Modeling and analysis of damaged rectangular steel plates subjected to a uniaxial compressive stress

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ABSTRACT: The objective of the present study is to investigate the effects of corrosion on the ultimate strength of unstiffened rectangular steel plates subjected to uniaxial compressive load. Two new and distinct corrosion models are proposed and investigated. A total of 3575 corroded plate surface geometries are generated by Monte Carlo simulation for different degrees of degradation, location and ages and nonlinear finite element analyses are carried out, using a commercial finite element code. Based on a regression analysis, empirical formulae to predict strength reduction due to corrosion have been derived demonstrating a good accuracy. The present work also evaluates the reliability of the nonlinear time variant randomly non-uniform corroded rectangular plate subjected to compressive load. Based on the derived relationship of the ultimate strength assessment of rectangular plate with surface geometries deteriorated by different degrees of corrosion, location and ages FORM techniques are applied to assess the structural reliability accounting for the global ship hull deterioration. The structural system, composed by two different correlated events, is evaluated as a series system.

1 INTRODUCTION

Corrosion has always been one of the major problems in marine industry. Many catastrophic situations have been caused by corrosion damage, even when all the design requirements are satisfied (Nakai et al., 2004, 2006).

Some studies from the last decades considered simplified models of general corrosion wastage, linearly increasing with time (Hart et al. 1986, Guedes Soares 1988a and Shi 1993). More recent studies demonstrated the nonlinear time dependent corrosion models are more appropriate (Guedes Soares et al. 2009) and that corroded surfaces could be modelled by random fields (Teixeira and Soares, 2008, Silva et al., 2011b, a).

Many models and studies have been carried out to predict the behaviour of structural elements affected by corrosion degradation in a deterministic way, focusing their attention on pitting corrosion as one of the most hazardous forms. Paik et. al. (2003, 2004, 2005) investigated the ultimate strength of plate elements with pit corrosion wastage under axial loads and in-plane shear loads. They derived a closed-form solution to estimate the ultimate strength of pitted plates by idealizing corrosion pits as a cylindrical shape and by varying the degree of pits and intensity in a systematic way.

Duo et al. (2007) idealized corrosion pits as cylindrical cones and investigated the influence of localized corrosion on the ultimate strength. Although over 256 nonlinear finite element analyses were conducted in a systematic way it was assumed that corrosion was constrained to a rectangular area on the plate. Jiang and Guedes Soares, (2008, 2009, 2010) and Saad-Elddeen and Guedes Soares, (2009) focused their attention on the influence of scattered pitted plates on the collapse strength by using the mathematical model proposed by Daidola et al., (1997), who developed a method to estimate the residual thickness of pitted plates.

The ultimate strength assessment of corroded steel plates conducted in this study proceeds from a Monte Carlo simulation of corroded surfaces based on a quasi-random distribution of plate thickness, by applying the approach of corrosion wastage developed by Guedes Soares and Garbatov (2009), and also based on the idealization of corrosion as a superposition of random indentations. By applying nodal thicknesses on the finite element model and using non-linear finite element analysis, the ultimate strength of a steel plate has been evaluated.

Structural analysis and design have traditionally been based on deterministic methods. However, uncertainties in the loads, strengths and in the modeling of the structures require that probabilistic methods to be used. A structure is usually required to have a satisfactory performance in the expected lifetime, i.e. it is required that it does not collapse or becomes unsafe and that it fulfils certain functional requirements.

The reliability estimated as a measure of the safety of a structure can be used in a decision making process. A lower level of the reliability can be used as a constraint in an optimal design and maintenance problems. The lower level of the reliability can be obtained by analyzing similar structures designed based on current practice or it can be determined as the reliability level giving the largest utility (benefits – costs and risk) when solving a decision problem where all possible costs and benefits in the expected lifetime of the structural components are taken into account.

The main steps in a reliability analysis include: select a target reliability level; identify the significant failure modes of the structural component; decompose the failure modes in systems of parallel systems of single components (only needed if the failure modes consist of more than one component); formulate failure functions (limit state functions) corresponding to each component in the failure modes; identify the stochastic variables and the deterministic parameters in the failure functions and further specify the distribution types and statistical parameters for the stochastic variables and the dependencies between them; estimate the reliability of each failure mode; compare with the target reliability; evaluate the reliability result by performing sensitivity analyses.

The failure modes (limit states) are divided in: ultimate limit states - correspond to the maximum load carrying capacity,
which can be related to e.g. formation of a mechanism in the structure, excessive plasticity, rupture due to fatigue and buckling; conditional limit states - correspond to the load-carrying capacity if a local part of the structure has failed. The conditional limit states can be related to e.g. formation of a mechanism in the structure, exceeding of the material strength or buckling; serviceability limit states - related to normal use of the structure, e.g. excessive deflections, local damage and excessive vibrations.

The fundamental quantities that characterize the behavior of a structure are the basic variables, denoted as \(x = (X_1, X_2, ..., X_n)\), where \(n\) is the number of basic stochastic variables. Typical examples of basic variables are loads, strengths, dimensions and materials.

The reliability analysis presented here is using FORM/SORM techniques to identify a set of basic random variables, which influence the failure mode or the limit-state under consideration. The limit-state function is formulated in terms of the \(n\) basic variables given as:

\[
g(X) = g(X_1, X_2, ..., X_n)
\]

(1)

This function defines a failure surface when equals to 0. It defines an \((n-1)\) dimensional surface in the space of \(n\) basic variables. This surface divides the basic variable space into a safe region, where \(g(x) > 0\) and an unsafe region where \(g(x) < 0\). The failure probability of a structural component with respect to a single failure mode can formally be written as:

\[
P_f = P[g(x) \leq 0] = \int_{g(x)=0} f_x(x) dx
\]

(2)

where \(f_x(x)\) is the joint probability density function of the \(n\) basic variables and \(P_f\) denotes probability of failure. The \(n\)-dimensional integral is defined over the failure region.

