Static and dynamic behavior of Rubbercork Composite materials

Ivo Machado Guelho

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Júri

Presidente: Professor Luís Manuel Varejão de Oliveira Faria
Orientador: Professor Mihail Fontul
Co-Orientador: Professor Luís Filipe Galrão dos Reis
Vogal: Professor Miguel António Lopes de Matos Neves

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“Genius is one percent inspiration and ninety-nine percent perspiration.”

Thomas A. Edison
I want to express my sincere acknowledgments,

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Resumo

Este documento apresenta o estudo do comportamento estático e dinâmico de diferentes materiais compósitos de cortiça com borracha de modo a caracterizar o seu comportamento mecânico. Reconhecendo a importância destes materiais como sistemas de amortecimento passivo, modelar e compreender de que modo estes materiais dissipam energia torna-se essencial para uma aplicação mais precisa.

Os amortecedores passivos fazem usufruto do desempenho dos seus próprios materiais constituintes de forma a dissipar a energia proveniente de vibrações ou choques. A capacidade de absorção das vibrações ou choques é determinada pela sua capacidade de amortecimento, a qual é medida através do factor de perda.

Este trabalho apresenta um estudo baseado na análise modal com o intuito de determinar o comportamento dinâmico dos materiais e a sua capacidade de absorção de energia. De modo a alcançar estes objectivos, uma cadeia de medição foi especialmente concebida de forma a simular um sistema modal com um grau de liberdade.

Assim, foram realizados ensaios quasi-estáticos numa máquina electromecânica e ensaios de vibração num vibrador electrodinâmico. Os resultados obtidos a partir dos testes de vibração permitiram estimar todos os parâmetros modais necessários na parameterização modal. Contudo, a precisão na determinação dos parâmetros modais pode ser particularmente afectada consoante o método escolhido ou mesmo pelas características das funções da resposta em frequência utilizadas. Assim sendo, neste trabalho foram utilizados três métodos na extração dos parâmetros modais e no cálculo das propriedades dinâmicas dos materiais: o método da meia-potência, o método da aproximação do círculo de Nyquist e o método dos polinómios racionais fraccionários.

O objectivo principal deste trabalho foi alcançado quando as propriedades dinâmicas dos materiais tais como, o factor de perda, os módulos elásticos de armazenamento e de perda, a rigidez complexa, entre outras, foram determinadas e quando todos os parâmetros modais inerentes às equações analíticas que representam o comportamento dinâmico dos materiais puderam ser estabelecidas.

Palavras-Chave: Aglomerado de cortiça com borracha, vibração, análise modal, módulo complexo, ensaios experimentais.
The present thesis studies the static and the dynamic response of different types of rubbercork composite materials in order to characterize their mechanical behavior. Recognizing their importance as passive damping device systems, to model and comprehend their energy dissipation mechanisms becomes essential for a better product development and suitable application.

Passive dampers make use of their inherent materials performance in order to absorb the vibration energy and therefore providing passive energy dissipation. This vibration reduction ability of a device is determined by its damping capacity which is measured by the loss factor and the storage modulus of the material.

This work presents a modal analysis based study to determine the materials dynamic behavior and their energy absorbing capacity. To accomplish these purposes, a measuring chain was specially designed to simulate the single degree of freedom modal system.

Afterwards, experimental data were obtained by doing quasi-static compression tests in a servo-hydraulic machine and vibration tests in an electrodynamic shaker. All the acquired data produced in vibration tests permitted to estimate all modal parameters necessary in the modal parameterization. Although, the accuracy in determine the modal parameters can be particularly affected by the chosen method or even by the characteristics of the Frequency Response Functions used. Therefore, three methods were used. The well-known Half-Power bandwidth method used to determine natural frequency and damping ratio and two other methods based on the identification and extraction of modal parameters. They were the Nyquist Circle-Fit method and the Fraction Rational Polynomial method.

The main goal of this work was achieved when the dynamic properties of the materials such as loss factor, elastic moduli (storage and loss modulus) and the complex dynamic stiffness, among others, were determined and when analytical equations representing the dynamic behavior of the materials were established.

**Keywords:** Rubbercork composites, vibration, modal analysis, complex modulus, experimental tests.
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<tr>
<td>A</td>
<td>Area</td>
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<tr>
<td>B</td>
<td>Complex modal constant</td>
</tr>
<tr>
<td>C</td>
<td>Complex modal constant</td>
</tr>
<tr>
<td>c</td>
<td>Viscous damping</td>
</tr>
<tr>
<td>D*</td>
<td>Damping</td>
</tr>
<tr>
<td>D, d</td>
<td>Diameter</td>
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<tr>
<td>E</td>
<td>Longitudinal modulus of elasticity or Young’s modulus</td>
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<tr>
<td>E*</td>
<td>Complex elasticity modulus</td>
</tr>
<tr>
<td>E’</td>
<td>Storage modulus</td>
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<tr>
<td>E”</td>
<td>Loss modulus</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
</tr>
<tr>
<td>h</td>
<td>Hysteretic damping</td>
</tr>
<tr>
<td>i, j</td>
<td>Imaginary unit ((i^2=j^2=-1))</td>
</tr>
<tr>
<td>k</td>
<td>Stiffness</td>
</tr>
<tr>
<td>K*</td>
<td>Complex stiffness modulus</td>
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<tr>
<td>K’</td>
<td>Storage stiffness modulus</td>
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<td>K”</td>
<td>Loss stiffness modulus</td>
</tr>
<tr>
<td>L</td>
<td>Thickness</td>
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<tr>
<td>R, r</td>
<td>Radius; Vibration mode</td>
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<tr>
<td>T</td>
<td>Transmissibility</td>
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<tr>
<td>t</td>
<td>Time variable</td>
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<td>Potential energy</td>
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<tr>
<td>x, y</td>
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<tr>
<td>α”</td>
<td>Accelerance</td>
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<td>Frequency ratio</td>
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<td>Phase lag</td>
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<tr>
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<td>Strain</td>
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<td>Loss factor</td>
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<td>Phase angle</td>
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<td>Stress</td>
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<tr>
<td>ϕ</td>
<td>Phase angle</td>
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<td>Frequency</td>
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<tr>
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<td>ωₙ</td>
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Chapter 1

1 Introduction

Composite materials are engineered materials composed from two or more materials with significant different physical or chemical properties. They present different mechanical properties derived from their constituents and the interaction between them. That's equally for nano and macro-composites making such distinctions only a matter of scale.

Composites may consist of matrixes with mixture embedded fibers, particles or even layers of distinct materials.

Cork materials are used by the humanity for over 5,000 years. However, only in the final of the 19th century cork agglomerates started to be developed [1].

The rubber-cork composite materials specifically started to be developed in the 1960's and they were initially used in oil sealing applications in the automotive industry [1]. The R&D over the years has shown that it has a much broader application, and nowadays it is applied in acoustic, heat, electric and vibration isolation fields (Figure 1.1).

![Figure 1.1 – Rubber-cork anti-vibration pad in a wind power transformer.](image)

By being agglomerates of two viscoelastic materials, rubber and cork, and due to their use in passive vibration and noise control it is important to well-known them for suitable development and appropriate applications. Their properties can vary depending on granulates grain size, cork density or even in the type of rubber used to make the agglomerate. When manufactured, these composites can also derivate from a variety of binders from Nitrile and Neoprene to Silicone and Epichlorohydrin for high heat applications. The physical and chemical characteristics of the binders determine the strength of agglomerate and therefore its applications [3]. They are well recognized by combining resilience and compressibility of cork with the high mechanical strength and dimensional stability of rubber.
Nowadays, with a wider range of applications, the characterization of these composites becomes essential to get better performances in each application. Therefore, by being a “reinvented” material has become quite motivating to develop this work.

The possibility of producing good results in characterizing these composites without using a dynamic mechanical analyzer machine (DMA), which is the commonly used, with a much more expensive cost associated, was the main purpose. The partnership with the Portuguese company Amorim Cork Composites, S.A., world leader in cork and cork agglomerates, brought also an extra motivation on the development of this work.

Currently, one of the most important applications of these materials in engineering is as energy dissipation mechanisms, namely as passive damping devices. Thus, this thesis tries to characterize these composite materials and determine their vibration absorbing ability by determine their damping. In order to do that, the present work presents a modal analysis based study.

The modal analysis is a strong engineering tool that appeared in the 1940’s and provided better comprehension about the dynamic properties of structures under dynamic excitation. Extracting modal parameters like natural frequencies, loss factors, phase angles and residues (inherent to the contribution of other modes) are essential in modal testing, since the analytical frequency response function curves (FRFs) must fit with the curves obtained from the experimental tests. The accuracy of this extraction can be particularly affected by the chosen method or even by the characteristics of the Frequency Response Functions used. Thus, three methods were used: the well-known Half-Power bandwidth method used to determine natural frequency and damping ratio and two other methods based on the identification and extraction of modal parameters. They were the Circle-Fit method and the Fraction Rational Polynomial method.

This thesis pays particularly attention in understanding how efficient the dynamic behavior of these materials can be described by the simplest modal analysis system, the single degree of freedom system and how accurate the extraction techniques used in extracting modal parameters are.

This thesis has 6 main chapters. In chapter one is made a brief introduction to this work, explaining the framework, its importance and applications. At the end, the basic guidelines of this work are exhibited. A literature review is presented in chapter two with a brief historical evolution, the methods adopted in this work, some of the work already existent in this field and the problems encountered in characterizing these materials.

The third chapter enounces some important theoretical considerations necessary to better understand the present work.
Following it, chapter 4 presents about experimental procedure and describes all the methodologies and techniques used to manufacture the experimental specimens.
In chapter 5 all the results obtained from the experimental tests and from the analytical
Finally, in chapter 6 the main conclusions and some final remarks are advanced concerning new developments that are considered to be crucial to improve the investigations in this field.
2 Literature Review

Many studies have been made to characterize viscoelastic materials. Models representing their static and dynamic behavior have been proposed. Two well-known models were proposed by James Clerk Maxwell, the so called “Maxwell-model” and by Kelvin Thomson and by Woldemar Voigt the “Kelvin-Voigt” model [4]. The simplest one for one-dimensional viscoelastic materials is the Maxwell model, proposed in 1867, known also as “Maxwell fluid”, which represents a “Newtonian” fluid damper with a spring connected in series. The Kelvin-Voigt model configures those two components but in the parallel form.

In 1968, Snowdon [5] presented a study of the frequency dependence for some rubber-like materials. Concepts like complex elastic modulus and complex shear modulus are explained and transmissibility and phase equations for different spring and dashpot combinations are deduced.

Characterizing composite materials containing different viscoelastic materials may not be very easy. As viscous and elastic materials their mechanical properties vary with temperature, frequency, humidity, in addition to all the parameters of the agglomerating process.

Mano [6], and Fortes et al. [7] present a study to characterize the viscoelastic properties of cork based on dynamic mechanical spectroscopy. Dynamic mechanical analysis (DMA) or Dynamic Mechanical Spectroscopy is a useful method used to study and characterize materials in most cases viscoelastic materials and their dynamic behavior.

DMA analysis also called DMTA (Dynamic Thermomechanical Analysis) consists in applying a low frequency oscillatory force and then measuring the Frequency response function (FRF) curves obtained. Normally, DMA analyzers can perform tests from 0.1Hz to 10Hz and gather information like frequency dependence, humidity dependence and since the machine has furnace and a temperature enclosure, allows also determine the temperature dependence of the materials. The capability of application of nitrogen permits also to locate the glass transition temperature of the material at negative temperatures.

In the 1940’s, a strong engineering tool called modal analysis had strong development and provided better comprehension about the dynamic properties of structures under dynamic excitation [8].

Modal analysis consists in studying the structural dynamic properties of a system in terms of its modal parameters.

Extraction techniques for modal parameters have been developed for many years and they can be executed in time-domain or frequency domain. This work is focused only in frequency domain systems.

One goal of this thesis is to study the accuracy of the methods used in estimating damping and the dynamic properties of the systems.
As was mentioned before, three modal parameter extraction techniques were chosen. They are the Half-Power Bandwidth method, the Nyquist Circle-Fit method and the Rational Fraction Polynomial method. The Half-Power bandwidth method is well-known method and provides good results for light damped systems. This method is referred in almost vibration books such as Maia [9] or da Silva [10]. Once called the Kennedy and Pancu method [11], after Klosterman investigation in developing new and more efficient techniques for systems with non-proportional damping, started to be called Circle-fit method. This is presented in some literature such as Maia [9] and Ewins [12]. It consists in evaluating the circle produced by the real and the imaginary parts of receptance by using the Argand plane.

The Rational Fraction Polynomials method was introduced in 1982 by Richardson and Formenti [13] in a Conference in Orlando. This method consists in finding the unknown coefficients of two polynomials in order to obtain the best fit to the experimental receptance curve. The fitting is made based in a least-square error criterion. Solving the analytical equation, by calculating the roots (poles and zeros) of both polynomials (numerator and characteristic polynomial), all the necessary parameters become easily to determine.

In determining the dynamic properties of the materials, Nashif et al. [14] showed that from three major parameters, i.e., the storage modulus, the complex dynamic stiffness and loss factor were enough to characterize a hysteretic damping system.

In [15], Imregun has made a comparison between two different modal analysis techniques. In a single-degree-of-freedom (SDoF) and a multi-degree-of freedom system (MDoF) applied to frequency response function measurements taken on a lightly damped linear structure. For the SDoF, the circle-fit and the line-fit were used to identify the modal properties, which gave very similar results for most of the cases. Due to noise it is not always possible to fit a reliable circle of the FRF data. It has been said that the circle-fit method gives reliable results when there are enough data points around resonance and that damping is not too low. It is also said that this method should not be used when the data contains non-negligible noise around resonance.

Fahey et al. in [16] compared different time-domain modal estimation techniques. These techniques were the Complex Exponential Algorithm, Pisarenko’s Harmonic Decomposition, Ibrahim’s Time Domain method, and the Eigensystem Realization Algorithm. They developed a numerical example in order to compare and contrast them. They say that there is a difference between the frequency-domain techniques and the time-domain techniques with respect to the system-damping ratio. This difference is that in the time-domain techniques generally work better than the frequency-domain techniques when the system damping is less than 0.5 percent. But, the frequency-domain techniques, generally gives more reasonable results when the damping is greater than 4.0 percent.

