Block Transmission Techniques For the Uplink of Future Mobile Broadband Systems

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Abstract—This paper deals with CP (Cyclic Prefix)-assisted block transmission solutions for future mobile broadband systems, in the context of a SC-FDMA (Single Carrier - Frequency Division Multiple Access) uplink. Two alternative choices are considered regarding the subcarrier mapping rule: a "localized" subcarrier mapping where user's data occupy a set of consecutive subcarriers (Rule R1); a "distributed" subcarrier mapping where user's data occupy a set of uniformly spaced subcarriers (Rule R2). Detailed performance evaluations, in this thesis, involve the consideration of two iterative FDE (Frequency Domain Equalization) receiver techniques, with different complexity levels, which can be regarded as extensions of iterative receiver techniques proposed previously within a single user context. A selected class of multipath radio channels, providing a range of channel time dispersion levels, is assumed for performance evaluation purposes, and a set of matched filter bounds on receiver performance plays a relevant role in "achievable performance” comparisons; a "normalized frequency-domain channel autocorrelation function”, easily derived from the channel model assumptions, is also used for discussing numerical performance results. Both the impact of the mapping rules and that of the iterative receiver techniques considered here are evaluated in detail. This thesis also studies the performance degradation due to a channel impulse response longer than the CP; such degradation is related to an "ICI effect" (possibly involving a multi-user interference component) which is inherent to the insufficient-CP conditions. The performance advantages under rule R2 are emphasized for a low or moderate channel time dispersion, and both specific iterative receiver techniques; for a higher time dispersion, receiver performances become very similar with both rules, at the BER values of practical interest (say, BER=10^-3). Having in mind that rule R2 has a power efficiency advantage regarding transmitter implementation (due to the reduced envelope fluctuations), we can conclude that this rule provides a clear overall power efficiency advantage, regarding both transmitter and receiver issues, practically for the entire range of channel time dispersion levels.

Keywords: Mobile Broadband Communication Systems, SC-FDMA, FDE, Iterative Receiver Techniques, Performance Evaluation.

I. INTRODUCTION

In recent years, appropriately designed CP - assisted block transmission schemes were proposed and developed for broadband wireless systems, which have to deal with strongly frequency-selective fading channel conditions. These schemes take advantage of current low-cost, flexible, FFT- based signal processing technology with both OFDM (Orthogonal Frequency Division Multiplexing) and SC/FDE (Single Carrier / Frequency Domain Equalization) alternative choices [1]. Mixed air interface solutions, with OFDM for the downlink and SC/FDE for the uplink as proposed in [2], [3] are now widely accepted. The main reason for replacing OFDM by SC/FDE, with regard to uplink transmission, is the lower envelope fluctuation of the transmitted signals when data symbols are directly defined in the time domain, leading to reduced power amplification problems at the mobile terminals. "Clipped and filtered" OFDM schemes have also been considered for low peak-to-mean envelope power ratio uplink transmission, in recent years. However, the SC/FDE alternative was shown to provide clearly better overall power efficiency when considering advanced iterative techniques of similar complexity in both cases [4]. Regarding broadband wireless communication systems based on CP-assisted block transmission, also became widely accepted, in recent years, multiple access techniques based on the assignment of disjoint subsets of subcarriers to the several simultaneous users, within the set of subcarriers provided by the system. OFDMA (Orthogonal Frequency Division Multiple Access) is a recommended choice for downlink transmission, where frequency-domain data symbols (e.g. from a QAM or PSK alphabet) are used; SC-FDMA is a recommended choice for uplink transmission, where time-domain data symbols (from the same QAM or PSK alphabets) are adopted. It should be noted that current working assumptions for the air interface in the 3GPP (3rd Generation Partnership Program) Project follow this hybrid approach [5]. In this paper, we consider SC-FDMA uplink block transmission using two alternative choices in which concerns the subcarrier mapping rule: a "localized" subcarrier mapping where user’s data occupy a set of consecutive, adjacent sub-carriers; a distributed subcarrier mapping where user’s data occupy a set of uniformly spaced subcarriers. Through the latter mapping rule, the multiple access scheme corresponds to a proposal in [6] (later adopted by other authors, under several acronyms), known to provide a lower envelope fluctuation. The main goal of this paper is to see if this rule also provides a "receiver performance” advantage. Our performance evaluations involve the consideration of iterative receiver techniques with two levels of complexity, which can be regarded as an extension of the iterative receiver approach proposed in [7], [8] regarding a single-user context. A selected class of multipath radio channels, providing a range of channel time dispersion levels, is considered for performance evaluation purposes, and a set of matched filter bounds on receiver performance plays a relevant role in "achievable performance” comparisons. Both the impact of the mapping rules and that of the iterative
receiver techniques considered here are evaluated in detail.