In practical applications, the reliability cannot be evaluated in the exact manner as given by Eqn(2). This is because enough statistical data is usually not available to develop the \(n\)-dimensional joint density function of the basic variables. Secondly, even when the joint density function is available, analytical or numerical integration is possible only for a few simple cases. The FORM/SORM methods provide a way of evaluating the reliability efficiently with reasonably good accuracy, which is adequate for practical applications (Hasofer and Lind (1974), Rackwitz and Fiessler (1978), Ditlevsen (1979a) and Hohenbichler and Rackwitz (1986)).

The time variant formulation of ship reliability results from modeling the problem with stochastic processes that represent the random nature of the load and strength parameters (Guedes Soares, C. and Garbatov, Y., 1996).

Engineering systems such as ship structures are designed to ensure an economical operation throughout the anticipated service life in compliance with given requirements and acceptance criteria. Deterioration processes such as fatigue crack growth and corrosion are always present to some degree and depending on the adapted design philosophy in terms of degradation allowance and protective measures the deterioration processes may reduce the performance of the system beyond what is acceptable.

In order to ensure that the given acceptance criteria are fulfilled throughout the service life of the engineering systems it is necessary to control the development of deterioration and, if required, to install corrective maintenance measures resulting in inspection in the most relevant and effective means of deterioration control.

The initial formulations of the time variant approach to ship structural reliability were developed in connection with the fatigue problem, in particular to be able to deal with the time degradation of reliability by Guedes Soares and Garbatov (1996a) with the improvements made by maintenance actions by Guedes Soares and Garbatov (1996b) and for corrosion deterioration by Guedes Soares and Garbatov (1998).

The objective of the present study is to investigate the effects of corrosion on the ultimate strength of unstiffened rectangular steel plates subjected to uniaxial compressive load. It also deals with the reliability assessment of an ageing steel plate, subjected to compressive load based on FORM approach. The structural reliability system is modeled as a series system and the correlation between the failure modes are accounted for.

2 CORROSION MODELING

2.1 Random, Non-Uniform and Time Dependent Corrosion Wastage

Three fundamental approaches can be applied for corrosion deterioration modelling. The conventional one is considering that corrosion grows linearly, which may leads to a very big overestimation of the corrosion deterioration. The second one is based on the results of experiments in specific environment conditions, which suggest laws of corrosion deterioration as a function of specific parameters. The corrosion model can be developed by considering all those laws derived from experiments accounting for in the environmental specific conditions. This approach involves one difficulty in generalizing results from laboratory tests to full-scale conditions. The other difficulty is related to the general lack of data on the environmental conditions, which affect corrosion in full-scale. The third approach, which is the one that is adopted here, is to consider that a model should provide the trend that is derived from for the dominating mechanism and then it should be fitted to the field data. Guedes Soares and Garbatov (1999) developed a model for the non-linear time-dependent function of general corrosion wastage. This time-dependent model separates corrosion degradation into three phases. In the first one there is no corrosion because the protection of the metal surface works properly. The second phase is initiated when the corrosion protection is damaged and corresponds really to the start of corrosion, which decreases the thickness of the plate. The third phase corresponds to a stop in the corrosion process and the corrosion rate becomes zero.

The model of Guedes Soares and Garbatov (1999) is based on the solution of a differential equation of the corrosion wastage, which leads to:

\[
d(t) = \begin{cases} 0, & t \leq \tau_e \\ d_e \left(1 - \exp\left[-\frac{t-\tau_e}{\tau_p}\right]\right), & t \geq \tau_e \end{cases}
\]

(3)

The parameters of the corrosion depth as a function of time were determined under the assumption that it is approximated by the exponential function given in Eqn (3) as is given for ballast tanks of tanker deck by Garbatov et al., (2007). The long-term corrosion wastage for deck plates of ballast tanks has been defined as \(d_e = 1.85\) mm. The time without corrosion is \(\tau_e = 10.54\) years and the transition period \(\tau_p = 17.54\) years.

The standard deviation, defined as a function of time, has been fit as \(StDev(t) = a \exp(-t/b)\), where \(a\) and \(b\) have been evaluated in 0.384 and 0.710 respectively.

Based on the analysis performed by Garbatov and Guedes Soares, (2008) the corrosion wastage depth is fit by the Lognormal distribution.
The mean value and the variance of the log-normal distribution for the corrosion wastage of deck plates of ballast tanks are -0.544 and 0.919. The model just presented is used for deteriorated plate surface modelling.

The corroded plate surface is modelled as random plate thickness that results in the random vertical position of the coordinates of corroded surface for equally spaced reference points positioned along the x and y direction of the plate. These reference points are defined by a Monte Carlo simulation as being the nodes of the finite element model on the plate.

The plate thickness, \( Z_{\text{corr}} \), at any reference point with coordinates \( x, y \) for the corroded plate surface, is defined by the random thickness of the intact plate surface, \( Z_{\text{int}} \), affected by the random vertical reduction resulting from the corrosion depth, \( Z_{\text{corr depth}} \)

\[
Z_{\text{corr}} = Z_{\text{int}} - Z_{\text{corr depth}}
\]

where \( Z \) are the matrixes of the corroded and intact surface corrosion depth.

