Correlation between roughness and friction angle of open discontinuity surfaces

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Abstract – This paper was conducted with the aim of study the variation of rock discontinuity friction angle with the discontinuity surfaces roughness. For this approach, the surfaces of two rock types were measured, marly limestone and micaceous schist, in order to quantify their roughness. Shear tests were also performed to obtain the shear strength parameters of the discontinuities. The quantification of rock discontinuities roughness was performed using the arithmetical average roughness parameter, $R_a$, and the surface roughness parameter, $R_s$. These parameters were related with the angle corresponding to the asperities component, $i'$, derived from the peak friction angle, $\phi_v + i'$, in joint shear tests.

The developed study, allowed the establishment of correlations between the experimental results and the strength characteristics of discontinuities.

Keywords – roughness, asperities, discontinuities, friction angle, shear strength.

1. Introduction

At shallow depth, where stresses are normally low, the behaviour of the rock mass is mainly controlled by sliding on the discontinuities. In order to analyze the stability of this system of individual rock blocks, it is necessary to understand the factors that control the shear strength of the discontinuities which separate those blocks (Hoek, 2007). The significant influence of roughness on discontinuity surface shear behavior implies that its knowledge and characterization are important to predict the resistant behavior of rock masses. With the purpose of defining the discontinuities mechanical behavior based on the knowledge of rocky material and surface geometry characterization has been established several physical models (e.g. Patton, 1966 e Barton, 1973).

Despite these efforts, continues to be necessary to use tests at various stages of an engineering project to characterize or at least assess the discontinuities behavior (Resende, 2003), mainly due to difficulty of roughness characterizing, by its three-dimensional and not repetitive nature.

2. Discontinuity shear strength

Patton (1966 apud Yang et al.,2010; Hoek, 2007; Grasselli, 2001) was the first researcher in rock mechanics to relate the shear behavior of joints to normal load and roughness. His work is based on an idealized model of a joint in which roughness is represented by a series of constant-angle triangles or saw-tooth. Based on the Mohr-Coulomb friction criterion, this model characterizes the joint
behavior by a single surface parameter that is the average roughness angle \((i)\) (Kemthong, 2006; Wyllie e Mah, 2004), also known as dilatancy angle (arc tangent of the ratio between vertical and shear displacement of the sample during the shearing). Patton proposed the following bilinear shear strength criterion:

\[
\tau_p = \sigma_n \tan(\phi_b + i) \quad \text{for} \quad \sigma_n < \sigma_t \tag{1}
\]

\[
\tau_p = c + \sigma_n \tan \phi_b \quad \text{for} \quad \sigma_n \geq \sigma_t \tag{2}
\]

Where \(\tau_p\), \(\sigma_n\), \(c\), \(i\), \(\phi_b\) are peak shear strength, normal stress, cohesion, peak dilatancy angle and basic friction angle, respectively, and \(\sigma_t\) is some stress that demarcates the two regimes.

Barton (1973, apud Asadollahi e Tonon, 2010; Yang et al., 2010; Hoek, 2007; Wyllie e Mah, 2004, Barton e Bandis, 1990) studied the behaviour of natural rock joints and proposed that equation (1) could be re-written as:

\[
\tau = \sigma_n \tan \left(\phi_b + JRC \log_{10} \left(\frac{JCS}{\sigma_n}\right)\right) \tag{3}
\]

In which \(JRC\) and \(JCS\) are joint roughness coefficient and joint compressive strength, respectively.

Conceptually \(\phi_b\) refers to a flat smooth unweathered surface and can be considered as a material constant, residual friction angle \(\phi_r\) refers to the residual condition of the natural surface of discontinuity which is reached after a large shear displacement. For unweathered rock fractures, \(\phi_r\) is equal to \(\phi_b\) (Asadollahi e Tonon, 2010; Kemthong, 2006).

The global shear strength of discontinuities is function of a peak friction angle \((\phi_r + i)\), reducing to \(\phi_r\) at higher normal stresses (Brady e Brown, 2005; Barton e Bandis, 1990). \(JRC\) is related to the \(i\) value by the following equation:

\[
i = JRC \log_{10} \left(\frac{JCS}{\sigma_n}\right) \tag{4}\]

3. Surface roughness

Surface topography of a rough joint is consisted of asperities which occur in many scales and can be categorized into primary (waviness) and secondary (irregularities) categories (Patton, 1966 apud Yang, 2010). Roughness is defined as a measure of the inherent surface irregularities and waviness of the discontinuity relative to its mean plane (Brady e Brown, 2005). Roughness degree can be quantified in terms of value \(i\), which is defined as a measure of the asperities inclination on the rock surface (Wyllie e Mah, 2004).