II. CP-ASSISTED BLOCK TRANSMISSION FOR BROADBAND WIRELESS COMMUNICATIONS

The adoption of a CP-assisted block transmission scheme is a common practice within broadband wireless communication systems, so as to cope with strongly time-dispersive effects of multipath propagation without having to resort to complex (expensive) implementations. When the channel impulse response is not longer than the selected CP, there is no inter-block interference (IBI); moreover, the linear convolution inherent to the time-dispersive channel becomes equivalent to a circular convolution in what concerns the useful part of each block. The impact of the corresponding multiplications in the frequency domain can then be easily compensated by appropriate multiplications, in the frequency-domain equalization (FDE) unit of a low-cost receiver. Both transmitters and receivers take advantage of the FFT algorithm for the required DFT (Discrete Fourier Transform) and IDFT (Inverse DFT) computations. With conventional OFDM for single-user CP-assisted block transmissions (fig. 1.a)), each length-\(N\) block of data symbols (QAM, PSK)

\[
S = [S_0, S_1, \cdots, S_{N-1}]^T \tag{1}
\]

is directly defined in the frequency domain. With the SC/FDE alternative (fig. 1.b)), each length-\(N\) block of data symbols (QAM,PSK)

\[
s = [s_0, s_1, \cdots, s_{N-1}]^T \tag{2}
\]

is directly defined in the time domain. For both OFDM and SC/FDE, the extended block through CP insertion as is follows:

\[
s_T = \begin{bmatrix}
    s_{N-L_s}, & s_{N-L_s+1}, & \cdots, & s_{N-1}; & s_0, & s_1, & \cdots, & s_{N-1}
\end{bmatrix}_{CP}^T \tag{3}
\]

(In the OFDM case, \(s = F_N^{-1} S\), where \(F_N^{-1}\) denotes the \(N \times N\) IDFT matrix). When space diversity, with \(Q\) branches,

\[
s_T = \begin{bmatrix}
    s_{N-L_s}, & s_{N-L_s+1}, & \cdots, & s_{N-1}; & s_0, & s_1, & \cdots, & s_{N-1}
\end{bmatrix}_{CP}^T \tag{3}
\]

is employed at the receiver side, \(Q\) CIR (Channel Impulse Response) vectors (\(q = 1, 2, \cdots, Q\))

\[
h^{(q)} = [h_0^{(q)}, h_1^{(q)}, \cdots, h_{N-1}^{(q)}]^T \tag{4}
\]

\(h_n^{(q)} = 0\) for \(L < n \leq N - 1\), with \(L \leq L_s\), \(Q\) CFR (Channel Frequency Response) vectors

\[
H^{(q)} = F_N h^{(q)} = \begin{bmatrix}
    H_0^{(q)}, & H_1^{(q)}, & \cdots, & H_{N-1}^{(q)}
\end{bmatrix}^T \tag{5}
\]

and \(Q\) receiver input noise vectors \(\nu^{(q)} = \begin{bmatrix}
    \nu_0^{(q)}, & \nu_1^{(q)}, & \cdots, & \nu_{N-1}^{(q)}
\end{bmatrix}^T\) can be used to describe the CP-assisted block transmission over the time-dispersive, noisy channel. With \(N_k^{(q)} (k = 0, 1, \cdots, N - 1)\) denoting the components of \(N^{(q)} = F_N \nu^{(q)}\), we can write

\[
Y_k^{(q)} = H_k^{(q)} S_k + N_k^{(q)} \tag{6}
\]

\((k = 0, 1, \cdots, N - 1, q = 1, 2, \cdots, Q)\)

![Fig. 2. FDE with Q inputs \((k = 0, 1, \cdots, N - 1)\).](image)

In the space diversity context, Fig. 2 describes the required FDE procedures, on a subchannel-by-subchannel basis \((k = 0, 1, \cdots, N - 1)\) [2]; the \(C_k\) coefficients can be computed according to either the ZF (Zero Forcing) criterion or the MMSE (Minimum Mean Squared Error) criterion, as follows:

\[
C_k = \begin{cases} 
    \frac{1}{\sum_{q=1}^Q |H_k^{(q)}|^2}, & \text{for ZF} \\
    \frac{1}{\alpha + \sum_{q=1}^Q |H_k^{(q)}|^2}, & \text{for MMSE}
\end{cases} \tag{7}
\]

where \(\alpha = \sigma_v^2/\sigma_n^2\) (\(\sigma_v^2 = E\left[|\nu_0^{(q)}|^2\right]\) and \(\sigma_n^2 = E\left[|s_n|^2\right]\)), since \(E\left[|\nu_0^{(q)}|^2\right] = E\left[|s_n|^2\right] = 0\).

In this paper, we consider a class of multipath radio channels, all of them characterized by a given set of path “power gains” \(\{P_i\}\) and a corresponding set of path delays \(\{\tau_i\}\). We assume that \(\tau_i = i\tau\) with \(\tau = N_s T_s\), where \(T_s\) is the symbol duration and \(N_s\) is an integer that can be adjusted according to the intended level of time dispersion effects on the transmitted signals. We also assume that \(P_i = \exp(-\beta i)\), for \(i = 0, 1, \cdots, 10\), and zero otherwise, with a selected non-negative constant \(\beta\), and an independent Rayleigh fading for each path and each receiver branch.