This convention is used to derive the formulation that describes the vertical position of the surface of the non-linear corroded plate by the Monte Carlo (Binder and Heermann, 2010) simulation resulting in randomly distributed plate thicknesses for randomly defined reference nodes at a specific year based on Eqn (4) and applying the corrosion degradation levels as defined by Eqn (3) and (4).

The vertical random coordinates (corrosion depth) of the corroded and intact plate surfaces (depth) and corrosion depths are modelled by a log-normal distribution. The intact plate surface coordinates and corresponding corrosion depths are considered as not correlated.

The modelling of the corroded plate surface in the finite element analysis is made according to the previous procedure, using the simulated random thickness at the reference nodes to adjust the plate thickness at the nodes according to Eqn(4). Corrosion plate reduction is applied symmetrically on both sides of plate symmetrically.

The mean value and the standard deviation of the corrosion depth are considered as the ones related to the deck plate of ballast tanks of tanker ship. The mean value and the standard deviation of the intact plate thickness are considered as 10mm and 1 mm respectively.

2.2 Random and Non-Uniform Dented Corrosion Wastage

Pitting corrosion can appear in a wide variety of shapes and locations depending on the environmental conditions as well as the location of the structural component. The objective here is to simulate a wide variety of pitting shapes and locations towards a more realistic modeling of corroded surfaces. For instance, the forms pitting can be described as a superposition of elliptical paraboloidal indentations.

A parallelepipedic shape with a single dent may be mathematically described by modeling its upper surface, \( S_{\text{sup}} \), in the global Cartesian space \( x, y, z \) as

\[
S_{\text{sup}}(x,y) = \begin{cases} h_0 & \theta \geq h_0 \\ \theta & \theta < h_0 \end{cases}
\]

where \( \theta \) is a function describing the indentation shape, and \( h_0 \) is the solid’s height, in \( z \)-direction, or thickness.

Figure 1 - Parallelepipedic solid with two indentations in the global Cartesian space

Some authors e.g. Paik et. al. (2003, 2004, 2005), Saad-Elddeen and Guedes Soares (2009) and also Jiang and Guedes Soares (2009) have been testing this approach by using conical, pyramidal and cylindrical indentations in order to simulate and test damaged structures. It can be argued that the elliptic paraboloids may be geometries closer to realistic cases, but it’s undoubtedly a more versatile mathematical entity able to better describe the cross sectional forms of pitting corrosion.

An elliptical paraboloidal surface is a quadratic surface, of which the general equation can be expressed as:

\[
\Theta(x,y,x_0,y_0,a_x,a_y) = \left( \frac{x-x_0}{a_x} \right)^2 + \left( \frac{y-y_0}{a_y} \right)^2 + z_0 > 0
\]

where \( a_x, a_y \) are the shape coefficients that govern the aperture and depth; \( x_0, y_0 \) are the planar coordinates of the dent’s centre and; \( z_0 \) is the distance between the solid’s lower surface and the center of the dent’s crater. It is noteworthy that \( z_0 \) must be always greater than zero, negative values of \( z_0 \) lead to complete perforations, which are to be avoided. On the other hand for \( z_0 > h_0 \) there will be no perforation whatsoever.

Eqn (6) holds for a single dent but as mentioned earlier, multiple dents are to be found in the same surface. Therefore, denoting the \( n \)-th indentation by \( \Theta^{(i)} \) and the correspondent individual perforated surface by \( S_{\text{sup}}^{(i)} \) a surface with multiple indentations, \( S \) may be described as the intersection, or superposition, of all the \( n \) individual indented upper surfaces

\[
S = S_{\text{sup}}^{(1)} \cap S_{\text{sup}}^{(2)} \cap \cdots \cap S_{\text{sup}}^{(n)} = \bigcap_{i=1}^{n} S_{\text{sup}}^{(i)}
\]

The corroded plate surfaces are modelled using the mathematical procedure described above (see Eqn.(5) to Eqn(7)), where the variation of thickness is due to random variation of the various parameters of the indentations, \( x_0, y_0, z_0, a_x, a_y \), and also by a random variation of its number, \( n \). Defining the \( xy \) plane as being the plate’s plane, the outcome of Eqn. (7) is then the corroded plate surface thickness. For a plate of a length \( L \) and width, \( b \) the shape and location parameters of the indentations are defined as a random number uniformly distributed lying in the following intervals:
The numbers in former equations are pseudorandom numbers, with double precision. The number of indentations \( n \) is also a uniformly distributed pseudorandom number, but since it represents a quantity it is an integer from 1 to 800.

The degree of degradation, DOD, to measure the plate’s deterioration, which also stands for the degree of dents, is defined by:

\[
DOD\% = \frac{V_0 - V_r}{V_0} \times 100\%
\]

where \( V_0 \) is the intact plate volume, \( L \times b \times h_0 \), and \( V_r \) denotes the corroded volume, defined as:

\[
V_r = \int_0^b \int_0^h S(x, y) \, dx \, dy
\]

The material loss caused by corrosion degradation can also be assessed by the concept of equivalent thickness, defined as:

\[
h_{eq} = \frac{V_r - V_o}{h_0 (1 - DOD)}
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The scope of the Monte Carlo simulation of the corroded plate surfaces is limited by the usability of the plate, i.e., it is not worth considering damaged specimens with high levels of degradation. Hence, only five classes of degradations are considered here, \([0,5\%], [5,10\%], [10,15\%], [15,20\%] \) and \([20,25\%] \), regarding that the last of these classes are already out of the range of usability as defined by Classification Societies (e.g. DNV (2001)).

The model can’t constrain the location of the indentations. This means that an indentation can re-perforate a region already indented. Thus, the number of dents, \( n \), can only be presented as the maximum number of dents.

