sea states, only relative νr is presented in picture (c) of Fig. 8.

Apparently, 0.1 could work as a threshold value again to identify the change trend. When initial steepness is less than 0.1, the relative spectral width extends to one. But after that turning point, the relative wave spectral width decreases quickly and reaches its minimal value near 0.15. And then, the relative wave spectral width increases but still smaller than 1.0. Whatever, it could be noted that the averaged decrease is nearly 8% in most cases. The last thing that needs to be emphasized is that all the wave spectral widths ν are in the range from 0.2 to 0.3 in the experiment which could be considered as narrow band spectra. It is very strange that the maximal decrease is present in the intermediate sea state rather than the most serious one.

3.3 Statistical Analysis

3.3.1 Skewness and Kurtosis
Skewness and kurtosis are two important parameters in statistics analysis. In the experiment the most common distribution appeared is presented in picture (a) of Fig. 12 where the peak value is located smaller than the mean value zero because of the positive skewness and the shape is much sharper due to the large positive kurtosis.

In the laboratory experiment, only in the case of very small initial steepness, the value of kurtosis is negative as presented in the first picture of Fig. 11. When the initial steepness is intermediate or large, kurtosis is positive and increases to a constant value at about 60m from the wave maker. The same spatial variation was presented by Petrova et al. (2008). At the position near to wave maker, the third order nonlinearity effect is not very strong because the value of kurtosis is very small there. What is more, the largest kurtosis is presented in the sea state with intermediate initial steepness rather than the largest one. That is also what happens to the decrease of spectral width parameter discussed in section 3.2.5. But the value of skewness is almost positive everywhere and its spatial variation is very small along the wave basin which means that the second order nonlinearity effect is constant in the wave tank. Moreover, larger skewness corresponds to the larger initial steepness, that is the severer sea state.

To further describe the above conclusions clearly, it is feasible and necessary to study the coefficients of kurtosis and skewness of the outmost gauge as a function of the initial steepness as presented in picture (b) and (c) of Fig. 12. Apparently, the relationship between the coefficient of skewness and initial
steepness is linear. As for the coefficient of kurtosis, the largest value is in the interval from 0.1 to 0.15. When the initial steepness is small, its value is small, while the initial steepness is larger than 0.15, it almost extends to a constant with a little decreased trend.

If we research the coefficients of skewness, kurtosis as a function of the wave steepness for each gauge, almost the same conclusions could be drawn. In (a) of Fig. 13, although the theoretical Eq.(13) overestimates the coefficient of skewness, the linear relationship still holds and the fitted lines almost have the same slope with the theoretical one. One thing should be kept in mind is that part of the decrease of coefficient of skewness is due to filtering wave series. In picture (b) of Fig. 13, Eq.(14) does not predict the coefficient of kurtosis well for the reason that the fitted curve decreases for the larger wave steepness.

The coefficients of skewness and kurtosis also have some relationship with each other as presented in picture (c) of Fig. 13. The mathematical model proposed here is much better than the five order polynomial function.

3.3.2 EDF of Wave Crest, Trough and Height
To see this exceedance distribution clearly, it is better to arrange the exceedance distributions of wave crest, trough and height together. Based on the experiment, it is strongly supported that GC approximations could predict wave crest and height much better than Rayleigh distribution in most cases as illustrated in Fig. 14. However, in some cases GC approximation also does not work and the same phenomenon is also discovered by Cherneva, Tayfun and Guedes Soares(2009). The discrepancy between observed data and theoretical models may be caused by the wave breaking and filtering.

3.3.3 Wave Period
The distributions of wave periods are considered in a statistical view. The theoretical distribution in Eq.(26) is only related to the wave spectral width. Obviously, the theoretical wave period distribution changes slowly because the spectral width does not decrease quickly when the sea states changes. However, the observed wave period changes rapidly and its shape becomes sharper and narrower as the wave steepness increases. Therefore, the wave period distribution should be related not only with the wave spectral width but also with the wave steepness. It seems that the wave periods are asymptotically uniformly distributed around the carrier wave period when the sea state is mild as described in picture (a) of Fig. 15. As the sea state becomes severer, the periods will concentrate on the peak wave period which is a little larger than the carrier wave period as presented in picture (b) of Fig. 15. It seems that the theoretical distribution agrees with the experiment data well in the sea state with intermediate steepness. And it will be much better if the wave periods are normalized with peak period rather than the mean wave period for the reason that the mode of the observed wave period will move backward and conform with the theoretical maximum value.