For the \(q\)th receiver branch \((q = 1, 2, \cdots, Q)\), the CIR components are then characterized as follows, through the use of independent, zero-mean, complex Gaussian variables \(Z_i^{(q)}\) with variance equal to 1:

\[
h_n^{(q)} = \sqrt{P_{n/N_s} Z_n^{(q)}}, \tag{8}
\]

if \(n \mod N_s = 0\), and zero otherwise \((n = 0, 1, \cdots, N - 1)\). Of course, \(E\left[h_n^{(q)}\right] = 0\), \(E\left[|h_n^{(q)}|^2\right] = P_{n/N_s}\) if \(n \mod N_s = \]
0 and zero otherwise, and \( E \left[ h_n(q) h_{n'}(q')^* \right] = 0 \) for \( n' \neq n \) and/or \( q' \neq q \).

Some performance results are provided in the following, when using either OFDM or SC/FDE, with \( N = 512 \), \( L_s = 128 \) and QPSK (Quaternary PSK) symbol constellation in both cases: \( s_n = \frac{s}{\sqrt{2}}(\pm 1 \pm j) \) for SC/FDE; \( S_k = \frac{s}{\sqrt{2}}(\pm 1 \pm j) \) for OFDM. The simulation results are shown in fig. 4, both without diversity and with a two-branch diversity, for a channel as described above, with \( \beta = 0.5 \) and \( N_s = 11 \); since \( L_s = 128 \), we can say that the CP is long enough to cope with the time-dispersive effects of the strongly frequency-selective Rayleigh fading channel. From these results - and also from other results obtained under similar, highly time-dispersive conditions - we can conclude that the SC/FDE choice gives better performance than the OFDM choice, with either \( Q = 1 \) or \( Q = 2 \), provided that the MMSE criterion is adopted. When using diversity, the BER performance improves dramatically in all cases, and, when using \( Q = 2 \) with both modulations, the SC/FDE choice typically exhibits a better performance even under the ZF criterion. It should be noted that the BER performances for the OFDM case, with both \( Q = 1 \) and \( Q = 2 \), are very close to those corresponding to flat Rayleigh fading conditions: these results can be easily explained, since the Rayleigh fading, in spite of being frequency-selective across the transmission bandwidth, is "approximately flat" at the subchannel level where the decisions are made.

As proposed in [2], when the wireless network includes fixed terminals (e.g. within the base stations (BS)) a realistic solution for system implementation is to choose an SC/FDE scheme, exhibiting low envelope fluctuations, for the uplink, and an OFDM scheme for the downlink. This means an implementation advantage for the mobile terminals (MT), where simple SC/FDE transmission and OFDM reception functions are carried out (see fig. 1). The "implementation charge" is concentrated in the fixed terminals (where increased power consumption levels are not so critical), concerning both the signal processing effort and the power amplification difficulties.

III. OFDMA AND SC-FDMA FOR FUTURE MOBILE BROADBAND SYSTEMS

OFDMA is a multiple access technique based on OFDM (see fig. 6), where different users occupy disjoint sets of subcarriers, so as to ensure orthogonality. In a system with \( J \) users and \( N \) subcarriers, like the one depicted in figure 6, this implies that \( \sum_{j=1}^{J} M_j \leq N \), when using \( M_j \) to denote the number of subcarriers occupied by user \( j \). The length-\( M_j \) block of frequency-domain input data concerning user \( j \) is given by \( S_{j}^{(in)} = [s_{j,0}^{(in)}, s_{j,1}^{(in)}, \ldots, s_{j,M_j-1}^{(in)}]^{T} \).

The so-called SC-FDMA multiple access technique (see Fig. 7) can be regarded as a modified OFDMA technique, by including a "DFT spreading" of the length-\( M_j \) input data block. Therefore, this data block becomes a "time-domain block", given by \( s_{j}^{(in)} = [s_{j,0}^{(in)}, s_{j,1}^{(in)}, \ldots, s_{j,M_j-1}^{(in)}]^{T} \). Through the \( M_j \)-point DFT, a frequency-domain representation of the data block concerning user \( j \) is then produced, according to

\[
S_j^{(in)} = F_{M_j} s_j^{(in)} = [s_{j,0}^{(in)}, s_{j,1}^{(in)}, \ldots, s_{j,M_j-1}^{(in)}]^{T},
\]

and - just as in the OFDMA case - this is followed by a selected procedure, where each of the \( M_j \) DFT outputs is mapped into one of the \( N \geq M_j \) orthogonal subcarriers that can be transmitted. The result of this subcarrier mapping is the length-\( N \) frequency-domain vector

\[
S_j = [s_{j,0}, s_{j,1}, \ldots, s_{j,N-1}]^{T}.
\]

Then, an \( N \)-point IDFT operation brings the data block information back to the time domain, leading to the length-\( N \) vector \( s_j = F_{N}^H S_j \), by inserting a length-\( L_s \) CP, the transmitted symbol block is obtained.

In a limited situation where \( M_j \) is increased to the value of \( N \), the resulting OFDMA system (see Fig. 6) behaves as a traditional OFDM system. Similarly, if a user occupies all the subcarriers available in the SC-FDMA system (see Fig. 7), a conventional SC/FDE system is obtained.
Two standards are being deployed for mobile broadband communications, WiMAX and 3GPP-LTE (also known as 4G)[9][5]. Although having different origins, they show many similarities, namely the fact that both rely on CP-assisted block transmission schemes (see table I). WiMAX uses OFDMA both for uplink and downlink; 3GPP-LTE uses OFDMA for the downlink, and SC-FDMA for the uplink.