In [9], N. M. M. Maia studied and analyzed some SDOF modal analysis methods. He gave a brief review of the Peak Amplitude, the Quadrature Response, the Maximum Quadrature Component, the Kennedy-
Pancu, the Nyquist Circle-Fitting, the Inverse method and Dobson method. Maia shows that the Dobson’s method is a more refined and more powerful version than the Inverse method. Also, he demonstrates that for practical use, Dobson’s method gives better results than the Inverse method and it works better than the Circle Fitting method.
3 Theoretical Analysis

To accomplish the main objective in determining the dynamic properties inherent of the rubbercork materials, an engineering tool called Modal Analysis was used. Regarding the fact that modeling mechanical systems can be very complex, in most cases a discrete approach turns out to be sufficient. Although Modal Analysis is largely based on the analysis of multi-degree-of-freedom (MDoF) systems with an inherent matrix theory background, this work tries to simplify the characterization of these viscoelastic materials by using the simplest vibratory system, the single-degree-of-freedom (SDoF) system.

3.1 Vibration Response

In a certain way, modal analysis can be described as a process where a structure and its movement can be analyzed in terms of its natural characteristics. These natural characteristics are the natural frequency, loss factor, vibration modes and their dynamic properties.

3.1.1 Single-degree of freedom system

A single degree of freedom system, SDoF, is a simple discretization of a physical model into a spring-mass-damper system whose motion can be described by a single variable $x$ (Figure 3.1).

Each one of these three elements (spring-mass-damper) represents a unique property of the system where: the spring represents the stiffness and is considered massless; the mass represents the inertia and is treated as an infinitely rigid body; the damping is represented by a dashpot which is considered as having no stiffness or mass.

Figure 3.1 shows a representation of an SDoF system with the two types of existing damping, the viscous and the hysteretic or structural damping.

![Figure 3.1 - Discretized representation of: a) SDoF system with viscous damping; b) SDoF system with hysteretic damping](image-url)

[9]
The application of the Second Newton’s Law to the previous models and summing all the forces on the masses we get the following differential equations of motion for the two damping types:

Viscous damping: \[ m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \] (3.1.1)

Hysteretic damping: \[ m\ddot{x}(t) + \frac{h}{\omega}\dot{x}(t) + kx(t) = f(t) \] (3.1.2)

Where the variable \( t \) represents time, \( \omega \) is the frequency, \( m \) is the mass, \( x(t) \) is the mass position through time, \( c \) is the damping, \( h \) is the hysteretic damping factor, \( k \) is the stiffness and \( f(t) \) is an external dynamic load. If an harmonic force is applied to the system, \( f(t) = F e^{j\omega t} \) \((j = \sqrt{-1})\), the response will be also harmonic and will take the form of \( x(t) = \bar{X} e^{j\omega t} \). Velocity and acceleration can also be deduced by deriving \( x(t) \). Therefore,

\[ f(t) = F e^{j\omega t} \] (3.1.3)
\[ x(t) = \bar{X} e^{j\omega t} \] (3.1.4)
\[ \dot{x}(t) = j\omega \bar{X} e^{j\omega t} \] (3.1.5)
\[ \ddot{x}(t) = -\omega^2 \bar{X} e^{j\omega t} \] (3.1.6)

By substituting the previous variables in equations (3.1.1) and (3.1.2), it comes that

\[ (k - \omega^2 m + j\omega c)\bar{X} = F \] (3.1.7)
\[ (k - \omega^2 m + jh)\bar{X} = F \] (3.1.8)

This is a particularly important step because now it is easy to write the ratio between the displace response and the input force as:

\[ \alpha(\omega) = \frac{\bar{X}}{\bar{F}} = \frac{1}{(k - \omega^2 m + j\omega c)} \] (3.1.9)
\[ \alpha(\omega) = \frac{\bar{X}}{\bar{F}} = \frac{1}{(k - \omega^2 m + jh)} \] (3.1.10)

Where \( \alpha(\omega) \) is the frequency response function (FRF) of the system. The FRF is the main function on which modal analysis will depend and although in theory the FRF is dictated only by the system, in reality
the accuracy of measured FRF data is critical to the success of modal analysis. Since this FRF uses the displacement as the response, normally is denoted as receptance or inertance of the system.

The response can also be given in the form of velocity $\dot{x}$ or acceleration $\ddot{x}$, thus

**Viscous damping:**

\[
\begin{align*}
\text{mobility} & \quad \alpha'(\omega) = \frac{\dot{x}}{F} = \frac{j\omega}{(k - \omega^2m + j\omega c)} \\
\text{accelerance} & \quad \alpha''(\omega) = \frac{\ddot{x}}{F} = \frac{-\omega^2}{(k - \omega^2m + j\omega c)}
\end{align*}
\]  

(3.1.11)  

(3.1.12)

**Hysteretic damping:**

\[
\begin{align*}
\text{mobility} & \quad \alpha'(\omega) = \frac{\dot{x}}{F} = \frac{j\omega}{(k - \omega^2m + jh)} \\
\text{accelerance} & \quad \alpha''(\omega) = \frac{\ddot{x}}{F} = \frac{-\omega^2}{(k - \omega^2m + jh)}
\end{align*}
\]  

(3.1.13)  

(3.1.14)

Equations (3.1.11) and (3.1.13) give the Mobility FRF’s for viscous and hysteretic damping respectively, and the equations (3.1.12) and (3.1.14) are the acceleration FRF’s.

From the Receptance equations an important concept in vibrations theory called Transmissibility, which describes the effectiveness of vibration isolation system, can be deduced:

\[
\begin{align*}
T_{\text{viscous}} = \frac{|\ddot{X}|}{X_s} &= \frac{1 + (2\zeta \beta)^2}{\sqrt{(1 - \beta^2)^2 + (2\zeta \beta)^2}} \\
T_{\text{hysteretic}} = \frac{|\ddot{X}|}{X_s} &= \frac{1}{\sqrt{(1 - \beta^2)^2 + \zeta^2}}
\end{align*}
\]  

(3.1.15)  

(3.1.16)

Where $\beta$ is the frequency ratio and $\zeta$ is the damping ratio defined as:

\[
\begin{align*}
\beta &= \frac{\omega}{\omega_n} \\
\zeta &= \frac{c}{c_{cr}} = \frac{c}{2m\omega_n}
\end{align*}
\]  

(3.1.17)  

(3.1.18)
In a passive system, at frequencies less than its system resonance, no isolation takes place, and the vibrations transferred through the isolation system increase from $\beta=0$ till $\beta=1$. At the resonance point the maximum of amplification occurs, where the transmissibility is greater than unity. Once past the crossover frequency, isolation occurs.

Another important property of the FRF’s on an SDoF system is the circularity of the Nyquist Plot in the Argand plane. By drawing the real and the imaginary parts of the mobility for viscous damping and the receptance for the hysteretic damping we find out that they are plotted as circles.

### 3.1.2 The Complex Young Modulus

Viscoelastic materials are characterized by having an intermediate behavior between perfectly elastic and perfectly viscous behavior. They are typically polymers, polymers solutions, amorphous materials or metals at very high temperatures. Regarding that perfectly elastic materials present an *in phase* ($\delta=0^\circ$) stress-strain response while the perfectly viscous materials show an *out phase* ($\delta=90^\circ$) lag, the viscoelastic materials behavior is evidenced when the materials are subjected to an oscillatory strain with frequency, where the stress-strain sinusoidal response curves show a phase lag or phase angle ($\delta$) between stress and strain.

![Figure 3.2 – Phase lag between stress and strain][17]

The stress and strain equations from Figure 3.2 can be defined as:

\[
\sigma(t) = \sigma_0 \sin(\omega t + \delta) \tag{3.1.19}
\]

\[
\epsilon(t) = \epsilon_0 \sin(\omega t) \tag{3.1.20}
\]

The equation (3.1.19) can also be written as:
\[ \sigma(t) = \sigma_0 \cos \delta \sin(\omega t) + \sigma_0 \sin \delta \cos(\omega t) \] (3.1.21)

From the preview equation we can distinguish two different components: an \textit{in phase}(\delta=0^\circ) stress with strain component

\[ \sigma_1(t) = \sigma_0 \cos \delta \sin(\omega t) \] (3.1.22)

and an \textit{out phase}(\delta=90^\circ) component

\[ \sigma_2(t) = \sigma_0 \sin \delta \cos(\omega t) \] (3.1.23)

Therefore, and by applying the Hooke’s law \((\sigma = E\varepsilon)\), the Young modulus can also be written in two components:

\[ E'(\omega) = \frac{\sigma_0}{\varepsilon_0} \cos \delta \] (3.1.24)

and

\[ E''(\omega) = \frac{\sigma_0}{\varepsilon_0} \sin \delta \] (3.1.25)

Where \(E'\) is the storage modulus, which represents the elastic energy stored in the material, while \(E''\) is the energy loss and it is called loss modulus, see Figure 3.3.

![Figure 3.3 – Storage and loss moduli representation](image-url)
By representing the storage and loss moduli in the Argand plane (Figure 3.3), the complex Young modulus $E^*$ can be defined as

$$E^*(\omega) = \frac{\sigma_0}{\varepsilon_0} e^{i\delta} = \frac{\sigma_0}{\varepsilon_0} (\cos \delta + i \sin \delta) = E'(\omega) + iE''(\omega)$$ (3.1.26)

The previous equation (3.1.26) permits to define a parameter called the loss factor $\eta$, which represents the damping capacity of the material, described as

$$\eta(\omega) = \tan \delta = \frac{\text{energy loss}}{\text{energy stored}} = \frac{E''(\omega)}{E'(\omega)}$$ (3.1.27)

And therefore the complex modulus stays as,

$$E^* = E'(1 + i\eta)$$ (3.1.28)

Now, and considering the Hooke’s law $\sigma = E . \varepsilon$, from equation (3.1.26) and regarding that $\sigma_0 = F/A$ and $\varepsilon_0 = x/L$, the complex Young’s modulus can be written as

$$E^*(\omega) = \frac{\sigma_0}{\varepsilon_0} (\cos \delta + i \sin \delta) = \frac{FL}{Ax} (\cos \delta + i \sin \delta) = K^* \frac{L}{A}$$ (3.1.29)

where $K^*$ is the complex stiffness, $L$ is the thickness and $A$ is the loaded area (fig. 3.17). This equation is particularly important because it relates the Young’s modulus with the complex stiffness of the material.

As explained before, rubbercork materials as being viscoelastic materials, can physically represented by two discrete elements, a spring and a damper. The spring force ($f = -kx$) concerning the elastic behavior is in phase with the displacement while the damping force presents a 90º phase lead. By considering a massless system we can write the motion equation as:

$$f = Kx_0 \cos(\omega t) + cx_0 \cos \left(\omega t + \frac{\pi}{2}\right)$$ (3.1.30)

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The previous equation can be represented in an Argand plane where the in phase part is considered the real part and the 90° phase lead represents the imaginary part. Therefore,

\[ f = Kx + j\omega x \overset{yields}{\longrightarrow} K^* = K + j\omega \]  

(3.1.31)

where \( K^* \) is called the complex dynamic stiffness and comprises the elastic and the viscous parts.

### 3.1.3 Natural Frequency and Resonance

In Figure 3.4, two typical transmissibility curves are represented for two different damped systems: The first one with a highly damped system (\( \zeta = 0.3 \)); and second one lightly damped (\( \zeta = 0.05 \)).

At low frequencies where the frequency ratio \( \beta \ll 1 \), it can be observed that the transmissibility is practically equal to 1. This means that the output force measured is equal to the force applied to the system. Another way to say is that the system will vibrate all equally with no phase lag between the input and the output.

When the system reaches the maximum transmissibility peak (\( \beta = 1 \)), it is called the system resonance. Resonance occurs when a system is able to store and easily transfer energy between two or more different modes (such as kinetic energy and potential energy).

In each cycle there is always energy loss through the damping of the material. When damping is small, the resonant frequency is approximately equal to the natural frequency of the system, which is a frequency of undamped vibrations. As damping increases, the resonance frequency may not correspond to natural frequency, the transmissibility at resonance decreases and the isolation region becomes smaller.

Of course, all real world systems have some level of inherent damping, but this demonstrates the important role that damping can play in vibration isolation. When a vibration isolation mount with very little damping is used at or near resonance (Figure 3.4 and Figure 3.5), the energy amplification can create many problems, ranging from a simple increase in noise levels to catastrophic damage.
3.1.4 Quality factor of oscillators

The quality factor Q is a parameter which represents how much an oscillatory system is under-damped or in other words, characterizes the bandwidth around the resonance frequency in a transmissibility curve. A low quality factor system \((Q < \frac{1}{2})\) is said to be overdamped \((\zeta > 1)\) when a system does not oscillate at all, but when displaced from its equilibrium steady-state output it returns to it by exponential decay, approaching the steady state value asymptotically. It has an impulse response that is the sum of two decaying exponential functions with different rates of decay. As the quality factor decreases the slower decay mode becomes stronger relative to the faster mode and dominates the system's response resulting in a slower system. A second-order low-pass filter with a very low quality factor has a nearly first-order step response; the system's output responds to a step input by slowly rising toward an asymptote.
A system with high quality factor \((Q > \frac{1}{2})\) is said to be underdamped. Underdamped systems combine oscillation at a specific frequency with a decay of the amplitude of the signal. Underdamped systems with a low quality factor (a little above \(Q = \frac{1}{2}\)) may oscillate only once or a few times before dying out. As the quality factor increases, the relative amount of damping decreases. A high-quality bell rings with a single pure tone for a very long time after being struck. A purely oscillatory system, such as a bell that rings forever, has an infinite quality factor. More generally, the output of a second-order low-pass filter with a very high quality factor responds to a step input by quickly rising above, oscillating around, and eventually converging to a steady-state value.

A system with an intermediate quality factor \((Q = \frac{1}{2})\) is said to be critically damped. As the overdamped system, the output does not oscillate, and does not exceed its steady-state output (i.e., it approaches a steady-state asymptote). Like an underdamped response, the output of such a system responds quickly to a unit step input.

When \(Q\) takes high values, it means that the system is lightly damped and only a few rate of energy is lost relative comparing the stored energy of the oscillator.