3.3.4 Joint Distribution
In Fig. 16, the theoretical joint distribution almost keeps the same shape for typical sea states for the reason that spectral width parameter is the only variable in Eq.(24). However, the observed distribution changes greatly as the wave steepness increases. For the mild sea state, this theoretical joint distribution does not work due to the assumption that the wave frequencies mainly concentrate on the carrier wave frequency.

As the sea state becomes serious, the theoretical joint distribution agrees with the observed data reasonably well. Therefore, as what has been mentioned in the period distribution, the theoretical joint distribution is not only related to the wave spectral width parameter but also connected with the wave steepness. The theoretical joint distribution is also based on the linear superposition which may be not suitable in the severest sea state due to the strong nonlinearity and thus maybe explain the deviation from the observed results in picture (c) of Fig. 16.

The position of the observed maximal probability stays in the bottom right corner of the theoretical location in most cases. It means that the normalization parameter should be changed and this topic could be further continued but is out of the scope of this thesis.

3.4 Conclusions
In the work 552 laboratory wave series have been analyzed. The spectral and statistical methods are used in this study. Several wave parameters like significant wave height, zero spectral moment, carrier wave frequency, etc., are researched. The main focus is on the spatial variation and their relationship with the initial wave steepness. The spatial variation is determined by the sea state and could be categorized into three groups by the value of initial steepness.

As it was discussed before, significant wave height, zero spectral moment and steepness almost present the same spatial variation in that large wave energy is characterized by large wave height which definitely will result in large steepness. The carrier wave frequency and spectral width parameters illustrate the same spatial variation due to the energy dissipation mainly caused by wave breaking. Considering the direct relationship between zero-up-crossing period and mean period, it is easy to understand that zero-up-crossing wave period must have the same critical value of initial steepness as carrier wave frequency which is the reciprocal of mean period. The coefficient of skewness almost keeps con-
stant along the wave basin and its value increases as the initial steepness increases. Kurtosis is negative and nearly constant only in two sea states with small initial steepness. In most cases, it is small near to the wave maker and increases to the maximal value approximately in the middle of the wave basin which will be kept constant in the later propagation.

When comparing the experiment data with the theoretical distribution models, the useful information are summarized into three groups denoted by small, intermediate and large initial wave steepness. Wave spectra calculated with Blackman-Tukey method and Cooley-Tukey method are almost the same. Compared with the standard JONSWAP spectrum, only in the sea state with small initial wave steepness, they coincide. For the large initial wave steepness, the discrepancy could be apparently observed. It is identified that the high frequency part is tailed from $\mu^{-4}$ to $\mu^{-2}$ as a result of wave breaking as the initial steepness increases. The observed distributions of wave crest, trough and height are compared with Rayleigh distribution and GC approximation. As a result, in most sea states GC model fits the empirical data better than Rayleigh distribution. Wave periods are also compared with the theoretical distribution which is the marginal distribution of joint distribution proposed by Longuet-Higgins. The experiment data fit the theoretical distribution well in the sea state with intermediate steepness. Based on our results it is suggested to normalize the wave periods with peak period considering that most energy is concentrated on the peak frequency. The same conclusion could be obtained for the joint distribution. The observed location of the maximal probability is always in the bottom left corner of the theoretical location.

Reference


Fig. 3 Wave spectra in three typical sea states

Fig. 4 Normalized wave spectra in three typical sea states

Fig. 5 Spatial variations of normalized wave spectra with initial steepness $s_0=0.1152$

Fig. 6 Spatial variations of significant wave heights

Fig. 7 Spatial variations of zero-crossing wave periods
Fig. 8 Relationships between initial steepness and relative wave parameters

Fig. 9 Spatial variations of $m_0$ and steepness

Fig. 10 Spatial variations of spectral width parameters

Fig. 11 Spatial variations of coefficients of skewness and kurtosis

Fig. 12 Relationships about coefficients of skewness and kurtosis
Fig. 13 Relationships about coefficients of skewness and kurtosis

Fig. 14 Exceedance distributions of G4 in three typical sea states

Fig. 15 Wave period distributions in three typical sea states

Fig. 16 Joint distributions in three typical sea states