The solution which has been adopted for future cellular systems (3GPP-LTE) can be regarded as an extension of the recommended solution in [2], for similar reasons. By using time-domain data symbols in the uplink - instead of frequency-domain data symbols - signal envelope fluctuations are significantly reduced, which simplifies the "power amplification problem" in the mobile terminal.

IV. SC-FDMA ISSUES: SUBCARRIER MAPPING RULES AND ACHIEVABLE RAW BER PERFORMANCE

A. Selection of Subcarrier Mapping Rules

Two subcarrier mapping rules considered here for SC-FDMA are described in the following:

a) Mapping rule R1: for a selected $K_j$,
\[
S_{j,k} = S_{j,k-K_j}^{in} \quad (11)
\]
if $k = K_j, K_j + 1, \cdots, K_j + M_j - 1$, and zero otherwise.

b) Mapping rule R2: for a selected $K_j' \in [0, m_j - 1]$, \(M_j = \frac{N}{m_j}\),
\[
S_{j,k} = S_{j,(k-K_j')/m_j}^{in} \quad (12)
\]

if $(k - K_j') \mod m_j = 0$, and zero otherwise.

R1 is a "localized" subcarrier mapping where user's data occupy a set of consecutive subcarriers, and R2 is a "uniformly distributed" subcarrier mapping (see fig.8). When using rule R2, the envelope fluctuation can be substantially reduced, for any $M_j$. It is easy to show that - due to the uniformly distributed subcarriers - for $k = 0, 1, \cdots, M_j - 1$ and $c \in \{0, 1, \cdots, m_j - 1\}$ we get $S_{j,m_j+k+c} = S_{j,k}^{in}$ if $i = 0$ and equal to zero if $i = 1, 2, \cdots, m_j - 1 (m_j = \frac{N}{m_j})$; therefore, we can conclude that $s_{j,n} = \frac{s_{j,n}^{in}}{m_j} \exp(-j2\pi n c/N)$, which means that $|s_{j,n}| = |s_{j,n}^{in}|/m_j$ (e.g. $|s_{j,n}^{in}| = \text{constant}$ (QPSK constellation) $\Rightarrow |s_{j,n}| = \text{constant}$).

B. Receiver Aspects

When the CP is long enough to cope with the channel time dispersion, the frequency-domain received samples concerning user $j$ (after removal of the CP-related samples and a length-$N$ DFT), at the $q$th branch of the BS receiver, are given by
\[
Y_j^{(q)} = S_j H_j^{(q)} + N_j^{(q)}, \quad (13)
\]
where \( N_k^{(q)} \) corresponds to independent, zero-mean, frequency-domain Gaussian noise terms with variance \( N N_0 \). Of course, this happens for \( L_s \geq L = 10 N N_0 \); if this condition is not met \( L_s < 10 N N_0 \), i.e. a CP duration \( L_s T_s \) below 10\( r = 10 N N_0 T_s \), where \( T_s \) is the duration of each symbol \( s_{j,n}^{(q)} \), then \( Y_j^{(q)} \) will also include a self-interference and multi-user interference terms resulting from the insufficient-CP conditions. For user \( j \) and branch \( q \), we get a length-\( M_j \), frequency-domain, input vector

\[
Y_j^{(in)}(q) = \left[ Y_{j,0}^{(in)}, Y_{j,1}^{(in)}, \cdots, Y_{j,M_j-1}^{(in)} \right]^T
\]

(14)

which is related to the set of \( M_j \) components \( Y_j^{(q)} \) of the length-\( N \) vector \( Y^{(q)} \) concerning user \( j \). Under the use of the appropriate mapping rule, eqn. (13) can be rewritten as

\[
Y_{j,k}^{(in)} = s_{j,k}^{(in)} H_{j,k}^{(in)} + N_{j,k}^{(in)}
\]

(15)

\( k = 0, 1, \cdots, M_j - 1 \). The signal processing operations described above (CP removal, length-\( N \) DFT and user "separation") are performed in the BS receiver, as depicted in Fig. 9.a. This user separation involves the appropriate subcarrier demapping for each of the \( J \) simultaneous users, allowing subsequent, separate FDE procedures for the several users. Fig. 9.b shows the well-known structure of a conventional FDE scheme for a Q-branch receiver [2], [10], in this case concerning user \( j \) (\( \circ \) denotes component-by-component vector multiplication).

### C. Achievable RAW BER Performances

A detailed, insightful evaluation of the iterative receiver techniques described in sec. II, in the SC-FDMA context considered in this paper, can benefit from a complementary evaluation of the corresponding performance bounds. Some bounds can be obtained through a simple and fast computation, under the channel assumptions of sec. II. This is the case with the MFB concerning "raw" BER performance, i.e., the error rate for direct decisions on the coded bits, without decoding. Under the ideal "ISI-free" assumption of a single user \( j \) and a QPSK transmission with a single symbol per block, we get an input vector \( s_{j,n}^{(in)} \) with

\[
\frac{E_b}{N_0} = \frac{M_j \sigma^2}{2 \eta N N_0} \sum_{n=0}^{M_j-1} P_n,
\]

(22)

where \( \eta = \frac{N}{N_0 + \Delta} \). Therefore,

\[
P \left( \frac{E_b}{\sqrt{MSNR}} \right) = \frac{2\eta}{M_j \sum_n P_n} \frac{E_b}{\sqrt{N_0}} \sum_{n=0}^{M_j-1} P_n \sum_{q=1}^{N} \left| H_{j,k}^{(in)} \right|^2.
\]

(23)
It should be noted that a similar analytic performance evaluation approach can be adopted for the conventional FDE (MMSE) receiver technique, with

$$C_{j,k} = \frac{1}{\alpha_j + \sum_{q=1}^{Q} |h_{j,k}^{(q)}|^{2}}$$  \hspace{1cm} (24)

where

$$\alpha_j = \frac{\sum_{n=0}^{\gamma_j} P_j}{2N}$$

in the SC-FDMA context.