3 ULTIMATE STRENGTH ASSESSMENT

3.1 Plates without corrosion

The plate studied herein is a rectangular panel defined with \( z \) being the perpendicular axis to the plate’s plane. \( L \) is the plate length in \( y \) direction, \( b \) is the width along the \( x \)-axis and \( h_0 \) is the intact thickness. It has an elasticity modulus, \( E \) of 205.8 GPa and a Poisson coefficient \( \nu = 0.3 \).

The support conditions of the plate are as follows: there is no displacement along \( z \)-axis in all the edges; the rotation along an axis parallel to \( x \)-axis in the edges \( y=0 \) and \( y=L \) is constrained; there is no displacement along \( y \)-axis in the edge \( y=0 \) and; the point \((x=b/2, y=0)\) is clamped to ensure symmetry. These boundary conditions are a compromise between the simple supported case and the applicability to the finite element model. The clamped point ensures a fixed reference point for an accurate extraction of the displacement, while the constrained rotation on the resistant edges ensures that the applied forces are always perpendicular to the resistant edges.

The initial imperfections, which simulate the presence of manufacture and welding defects, are considered in this study as proposed by Smith et al. (1988):

\[
w(x, y) = w_0 \sin \left( \frac{x}{b} \pi \right) \sin \left( \frac{y}{L} \pi \right)
\]

\[
w_0 = 0.1 h_0 \beta_{\rho,0}^2
\]

where, \( x, z, y \) are the plate’s coordinates system, \( w_0 \) is the maximum out-of-plane deflection, \( \beta_{\rho,0} \) denotes the intact plate slenderness as proposed by Faulkner (1975) and \( E \) and \( \sigma_{yp} \) are Young modulus and yield stress respectively.

The finite element geometry is modelled using around 2520 rectangular elements and 2627 nodes leading to 15762 degrees of freedom. The defined mesh size proves a good quality of results and it is not too dense to avoid endless calculations.

The plate is modelled by nonlinear shell elements SHELL181, with four nodes, each having six-dof-grees of freedom. This element permits to use nodal properties for introducing thickness on every node and still accounts the non-linearity associated with the change of the nodal thickness. Another advantage of this element is the ability to set up non-linear and/or multi-linear material properties (ANSYS, 2009).

The stress-strain curves of the various plates thicknesses used for FE analysis are shown in Figure 2.

The axial load is applied on the edge \( y=L \) and the average stresses are calculated based on the reaction forces in the edge \( y=0 \).

\[
ASR = \frac{\sum R_{y,i}}{A_0 \sigma_{yp}}
\]

where \( ASR \) is the average stress ratio, \( R_{y,i} \) is the reaction forces in \( y \) direction, at the \( i^{th} \) node, which has the coordinates: \((x_0, 0, 0)\), \( k \) is the number of nodes at \( y=0 \), \( A_0 = h_0 b \) is the sectional area of plate at \( y=0 \) and \( \sigma_{yp} \) is the yield stress point of the material.

\[
\frac{\mu}{\varepsilon_{yp}} = \frac{U_{y,p}}{L \varepsilon_{yp}}
\]

where \( \varepsilon \) is plate strain, \( \varepsilon_{yp}=0.001714 \) is the yield strain of material, \( U_{y,p} \) is the displacement in \( y \) direction at point \((b/2, L, 0)\) and \( L \) is the plate length. The non-dimensional collapse strength of the studied plate is defined as \( \sigma_{n,0}/\sigma_{yp}=0.6972 \).

![Figure 2 Stress-strain response of plates with uniform thickness](image-url)

The nodes lying on the edge \( y=0 \) of the plate are kept constant throughout this study, forming an artificial grip where the stresses are measured. This will ensure that even if thickness is changed in any other node of the plate, the edge \( y=0 \) is still referent to all possible distributions of thickness.

By assuming that the volume of plates may be evaluated as \( V_0 = Lbh \), where \( V_0 \) is the volume of a plate with thickness
h ≤ h_0. Hence, the relation between the collapse strength of plates as a function of thickness can be expressed as:

$$\frac{\sigma_u}{\sigma_{up}}(h) = -1.0541 \left( \frac{h_0 - h}{h_0} \right) + \frac{\sigma_{u,0}}{\sigma_{up}} \tag{17}$$

where the coefficient of acceptance, R^2, of this linear regression analysis is 0.999. In fact, for the case where there is no artificial grip, the absolute value of the slope of the linear regression will be 0.4656, meaning an increase of the collapse strength of thinner plates when compared with the present case. Not taking into consideration for the artificial grip will lead to erroneous comparisons between the ultimate strength of thinner and corroded plates.

### 3.2 Plates with random and non uniform corrosion wastage

The application of the corrosion model of Section 4.1 to the plate in discussion resulted into 570 successful non-linear finite element analyses.

The results of ultimate strength with respect to time reveal that a nonlinear curve is the best fitted one to the collected data. Following this tendency an exponential equation is used to define to the ultimate strength ratio as:

$$E \left[ \frac{\sigma_u}{\sigma_{up}}(t) \right] = \left\{ \begin{array}{ll} \frac{\sigma_{u,0}}{\sigma_{up}} \exp \left( \frac{t - \tau_c}{\tau_{U}} \right)^{-n} \exp, & t \leq \tau_c \\
\sigma_{u,0} \exp \left( \frac{t - \tau_c}{\tau_{U}} \right)^{-n} \sigma_{u,0} \exp, & t \geq \tau_c \end{array} \right. \tag{18}$$

where \(t\) is time in years, \(\tau_c^* = 10.54\) years is the coating life, \(\sigma_{u,0}/\sigma_{up}\) is the ultimate strength ratio when \(t = \tau_c\), \(\tau_{U}\) is the transition time to be adjusted and it has time unit and \(n_{U}\) is a non-dimensional parameter, which represent the time decay capacity of the ultimate strength of the plate.