A natural way to assess the roughness of a surface is to run a finger across it. Mechanical profilometers work in the same way: a contacting-probe moves across the surface measuring the height of the surface along a line. The principle is to measure the vertical displacement of the stylus as it moves across the surface (Grasselli, 2001). Since the first assessments of surface roughness were
introduced in rock mechanics, both because of the difficulty to obtain data as to the purpose of simplifying the problem of evaluating, have been used linear profiles and roughness is normally sampled by linear profiles taken parallel to the direction of sliding, that is, parallel to the dip (dip vector) (Wyllie e Mah, 2004). Over the last years, increasing attention has been given to the characterization of 3D surface roughness and its connection with the mechanical behavior of discontinuities (e.g. Belem et al., 2009, 2000 e 1997; Jiang et al., 2006; Lee et al., 2006; Grasseli et al., 2002; Grasselli, 2001; Homand et al., 2001; Fardin et al., 2004 e 2001), however, the characterization of linear profiles and roughness estimation using 2D approaches continues to be important for applications such as the empirical predictions of shear strength (Rasouli e Harrison, 2010; Tatone e Grasselli, 2010).

3.1. Roughness parameters

The profile parameters are calculated from individual profiles obtained along a direction (X and/or Y), as illustrated in Figure 1.

Figure 1 – Two-dimensional graphs of the shape of the surface in the sectioning plane created by the profiling instrument, along X and Y directions.

The amplitude parameter and the surface roughness parameter used in this study were:

- **Average Roughness** ($R_a$) is the area between the roughness profile and its mean line, or the integral of the absolute value of the roughness profile height over the profile length (Palma, 2006):

$$R_a = \frac{1}{L} \int_0^L |z| \, dx = \frac{A}{L} \quad (5)$$

Where $L$ is the profile length and $z$ is the height of the irregularities.

- **Surface roughness coefficient superficial** ($R_s$) is defined as the ratio between the actual area of the surface ($A_s$) and the projection area of the surface ($A_0$) (El Soudani, 1978 *apud* Belem et al., 2007; Lee et al., 2006; Fifer Bizjak, 2010):

$$R_s = \frac{A_s}{A_0} \quad (6)$$

3
4. Laboratory tests

It was used in this study discontinuities samples, induced by mechanical shock, of two rock types: marly limestone (C – 5 specimens) and micaceous schist (X – 5 specimens), both taken on core samples. The experimental work consisted in the measurement of the sliding surfaces roughness and were also performed joint shear tests with normal stresses ($\sigma_n$) of 0.15, 0.30, 0.60 e 1.20 MPa applied to characterize the shear behavior of these surfaces and obtain the strength parameters $\phi_p$, and $\phi_r$.

4.1. Experimental method for the characterization of the surface roughness

For the measurement of roughness was used a contact rugosimeter composed of a digital system for linear measurement (absolute linear scale AT 715 Mitutoyo), with a resolution of 0.001 mm, and a digital read out (KA counter Mitutoyo). For each surface was defined a grid spaced of 5 mm for both directions, X e Y (Figure 2). Depending on the size of the sample it was obtained about 15 profiles on X direction and 12 on Y direction for each sample of schist and about 12 profiles on each direction for limestone samples.

Figure 3 presents a sketch of the methodology used to obtain the roughness parameters, after data acquisition.

5. Results and Data Analysis

Table 5.1 gather the results obtained for each sample, taking into account the weighted average of all values of $R_{ax}$ e $R_{ay}$ ($\bar{R}_{ax}$, $\bar{R}_{ay}$), from the length of the measured profiles, and the maximum value ($R_{ax(max)}$ e $R_{ay(max)}$) of all $R_a$ calculated in each direction.
Table 1 – Average and maximum values of $R_\alpha$ parameter.