In this case, we can express the FDE output samples as

$$\tilde{s}_{j,n} = \gamma_j s_{j,n} + \text{uncorrelated noise-like term (Gaussian noise + residual ISI)}$$

where, according to (19) and (24),

$$\gamma_j = \frac{1}{M_j} \sum_{k=0}^{M_j-1} \frac{1}{\alpha_j + \sum_{q=1}^{Q} |h_{j,k}^{(q)}|^{2}}$$  \hspace{1cm} (25)

(0 < \gamma_j < 1, and \gamma_j \to 1 when \alpha_j \to 0). At the FDE (MMSE) output, it can be shown that the signal-to-noise ratio in the \(\tilde{s}_{j,n}\) samples is then given by

$$\text{SNR} = \frac{\gamma_j,\text{MMSE}}{1 - \gamma_j,\text{MMSE}} = \frac{1}{\alpha_j} \left( \frac{1}{\sum_{k=0}^{M_j-1} \frac{1}{\alpha_j + \sum_{q=1}^{Q} |h_{j,k}^{(q)}|^{2}}} \right).$$  \hspace{1cm} (26)

Consequently, by assuming that the residual ISI is zero-mean and quasi-Gaussian, we can write

$$P\left( e | \{ H_{j}^{(q)}; q = 1, 2, \ldots, Q \} \right) \approx Q \left( \sqrt{\text{SNR}} \right)$$  \hspace{1cm} (27)

with SNR according to (26), where

$$\alpha_j = \frac{\sum_{n=0}^{\gamma_j} P_j}{2N^{2}}$$

The SC-FDMA receiver performance was evaluated, using the channel model of sec. II, for both rule R1 and rule R2, with \(N = 512\), \(L_s = 128\), and \(M_j = 64\).

The numerical results depicted in Figs. 10 and 11 (for \(\beta = 0\) and \(\beta = 0.5\), respectively) correspond to performances when a two-branch receiver structure (\(Q = 2\)) is adopted. These figures indicate the required \(E_b/N_0\) (\(dB\)) by \(E_b/2\) (the channel bit energy), per receiver branch, so as to ensure a raw BER equal to \(10^{-4}\), for both Rule 1 (64 contiguous subcarriers) and Rule 2 (64 uniformly spaced subcarriers). These MFB and FDE(MMSE) performances have been semi-analytically computed by using (23) and (27) respectively, for each channel realization, and by averaging over the set of channel realizations. In the FDE (MMSE) case, figs. 10 and 11, show that these semi-analytical results are practically identical to simulated results. By comparing figs. 10 and 11, we can predict a performance improvement, in general, when \(\beta\) is decreased (this has been confirmed with other results not shown here, for other values of \(\beta\)). By increasing the delay spread through an increased \(N_S\), improved performance results are also achieved, in general, with the "localized" option (rule R1), but there is a stabilization for \(8 < N_S < 12\). With the "uniform" option (rule R2), performances are very similar for practically all values of \(N_S\) (if \(N_S \leq 12\), except for \(N_S = 8\); these very similar performances are also similar to those concerning rule R1, for \(8 < N_S \leq 12\)).

Figs 10 and 11 also indicate that at BER=\(10^{-4}\), there is a "gap" exceeding 2dB between the achieved "FDE (MMSE) BER Performance" and the ideal "MFB BER Performance" when \(\beta\) is small (say \(0 \leq \beta \leq 0.5\)), for \(1 \leq N_s \leq 11\); from these and other results with \(\beta \geq 0.5\), we can conclude that reducing \(\beta\) leads to an increased performance gap [for \(\beta \gg 1\) (i.e., under quasi-flat Rayleigh fading conditions) practically no gap would be noticed, but at the cost of a very poor performance]. Suitable iterative FDE receiver techniques will be presented in Sec. V and evaluated in Sec. VI, in the SC-FDMA context, so as to reduce significantly the above-mentioned performance gap and also the corresponding gap for coded transmission.