The quality factor of oscillators varies substantially from system to system. Systems for which damping is important, like passive damping devices, should be special designed to accomplish the purpose of its application.

### 3.1.5 Damping

Here, the word damping describes the vibrational energy absorbed by a system and converted as heat or sound, resulting in an overall energy loss. By looking to the transmissibility curves, two major effects can be seen when increasing the damping of a system: a lower transmissibility occurs at the resonance frequency and the isolation region is “pushed” for high frequencies (Figure 3.4). The damping behavior of a system is usually measured by its damping ratio \(\zeta\). There are different kinds of harmonic oscillator systems and they can be divided as:

- **Overdamped** \((\zeta > 1)\): The system returns to equilibrium without oscillating. Larger values of the damping ratio \(\zeta\) return to equilibrium slower.
- **Critically damped** \((\zeta = 1)\): The system returns to equilibrium as quickly as possible without oscillating.
- **Underdamped** \((0 < \zeta < 1)\): The system oscillates (at reduced frequency compared to the *undamped* case) with the amplitude gradually decreasing to zero.
- **Undamped** \((\zeta = 0)\): The system oscillates at its natural resonant frequency \(\omega_0\).
Almost every systems function as underdamped systems. In this work, the material damping will be determined by its loss factor $\eta$, as it will be explained in next subchapter 3.1.6. The damping will vary with the percentage of cork granulates and the rubber type used in the composite. The Figure 3.6 exhibits some transmissibility curves for different kinds of rubber, where it can be easily seen how the damping varies.

![Figure 3.6 – Transmissibility curves for different types of rubber.](image)

### 3.1.6 Measuring damping

There are many methods to estimate the material damping, although many of them are too much complex or will not give an accurate result. Before choosing a method to measure damping, a model representing the system should be proposed in order to evaluate which parameters should be measured and calculated.

Damping can be defined as the ratio between the lost energy in complete cycle of motion, $W$, and the maximum of potential energy,

$$D' = \frac{\Delta U}{U_{\text{max}}}$$

The usual way to measure damping is through the damping ratio. However, it can also be measure in terms of loss factor $\eta$, described previously in (3.1.27). The loss factor measures ratio between the energy lost in a cycle $\Delta U$ and the peak of energy stored in a system $U_{\text{max}}$, in radians per second. Thus,
\[ \eta = \frac{\Delta U}{2\pi U_{\text{max}}} \]  

(3.1.33)

For a viscous damping model it comes:

\[ \eta = \frac{2\omega \xi}{\omega_n} \]  

(3.1.34)

Where for \( \omega = \omega_n \) gives an important relation:

\[ \eta = 2\xi \]  

(3.1.35)

However, theoretically this relation is only valid for low damping materials where \( \leq 0.1 \), see [10].

Similarly, from the above equation (3.1.35) and recognizing that \( h = c\omega \) for hysteretic damping, it follows:

\[ \eta = \frac{h}{k} \]  

(3.1.36)

As referred before, measuring damping can be made by two ways: from time-response record or by using frequency-response function of the system. Three distinct methods were used in measuring damping, all centered in FRF systems. The Half-Power bandwidth, the Circle-Fit and the RFP methods, are presented next.

### 3.1.7 Modal parameters identification methods

Only some modal extraction methods can be applied to a FRF at once. These methods are denominated as single input/single output (SISO). Other methods permit that several FRF’s can be analyzed simultaneously using a punctual excitation. They are called single input/multiple outputs (SIMO). Finally, there are some methods that can process simultaneously several FRF’s with several responses to several excitations. They are called multiple inputs/multiple outputs (MIMO).

Figure 3.7 shows all the methods existent in frequency-domain, time-domain and also for tuned-sinusoidal methods, as described in [9].
In this work, it is going to be used the SISO method for a single-degree of freedom system, measured in frequency domain.

3.1.8 Half-Power Bandwidth Method
The half-power bandwidth method is a simple and fast technique to determine the damping ratio and therefore the loss factor of low-damped systems. Considering a SDoF system, the basic principle of this method consists in: first, to find the value of the quality factor, \( Q \), which represents the peak magnitude of receptance \( \alpha(\omega) \), at the resonant frequency; second, locate the magnitude values where the magnitude is \( 1/\sqrt{2} \) times of the amplification factor and get the two frequencies \( \omega_1 \) and \( \omega_2 \) (Figure 3.8); finally, to create a band around the resonance frequency with those two frequencies.
The equation (3.1.9), which represents the receptance of the FRF of a SDoF viscous system can also be written as

\[
|\alpha(\omega)| = \frac{\omega_n^2}{\left((\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2\right)^{1/2}} \tag{3.1.37}
\]

From Figure 3.8 we can rewrite equation (3.1.37) as

\[
|\alpha(\omega)|_{1,2} = \frac{|\alpha(\omega)|_{\text{max}}}{\sqrt{2}} \tag{3.1.38}
\]

And noting that \( Q = \frac{1}{2\zeta} \), solving equations (3.1.37) and (3.1.38) turns out:

\[
\frac{\omega_n^2}{\left((\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2\right)^{1/2}} = \frac{1}{\sqrt{2} \cdot 2\zeta} \tag{3.1.39}
\]

After solving equation (3.1.39), it gives that the damping ratio is given by

\[
\zeta = \frac{1}{2} \frac{\Delta \omega}{\omega_r} \tag{3.1.40}
\]

It is also important to note that for logarithmic amplitude scales the half-power amplitudes of \( \omega_1 \) and \( \omega_2 \), represent 3 dB lower than the peak amplitude (Figure 3.9). This 3 dB \((20 \log_{10} \sqrt{2})\) bandwidth corresponds to the width of a filter transfer function of the half-power level.
3.1.9 Circle-Fit Method

The Circle-Fit method is no more than a curve fitting process in the Argand plane which allows determining the analytical parameters for the frequency transfer function from the analysis of the Nyquist plot.

By plotting the real and imaginary parts of the receptance FRF for structural damping systems, it can be easily verified that an exact circle are plotted in the Argand plane (Figure 3.10-a). Similarly, for viscous damping systems the mobility FRF also turns out to be a circle (Figure 3.10-b).

In most cases, the circle is rotated and displaced from its place, as shown in the Figure 3.11. This is explained by the fact that other vibration modes are also present despite its influence can be neglected for values around the resonance frequency.
Modal parameter extraction

Modal extraction begins by first, determining and understanding the exact response curve that represents the modal system involved which represents the real structure. Substituting equation (3.1.36) in equation (3.1.10) and regarding that \( \omega_c^2 = \frac{k}{m} \), the receptance can be written as

\[
\alpha_{jk}(\omega) = \frac{x(\omega)}{f(\omega)} = \sum_{r=1}^{n} \frac{rA_{jk}}{(\omega_c^2 - \omega^2 + i\eta \omega_c^2)}
\]  

(3.1.41)

Where \( rA_{jk} \) represents the curve gain and \( r \) denotes the vibration mode.

The previous equation can also be written by substituting the complex constant gain \( rA_{jk} \) by a real modulus \( rC_{jk} \) with a phase angle \( e^{i\phi_{jk}} \). This angle symbolizes the rotation of the circle (Figure 3.11).

\[
\alpha_{jk}(\omega) = \frac{x(\omega)}{f(\omega)} = \sum_{r=1}^{n} \frac{rC_{jk}}{(\omega_c^2 - \omega^2 + i\eta \omega_c^2)} + \frac{1}{K_{jk}} - \frac{1}{\omega M_{jk}}
\]  

(3.1.42)

By considering the circle rotation, we should also account the mass and stiffness residues \( \left( \frac{1}{K_{jk}}; \frac{1}{\omega M_{jk}} \right) \) created by those modes outside the frequency range measured.
By being two complex constants, they can be grouped into a single constant, also complex, \( rB_{jk} \) (represented in Figure 3.11 as \( rD_{jk} \)) to help the calculus. Thus,

\[
\alpha_{jk}(\omega) = \frac{x(\omega)}{f(\omega)} = \sum_{i=1}^{n} \left( \frac{rC_{jk}}{\omega_i^2 - \omega^2 + i\eta \omega_i^2} \right) + rB_{jk} \tag{3.1.43}
\]

**Locating the Resonance Frequency**

Simplifying the resonance frequency to the point where the magnitude of receptance is a maximum is in fact a very poor estimation. Is even much poor when the systems are highly damped and the peak is not well defined.

A useful technique to determine the real resonance frequency value is to construct a finite difference table (Figure 3.12) with several receptance phase angle values around the first estimated frequency.

![Figure 3.12 – Finite difference table](image)

Thus, it should be easy to determine the resonance location by the changing of sign of \( \Delta^2 \gamma \), where the value \( \gamma \) is 2 times the phase angle (Figure 3.13).

![Figure 3.13 – Natural frequency location](image)
It can be easily witness that when arriving to the resonance, the phase angle quickly drops 180°, which is a characteristic for resonant systems in a Bode plot. As less damped a system is, the quicker the phase angle will drop at the resonance frequency (Figure 3.14).

![Bode plot of magnitude and phase](image)

Figure 3.14 – Typical Bode plot of magnitude and phase.

Estimating the loss factor can now be made by taking two points a and b, one above and other below the resonance frequency (Figure 3.15)

![Auxiliary frequencies in determining the loss factor](image)

Figure 3.15 – Auxiliary frequencies in determining the loss factor[9].
In Maia [9] and Ewins [12] the loss factor was derived and by trigonometry relations, was found to be

\[ \eta_r = \frac{\omega_r^2 - \omega_p^2}{\omega_r^2} \cdot \frac{1}{\tan(\Delta \theta_p) + \tan(\Delta \theta_n)} \]  \hspace{1cm} (3.1.44)

It is also known that the best results should be obtained when the angles take similar values and when they are not too small (not too close the resonance frequency).

After \( \omega_r \) and \( \eta_r \) being determined, the complex constants \( rC_{jk} \) and \( rB_{jk} \) should now be determined.

Paying attention to equation (3.1.43) we can see that \( rC_{jk} \) represents the function gain or magnitude and will directly affect the circle radius as well as its rotation. On the other hand, the complex constant \( rB_{jk} \) representing the residues, will only affect the translation between the circle and the origin.

In line with what was described above and regarding Figure 3.11 the circle diameter is given by:

\[ D = \frac{C_r}{\omega_r^2 \cdot \eta_r} \]  \hspace{1cm} (3.1.45)

This can be seen in Figure 3.11 where the phase angle is also represented as being as

\[ r\phi_{jk} = \tan^{-1} \left( \frac{x_D - x_0}{y_D - y_0} \right) \]  \hspace{1cm} (3.1.46)

The coordinates \((x_0, y_0)\) represent center of the circle and the coordinates \((x_D, y_D)\) are the displaced values from the origin. This coordinates relative to the translation of the circle are given right after the resonance frequency being located (Figure 3.11).

### 3.1.10 Rational Fraction Polynomials method

The process of identifying parameters from an FRF function is commonly called curve fitting or parameter estimation.

Rational Fraction Polynomial method (RFP method), performs curve fitting on the measurement (FRFs) curves in order to identify the modal parameters (natural frequencies, damping ratios, loss factors, etc.) of the predominant modes of vibration of the structure, and can also be used to identifying poles, zeros and resonances of combined electro-mechanical-acoustic systems.

By assuming that the frequency response measurements are taken from a linear, second order dynamical system, it can be represented by the ratio of two polynomials [13].
Figure 3.16 – Curve fitting using Rational Fraction Polynomials.

By curve fitting (Figure 3.16) the experimental curve, the analytical equation (3.1.47) with the polynomials coefficients can be determined in a least squared error criterion. Modeling the formulated problem in Laplace domain derives in

\[
H(\omega) = \frac{\sum_{k=0}^{m} a_{k} s^{k}}{\sum_{k=0}^{n} b_{k} s^{k}} \rightarrow s = j\omega
\]  
(3.1.47)

This is called the Rational Fraction Form and is only useful when poles are located along the damping axis (non-resonant systems). For resonant systems, the FRF analytical curve can be represented in a partial fraction form. This form describes the FRF in terms of parameters, giving for \(n\)-degrees of freedom \(n\)-pole pairs and clearly differentiates the residues \(r_{k}\) associated to each pole pair.

\[
H(\omega) = \sum_{k=1}^{n} \left( \frac{r_{k}}{s - p_{k}} + \frac{r_{k}^*}{s - p_{k}^*} \right)
\]  
(3.1.48)

\[k^{th}\) pole \(\rightarrow p_{k} = -\sigma_{k} + i\omega_{k}\]

(3.1.49)

After gathering all the polynomial coefficients, the natural frequencies \(\omega_{n}\) of the \(p_{k}\) pole are [20]:
\[ \omega_n(p_k) = |p_k| = \sqrt{\sigma_k^2 + i\omega_k^2} \] (3.1.50)

where the real part of the pole, from [9] and [13], is equal to

\[ \sigma_k = (-\zeta \omega_n)_k \] (3.1.51)

It is now time to rewrite equation (3.1.48) in order to damping ratio. By substituting equation (3.1.50) in equation (3.1.51), the damping ratio becomes,

\[ \zeta(p_k) = \frac{-\sigma_k}{|p_k|} \] (3.1.52)

Since that this model was established for viscous damping models, we should use the equation \( \eta \approx 2\zeta \) for lightly damped materials.

### 3.1.11 Modal fitting curve analysis

In order to help validate the loss factors when the other methods fail, iterations were made by using the receptance equation (3.1.10) for a hysteretic model. These iterations were made by making a substituting the loss factor values in the referred equation that could better approximate the analytic curve to the experimental one.

The fit was made using the Least squares method. It consists in adjusting the parameters of a model function to best fit a data set. A simple data set consists of \( n \) points (data pairs \((x_i, y_i)\), \( i=1, \ldots, n \), where \( x_i \) is an independent variable and \( y_i \) is a dependent variable whose value is found from the real curve. The analytical function takes the shape of \( f(x, \beta) \), where the \( m \) adjustable parameters are held in the vector \( \beta \). In this case the varying parameter is the loss factor \( \eta \).