<table>
<thead>
<tr>
<th>Amostras</th>
<th>$R_{\alpha X}$</th>
<th>$R_{\alpha Y}$</th>
<th>$R_{\alpha X(\text{max})}$</th>
<th>$R_{\alpha Y(\text{max})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.558</td>
<td>1.300</td>
<td>1.444</td>
<td>1.517</td>
</tr>
<tr>
<td>C2</td>
<td>1.470</td>
<td>1.531</td>
<td>3.782</td>
<td>3.070</td>
</tr>
<tr>
<td>C3</td>
<td>0.720</td>
<td>0.892</td>
<td>1.536</td>
<td>1.861</td>
</tr>
<tr>
<td>C4</td>
<td>0.470</td>
<td>1.982</td>
<td>1.269</td>
<td>2.545</td>
</tr>
<tr>
<td>C5</td>
<td>1.336</td>
<td>1.710</td>
<td>2.433</td>
<td>3.961</td>
</tr>
<tr>
<td>X1</td>
<td>0.323</td>
<td>0.475</td>
<td>0.584</td>
<td>0.595</td>
</tr>
<tr>
<td>X2</td>
<td>0.587</td>
<td>1.447</td>
<td>1.742</td>
<td>2.078</td>
</tr>
<tr>
<td>X3</td>
<td>0.994</td>
<td>1.237</td>
<td>1.948</td>
<td>1.601</td>
</tr>
<tr>
<td>X4</td>
<td>0.677</td>
<td>0.800</td>
<td>1.071</td>
<td>0.901</td>
</tr>
<tr>
<td>X5</td>
<td>0.591</td>
<td>1.094</td>
<td>0.901</td>
<td>1.399</td>
</tr>
</tbody>
</table>

Figure 4 shows the representation of 3D surfaces and the results for $R_s$ parameter are presented in Table 2.

![Figure 4](image)

**Figura 4 – Marly limestone surface (C4) and mica schist surface (X5), with direction of shearing.**

Table 2 – $R_s$ parameter.

<table>
<thead>
<tr>
<th>Amostras</th>
<th>$A_5$ (mm$^2$)</th>
<th>$A_0$ (mm$^2$)</th>
<th>$R_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>3112.751</td>
<td>3016.274</td>
<td>1.032</td>
</tr>
<tr>
<td>C2</td>
<td>3090.174</td>
<td>2930.187</td>
<td>1.055</td>
</tr>
<tr>
<td>C3</td>
<td>3012.359</td>
<td>2904.556</td>
<td>1.037</td>
</tr>
<tr>
<td>C4</td>
<td>3007.248</td>
<td>2898.257</td>
<td>1.038</td>
</tr>
<tr>
<td>C5</td>
<td>3103.179</td>
<td>2878.644</td>
<td>1.078</td>
</tr>
<tr>
<td>X1</td>
<td>3457.402</td>
<td>3415.511</td>
<td>1.012</td>
</tr>
<tr>
<td>X2</td>
<td>4005.635</td>
<td>3910.408</td>
<td>1.024</td>
</tr>
<tr>
<td>X3</td>
<td>3288.692</td>
<td>3158.464</td>
<td>1.041</td>
</tr>
<tr>
<td>X4</td>
<td>3374.269</td>
<td>3304.609</td>
<td>1.021</td>
</tr>
<tr>
<td>X5</td>
<td>3892.535</td>
<td>3810.514</td>
<td>1.022</td>
</tr>
</tbody>
</table>

According to the Equation 1 the angle $i'$, for this study, was calculated derived from the peak friction angle and was related to the roughness parameters, as well as the $JRC$ obtained by the Equation 4. Figure 5 shows the obtained relationships between $R_s$ and $i'$, $JRC$, respectively.
In order to assess the influence of the measurement direction on value of the roughness it was tested the relationship between $JRC$, $R_{aX}$, $R_{av}$, $R_{ax(max)}$ and $R_{ay(max)}$. The best results were find to the maximum values for profiles taken parallel to the direction of sliding ($R_{ay(max)}$). Figure shows the linear regression relationship of this parameter with the $i'$ and $JRC$.

It was obtained the mathematical equation that represents the best linear numerical relationship between the set of data pairs of the variables $i'$ and $R_{ay(max)}$ with a correlation coefficient $R = 0.82$:

$$i' = 2.3829R_{ay(max)} + 2.8392; \quad R^2 = 0.68$$  \hfill (7)

**Conclusions**

- $R_a$ values measured in the Y direction are higher than in the X direction for both types of rock, due to the steeper geometry, resulting from how the fracture was caused. The limestones have ripples of larger amplitudes in the Y direction due to sub-type fracture. While the schists have a higher degree of variability, and the differences between directions are not as big as the rock breaks according to the weakness surface (foliation), yet the higher values correspond to the profiles measured parallel to the direction of the schistosity plans.
• There is a strong correlation between the coefficient of surface roughness and peak and residual friction angle measured in the laboratory.

• Studying the variation of surface roughness with \( R_s \) and \( JRC \), the correlation decreases (59%) indicating a slightly higher correlation (63%) of this parameter with the maximum value of average roughness of each profile. The JRC is indeed a one-dimensional parameter and is particularly sensitive to higher amplitudes.

• For the parameter \( i' \) the trend verifies and there is an increased correlation of 63% with the surface roughness to 68% with the roughness parameter in the direction \( Y \).

• The obtained correlations emphasize the importance of surface roughness on the shear strength of the open surfaces of discontinuity.

References