The results shown above on FDE (MMSE) and MFB performances can be easily understood by having in mind the correlation of fading effects throughout the transmission band. One can resort to the "normalized frequency-domain channel autocorrelation function" given by

$$E[H_{j}H_{j-k}^{*}] = \sum_{n=0}^{N-1} E[h_{n}]^2 \exp(-j2\pi nk/N)$$

$$E[|H_{j}|^2] = \sum_{n=0}^{N-1} E[|h_{n}|^2]$$

$$R_H(k) = \frac{\sum_{n=0}^{N-1} E[h_{n}]^2 \exp(-j2\pi nk/N)}{\sum_{j} P_j}$$  \hspace{1cm} (28)
For $N_S = 1$, $|R_H(k)| = \frac{|1 - \exp(-11\alpha(k))|}{1 + \exp(-11\alpha(k))}$, with $\alpha(k) = \beta + j\frac{\pi}{2}k(k = 0, 1, \ldots, 511)$, as shown in Fig. 12(a). Fig. 12(b) shows $|R_H(k)|$ for $N_S = 8$.

The best performance results on MFB, in Figs. 10 and 11 are concerned to the smallest correlation of fading effects within the set of $M_j = 64$ allocated subchannels. High correlation levels (for example, when $N_S = 1$ and rule R1 is adopted) lead to poor performances. The small "accident" for $N_S = 8$, in the performance curves regarding rule R2 (which means, for example, the use of subchannels $0, 8, 16, \cdots, 504$), can be easily explained by resorting to Fig. 12(b) (peaks at $k = 64, 128, \cdots, 448$).

\[ G^{(i)}_{j,k} = E \left[ \hat{s}^{(i)}_{j,k} \right] - C^{(i)}_{j,k} \sum_{q=1}^{Q} \left[ H^{(q),(i)}_{j,k} \right]^2, \]

having in mind that $Y^{(q),(i)}_{j,k} = S^{(n)}_{j,k} H^{(q),(i)}_{j,k}$ + Gaussian Noise. Therefore,

\[ G^{(i)}_{j,k} = \left[ \gamma^{(i)}_{j,k} - C^{(i)}_{j,k} \sum_{q=1}^{Q} \left[ H^{(q),(i)}_{j,k} \right]^2 \right] S^{(n)}_{j,k}, \]

when using $S^{(n)}_{j,k}$,

\[ \mathbf{F}_{M_j} \left[ \begin{array}{c} S^{(n)}_{j,0} \\ \vdots \\ S^{(n)}_{j,M_j-1} \end{array} \right] \]

denote the frequency-domain version of the currently available estimate of the data symbol vector, derived from the preceding iteration. This confirms the results of [7], [8] on the additive vector used for soft cancelation of ISI. As to the $C^{(i)}_{j,k}$ coefficients, the results of [7], [8] can also be extended to our SC-FDMA context:

\[ C^{(i)}_{j,k} = \alpha_j + K_F^{(i)} \left( 1 - \beta_j^{(i)} \right)^2 \sum_{q=1}^{Q} \left[ H^{(q),(i)}_{j,k} \right]^2, \]

where $\alpha_j = \frac{N\sigma}{M_j \sigma_i^2}$ and $K_F^{(i)}$ is a normalization factor which leads to $\gamma^{(i)}_{j,k} = 1$. As to $\beta_j^{(i)}$ (and also to $S^{(n)}_{j,k}$), required to compute $C^{(i)}_{j,k}$, we need to use the Log-Likelihood Ratio (LLR) values concerning the coded bits, as provided by the preceding iteration [7], [8]. Since there is no "decoding aid" within the iterative FDE technique of Fig. 13, the required LLR values are simply those previously obtained, through soft bit demapping, from the equalized samples $\hat{s}^{(n),(i-1)}_{j,k}$. For $i = 1$, with no a priori information on the coded bits, the LLR values are equal to zero, which leads to $G^{(i)}_{j,k} = 0$ and $\beta_j^{(i)} = 0$ in (31); this means that a conventional FDE under the Minimum Mean Squared Error (MMSE) criterion is actually carried out in the first iteration. After several iterations and/or for high SNR, typically $\beta_j^{(i)} \approx 1$ and $S^{(n)}_{j,k} \approx \hat{s}^{(n),(i-1)}_{j,k}$, leading to approximately constant $C^{(i)}_{j,k}$ coefficients (quasi-matched filtering conditions) and a nearly perfect soft cancelation of residual ISI after that quasi-matched filtering. Decisions on the coded bits are directly made at the soft demapper output, based on the signs of the corresponding LLR values, after a certain number of FDE iterations. BER performances very close to the MFB derived in sec. IV are then achievable, even for 3 iterations only. It should be noted that we can confirm the MFB performance semi-analytically obtained (as explained in sec. IV) through a Monte Carlo simulation for the receiver technique of Fig. 13, by assuming ideal channel estimation and one iteration only, with $\beta_j^{(i)} \approx 1$ in the computation of $C^{(i)}_{j,k}$ (eq. (31)) and ideal $G^{(i)}_{j,k}$ coefficients, i.e., $\tilde{S}^{(n),(i-1)}_{j,k} = S^{(n)}_{j,k}$ in (30). This iterative FDE technique can then be regarded as an extension of the simplest technique proposed in [7], [8], which does not employ any decoding aid in the several FDE iterations and is closely related to an earlier proposal [11].