The goal is to find the parameter values for the model which “best” fits the data. The least squares method finds its optimum when the sum, \( S \), of squared residuals is a minimum.

\[ S = \sum_{i=1}^{n} r_i^2 \] (3.1.53)

The difference between the actual value of the dependent variable and the value predicted by the model is called residue,
\[ r_i = y_i - f(x_i, \beta) \] (3.1.54)

### 3.1.12 Shape factor

Shape factor is a dimensionless value that represents the ratio between a loaded area and the free area in a compressive test, see Figure 3.17. Snowden [5] and Nielsen and Fuglsang [21] presented an equation to determine an apparent elastic modulus (equation (3.1.55)), which regards the contribution of this shape factor (SF). This geometric consideration should be taken into account by the designers regarding the materials purpose.

Apparent elastic modulus \[ E_a = E(1 + 2S^2) \] (3.1.55)

Therefore, the shape factor must be determined from the equation (3.1.56),

\[ SF = \frac{\text{loaded area}}{\text{free area}} \] (3.1.56)

Which for a disk with diameter \( D \) and thickness \( l \), leads to equation (3.1.57).

\[ SF = \frac{\pi D^2}{4\pi rl} = \frac{D}{4l} \] (3.1.57)
4 Materials and experimental setup

All the materials involved in this work were provided by the Amorim Cork Composites, with the following commercial designations: VC1001, VC2100, VC6400, VC5200, polychloroprene rubber (neoprene) and NL20 (natural cork).

Two types of experimental procedures were performed: the quasi-static uniaxial compressive tests and the vibration tests.

The compressive tests were carried out in the Mechanical Analysis Laboratory and the vibrations experiments were performed at Vibrations Laboratory both in the Mechanical Engineering Department (DEM), at Instituto Superior Técnico (IST).

To accomplish the main objective, to characterize the mechanical behavior of these materials, primarily quasi-static compressive experiments were carried out to determine the Young's modulus, among other parameters, from the experimental stress-strain curves.

The typical theoretic stress-strain curves for foam materials present identical behavior to the rubbercork composites, exhibiting three distinguish regions: a linear elastic region, a plateau zone and finally a densification region, see Figure 4.1. Usually, till 5 % the viscoelastic materials present a linear elastic behavior, with a slope equal to the Young modulus. As the load increases, the cells begin to collapse by elastic buckling, plastic yielding or brittle crushing, depending on mechanical properties of the cell walls. Collapse progresses at roughly constant load, giving a stress plateau, until the opposing walls in the cells meet and touch, when densification occurs, causing an abruptly increase in stress [22].

![Figure 4.1 – Typical foam stress-strain curve (quasi-static compressive test)](image)

For this work only the first region, the fully elastic region, will be object of interest in order to determine the static elastic modulus (Young's modulus) by the Hooke's law ($\sigma = E \cdot \varepsilon$).
Afterwards, vibration tests were performed to a distinct group of specimens and sets of FRF curves were recorded allowing the posterior application of modal analysis and the determination of the dynamic properties of the materials.

### 4.1 Materials

The materials chosen to perform tests were the commercial VC1001, the VC2100, the VC5200, the VC6400, Polychloroprene rubber and NL20 cork.

In Table 4.1 are present all the materials involved in this thesis as well as the estimated density from the specimens ($\rho = \frac{m}{V}$).

<table>
<thead>
<tr>
<th>Material</th>
<th>Rubber type¹</th>
<th>Cork granulates [%]</th>
<th>Density [kg/m³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polychloroprene</td>
<td>CR</td>
<td>0</td>
<td>960</td>
</tr>
<tr>
<td>VC1001</td>
<td>NR / SBR</td>
<td>10</td>
<td>450</td>
</tr>
<tr>
<td>VC6400</td>
<td>NR / CR / SBR</td>
<td>20</td>
<td>900</td>
</tr>
<tr>
<td>VC2100</td>
<td>NBR / CR</td>
<td>30</td>
<td>925</td>
</tr>
<tr>
<td>VC5200</td>
<td>CR / SBR / NBR</td>
<td>36</td>
<td>600</td>
</tr>
<tr>
<td>NL20</td>
<td>-</td>
<td>100</td>
<td>200</td>
</tr>
</tbody>
</table>

Table 4.1 – Rubbercork materials composition and density.

Table 4.2 and Table 4.3 present the datasheets from the material supplier with some mechanical properties and suitable temperature and load range applications for these materials.

Table 4.2 - NL20 mechanical properties.²

<table>
<thead>
<tr>
<th>Property</th>
<th>Method</th>
<th>Unit</th>
<th>NL20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>ASTM C271</td>
<td>Kg/m³</td>
<td>200</td>
</tr>
<tr>
<td>Compressive Strength</td>
<td>ASTM C365</td>
<td>MPa</td>
<td>0.5</td>
</tr>
<tr>
<td>Compressive Modulus</td>
<td>ASTM C365</td>
<td>psi</td>
<td>60</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>ASTM C297</td>
<td>MPa</td>
<td>0.7</td>
</tr>
<tr>
<td>Shear Strength</td>
<td>ASTM C273</td>
<td>MPa</td>
<td>6.9</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>ASTM C273</td>
<td>psi</td>
<td>5.9</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>ASTM E1530</td>
<td>W/mK</td>
<td>0.044</td>
</tr>
<tr>
<td>Loss Factor (at 1KHz)</td>
<td>ASTM E756</td>
<td>—</td>
<td>0.043</td>
</tr>
</tbody>
</table>

¹ Rubbery materials abbreviations: NR - Natural rubber; CR - Polychloroprene rubber; SBR - Styrene butadiene rubber; NBR - Acrylonitrile butadiene rubber.

² Table 4.2 and Table 4.3 present the datasheets from the material supplier with some mechanical properties and suitable temperature and load range applications for these materials.
4.2 Experimental Procedure

In order to characterize the rubbercork agglomerates, the process started by constructing models that could better describe the Single Degree of Freedom model for the vibrations tests and at the same time considering the ASTM 5992-96 and the ISO 9052-1 standards, which were the main standards found that could be helpful in this work.

The first standard is a guideline for Rubberlike materials called “Dynamic Testing of Vulcanized Rubber and Rubber-Like Materials Using Vibratory Methods” and shows the constructive aspects to be considered when developing vibration tests to rubberlike materials.

The ISO 9052-1 standard is related to the measure vibration in materials used under floating floors in houses. It gives important information about the relation of weight that should be considered when constructing the measuring chain and presents alternative ways of constructing the model to better describe the single-degree of freedom system (Figure 4.2).
4.2.1 Specimens construction

Considering the standards, and regarding the materials thickness provided by Amorim, all the rubbercork specimens (8 for compression tests and 21 for vibration tests) were cut in disk plates with 80 mm of diameter with 10 and 20 mm of thickness. Then, they were bounded between two Ck45 steel disk plates with the same diameter and with 10 mm of thickness, see Figure 4.4.

All the fifty-eight steel disks were cut from a steel rod with a hydraulic band saw, see Figure 4.3 – a). Posteriorly, due to lack of dimensional control consequence of the vibrations produced in cutting, they had to be faced in a lathe (Figure 4.3 – b) and c)) at Laboratório de Técnicas Oficinas (LTO).
The Ck45 steel has a Young modulus of 190 to 210 GPa and therefore a high stiffness value. The choice of this material for the disks was particularly important because it should prevent steel resonance from being nearby the frequency range tests and should also guarantee significant stiffness that shall prevent bending and assure that won’t be any extra vibration mode.

The specimens were bounded between two Ck45 steel disks with “Sika Tack-Panel” which is a polyurethane adhesive system designed for the economic, concealed fixing of ventilated facade panels.

4.2.2 Hardware and Equipment

Setup of the Quasi-Static tests
The quasi-static uniaxial compressive tests were performed in an electro-mechanical testing machine (INSTRON 5566) available at the Mechanical Laboratory with a 50kN load cell capacity. The tests were made at room temperature of 23°C with a fixed loading speed of 1mm/min to ensure a quasi-static response.
These quasi-static compression tests intended to be illustrative of the different regions of the stress-strain curves relative to these materials and considering that they will be destructive tests, 8 specimens (1 for each material) were specially prepared to perform these tests. Another purpose of these tests is to show how the shape factor affects the Young’s modulus. All the compressive stress-strain curves plotted can be seen in Chapter 5.

Setup of the Vibration tests
The vibration tests were performed in the Laboratory of Vibrations also in the Mechanical Department in Instituto Superior Técnico.
The equipment used to do the tests was:
- a permanent magnetic vibration exciter (shaker) - Brüel & Kjaer Type 4809 [23];
- a data acquisition equipment - Brüel & Kjaer Type 3560-C;
- a power supply unit - Brüel & Kjaer Type 2827;
- a power amplifier - Brüel & Kjaer Type 2712 [24];
- a generator module, 4/2 ch. Input/Output - Brüel & Kjaer Type 3109;
- two accelerometers - Brüel & Kjaer Type 4508B [25];
- a force transducer - Brüel & Kjaer Type 8201 [26];
Experimental setup

The setup taken in this work in order to gather the FRF curves and characterize the dynamic properties of the materials is presented in Figure 4.6. It consists in a shaker which will provide an external force to the system. This will be measured by the force transducer and for the accelerometer 1, which will give the input force and the acceleration of the lower disk, respectively. Then, an accelerometer 2 will gather the acceleration in the superior disk and also gives to the data acquisition and posteriorly analyzed by analysis software.

![Experimental setup](image)

Figure 4.6 – Experimental setup.

All the tests were performed at room temperature of 23ºC. The tests were accomplished by transmitting a random sine wave from the shaker to the bottom of the Ck45 steel plate, as shown in Figure 4.6.
5 Results and Discussion

5.1.1 Static Behavior

VC1001 Rubercork (10mm of thickness)

Despite the only 10% of cork granulates in its composition, the VC1001 is a low density material (450kg/m³) due to its manufacture method. Presenting cork granulates with 3mm of grain size and small gaps between cork and rubber, gives them lower density and extra room for elastic buckling. As shown in Figure 5.1 this composite is a very elastic material behaving almost linearly elastic till 40% of compressive strain.

In Figure 5.1 two different regions are well identified: an almost plateau region till 40% of strain and the densification region from 60-80% of strain. Till the first 10% of strain it can be seen that the material presents linear elastic behavior. By making a tendency line till 8,5% of strain, the elastic modulus of the material is 0,71 MPa with a stress-strain correlation of 99,92%.

Concluding the test, the elastic recovery of the material was measured, and after 1 minute of recovery the result was a 98%.

Figure 5.1 – Compression test in a VC1001 specimen with 10mm of thickness.
VC1001 Rubbercork (20mm of thickness)

In Figure 5.2 it is represented the stress-strain curve for the VC1001 with 20 mm of thickness. The Young’s modulus in this specimen has increased to 1.12 MPa. At 75% of strain, the experimental curve (blue line - Figure 5.2) had an unexpected behavior which was caused by the loss of adhesion of the bounded surfaces and the consequent slip of the material from its position to outside geometry bounds. Finally, the elastic recovery of the material measured 1 minute after the compression tests was 99%.

Figure 5.2 - Compression test in a VC1001 specimen with 20mm of thickness.
The Figure 5.3 presented next shows the influence of the shape factor in a compressive test for the VC1001 material. It can be seen that increasing the shape factor, by decreasing thickness, the nominal stress necessary to produce the same strain in the densification region increases significantly. This is explained because when there’s no more room inside the specimen for the cells rearrangement, they expand to the sides, to the free unloaded areas of the specimen. Since that for the 20 mm specimen the free unloaded area is two times higher, it is natural that it will allow more outward bulging on the specimen sides, allowing more barrel effect.

It can also be seen from Figure 5.3 that despite the specimen with 20 mm of thickness had suffer higher strain from compression (more 5% at 8 MPa) it had a higher elastic recovery (more 1%).

Figure 5.3 – VC1001 Stress-Strain compression curves comparison for different shape factors.
Polychloroprene rubber

Then, compression tests were performed to a Polychloroprene rubber specimen. This test was especially created in order to understand, by compassion with the VC2100 material, the influence of introducing 30% of cork granulates in a polychloroprene matrix composite. The results illustrated in Figure 5.4 show that the material is linear elastic almost till 20% of strain, with almost no defined plateau region where the polymer start to be smashed right after the elastic region. Amplifying the elastic region it can also be noted that the nominal stress values are verified 99,98% of times till 13% of strain. The static elastic modulus for this material is 13,03 MPa. The elastic recovery of the material measured 1 minute after the compression tests was 97,8%.

Figure 5.4 - Compression test in a polychloroprene specimen.
VC2100 Rubbercork

Agglomerating cork granulates with polychloroprene rubber (VC2100) causes a decrease in Young’s modulus (7.62 MPa) in the linear elastic region. However, in Figure 5.5 it can also be seen that right after the linear elastic region the material starts to be crushed causing an almost half final strain value, when compared to the polychloroprene rubber. Once more, almost at the final (7.6 MPa) the adhesive failed its purpose and the curve took an unexpected path. Finally, the elastic recovery of the material measured 1 minute after the compression tests was 98%.

Figure 5.5 - Compression test in a VC2100 specimen with 10mm of thickness.
Comparing now the two specimens presented before (polychloroprene rubber and VC2100), it can be seen from Figure 5.4 and Figure 5.5 that the introduction of cork granulates decreases the linear elastic region of the material and also increases very much the stiffness (Figure 5.6) right after 25-30% of strain. The inexistence of a plateau region gives a direct passage between the completely elastic region and the crushing cells region (densification). This stiffness increase gives the VC2100 rubber composites much less capacity to strain.

Figure 5.6 – Polychloroprene and VC2100 Stress-Strain compression curves comparison.
**VC5200 Rubbercork**

Regarding the VC5200 rubbercork composite, it presents a static elastic modulus of 6.56 MPa, with a very short linear elastic region (1 to 2.5%). The plateau region where the cells are rearranged goes from 3 to almost 40%, value from what the material starts to be smashed till a final value of 60% of extension. The elastic recovery of the material 1 minute after the compression tests was 92.9%.

![Compression test in a VC5200 specimen with 10 mm of thickness.](image)

\[ y = 6.5614x - 0.062 \]
\[ R^2 = 0.9968 \]
VC6400-NR Rubbercork (Natural Rubber)

The VC6400 with natural rubber composite shows from Figure 5.8 a completely linear elastic region from 5 to 8% with an elastic modulus of 8.77MPa. By being a high density material it shows also that the material starts to be crushed in an early stage (20-30% of strain).