However, the parameters \(\tau_{U}\) and \(n_{U}\) depend on the plate ultimate strength. For the studied plate, the parameters that best fit Eqn (18) are \(\tau_{U} = 49.92\) years and \(n_{U} = 1.42\).

The R^2 value has been evaluated to check the accuracy of the regression analysis showing a good agreement between the calculated and predicted values of ultimate strength ratio, \(R^2 = 0.9769\).

The standard deviation, as a function of time, has been defined as

$$StDev \left[ \frac{\sigma_u}{\sigma_{up}}(t) \right] = \left\{ \begin{array}{ll} 0, & t \leq \tau_{c}^* \\
\left[ a_2 \ln(t) + b_2 \right] \exp, & t \geq \tau_{c}^* \end{array} \right. \tag{19}$$

where \(a_2\) and \(b_2\) are defined based on the regression analysis resulting in 0.0156 and -0.0411 respectively; and \(\tau_{c}^*\) is the time from which the standard deviation is not zero. The R^2 value for this regression analysis has been calculated as 0.871.

The ultimate strength was also evaluated with respect to the plate slenderness, by using the equivalent thickness of the plate in Eqn(14), and with respect to the degree of degradation (Silva et al., 2011b).

This Monte Carlo simulation has been used to establish the evolution in time of the degree of degradation, Eqn(9). The mean values of DOD, as a function of time, are fitted as:

$$E[DOD(t)] = \left\{ \begin{array}{ll} 0, & t \leq \tau_c \\
\text{dod}_c \left[ 1 - \exp \left( \frac{t - \tau_c}{\tau_{dod,c}} \right)^{n_{dod,c}} \right], & t \geq \tau_c \end{array} \right. \tag{20}$$

where \(\text{dod}_c = 1\) is the degree of degradation in the infinite, \(\tau_{dod,c} = 119.1\) years is the transition period and \(n_{dod,c} = 1.23\) is a non-dimensional parameter. The coefficient of acceptance, R^2, of this non-linear regression analysis has been evaluated as 0.989. While the standard deviation was modelled as:

$$StDev[DOD(t)] = \left\{ \begin{array}{ll} 0, & t \leq \tau_{c}^* \\
\left[ a_{dod,c} \ln(t) + b_{dod,c} \right], & t \geq \tau_{c}^* \end{array} \right. \tag{21}$$

where the parameters \(a_{dod,c}\) and \(b_{dod,c}\) have been evaluated as 0.0055 and -0.0142 respectively and \(\tau_{c}^*\) is the time from which the standard deviation begins to be significant. The R^2 parameter of this adjustment is 0.925.

### 3.3 Plates with random indentations

The Monte Carlo simulation of the corroded surfaces generated 601 surfaces per class of DOD. From the 3005 analyzed plate surfaces by FEM, only 2811 successfully converged into a final solution. The nonlinearities associated with this type of problem didn’t allow the acceptance of all computations.

The classes of degree of degradation as defined for the Monte Carlo are too widespread. In order to better process the results from FE analyses, they are separated per bins of 1% DOD (i.e., \([0,1\%], [1\%, 2\%], \ldots, [24\%, 25\%]\)).

The mean values of each category of DOD were fitted, by regression analysis, using a polynomial function, Eqn(22). Although this type of adjustment could be somewhat arduous, having six degrees of freedom allows an accurate representation of the mean values trend, in the evaluated domain. The Person correlation, R^2, has been evaluated as 0.998, which clearly reveals an almost exact adjustment. Figure 3 shows the calculated mean values of each category as well as the predicted, or adjusted, mean value.

$$E \left[ \frac{\sigma_u}{\sigma_{up}}(DOD) \right] = \frac{\sigma_{u,0}}{\sigma_{up}} c_1 DOD + c_2 DOD^2 + c_3 DOD^3 + c_4 DOD^4 + c_5 DOD^5 + c_6 DOD^6, \quad 0 \leq DOD \leq 0.25 \tag{22}$$

![Figure 3 - Ultimate strength ratio, \(\sigma_u/\sigma_{up}\), as a function of the degree of degradation](image-url)

where the coefficients that best fit the former equation to the observed mean are:

$$c_1 = -8.656555, \quad c_2 = 151.731184, \quad c_3 = -1685.387707, \quad c_4 = 10048.636742, \quad c_5 = -29926.527275, \quad c_6 = 35000.877908 \tag{23}$$

The high degree of significance of the coefficients is justified by the extreme sensibility of spline functions. It must be
noted that Eqn (22), with these coefficients, is meaningful only in the considered range of DOD, since these types of functions usually present undesired behaviours outside of the adjustment interval. The standard deviation of each category of DOD has been fitted to:

\[
\text{StDev} \left[ \frac{\sigma_u}{\sigma_{y, p}}(\text{DOD}) \right] = a_d \text{DOD} + b_d, \quad 0 \leq \text{DOD} \leq 0.25
\]

(24)

where \( a_d = -0.1280 \) and \( b_d = 0.0606 \) and the \( R^2 \) coefficient has been evaluated as 0.894.

### 3.4 Discussion

In what concerns uniform thickness reduction as a procedure to account for corrosion degradation, it has been proven that the reduction of the ultimate strength of corroded plates is far from being linear. Although the linear approach has been common, the uniform reduction of thickness approach reveals that for a volume loss (due to corrosion) of 16%, the decrease of the ultimate strength of the studied plate is approximately 24%.