V. ITERATIVE RECEIVER TECHNIQUES

A. SIMPLIFIED ITERATIVE FDE TECHNIQUE

In the following, we consider a low-complexity, iterative FDE technique which can be regarded as a simplified version of the Turbo FDE technique presented in [7], [8], appropriately extended from a single-user context to the SC-FDMA context of this paper. The main simplification consists of not using a decoding aid within the iterative FDE procedures; the additional simplification consists of not trying to compensate for possible insufficient-CP conditions. With regards to user $j$, this FDE scheme, depicted in Fig. 13, should be compared with the conventional FDE scheme of Fig. 9.b. Instead of a fixed vector $C_j$, for an element-by-element multiplication, after maximal-ratio combining, we have a vector $C^{(i)}_{j,k}$ which is upgraded from iteration to iteration ($i$ is the iteration number). Additionally, there is a soft cancelation of residual ISI after that multiplication, through the additive vector $G^{(i)}_{j,k}$, also upgraded from iteration to iteration. Using

\[ C^{(i)}_{j,k} = \gamma^{(i)}_{j,k} S^{(n)}_{j,k} + \text{Gaussian Noise} + \text{Resid. ISI} \]

with $\gamma^{(i)}_{j,k}$ in accordance with (19), when replacing the fixed $C^{(i)}_{j,k}$ coefficients by the iteratively adjusted $C^{(i)}_{j,k}$ coefficients. The iteratively adjusted vector $G^{(i)}_{j,k}$, for soft cancelation of estimated residual ISI at iteration $i$, obviously has components given by

\[ G^{(i)}_{j,k} = E \left[ \hat{s}^{(i)}_{j,k} \right] - C^{(i)}_{j,k} \sum_{q=1}^{Q} \left[ H^{(q),(i)}_{j,k} \right]^2. \]
Fig. 13. Simplified iterative FDE receiver technique.

B. Turbo FDE Technique

By considering a true turbo FDE technique, as depicted in Fig. 14, we can take advantage of the "full information" on the coded bits which are provided by the Soft-In Soft-Out (SISO) decoder. This full information includes an "extrinsic information" contribution (to be added to the LLR’s of the coded bits at the decoder input), thereby helping the FDE process through the resulting upgraded values [7], [8].

![Fig. 14. Turbo FDE receiver technique.](image)

VI. PERFORMANCE EVALUATION

A. Raw BER Performance for the Simplified Iterative Receiver Technique

The simplified iterative receiver technique of Fig. 13, which does not use a decoding aid within the iterative FDE procedures, was evaluated for both mapping rules using the channel model described in sec. II. We assumed \( N = 512 \), \( L_S = 128 \), and a range of channels, each of them characterized by a specific value of the parameters \( N_S \) and \( \beta \) (the time dispersion effects of the channel become higher when \( N_S \) increased and/or \( \beta \) is decreased).

The numerical results depicted in Figs. 15 and 16 correspond to performances regarding user \( j \), when \( M_j = 64 \), and a two-branch receiver structure (\( Q = 2 \)) is adopted. These figures indicate the required \( E_b/N_0 \) \((dB)\) (with \( E_b \) denoting the channel bit energy), per receiver branch, so as to ensure a raw BER equal to \( 10^{-4} \), for both rule R1 (64 "localized" subcarriers) and rule R2 (64 "uniformly" spaced subcarriers), in the following cases: performance under decisions just after 1, 2 or 3 iterations; MFB performances. These MFB performances have been semi-analytically computed by using (23), for each channel realization, and by averaging over the set of channel realizations. These results could be confirmed strictly by simulation, when assuming an FDE receiver structure, as in Fig. 13, with an iteration only, under a matched-filter assumption (\( C_{j,k} = \text{Constant} \)) and a perfect cancellation of residual ISI \( \left( S_{j,k}^{(m)} \right. \) instead of \( S_{j,k}^{(m)} \) in (30)).

The results of Figs. 15 shows that significantly improved performances are obtained by replacing a conventional FDE (MMSE) technique (first iteration) by the simplified iterative FDE technique of sec. V-A; they also show, in all cases, that three iterations are enough to get a good approximation to the MFB performance.

By increasing the delay spread through an increased \( N_S \), improved performance results are also achieved, in general, with the "localized" option (rule R1), but there is a stabilization for \( 8 < N_S < 12 \). With the "uniform" option (rule R2), performances are very similar for practically all values of \( N_S \) (if \( N_S \leq 12 \), except for \( N_S = 8 \); these very similar performances are also similar to those concerning rule R1, for \( 8 < N_S \leq 12 \). For \( N_S > 12 \), there is some performance degradation, as expected, since the length of the channel impulse response turns out to exceed the CP length, leading toICI between neighbouring subchannels (for rule R2, however, the performance degradation is very small, since all used subchannels are not adjacent). Exceptions occur when \( N_S \) is a multiple of 8, as explained through "correlation analysis" at the end of section IV.

B. BER Performance for the Turbo FDE Technique

The Turbo FDE receiver technique of Fig. 14 was used here, for the transmission assumptions already assumed in Sec. VI-A, a 16-state, rate-1/2 convolutional code, characterized by \( G(D) = \left[ \frac{1+D^2+D^3+D^4}{1+D+D^2} \right] \), and a low-complexity Max-Log-MAP SISO decoding algorithm. The resulting performances are depicted in Figs. 16, in this case regarding a BER equal to \( 10^{-5} \). The conclusions are very similar to those of Sec. VI-A, but at much improved performance levels in all cases, as expected, thanks to the impact of the SISO decoding.