The elastic recovery measured 1 minute after the compression tests was 94%.

Figure 5.8 - Compression test in a VC6400NR specimen with 10 mm of thickness.
VC6400-CR Rubbercork (Polychloroprene rubber)

On the other hand the VC6400 with chloroprene rubber as high density material with a stiffer rubber in its composition shows a linear elastic region from 4 to 7% of strain but no signal of any plateau region were the cells are being smashed right after the first region (Figure 5.9). The material is being crushed right from the beginning to the end of the test. The amplification of the linear elastic region in Figure 5.9 shows an increase of the elastic modulus (10.48 MPa) when comparing to the VC6400 with natural rubber (8.77 MPa).

The elastic recovery of the material measured 1 minute after the compression tests was 95%.

Figure 5.9 - Compression test in a VC6400CR specimen with 10 mm of thickness.
VC6400-SBR Rubbercork (Styrene butadiene rubber)

The VC6400SBR and the VC6400CR composite materials show similar values of Young’s modulus, 10.38 MPa and 10.48 MPa respectively (Figure 5.9 and Figure 5.10).

The elastic recovery of the material 1 minute after the compression tests was the same as the VC6400CR, 95%.

Figure 5.10 - Compression test in a VC6400SBR specimen with 10 mm of thickness.
To better illustrate and allow the comparison of the different VC6400 materials, Figure 5.11 encloses all in one single graphic.

It can be noted that there is almost no difference between the specimens VC6400CR and the VC6400SBR. Although there’s not much difference between the natural rubber specimen and the others, it presents a small region (10-30% of strain) where the materials cells are suffering structure rearrangements and only after 30% of strain they start being smashed against each other.

![Stress-Strain compression curves comparison](image-url)

Figure 5.11 – VC6400 Stress-Strain compression curves comparison.
The NL20-cork specimen due to its high density presents a high Young’s modulus value (16.3 MPa) in the linear elastic region, which goes from 0.7 to 2.5% of strain (Figure 5.12). Then, it presents a large plateau region were the cork cells deform elastically and plastically. At the end of the quasi-static tests, the elastic recovery of the material 1 minute after the compression tests measured was 84.2%.

Figure 5.12 - Compression test in a NL20-cork specimen with 10 mm of thickness.
5.1.2 Dynamic behavior

Next, the results from the vibration tests performed in the shaker are presented based in the inherent vibration theory explained in Chapter 3.

The “VC1001” Rubercork

As explained before, the vibration tests started with the application of a random sine to the shaker and by the acquisition of the accelerance and phase curves (Figure 5.13 and Figure 5.14) through a data acquisition box.

Subsequently, by using equations (3.1.10) and (3.1.13) we were able to plot also the FRF curves of Receptance and Mobility.

In Figure 5.13, it is shown the typical frequency response curves for a VC1001 rubercork material with 20mm of thickness.

![Figure 5.13 – Accelerance, mobility and receptance of VC1001 (specimen 5) with 20mm of thickness.](image)
The Figure 5.15 shows the representation in the Argand plane of the points near resonance that better match to a circle (blue line). The black line represents the estimated circle-fit to the points.

As illustrated in Figure 5.16, the best values of the loss factor \((\eta)\) are determined when the angles \(\theta_a\) and \(\theta_b\) (Figure 3.15) are not too small and have similar values. Usually, the first 15 degrees around the natural frequency should be disregarded. The loss factor \(\eta\) should be determined by picking the mean values of the matrix where the gradient is minimal. The top view Figure 5.16 elucidates better the direction where the gradient of \(\eta\) is minimum. Normally, picking values in a 45 degrees vector gives good loss factor values.
After determine the values of $\eta$ for the Nyquist plot from equation (3.1.44), all the modal parameters were able to be determined. These values are presented in Table 5.2 and they allowed the determination of the Circle-fit receptance curve from equation (3.1.43) as well as the analytic receptance curve from modal analysis - equation (3.1.10).

All the modal parameters from the RFP method were also determined (Table 5.3) and the estimated polynomial for this particular specimen is plotted in Figure 5.18.
Figure 5.18 – Experimental and RFP numerical curve of VC1001 (test specimen 5).

Subsequently, all the values of loss factor for the different techniques used are expressed in the next table (Table 5.1). Looking closely at the table it can be seen that the RFP method fails for specimens 6, 7 and 8 where the thickness is 10mm.

Table 5.1 – VC1001: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L[mm]</th>
<th>$f_n[Hz]$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Half-Power</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>279</td>
<td>0,235</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>272,5</td>
<td>0,226</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>248,5</td>
<td>0,246</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>244,5</td>
<td>0,240</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>284,5</td>
<td>0,256</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>308</td>
<td>0,223</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>397,5</td>
<td>0,243</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>292,5</td>
<td>0,250</td>
</tr>
</tbody>
</table>

In Figure 5.19 is represented the Transmissibility curve of the material where it can be seen that the attenuation region starts around 420Hz (crossover frequency) with the curve presenting a slope of -44,3dB/decade.
It should be noted that all the transmissibility curves are only represented after 10Hz since there is only noise at low frequencies (till 10-20Hz).

All the modal parameters for all the VC1001 specimens estimated from the Circle-fit method are presented in Table 5.2 as well as the storage stiffness and the elastic moduli calculated from those parameters. In Table 5.3 are the elastic moduli but this time obtained from the RFP method. Comparing Table 5.2 and Table 5.3 it can be realized that both methods produce similar results for the first 5 specimens, where the loss factor is identical. For specimens 6, 7 and 8 the RFP take unrealistic values of $\eta$ when comparing with the other methods.
**Modal parameters extraction of VC1001**

Table 5.2 – VC1001 modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$C_{jk}$</th>
<th>$\phi_{jk}$ [º.]</th>
<th>$B_{jk}$</th>
<th>$f_n$[Hz]</th>
<th>$\eta$</th>
<th>$K'$ [N/m]</th>
<th>$E'$ [MPa]</th>
<th>$E''$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,36</td>
<td>-3</td>
<td>3,5x10^{-7}+3,8x10^{-6}i</td>
<td>279</td>
<td>0,195</td>
<td>1,50x10^6</td>
<td>6,00</td>
<td>1,17</td>
</tr>
<tr>
<td>2</td>
<td>1,17</td>
<td>13</td>
<td>4,3x10^{-8}+1,3x10^{-8}i</td>
<td>279</td>
<td>0,178</td>
<td>1,57x10^6</td>
<td>6,24</td>
<td>1,11</td>
</tr>
<tr>
<td>3</td>
<td>1,39</td>
<td>14</td>
<td>-6,6x10^{-9}+2,6x10^{-10}i</td>
<td>254,5</td>
<td>0,204</td>
<td>1,25x10^6</td>
<td>4,95</td>
<td>1,01</td>
</tr>
<tr>
<td>4</td>
<td>1,29</td>
<td>14</td>
<td>6,1x10^{-9}+3,2x10^{-9}i</td>
<td>250</td>
<td>0,182</td>
<td>1,21x10^6</td>
<td>4,80</td>
<td>0,87</td>
</tr>
<tr>
<td>5</td>
<td>1,23</td>
<td>10</td>
<td>1,1x10^{-7}+2,7x10^{-8}i</td>
<td>291</td>
<td>0,186</td>
<td>1,69x10^6</td>
<td>6,72</td>
<td>1,25</td>
</tr>
<tr>
<td>6</td>
<td>1,29</td>
<td>9</td>
<td>1,0x10^{-7}+2,4x10^{-8}i</td>
<td>313,5</td>
<td>0,178</td>
<td>1,89x10^6</td>
<td>3,77</td>
<td>0,67</td>
</tr>
<tr>
<td>7</td>
<td>1,20</td>
<td>19</td>
<td>3,5x10^{-8}+1,7x10^{-8}i</td>
<td>411,5</td>
<td>0,187</td>
<td>3,31x10^6</td>
<td>6,60</td>
<td>1,24</td>
</tr>
<tr>
<td>8</td>
<td>1,40</td>
<td>18</td>
<td>-2,5x10^{-9}+6,5x10^{-10}i</td>
<td>300,5</td>
<td>0,215</td>
<td>1,78x10^6</td>
<td>3,54</td>
<td>0,76</td>
</tr>
</tbody>
</table>

Table 5.3 – VC1001 modal parameters and dynamic properties determined from the RFP method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues (x10^{-3})</th>
<th>$f_n$[Hz]</th>
<th>$\eta$</th>
<th>$E'$ [MPa]</th>
<th>$E''$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-24,6+277,2i</td>
<td>0,1526-0,0054i</td>
<td>278,3</td>
<td>0,176</td>
<td>6,02</td>
<td>1,06</td>
</tr>
<tr>
<td>2</td>
<td>26,6+274,2i</td>
<td>-0,1759+0,0338i</td>
<td>275,5</td>
<td>0,193</td>
<td>6,22</td>
<td>1,20</td>
</tr>
<tr>
<td>3</td>
<td>25,6+251,5i</td>
<td>-0,2216-0,0044i</td>
<td>252,8</td>
<td>0,202</td>
<td>4,96</td>
<td>1,00</td>
</tr>
<tr>
<td>4</td>
<td>24,4+247,4i</td>
<td>-0,2213+0,0052i</td>
<td>248,6</td>
<td>0,196</td>
<td>4,79</td>
<td>0,94</td>
</tr>
<tr>
<td>5</td>
<td>-29,1+286,7i</td>
<td>0,1805+0,0076i</td>
<td>288,2</td>
<td>0,202</td>
<td>6,70</td>
<td>1,35</td>
</tr>
<tr>
<td>6</td>
<td>50,3+297,2i</td>
<td>-0,4608-0,0466i</td>
<td>301,4</td>
<td>0,334</td>
<td>3,63</td>
<td>1,21</td>
</tr>
<tr>
<td>7</td>
<td>-67,3+403,1i</td>
<td>0,2943-0,0520i</td>
<td>408,6</td>
<td>0,329</td>
<td>6,38</td>
<td>2,10</td>
</tr>
<tr>
<td>8</td>
<td>-50,44+285,3i</td>
<td>0,4568-0,0434i</td>
<td>289,8</td>
<td>0,348</td>
<td>3,42</td>
<td>1,19</td>
</tr>
</tbody>
</table>
**The Polychloroprene rubber**

The polychloroprene rubber as any rubber it’s characterized by its high damping capacity. The Figure 5.20, obtained from the vibration tests, is an evidence of this fact where there is no well-defined resonance peak.

![Receptance](image1.png)

**Figure 5.20** – Experimental receptance curve of Polychloroprene Rubber.

![Phase lead](image2.png)

**Figure 5.21** – Experimental phase curve of Polychloroprene rubber.
Regarding the Argand plane representation of the experimental receptance curve (Figure 5.22), only values after 1700Hz (blue line) were taken into account in order to provide the best circle-fit minimizing the square of the error.

![Nyquist plot of Receptance](image)

Figure 5.22 – Nyquist circle-fit plot for Polychloroprene Rubber.

Figure 5.23 shows the loss factor values obtained from equation (3.1.44) and demonstrates that values don’t vary significantly. Therefore, the loss factor values should be accurate.

![Loss Factor for different data points](image)

Figure 5.23 - Loss factor of Polychloroprene rubber for different data points a) 3d view; b) top-view.
In Figure 5.24 are presented the experimental and numerical FRF receptance curves determined through the parameters inherent to the Circle-fit and RFP methods.

![Figure 5.24 - Experimental and numerical FRF curves of polychloroprene rubber, based on: a) The Circle-fit method; b) The analytical model.](image)

As it can be seen from Figure 5.25, the accuracy of the polynomial for this particular specimen is plotted in Figure 5.18.

![Figure 5.25 - Experimental and RFP numerical curve of polychloroprene rubber.](image)
Modal parameters extraction of Polychloroprene rubber

Table 5.4 - Polychloroprene Rubber: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen</th>
<th>L[mm]</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polychloroprene Rubber</td>
<td>9</td>
<td>10</td>
<td>1844</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0,675</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0,601</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0,585</td>
</tr>
</tbody>
</table>

Figure 5.26 shows the transmissibility curve for the polychloroprene rubber where, as was expect, confirms a lower transmissibility value at resonance (5,35dB) comparing to the other materials. It also shows that the isolation region starts at 2620Hz with the curve magnitude showing a slope of -62dB/decade.

In Table 5.5 and Table 5.6 all the modal parameters referents to the Circle-fit method and to the RFP method are exhibit. By paying some attention to Figure 5.26 and to Table 5.6, it can be seen that the natural frequency estimation by the RFP method is poor. On the other hand, the approach from equation (3.1.35) is only valid for light-damped systems causing has a great uncertain on the loss factor value measured from the RFP method.
Table 5.5 - Polychloroprene Rubber modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( C_{jk} )</th>
<th>( \phi_{jk} ) [(^\circ)]</th>
<th>( B_{jk} )</th>
<th>( f_n ) [Hz]</th>
<th>( \eta )</th>
<th>( K' ) [N/m]</th>
<th>( E' ) [MPa]</th>
<th>( E'' ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1.46</td>
<td>9</td>
<td>6.3x10^{-9}+5.3x10^{-9}i</td>
<td>1844</td>
<td>0.675</td>
<td>55.3x10^{6}</td>
<td>110</td>
<td>74.2</td>
</tr>
</tbody>
</table>

Table 5.6 - Polychloroprene Rubber modal parameters and dynamic properties determined from the RFP method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues (x10^{-4})</th>
<th>( f_n ) [Hz]</th>
<th>( \eta )</th>
<th>( E' ) [MPa]</th>
<th>( E'' ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>657.6+2086.8i</td>
<td>-0.0053-0.0196i</td>
<td>2188</td>
<td>0.601</td>
<td>114</td>
<td>68.4</td>
</tr>
</tbody>
</table>

The “VC2100” Rubercork

![Receptance](image)

Figure 5.27 - Experimental receptance curve of VC2100.
Figure 5.28 – Experimental phase curve of VC2100.

Figure 5.29 - Nyquist circle-fit plot for VC2100.
Figure 5.30 - Loss factor of VC2100 for different data points a) 3d view; b) top-view.