From the premise that, for corroded surfaces, thickness varies in every point, in this case, nodes of finite element mesh, the models of corrosion degradation presented here are arguably closer to real case scenario then uniform reduction approach. Figure 4 illustrates the closed form procedure achieved with this study as explained in previous section.

### 4 RELIABILITY ASSESSMENT

The reliability assessment conducted in the present study deals with the hypothesis of structural failure due to corrosion deterioration in the service life of ship structure. The corrosion degradation can affect the ship structural integrity either due to strength loss. The resulting events are modeled as series system assuming that the inspections on corrosion may be ineffective or the structure will collapse due to strength degradation accounting for the correlation between the referred failure modes (Silva et al., 2011a).

#### Corrosion deterioration failure mode, \( E_i(t) \)

The time dependent limit state function that reflects corrosion degradation along the years of a steel deck plate is based on the corrosion model proposed by Guedes Soares and Garbatov (1999) and the demands of classification societies for replacing any structural element affected by corrosion. The implicit limit state function is defined as:

\[
E_i(t) = \bar{D}_{cs} - \bar{D}_{corr}(t)
\]

(25)

where \( \bar{D}_{cs} \) is the demands of classification societies for replacing any corroded structural member of a ballast tank (IACS, 2006), which is modeled as a log-normal distribution with mean value and standard deviation of 2 mm and 0.1 mm respectively.

\( \bar{D}_{corr} \) is the corrosion depth, modeled as a log-normal distribution:

\[
\bar{D}_{corr}(t) \sim \text{LogNormal}\{d_{corr}(t); \text{StDev}_{corr}(t)\}
\]

(26)

where the mean value, \( d_{corr}(t) \), is given by Eqn. (3), and standard deviation is defined as \( \text{StDev}_{corr}(t) = 0.384 \ln(t) - 0.710 \), for \( t \) greater than \( \varepsilon \) (Guedes Soares and Garbatov, 1999)

Reliability index and probability of failure defined by the limit state function expressed by Eqn (25) is based on FORM and can be seen in Figure 5.

#### Ultimate strength failure modes, \( E_{2,3}(t) \)

The limit state function developed for ultimate strength of a deck plate subjected to compressive load for sagging condition is defined as:

\[
E_{2,3}(t) = \bar{x}_c \cdot \bar{S}M(t) \cdot \bar{\sigma}_u(t) - \bar{x}_{sw} \cdot \bar{M}_{sw} - \bar{x}_w \cdot \bar{M}_w
\]

(27)

where \( \bar{SM}(t) \) is the midship section modulus, \( \bar{\sigma}_u(t) \) is the critical failure stress, \( \bar{M}_{sw} \) is the still water bending moment, \( \bar{M}_w \) is the wave induced bending moment, \( \bar{x}_c \) is model uncertainty on ultimate strength, \( \bar{x}_{sw} \) is uncertainty in the model of predicting the still water bending moment, \( \bar{x}_w \) is the error in the wave bending moment due to linear sea keeping analysis and \( \bar{x}_c \) takes into account nonlinearities in sagging.

The full statistical descriptions of parameters involved in the limit state function presented in Eqn (27) are given here as example as (Mansour et al., 1993):

\[
\begin{align*}
\bar{x}_c & \sim N\{1;0.15\} \\
\bar{x}_{sw} & \sim N\{1;0.5\} \\
\bar{x}_w & \sim N\{0.9;0.14\} \\
\bar{x}_l & \sim N\{1.15;0.03\}
\end{align*}
\]

(28)

where \( N \) denotes the normal distribution function and the first and second indicator inside the brackets refer to the mean value and standard deviation respectively. However the values of the present statistical descriptors are considered here as an example to check the flexibility of the model, developed here.

The midship section modulus is defined based on the minimum requirements of Classification Societies. It is assumed here that the as built ship section modulus, accounting for the
additional corrosion plate thickness, \( \overline{SM}_{as \ b u i l t} \), is considered as 25% larger than the minimum required one. However, along the ship service life, the midship section will degrade and, consequently, the section modulus will be also reduced. The corrosion degradation of the midship section modulus along the service life is assumed here as an example to be linear and will be modeled as a function time by a reduction factor of \( \delta(t) \).

The midship section modulus is assumed here as a Log-Normal distribution function with the mean value at the beginning of the service life equals to 25% of minimum required by Classification Society and a covariance of 4% as:

\[
\overline{SM}(t) = [1 - \delta(t)] \overline{SM}_{as \ b u i l t} \\
\overline{SM}_{as \ b u i l t} = 1.25 \overline{SM}_{CS}
\]

where

\[
\delta(t) = \begin{cases} 
0 & t \leq \tau_c \\
25 - \tau_c & \tau_c \leq t \leq 25 \text{years}
\end{cases}
\]

\( \overline{SM}_{CS} \sim \lognormal(\overline{SM}_{as \ b u i l t}, 0.04 \cdot \overline{SM}_{CS}) \)  

where \( \delta_2 \) is the section modulus reduction factor at the 25th year of service life. Four severity of corrosion degradation have been analyzed here: \( \delta_2 = 0.00, 0.05, 0.10, 0.15 \) i.e. a section modulus reduction of 0%, 5%, 10%, 15% at the 25th year of service life respectively.