For obtaining the simulated MFB performances, we used an FDE receiver structure as in Fig. 14, in a similar way as for obtaining the MFB performances reported in Sec. VI-A with the receiver structure of Fig. 13.

![Fig. 15. Raw BER performances when using the simplified iterative FDE technique, for \( \beta = 0 \): solid line for rule R1 and dashed line for rule R2.](image)

C. Performances under Insufficient-CP Conditions in a Multiuser Context and Complementary Comparisons

When the CIR length exceeds the CP length there is some performance degradation, even in a single-user context, as
shown in the examples of Sec. VI.a and VI.b, for \( N > 12 \). In the following, we will show and discuss some performance results regarding insufficient-CP conditions in a multi-user context, which obviously leads to increased ICI levels, and therefore, to an increased performance degradation, for both rule R1 (a) and rule R2 (b); as to the channel, we assume \( \beta = 0 \) and either \( N_s = 18 \) or \( N_s = 22 \). Figures 17 to 21 show performance results for both the single-user context (dashed line) and a multi-user context (solid line), for selected "worst cases", regarding either rule R1 or rule R2, when two users are assumed (with \( M_2 = M_1 = 64 \) and the same received power level); the single-user MFB performance (dotted line) is also included. Fig. 17 is concerned to the raw BER performances when using the simplified iterative FDE technique of Fig. 13, for \( N_s = 18 \); Figs. 18 and 19 show the BER performances when the turbo FDE technique of Fig. 14 is employed, for \( N_s = 18 \) and \( N_s = 22 \), respectively.

The results of Fig. 17 confirm that the insufficient-CP conditions lead to a significant performance degradation at low values of the raw BER, even in a single-user context, when using rule R1; however, this is not the case when using rule R2, where close approximations to the MFB performance are still achievable. In the multi-user context, performance degradations are slightly increased, in all cases. It should be noted that, at high values of the raw BER (say, at BER~10^{-5}), practically there is no difference between performances under rule R1 and rule R2 (the ICI levels are then negligible when compared with the Gaussian noise levels).

The results of Figs. 18 (\( N_s = 18 \)) and 19 (\( N_s = 22 \)) confirm that the insufficient-CP conditions lead to small, very similar, performance degradations, at BER greater than 10^{-6}, with rules R1 and R2 (a bit higher in the multi-user context).

Figs. 20 and 21 have been devised to allow a direct comparison of BER performances when using either the turbo FDE receiver technique of Fig. 14 (solid line) or the "Simplified Iterative FDE" technique of Fig. 13, followed by "Viterbi Decoding" (dashed line). Two users have been assumed, with \( M_1 = M_2 = 64 \) and the same received power level, with either \( N_s = 18 \) (Fig. 20) or \( N_s = 22 \) (Fig. 21). These figures show that replacing the turbo FDE technique of Fig. 14 by the simplified iterative FDE technique of Fig. 13, plus the complementary soft-decision decoding, leads to some
for a selected class of multipath radio channels, providing resulting receiver performances have been evaluated in detail, for SC-FDMA uplink block transmission were considered, the raw BER performances, already reported, at BER $\approx 10^{-2}$ followed by Viterbi decoding, the required Eb/No value at the simplified iterative FDE technique (without decoding aid) different performance curves of Fig. 17: in fact, when using BER performance at BER $\approx 10^{-2}$, and for both rule R1 and rule R2. It is not surprising that both rules can lead to practically the same BER performance at BER $= 10^{-5}$, in spite of the quite different performance curves of Fig. 17: in fact, when using the simplified iterative FDE technique (without decoding aid) followed by Viterbi decoding, the required Eb/No value at BER $= 10^{-5}$ is a value which corresponds to the very similar raw BER performances, already reported, at BER $\approx 10^{-2}$.

VII. CONCLUSIONS

In this paper, where two alternative subcarrier mapping rules for SC-FDMA uplink block transmission were considered, the resulting receiver performances have been evaluated in detail, for a selected class of multipath radio channels, providing a range of time dispersion levels. Our detailed performance evaluations, involved the consideration of two iterative FDE receiver techniques, with different complexity levels. These detailed evaluations also involved some MFB performances derived by analytical means. The several MFB performances have been very useful for discussing the numerical performance results; a "normalized frequency-domain channel auto-correlation function", easily derived from the channel model assumptions, was also useful for this purpose. This thesis also studied the performance degradation due to a CIR longer than the CP: such degradation is related to an "ICI effect" which is inherent to the insufficient-CP conditions.

The effectiveness of both iterative FDE receiver techniques was emphasized: in fact, after a few iterations, they can practically remove the "gap" between the conventional FDE(MMSE) performance - achieved in the first iteration - and the MFB performance, in all cases where a "full CP" was adopted.

The performance advantages under rule R2 were emphasized for a low or moderate channel time dispersion, and both specific iterative receiver techniques; for a higher time dispersion, receiver performances where shown to become very similar with both rules, at the BER values of practical interest. Having in mind that rule R2 has a power efficiency advantage regarding transmitter implementation (due to the reduced envelope fluctuations), we can conclude that this rule provides a clear overall power efficiency advantage, regarding both transmitter and receiver issues, for practically the entire range of channel time dispersion levels.

REFERENCES