Figure 5.31 - Experimental and numerical FRF curves of VC2100 material, based on: a) The Circle-fit method; b) The analytical model.

The Figure 5.32 shows the RFP curve fitting for the VC2100.
Figure 5.32 - Experimental and numerical RFP curve of VC2100.

Table 5.7 – VC2100: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen</th>
<th>L [mm]</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VC2100</td>
<td>10</td>
<td>10</td>
<td>1577</td>
<td>0.426</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.386</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.355</td>
</tr>
</tbody>
</table>

As was expected, from above we testify that for materials with very high damping values the Half-Power method is a poor method unlike the Circle-fit and the RFP method.

Figure 5.33 shows the transmissibility curve for the VC2100 with the isolation region starting at 2340Hz.

Figure 5.33 - Transmissibility curve of the VC2100.

Once more, all parameters necessary to determine the numerical FRF receptance curve for the Circle-fit and the RFP methods were determined and are presented in Table 5.8 and
**Modal parameters extraction of VC2100**

Table 5.8 – VC2100 modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( C_{jk} )</th>
<th>( \phi_{jk} ) [º]</th>
<th>( B_{jk} )</th>
<th>( f_n ) [Hz]</th>
<th>( \eta )</th>
<th>( K' ) [N/m]</th>
<th>( E' ) [MPa]</th>
<th>( E'' ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1,73</td>
<td>-12</td>
<td>1,5\times10^{-8} +1,7\times10^{-6}i</td>
<td>1598</td>
<td>0,331</td>
<td>47,7\times10^6</td>
<td>94,8</td>
<td>31,4</td>
</tr>
</tbody>
</table>

Table 5.9 - VC2100 modal parameters and dynamic properties determined from the RFP method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues ( (x10^{-4}) )</th>
<th>( f_n ) [Hz]</th>
<th>( \eta )</th>
<th>( E' ) [MPa]</th>
<th>( E'' ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-313,0+1589,8i</td>
<td>0,0477-0,0279i</td>
<td>1620,4</td>
<td>0,386</td>
<td>90,8</td>
<td>35,1</td>
</tr>
</tbody>
</table>
The “VC5200” Rubercork

Figure 5.34 – Experimental receptance curve of VC5200 – test specimen 11.

Figure 5.35 – Experimental phase curve of VC5200 – test specimen 11.
Figure 5.36 - Nyquist circle-fit plot for VC5200 – test specimen 11.

Figure 5.37 - Loss factor of VC5200 for different data points a) 3d view; b) top-view.
Figure 5.38 - Experimental and numerical FRF curves of VC5200 material, based on:
   a) The Circle-fit method; b) The analytical model.

Figure 5.39 - Experimental and numerical RFP curve of VC5200 – test specimen 11.
Table 5.10 – SP52: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L[mm]</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
<th>Half-Power</th>
<th>Nyquist Circle-Fit</th>
<th>RFP</th>
<th>Analytic</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>10</td>
<td>1224</td>
<td>0,359</td>
<td>0,249</td>
<td>0,257</td>
<td>0,259</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>1221,5</td>
<td>0,375</td>
<td>0,234</td>
<td>0,242</td>
<td>0,255</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>1256,5</td>
<td>0,420</td>
<td>0,304</td>
<td>0,221</td>
<td>0,265</td>
<td></td>
</tr>
</tbody>
</table>

In Figure 5.40 is represented the transmissibility curve of the material where it can be seen that the attenuation region starts around 1880Hz (crossover frequency).

![Transmissibility curve of the VC5200 - specimen 11.](image-url)
Modal parameters extraction of VC5200

Table 5.11 – VC5200 modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( C_{jk} )</th>
<th>( \phi_{jk} [\text{º}] )</th>
<th>( B_{jk} )</th>
<th>( f_{u} [\text{Hz}] )</th>
<th>( \eta )</th>
<th>( K' [\text{N/m}] )</th>
<th>( E' [\text{MPa}] )</th>
<th>( E'' [\text{MPa}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>1,44</td>
<td>-2</td>
<td>2,0 \times 10^{-8} +4,5 \times 10^{-9}i</td>
<td>1224</td>
<td>0,249</td>
<td>25,9 \times 10^{6}</td>
<td>51,6</td>
<td>12,9</td>
</tr>
<tr>
<td>12</td>
<td>1,39</td>
<td>-1</td>
<td>1,9 \times 10^{-8}+4,4 \times 10^{-9}i</td>
<td>1221,5</td>
<td>0,234</td>
<td>26,0 \times 10^{6}</td>
<td>51,8</td>
<td>12,2</td>
</tr>
<tr>
<td>13</td>
<td>1,49</td>
<td>-10</td>
<td>1,8 \times 10^{-8}+1,3 \times 10^{-9}i</td>
<td>1256,5</td>
<td>0,304</td>
<td>26,4 \times 10^{6}</td>
<td>52,5</td>
<td>16,0</td>
</tr>
</tbody>
</table>

Table 5.12 – VC5200 modal parameters and dynamic properties determined from the RFP method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues (x10^{-4})</th>
<th>( f_{u} [\text{Hz}] )</th>
<th>( \eta )</th>
<th>( E' [\text{MPa}] )</th>
<th>( E'' [\text{MPa}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>156,4+1208,3i</td>
<td>-0,0463+0,0147i</td>
<td>1218,4</td>
<td>0,257</td>
<td>51,5</td>
<td>13,2</td>
</tr>
<tr>
<td>12</td>
<td>139,2+1230,4i</td>
<td>-0,0308-0,00380i</td>
<td>1238,2</td>
<td>0,225</td>
<td>51,9</td>
<td>11,7</td>
</tr>
<tr>
<td>13</td>
<td>137,1+1255,1i</td>
<td>-0,0226+0,0106i</td>
<td>1262,6</td>
<td>0,217</td>
<td>53,7</td>
<td>11,7</td>
</tr>
</tbody>
</table>

The “VC6400-NR” Rubercork

The vibration tests performed to the VC6400 with natural rubber presented the following experimental FRF receptance curve illustrated in Figure 5.41.

![Receptance](image)

Figure 5.41 - Experimental receptance curve of VC6400 – Natural Rubber.
Figure 5.42 - Experimental phase curve of VC6400 – Natural Rubber.

Figure 5.43 - Nyquist circle-fit plot for VC6400 – Natural Rubber.
Figure 5.44 - Loss factor of VC6400 for different data points a) 3d view; b) top-view.

The polynomial estimated from the modal parameters presented in Table 5.15 led to the fitting curve (red line) illustrated in Figure 5.46.

Figure 5.45 - Experimental and numerical FRF curves of VC6400NR material, based on: a) The Circle-fit method; b) The analytical model.
As we can see from Table 5.13, the loss factor value for the specimen number 17 is very different. The cause of this problem can be found in the points near the natural frequency, see Figure 5.47. In this case, the error propagates to the Circle-fit method since it uses precisely those values to determine the loss factor data points. Therefore, only the loss factor from the RFP method, supported by the analytical fitting approach, turns out to be useful.

Table 5.13 – VC6400: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( L [\text{mm}] )</th>
<th>( f_n [\text{Hz}] )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Half-Power</td>
<td>Nyquist Circle-Fit</td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td>0,420</td>
<td>0,275</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>0,396</td>
<td>0,289</td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td>0,270</td>
<td>0,237</td>
</tr>
<tr>
<td>17</td>
<td>12,5</td>
<td>0,196</td>
<td>0,290</td>
</tr>
<tr>
<td>18</td>
<td>12,5</td>
<td>0,227</td>
<td>0,217</td>
</tr>
<tr>
<td>19</td>
<td>12,5</td>
<td>0,238</td>
<td>0,206</td>
</tr>
</tbody>
</table>
Figure 5.47 - Experimental receptance curve of VC6400 – test specimen 17.

Modal parameters extraction of VC6400

Table 5.14 – VC6400 modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$C_{jk}$</th>
<th>$\phi_{jk}$ [°]</th>
<th>$B_{jk}$</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
<th>$K'$ [N/m]</th>
<th>$E'$ [MPa]</th>
<th>$E''$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>1.56</td>
<td>-7</td>
<td>$1.2 \times 10^8 + 8.1 \times 10^{10}i$</td>
<td>1512</td>
<td>0.275</td>
<td>43.0x10^6</td>
<td>85.7</td>
<td>23.6</td>
</tr>
<tr>
<td>15</td>
<td>1.56</td>
<td>-14</td>
<td>$1.3 \times 10^8 + 3.5 \times 10^{10}i$</td>
<td>1623</td>
<td>0.289</td>
<td>49.7x10^6</td>
<td>98.9</td>
<td>28.7</td>
</tr>
<tr>
<td>16</td>
<td>1.51</td>
<td>-6</td>
<td>$2.1 \times 10^8 + 3.2 \times 10^{9}i$</td>
<td>1350,5</td>
<td>0.237</td>
<td>34.8x10^6</td>
<td>69.1</td>
<td>16.4</td>
</tr>
<tr>
<td>17</td>
<td>1.43</td>
<td>-12</td>
<td>$9.7 \times 10^{9} - 1.2 \times 10^{9}i$</td>
<td>1428</td>
<td>0.290</td>
<td>38.3x10^6</td>
<td>95.3</td>
<td>27.7</td>
</tr>
<tr>
<td>18</td>
<td>1.74</td>
<td>-9</td>
<td>$2.1 \times 10^8 + 1.8 \times 10^{9}i$</td>
<td>1399</td>
<td>0.217</td>
<td>36.7x10^6</td>
<td>91.3</td>
<td>19.8</td>
</tr>
<tr>
<td>19</td>
<td>1.44</td>
<td>-5</td>
<td>$1.4 \times 10^{8} + 1.9 \times 10^{9}i$</td>
<td>1534,5</td>
<td>0.206</td>
<td>44.9x10^6</td>
<td>112</td>
<td>23.0</td>
</tr>
</tbody>
</table>

Table 5.15 – VC6400 modal parameters and dynamic properties determined from the RFP method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues ($x10^4$)</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
<th>$E'$ [MPa]</th>
<th>$E''$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>199.9+1446,6i</td>
<td>-0.6223-0.1728i</td>
<td>1460.3</td>
<td>0.274</td>
<td>85.7</td>
<td>23.5</td>
</tr>
<tr>
<td>15</td>
<td>-228.9+1494,5i</td>
<td>0.0339-0.0557i</td>
<td>1512</td>
<td>0.303</td>
<td>98.6</td>
<td>29.9</td>
</tr>
<tr>
<td>16</td>
<td>-138.5+1338,1i</td>
<td>0.0438+0.0050i</td>
<td>1345.2</td>
<td>0.206</td>
<td>69.6</td>
<td>14.3</td>
</tr>
<tr>
<td>17</td>
<td>-118.1+1409,1i</td>
<td>0.0219+0.0070i</td>
<td>1414</td>
<td>0.167</td>
<td>97.9</td>
<td>16.4</td>
</tr>
<tr>
<td>18</td>
<td>-142.6+1354,2i</td>
<td>0.0633+0.0166i</td>
<td>1361.7</td>
<td>0.210</td>
<td>91.4</td>
<td>19.1</td>
</tr>
<tr>
<td>19</td>
<td>-151.6+1541,9i</td>
<td>0.0292-0.0101i</td>
<td>1549.3</td>
<td>0.196</td>
<td>112</td>
<td>21.9</td>
</tr>
</tbody>
</table>
The NL20-Cork

The NL20 with 100% of cork was also subject to vibration tests producing the acquired frequency response function curves illustrated in Figure 5.48 and Figure 5.49.

Figure 5.48 - Experimental receptance curve of NL20 cork – test specimen 20.

Figure 5.49 - Experimental phase curve of NL20 cork – test specimen 20.
From Figure 5.51 it can be seen that the best loss factor values, where the gradient in minimum, are determined in a 45 degrees direction.
Table 5.16 – NL20: natural frequency and loss factors from four different extraction techniques.

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen</th>
<th>L [mm]</th>
<th>$f_n$ [Hz]</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Half-Power</td>
</tr>
<tr>
<td>NL20</td>
<td>20</td>
<td>10</td>
<td>1463</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>10</td>
<td>1390</td>
<td>0.105</td>
</tr>
</tbody>
</table>

Figure 5.52 - Experimental and numerical FRF curves of NL20-cork material, based on:

- a) The Circle-fit method;
- b) The analytical model.

Figure 5.53 - Experimental and numerical RFP curve of NL20.
Figure 5.54 presents the transmissibility curve of the NL20 material. It can be noticed that the material amplifies the imposed force almost 25 times at resonance frequency and the crossover frequency is around the 2250Hz with a curve slope of -48.8db/decade.

![Transmissibility curve of the NL20 specimen](image)

Figure 5.54 – Transmissibility curve of the NL20 - specimen 20.

**Modal parameters extraction of NL20 - cork**

Once more, all the modal parameters necessary to establish the receptance equation were deduced from the Circle-fit equations (3.1.44), (3.1.45) and (3.1.46) and are presented next in Table 5.17.

The method described in section 3.1.10 was used to determine the poles, the natural frequency and the correspondent loss factor, from the approximation \( \eta = 2\zeta \). All the results are exhibited in Table 5.18.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( C_{jk} )</th>
<th>( \phi_{jk} [\text{º}] )</th>
<th>( B_{jk} )</th>
<th>( f_n [\text{Hz}] )</th>
<th>( \eta )</th>
<th>( K' [\text{N/m}] )</th>
<th>( E' [\text{MPa}] )</th>
<th>( E'' [\text{MPa}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.22</td>
<td>8</td>
<td>-1.7x10^{-9} +6.5x10^{-10}i</td>
<td>1463</td>
<td>0.077</td>
<td>34.3x10^{6}</td>
<td>90.7</td>
<td>7.00</td>
</tr>
<tr>
<td>21</td>
<td>1.29</td>
<td>10</td>
<td>-2.1x10^{-9} +6.3x10^{-10}i</td>
<td>1390</td>
<td>0.100</td>
<td>38.0x10^{6}</td>
<td>81.9</td>
<td>8.22</td>
</tr>
</tbody>
</table>

Table 5.17 – NL20 modal parameters and dynamic properties determined from the Circle-fit method.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Poles</th>
<th>Residues (x10^{-3})</th>
<th>( f_n [\text{Hz}] )</th>
<th>( \eta )</th>
<th>( E' [\text{MPa}] )</th>
<th>( E'' [\text{MPa}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>-71.6+1456.5i</td>
<td>0.0488+0.0028i</td>
<td>1458,2</td>
<td>0.098</td>
<td>90.5</td>
<td>8.89</td>
</tr>
<tr>
<td>21</td>
<td>89.4+1397.5i</td>
<td>0.0487-0.0066i</td>
<td>1404,5</td>
<td>0.123</td>
<td>81.7</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Table 5.18 – NL20 modal parameters and dynamic properties determined from the RFP method.
Loss factor frequency dependence
Finally, for the VC1001 and for the VC5200 materials were performed additional vibration tests by coupling more mass to the measuring chain representative of the SDoF system. The Figure 5.55 shows two of the four tests made in the vibrations laboratory.
By picking randomly two specimens of different materials (VC1001 - specimen 5 and VC5200 - specimen 11) and adding consecutively Ck45 disk plates, vibration tests were performed with the double and the triple of the initial mass of the system, see Figure 5.55.