The still water bending moment is modeled as a normal distribution. It is assumed here that the value given by Classification Societies is the maximum value with a probability of exceedance of 5%. The large variability in the still water bending moment results in a coefficient of variation of 40%, which gives the mean value of the distribution to be 60% of \( \overline{SM}_{CS} \) thus (Guedes Soares and Moan, 1982, 1988)

\[
\overline{SM}_{SW} \sim N\{0.6 \overline{SM}_{CS}, 0.24 \overline{SM}_{CS}\}
\]

If the wave induced loads subjected to marine structure can be represented as a stationary Gaussian process (short-term analysis) then various methods may be used to define the probability density function of the maximum load. The extreme value distribution is based on up crossing analysis.

The wave induced bending moment given by Classification society’s rules is modeled as an extreme value following the distribution function (Mansor, 1990):

\[
F_w(\omega) = \exp\left\{-N_w \exp\left(-\frac{\omega^2}{2\sigma_w^2}\right)\right\}
\]

\[
\mu_w = M_{w,CS} = \sqrt{2\lambda_0 \ln(N_w)} + \frac{0.5772}{\sqrt{2\lambda_0 \ln(N_w)}}
\]

\[
\sigma_w = \sqrt{\frac{\lambda_0}{2 \ln(N_w)}}
\]

where \( \mu_w \) is the mean value of the distribution and \( \sigma_w \) is the standard deviation. \( N_w \) is the number of wave bending moment peaks and \( \lambda_0 \) is the mean square of the wave bending moment process. The value given by Class society’s rules is assumed to be the mean value and choosing \( N_w \) to be 1000, which is equivalent to a 3 hours storm, gives a coefficient of variation of 9%.

The plate analyzed here, with a thickness of 10 mm, is a part of deck of a ship with a length of 128 m between perpendiculars, 19.2 m beam, 10.4 m depth, moulded draught of 8.5 m and a block coefficient of 0.704. For the ship used in the present study the required design bending moments and section modulus are (DnV, 2001):

\[
M_{SW,CS} = 289910 \text{ [kNm]} \\
M_{w,CS} = 490617 \text{ [kNm]}
\]

The two hypotheses for the formulation of the time variant stochastic variable that regards the critical failure stress, of a deck plate with different types of wastage, \( \bar{\sigma}_w(t) \), and \( \bar{\sigma}_w(t) \), form two distinct failure events \( E_1 \) and \( E_2 \) respectively. The critical failure stresses, \( \bar{\sigma}_w(t) \), are modeled as a log normal distribution function,

\[
\bar{\sigma}_w(t) \sim \lognormal(\sigma_{w,\bar{\sigma}}(t), \sigma_{w,\bar{\sigma}}(t), \sigma_{w,Stdev}(t))
\]

where \( \sigma_{w,\bar{\sigma}}(t) \) is defined by Eqn(18) and the correspondent standard deviation, \( Stdev_{\bar{\sigma}}(t) \), is given by Eqn (19). For the case of plates with random indentations the ultimate strength ratio, \( \frac{\sigma_{w,\bar{\sigma}}(t)}{\sigma_{w,Stdev}(t)} \), is given by Eqn(22) and the corresponded standard deviation given by Eqn(24), the time dependency of those equation is established by use of Eqn(20) in a deterministic sense. For both cases, the FORM results are shown in Figure 6 and Figure 7. As an example, only the case where \( \delta_2 = 0 \) is shown here.

![Figure 6 - Reliability index and correspondent probability of failure of \( E_1(t) \) for \( \delta_2 = 0 \)](image-url)

As can be observed in Figure 6, there is an abrupt decrease in the reliability index from the 13th to the 14th year of service life, in comparison to what has been observed in the corrosion degradation failure mode, \( E_1(t) \) as can be seen in Figure 5. This can be explained with the fact that the structure will only respond to the effects of corrosion degradation a few years after the coating time. Moreover, prior to the 13th year, the uncertainties on structural collapse are still considerably low, when compared to the ones present on following years.

From the FORM results with respect to the limit state function, \( E_1(t) \), it must be pointed that this failure event is much more conservative than the one of \( E_1(t) \). With coating protection system vanished, the reliability index, \( \beta_1 \), of the present failure mode is about 60% of the one evaluated in event \( E_1(t) \). This is due to presence of lower mean values of the critical stress. In addition, both expected value and standard deviation
of this stochastic variable are decreasing with time, leading to a loss of the significance in the limit state function of the term that regards the plate’s critical stress in Eqn (27). Thus, considering the data in Figure 7, the reliability of the failure event \( E_i(t) \) is only meaningful until the reliability index, \( \beta_i \), reaches its lowest value (see Figure 7).

\[
\beta_i = \Phi^{-1}(1-R_i(t))
\]

\( E_i(t) = g_i(x) \leq 0 \) (38)

The occurrence of any failure event \( E_i \), for series system of events, will cause the failure of the entire system. The failure event of the entire system, \( E \), is a union of all possible failure modes, which can be expressed as follows:

\[
E = E_1 \cup E_2 \cup \ldots \cup E_n
\]

(39)

The failure probability can be computed either by use of Morgan’s law, or by solving the multidimensional integral based on Eqn (2).

The \( n \)-dimensional integral presented in Eqn (2) can be solved only for some limited case and to overcome this problem some bounding techniques may need to be applied, such as the ones proposed by Cornell (1969) and more recently by Ditlevsen (1979b). Ditlevsen proposed “narrow” bounds on the probability of failure of a series system, Eqn (40).

\[
P(E) \geq P(E_1) + \sum_{j=2}^{n} \max \left\{ P(E_j) - \sum_{i=1}^{j} P(E_i), 0 \right\}
\]

(40)

where the joints probabilities of failure, \( P(E_j) \) can be expressed (Ang and Tang, 1984) as:

\[
P(E_j) = \Phi_\beta(-\beta_j \rho_j \beta_i \sqrt{1-\rho_i^2})
\]

(41)

with \( \beta_i \) and \( \beta_j \) are the reliability indices corresponding to the \( i \)th and \( j \)th failure modes, respectively, \( \rho_i \) is the correlation coefficient between the \( i \)th and \( j \)th failure modes and \( \Phi(\cdot) \) are the probability density and cumulative distribution function, respectively, of the 2D standard normal distribution.