Figure 5.55 – a) Vibration test of VC1001 specimen with three times the initial mass; b) Vibration test of VC5200 specimen with two times the initial mass.

The results obtained from the VC1001 specimen tests are illustrated in Figure 5.56, where the loss factor variation with frequency is represented. The results clearly show low frequency dependence of the loss factor, and respectively of the storage modulus (E'), in this material for this frequency range.

Figure 5.56 – Loss factor frequency dependence in a VC1001 material.
The Figure 5.57 shows almost the same results for the VC5200 rubbercork material with no significantly change in loss factor (5%).

![Graph showing loss factor frequency dependence in a VC5200 material.](image)

**Figure 5.57 -** Loss factor frequency dependence in a VC5200 material.

It can be concluded from Figure 5.56 and Figure 5.57 that these materials for the tested frequency range don't show any significant change in loss factor however, they should be tested for larger frequency spectrum in order to characterize them completely.

Finally, it should be noted that all the tests were made at room temperature and at constant humidity.
6 Conclusions and future work

This work came to fill a lack of information that was identified in other research group's investigations, namely a better characterization of the dynamic behavior of the rubercork composite materials. For the first time, the dynamic analysis was extended to high frequencies (3200 Hz) with consistent results. The interest of the company who provided the materials in this study was high due to the potentially introduction of the products in the market and due to their short capacity in presenting materials data sheets with dynamic mechanical properties from low to high frequencies, which allowed the development of this investigation in a proper and adequate environment.

Regarding the compression tests performed in this work, the main conclusions were that:
The VC1001 material presents the lowest Young's Modulus (0.71 MPa) while the NL20 cork has the highest elastic modulus of 16.33 MPa.
Comparing the two VC1001 specimens (10 and 20 mm), due to a higher unloaded area in the 20 mm specimen, allowing more outward bulging on the specimen sides, the nominal stress necessary to produce the same strain in the densification region is lower than the 10 mm specimen.
The VC2100 is the stiffer rubercork material, presenting only 45% of strain at 8MPa.

Concerning the vibrations test it was concluded that:
By increasing the material shape factor, e.g., by decreasing thickness, it was observed that the natural frequency increases, which can be explained by the implicit increase of the material stiffness. Therefore, if in a certain application the objective is to increase the isolation region, decreasing the shape factor can be a solution. This can be made by two ways: decreasing the loaded area (through diameter) or increasing the free areas (through thickness). Another solution for shifting the resonance frequency to lower values can be by changing the type of rubber in the agglomerate. Rubbers with lower stiffness and consequently lower Young's modulus (like natural rubber) will do the work.
The VC1001 despite being 90% of high density rubber (polychloroprene), their manufacture process provides them small "interstices" between the rubber and the cork. These spaces give them lower density and the resonance frequency is "pushed" to lower frequencies presenting then an increased isolation region.
From the specimens 9 and 10 it can be also concluded that the introduction of 30% of cork in a chloroprene rubber based composite brought the resonance frequency to lower frequencies (around 250Hz), reducing in this manner the stiffness and the elastic moduli and at the same time the damping of the material.
The natural rubber presented in specimens 16, 18 and 19 brings the resonance frequency for lower values comparing to the other specimens containing chloroprene rubber and styrene butadiene rubber. The loss factor also decreases with natural rubber, showing that chloroprene rubber provides more damping capacity.
It was also concluded that the loss factor for VC1001 and VC5200, in the frequency range measured, do not vary with frequency when at room temperature and with constant humidity (Figure 5.56 and Figure 5.57).

Concerning the methods used, it can be said that the Circle-fit method was the most reliable implemented method. The Half-Power method as expected is only useful for very lightly damped systems, were the resonance peak can be well distinguished. The Circle-fit method just reveals problems when the FRF also have problems precisely around the resonance frequency, just like Imregun exposed in [15]. The RFP method turns out to be very sensitive to any existent noise and it is always necessary to provide the right points around the resonance in order to obtain good results. The iterative method using the hysteretic model equation was in most cases correct and useful in validating the values from the other methods.

To complete the characterization of these materials, it would be interesting to perform DMA tests in order to study their humidity and temperature dependency, and compare the results at 23°C coincide with the values determined in this work. Determining also the glass transition temperature of these viscoelastic materials would be also important for the materials global characterization.
7 References


22. **Avalle, M., Belingardi, G. e Montanini, R.** Characterization of polymeric structural foams under compressive impact loading by means of energy-absorption diagram, Sant' Agata - Messina.

23. **Kjaer, Brüel &.** Vibration Exciter: Types 4809 [online] [Download on 10-10-2011]. Available at:<http://wwwcascina virgo.infn.it/EnvMon/List/Shakers/mediumShaker_BK4809.pdf>.


34. **Richardson, Mark H. e Formenti, David L.** Global curve fitting of frequency response measurements using the rational fraction polynomial method, San Jose, California, 1985.


8 APPENDIX

8.1 Matlab code

The code presented next was developed in order to implement the 3 modal parameters identification techniques and also to work the data acquired from the vibration tests.

The Half-Power, the Circle-fit and the RFP methods as well as an analytical interpolation of the experimental data from the modal analysis model equation (3.1.10).

clc
clear all
close all

%Caixa de diálogo que pergunta qual o ficheiro excel que o utilizador pretende abrir
[filename, titulo] = uigetfile('*.xlsx', 'Escolha o ficheiro excel:');
[typ, desc, fmt] = xlsfinfo(filename);
[linhas colunas] = size(desc);

%Caixa de diálogo que pergunta qual a folha de excel que o utilizador pretende abrir
[s, v] = listdlg('PromptString', 'Escolha a folha de excel:',
                 'SelectionMode', 'single',
                 'ListString', desc);
sheet = desc(v, s);

%Leitura do ficheiro excel
A = xlsread('1001.xlsx', '1001_1', 'B3:M6403');
A = xlsread(filename, sheet, 'B3:M6403');
[linhas_A colunas_A] = size(A);
rows_A = linhas_A;

A(:, 4) = [];
Amplitude = A(:, [1 11]);
Fase = A(:, [1 4]);
incremento_f = A(2, 1) - A(3, 1);

%Gama de valores que abrange o pico de ressonancia
[zz vv] = find(A == max(A(400:rows_A, 5)));
Aux0 = zz;
T = Aux0 - 200;
U = Aux0 + 200;

%Amplitude Máxima obtida da Acelerância
QQ = max(A(T:U, 5));
%Encontra a linha e coluna da Amplitude máxima na Acelerância
[rr tt] = find(A==max(A(T:U,5))); WnA=A(rr,1);

%Amplitude Máxima obtida da Receptância
Q=max(Amplitude(T:U,2));

%Encontra a linha e coluna da Amplitude máxima na Receptância
[l c] = find(Amplitude==max(Amplitude(T:U,2))); WnR=Amplitude(l,1);

%Verifica se a frequência de ressonância da receptância está dentro de uma banda de xHz e encontra a frequência de ressonância real
Banda_A=WnA+20; Banda_B=WnA-20;
if ((Banda_B<WnR)&&(WnR<Banda_A))
    Wn=Amplitude(l,1);
    auxiliar=1;
else
    Wn=A(rr,1);
    auxiliar=2;
end

%Determinação do valor corrigido de Wn
% 1) Pela mudança de sinal da 2ª derivada de gama
banda=50;

a1=A((Wn*2+1)-banda:(Wn*2+1)+banda,1);
gamas=Fase((Wn*2+1)-banda:(Wn*2+1)+banda,2);

deltas=zeros(banda*2,1);
for i=1:(banda*2)
deltas(i,1)=gamas(i+1,1)-gamas(i,1);
end
deltas_quadrado=zeros(banda*2-1,1);
for j=1:(banda*2-1)
deltas_quadrado(j,1)=deltas(j+1)-deltas(j,1);
end
```matlab
maximo_deltas = max(deltas(:,1));
[linha1 coluna1] = find(deltas == max(deltas(:,1)));

if (max(deltas(:,1)) > 160)
    Wn = a1(linha1+1,1);
else
    end

% 2) Pelo polinómio das diferenças divididas de Newton

x1 = A((Wn*2+1) - banda:(Wn*2+1) + banda,1);
y1 = Fase((Wn*2+1) - banda:(Wn*2+1) + banda,2);
x = (x1.^2);
y = (y1);

% x = [-5, 0, 7, 12, 25];
% y = [-12, 4, 15, 8, 2];
% x = X; y = Y;
n = length(x);
if length(x) ~= length(y)
    error('X and Y must have the same dimension')
end

% inserting x into 1st column of DD-table
DD(1:length(x),1) = x;

% inserting y into 2nd column of DD-table
DD(1:length(y),2) = y;

% creates divided difference coefficients
for j = 1:n-1
    for k = 1:n-j % j goes from 1 to n-1
        DD(k,j+2) = (DD(k+1,j+1) - DD(k,j+1)) / (DD(k+j,1) - DD(k,1));
    end
end

% Create a matrix Prod, that has DD(1,3)*(x-x(1)) in the first row,
% DD(1,4)*(x-x(2)) in the second and so on.
% We first create a n-1 by n matrix of zeros. If we have n points, we will
% get a n-1 degree polynomial.
Prod = zeros(n-1,n);
p = conv(1,[1,-x(1)]); % creates a poly (x-x(1))

for i = 1:n-1
    Prod(i, (n-(length(p)-1)):n) = DD(1,i+2)*p;
p = conv(p,[1,-x(i+1)]);
end
```

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% sum the columns of the matrix Prod to produce the polynomial P
P = sum(Prod,1);

% add y(1) to the last element of P, which is a_0 in a polynomial
P(1,n)=P(1,n)+y(1);

% plot polynomial p for values x_eval
%figure (33)
x_eval = min(x):0.8:max(x);
y_eval = polyval(P,x_eval);
plot(x_eval,y_eval); hold on;

% plot points used
for i = 1:n
plot(x(i),y(i),'or');
end

Coefficients = DD(1,3:end);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Aux11=rr-ampl;
end
[aa bb] = find(Aux1==min(Aux1(l-Aux11:1,1)));
[cc dd] = find(Aux2==min(Aux2(1:Aux11,1)));
dd;
w1= Amplitude(aa,1);
w2= Wn+Amplitude(cc,1);
Qraiz2=Q/sqrt(2);
CoefAmort=0.5*(w2-w1)/Wn;
LF_Wn=2*CoefAmort;
if (auxiliar==1)
    NN=1;
    MM=rows_A;
else
    NN=rr;
    MM=rows_A;
end

%==========================================================================
%##########################################################################
%Circle-Fit Newton-Gauss
%Faz um fit aos pontos a partir de Wn, visto que são aquele que mais se
%aproximam a uma circunferência
%==========================================================================
xx1=A(NN:MM , 9);
yy1=A(NN:MM , 10);
a=[xx1_yy1_ones(size(xx1))]\-(xx1.^2+yy1.^2);
x1c = -.5*a(1);
y1c = -.5*a(2);
RR = sqrt((a(1)^2+a(2)^2)/4-a(3));
Circ_fiit=zeros(360,2);
for bbb=1:360
    c=bbb*pi/360;
    Circ_fiit(bbb,1)=x1c+RR*cos(c);
    Circ_fiit(bbb,2)=y1c+RR*sin(c);
end
%==========================================================================
%##########################################################################
%Calcula a distância de todos os pontos da curva ao Circle-fit-Wn
PP=MM-NN;
distpontoss=zeros(PP,360);
for kkk=1:360;
    for jj=1:PP;
        distpontoss(jj,kkk)=sqrt(((Circ_fiit(kkk,1)-xx1(jj,1)))^2+((Circ_fiit(kkk,2)-yy1(jj,1)))^2);
    end
end
%==========================================================================
% Variável auxiliar que calcula a distância mínima de cada ponto da curva real à circunferência teórica (circle-fit)
Aux8=zeros(PP,1);
for tt=1:PP
    Aux8(tt,1)=min(distpontoss(tt,1:360));
end

% FOLGA
% Determina quais os valores da curva real que serão utilizados no fit. Para tal utiliza os pontos dentro da "folga" que é 10x superior à distância média dos pontos do 1º fit (Circle-fit-Wn)
dist_media=sum(Aux8)/(PP);
folga=dist_media*10;

xx2=A(:, 9);
yy2=A(:, 10);

PPP=rows_A;
distpontox=zeros(PPP,360);
for hhhh=1:360
    for iiii=1:PPP
        distpontox(iiii,hhhh)=sqrt((Circ_fiit(hhhh,1)-xx2(iiii,1))^2+(Circ_fiit(hhhh,2)-yy2(iiii,1))^2);
    end
end

Aux9=zeros(PP,1);
for ttt=1:PP
    Aux9(ttt,1)=min(distpontox(ttt,1:360));
end

Aux10=0;
for tttt=1:PP
    if ((Aux9(tttt,1)/folga)<2)
        Aux10=tttt;
        break
    end
end

if (Aux10==tttt)
    Aux10=tttt;
else if (auxiliar==1)
    Aux10=1-200;
else
    Aux10=rr-200;
end

N=Aux10;
M=rows_A;
P=M-N;
x=A(N:M , 9);
y=A(N:M , 10);