Eqn (40) represents the narrow bounds for the system probability of failure, and they still regard the joint failure modes. Ditlevsen (1979a) also proposed a method for bounding joint probability of failure, \( P(E_j) \) as:

\[
\max \left\{ P_a, P_b \right\} \leq P(E_j) \leq P_a + P_b
\]

(43)

where

\[
P_a = \Phi(-\beta_j) \Phi(-\beta_i \rho \sqrt{1-\rho_i^2})
\]

(44)

\[
P_b = \Phi(-\beta_j) \Phi(-\beta_i \rho \sqrt{1-\rho_i^2})
\]

Although, Ditlevsen, 1979 bounds demonstrated to be more practical and usable than the exact expressions.

4.2 Series system composed by two failure modes

The probability of failure of the series system, composed by the two events may be defined as:

\[
P(E) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 E_2)
\]

(45)

Taking into consideration that, in the present problem, \( P(E_1) \) is always greater than \( P(E_2) \) then the bounds for the system probability of failure, expressed by Eqn (40) to (44), may be written as:

\[
P(E) \geq P(E_1) + \max \left\{ P(E_2) - \left[ P_a + P_b \right], 0 \right\}
\]

(46)

where

\[
P_a = \Phi(-\beta_j) \Phi(-\beta_i \rho \sqrt{1-\rho_i^2})
\]

(47)

For the two events problem, \( P_a + P_b \) is always less than \( P(E_1) \), then one can easily see that Eqn (46) becomes equivalent to Eqn (45). Eqn (41) is an exact solution for the joint probabilities of failure \( P(E_j) \) and a numerical integration is needed to be performed.

For the evaluation of the joint probability of failure, \( P(E_1 E_2) \), only the case where \( \delta_i = 0 \), Eqn (30), will be considered here. As an example, for a correlation factor, \( \rho = 0.4 \) between the studied events, the Ditlevsen bounds, Eqn (43), for the joint probability of failure is shown on Figure 8 and Figure 9. These figures show how the joint probabilities of failure, \( P(E_1) \) and \( P(E_1), \) rely inside the bounds, which represent the most and less conservative probability of failure. The figures also include the case where the events are mutually exclusive.
The corrosion degradation failure mode, $E_1$, presented a decrease from the 11th year to 25th year of service life of 58%, while the reliability index of the ultimate strength failure mode, $E_2$, decreases in the same period, by 61.48%, 68.87%, 76.81% and 85.33% for $\delta_{11} = 0$, 0.05, 0.10 and 0.15 respectively. In opposition, the failure event defined by $E_3$ is a less reliable event when compared to $E_2$.

![Figure 8 - Joint probability of failure, $P(E_1)$, for $\rho = 0.4$, ($\delta_{25}$=0)](image)

The severity of the corrosion wastage increases the system probability of failure to nearly 50% at the 24th, 22nd, 21st and 20th years of service life for $\delta_{25} = 0$, 0.05, 0.10 and 0.15 respectively.

![Figure 9 - Joint probability of failure, $P(E_3)$, for $\rho = 0.4$, ($\delta_{25}$=0)](image)

5 CONCLUSIONS

The present study developed two new approaches to model corroded plate surfaces. A Monte Carlo simulation of the corroded plate surfaces was conducted in order to account for the randomness associated to corrosion deterioration.

For the purpose of demonstration, a rectangular steel plate was studied, and a considerable large set of non-linear finite element analyses were conducted. The plate was discretized with two-dimensional finite elements and the corrosion degradation effects were taken into account by establishing nodal properties.

The model is a compromise between the accuracy of corrosion discretization, computational capabilities and time, since the average computational effective time per plate was in between 5 to 10 minutes. Taking into account that the convergence of results wasn’t always a successful achievement the overall computational time increases considerably.

Nevertheless, the finite element model has proven to produce acceptable results, discouraging the use of tri-dimensional discretization for this type of problem. It may also be pointed out that the shell elements apply thickness on the mid surface, leading to a symmetrical introduction of nodal thickness. However, deck plates usually present corrosion on both faces, but if that is not the case shell elements have the ability to account with changes in the neutral axis.

The present study does not take into account any prior study and/or statistical analysis regarding the forms, locations and number of indentations such as Valor et al. (2007). The calibration of the parameters that control the shape of indentations is one of the next steps to take, as well as the inclusion of correlations between the pits.

Noteworthy is that the present study does not take into consideration maintenance. The plate is allowed to age and, the damaged caused to material evolves until a catastrophic stage is reached. Accounting with maintenance, inspections and also the inclusion of Bayesian updating methods are to be considered in future developments.

The parameters of regression analysis achieved here are intrinsically related with the properties of the plate under examination. Hence, the generalization of this procedure to other mechanical properties is of primary importance, as well as the realization of more corroded surfaces and shapes. The development of a self-sufficient and self-dedicated computational code, able to both generate and analyse corroded surfaces while avoiding conflicting solutions, is a way to achieve the former objectives.

This study opens the way to future comparison between the finite element modelling and full-scale test results, which must be performed in order to proof the accuracy of the present numerical model.

Since the plate is to be fitted as a component of a structural system, the applicability of the numerical models here presented to an entire structure model (such as ship tanks) must also be analysed.

6 REFERENCES