%2º Circle-fit
b=[x y ones(size(x))\-(x.^2+y.^2);
xc = -.5*b(1);
yc = -.5*b(2);
R  =  sqrt((b(1)^2+b(2)^2)/4-b(3));

%##########################################################################

%PLOTS
%==========================================================================

[amp1 amp2]=find(Amplitude==min(Amplitude(2:l,2))); if (auxiliar==1)
figure(1)
x1=Amplitude(50:end,1);
y1=20*log10(abs(Amplitude(50:end,2))); plot(x1,y1);
title('Receptance','FontSize',16);
xlabel('Frequency [Hz]')
ylabel('Magnitude [dB re. 1 m/N]')
pause else
figure(1)
x1=Amplitude(50:end,1);
y1=20*log10(Amplitude(50:end,2)); plot(x1,y1);
title('Receptance','FontSize',16);
xlabel('Frequency [Hz]')
ylabel('Magnitude [dB re. 1 m/N]')
pause end

figure(2)
x2=Fase(1:U+200,1);
y2=Fase(1:U+200,2);
plot(x2,y2);
title('Phase lead','FontSize',16);
xlabel('Frequency [Hz]')
ylabel('Phase [º]')
pause

figure(3)
plot(xx1,yy1)
hold on,
plot(x,y)
hold on,
t=(0:PP)*2*pi/PP;
plot( RR*cos(t)+xlc, RR*sin(t)+ylc ,'-r') ;
hold on,
plot( R*cos(t)+xc, R*sin(t)+yc ,'-k') ;
title('Nyquist plot of Receptance','FontSize',16);
xlabel('Re \alpha(w)')
ylabel('Im \alpha(w)')
pause

figure(4)
plot(x,y)
hold on,
t=(0:P)*2*pi/P;
plot( R*cos(t)+xc, R*sin(t)+yc ,'-k' );
title('Nyquist plot of Receptance','FontSize',16);
xlabel('Re \alpha(w)')
ylabel('Im \alpha(w)')
hold on,
%

%==========================================================================
%Descobre se Wn é o mínimo do círculo ou se há rotação (teta)%
%==========================================================================
Circ_fit=zeros(360,2);
for bb=1:360
    c=bb*2*pi/360;
    Circ_fit(bb,1)=xc+R*cos(c);
    Circ_fit(bb,2)=yc+R*sin(c);
end

%Mínimo teórico do círculo do fit
[ee dd] = find(Circ_fit==min(Circ_fit(:,2)));
Xn_fit=Circ_fit(ee,1);
Yn_fit=Circ_fit(ee,2);
%

%==========================================================================
%Encontra a distância de cada ponto do fit à curva real%
%==========================================================================
distpontos=zeros(P,360);
for k=1:360
    for j=1:P
        distpontos(j,k)=sqrt(((Circ_fit(k,1)-x(j,1)))^2+((Circ_fit(k,2)-
y(j,1)))^2);
    end
end
%

%==========================================================================
%Encontra a distância Mínima de cada ponto do fit à curva real%
%==========================================================================
Aux3=zeros(360,1);
distWn=zeros(360,1);
if (auxiliar==1)
for o=1:360
    Aux3(o,1)=min((distpontos(1:P,o)));
    distWn(o,1)=sqrt(((Circ_fit(o,1)-A(l,9)))^2+((Circ_fit(o,2)-A(l,10)))^2);
end
else
    for oo=1:360
        Aux3(oo,1)=min((distpontos(1:P,oo)));
        distWn(oo,1)=sqrt(((Circ_fit(oo,1)-A(rr,9)))^2+((Circ_fit(oo,2)-A(rr,10)))^2);
    end
end

[aaaa bbbb]= find(distWn==min(distWn(:,1)));
text(Circ_fit(aaaa+90,1), Circ_fit(aaaa+90,2), '\leftarrow W_b','FontSize',14);
end
end
end

end

text(Circ_fit(aaaa-90,1), Circ_fit(aaaa-90,2), '\leftarrow W_b','FontSize',14);
plot(rcx_1,rcy_1,'--g')
hold on,
plot(rcx_2,rcy_2,'--g')
p0=[0,0];
pl=[rx(2,1),ry(2,1)];
hold on,
vectarrow(p0,pl);
xlabel('Re \alpha(w)')
ylabel('Im \alpha(w)')
rc=[rcx_1, rcx_2; rcy_1, rcy_2];
%area(rc)
pause

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Encontra a posição de cada mínimo do circle-fit à curva real
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
posicao=zeros(360,1);
for ll=1:360
    for n=1:P
        if (distpontos(n,ll)==Aux3(ll,1))
            posicao(ll,1)=n;
        else
            end
    end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Determina todas as coordenadas dos pontos do circle-Fit da Meia Potencia
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
xya=zeros(90,2);
xyb=zeros(90,2);
tetan=aaaa-270;
for teta=1:90
    if (teten>0)
        xya(teta,1)=xc+R*sind(teta+tetan);
        xya(teta,2)=yc-R*cosd(teta+tetan);
        xyb(teta,1)=xc+R*sind(-teta+tetan);
        xyb(teta,2)=yc-R*cosd(-teta+tetan);
    elseif (teten<0)
        xya(teta,1)=xc+R*sind(teta+tetan);
        xya(teta,2)=yc-R*cosd(teta+tetan);
        xyb(teta,1)=xc+R*sind(-teta+tetan);
        xyb(teta,2)=yc-R*cosd(-teta+tetan);
    end
end
xya(teta,1)=xc+R*sind(teta-tetan);
xya(teta,2)=yc-R*cosd(teta-tetan);
xyb(teta,1)=xc+R*sind(-teta+tetan);
xyb(teta,2)=yc-R*cosd(-teta+tetan);
else
  xya(teta,1)=xc+R*sind(teta);
  xya(teta,2)=yc-R*cosd(teta);
  xyb(teta,1)=xc+R*sind(-teta);
  xyb(teta,2)=yc-R*cosd(-teta);
end
%==========================================================================
%##########################################################################
%Cálcula as distâncias de cada ponto do fit a Wa e Wb reais

distWa=zeros(P,90);
distWb=zeros(P,90);
for q=1:90
  for r=1:P
    %Aux3(o,1)=min((distpontos(1:3500,o)));
    distWa(r,q)=sqrt(((xya(q,1)-x(r,1)))^2+((xya(q,2)-y(r,1)))^2);
    distWb(r,q)=sqrt(((xyb(q,1)-x(r,1)))^2+((xyb(q,2)-y(r,1)))^2);
  end
end
%==========================================================================
%##########################################################################
%Cálcula as variáveis auxiliares Aux4 e Aux5 que representam os
%mínimos das distâncias entre o valor do fit e o valor real de Wa e Wb

Aux4=zeros(90,1);
Aux5=zeros(90,1);
for oo=1:90
  Aux4(oo,1)=min((distWa(1:P,oo)));
  Aux5(oo,1)=min((distWb(1:P,oo)));
end
%==========================================================================
%##########################################################################
%Determina a posição de Wa e Wb

posicaoWa=zeros(90,1);
for ml=1:90
  for n1=1:P
    if (distWa(n1,ml)==Aux4(ml,1))
      posicaoWa(ml,1)=n1;
    else
      end
  end
end

posicaoWb=zeros(90,1); %=1:90 zeros(size(posicaoWb))
for m2=1:90
for n2=1:P
    if (distWb(n2,m2)==Aux5(m2,1))
        posicaoWB(m2,1)=n2;
    else
        end
    end
end

%Coloca todos os valores de posicaoWa a 1's se posicaoWB(1,1)=1 ou
%posicaoWB(90,1)=P ou posicaoWB=1
if ((posicaoWB(1,1)==1)||(posicaoWB(90,1)==P)||(posicaoWa(90,1)==1)||(posicaoWa(90,1)==P))
    for m3=90:-1:1
        posicaoWa(m3,1)=1;
    end
end

%Matrix auxiliar com todos os valores de posicao Wa e Wb
Aux_pos=[posicaoWa;posicaoWb];

%Verifica os valores de posicaoWB se sao 1's e atribui valores para esses
%com o gradiente dos restantes
linha_a=0;
linha_b=0;
for m4=180:-1:1
    if (Aux_pos(m4,1)==1)
        grad_pos=gradient(Aux_pos(m4+1:180,1));
        grad_Aux_pos=-sum(grad_pos)/(180-m4);
        linha_b=m4;
        for m5=m4:-1:1
            Aux_pos(m5,1)=Aux_pos(m5+1,1)+grad_Aux_pos;
        end
        break
    end
end

posicaoWa(:,1)=Aux_pos(1:90,1);
posicaoWb(:,1)=Aux_pos(91:180,1);

%Determina os varios valores de eta ao longo de 90 graus
Wa=zeros(90,1);
Wb=zeros(90,1);
etar=zeros(90,1);
for zz=1:90
    Wa(zz,1)=(N+posicaoWa(zz,1))/(incremento_f^(-1))-incremento_f;
    Wb(zz,1)=(N+posicaoWb(zz,1))/(incremento_f^(-1))-incremento_f;
end
\begin{verbatim}
eta_r(zz,1)=(Wb(zz,1)^2-Wa(zz,1)^2)/(Wn^2)*(1/((tand(zz/2)+tand(z/2))));
end

eta_rr=zeros(90,90);
for ff=1:90
    for gg=1:90
        eta_rr(ff,gg)=(Wb(ff,1)^2-Wa(gg,1)^2)/(Wn^2)*(1/((tand(ff/2)+tand(gg/2))));
    end
end

figure(5)
surf(eta_rr), hold on,
title('Loss Factor for different data points','FontSize',16);
xlabel('\theta_b','FontSize',16)
ylabel('\theta_a','FontSize',16)
zlabel('\eta','FontSize',16)
shading flat
pause

RRR=sum(eta_rr(ff,gg));

% Eta_r médio
Aux13=zeros(90,90);
for i=1:90
    for j=1:90
        if (eta_rr(i,j)>1)
            Aux13(i,j)=eta_rr(45,45);
        else
            Aux13(i,j)=eta_rr(i,j);
        end
    end
end
eta_r_medioo=sum(sum(Aux13))/8100;

Aux14=zeros(90,1);
for l=1:90
    if (eta_r(l,1)>1)
        Aux14(l,1)=0;
    else
        Aux14(l,1)=eta_r(l,1);
    end
end

for m=1:90
    if (Aux14(m,1)==0)
        eta_r_medio=sum(Aux14(m:90,1))/(91-m);
bREAK
    end
end

banda_eta=0.1*eta_r_medioo;
if (eta_r_medioo-banda_eta<eta_r_medio<eta_r_medioo+banda_eta)
    eta=eta_r_medio;
\end{verbatim}
else
    eta=eta_r_medioo;
end

%==========================================================================
%Erro do fit à circunferência
%el
e1=zeros(P,1);
for jjj=1:P;
    e1(jjj,1)=(R-sqrt((x(jjj,1)-xc)^2+(y(jjj,1)-yc)^2))^2;
end

erro_1=sum(e1);

%==========================================================================
%e2
e2=zeros(P,1);
for hhh=1:P;
    e2(hhh,1)=(R^2-sqrt((x(hhh,1)-xc)^2+(y(hhh,1)-yc)^2))^2;
end

erro_2=sum(e2);

%==========================================================================
%Parâmetros Modais no Nyquist
if (eta_r_medioo-banda_eta<eta_r_medio<eta_r_medioo+banda_eta)
    rCjk=2*R*(2*pi()*Wn)^2*eta_r_medio;
else
    rCjk=2*R*(2*pi()*Wn)^2*eta_r_medioo;
end

Wn1=2*pi()*Wn;
x11=Amplitude(:,1);
y11=20*log10(Amplitude(:,2));
w=2*pi()*A(:,1);
f=A(:,1);

%rFijk=acosd((yc-ry(2,1))/R);%%%%%%%%%%%%%%%%%
rFijk=atand(abs(xc-rx(2,1))/abs(yc-ry(2,1)));%%%%%%%%%%%%%%%%
rDjk=rx(2,1)+ry(2,1)*sqrt(-1);%%%%%%%%%%%%%%%%
Recept=((rCjk*(cos(rFijk)+sin(rFijk)*sqrt(-1)))/(Wn1^2-w.^2+sqrt(-1)*eta_r_medio*(Wn1)^2))+(rDjk);
Receptancia=20*log10(abs(Recept));

figure(6)
plot(f(:,1),Receptancia(:,1),'r'), hold on,
legend
xlabel('Frequency (Hz)')
ylabel('Magnitude [dB re. 1 m/N]')
title('Receptance','FontSize',16)
% FRF real
plot(x11,y11)
pause
% Parâmetros Modais do método RFP
w=Amplitude(50:end,1);
FRF2=(Amplitude(50:end,2));

fprintf('Carregue para continuar');
pause

Initial=amp1;
Final=3*Wn;
N=5; % Number of modes

figure(7)
plot(w(:,1),20*log10(abs(FRF2(:,1))),'b'), hold on,
xlabel('Frequency [Hz]'),ylabel('Magnitude [dB re. 1 m/N]'),
omega=w(Initial:Final,1); % Frequency [rad s^\-1]
rec=FRF2(Initial:Final,1); % Magnitude of the receptance [m/N]
title('Receptance','FontSize',16);
[alpha,par]=rfp(rec,omega,N);
fn=par(:,1); %natural frequencies
xi=par(:,2); %damping ratios
C=[par(:,3),par(:,4)]; %modal constant (magnitude,phase)
Polos=par(:,5);
Residuos=par(:,6);
ff=fn; %natural frequencies in [Hz]
plot(omega,20*log10(abs(alpha)),'r'),hold on,
eta_rfp=2*abs(xi(3,1));
pause

% Verificar veracidade do valor de eta=2*csi
massa = xlsread(filename, sheet , 'T5');
[aaaa bbbb]= find(distWn==min(distWn(:,1)));
% Analytical FRF
Wn1=2*pi()*Wn;
x11=Amplitude(:,1);
y11=20*log10(Amplitude(:,2));
w=2*pi()A(:,1);
f=A(:,1);
Aux22=zeros(linhas_A-1,1);
Aux23=zeros(linhas_A-1,1);
ETA_min=eta-0.1;
ETA_max=eta+0.1;
banda_ETA=ETA_max-ETA_min;
G=zeros(linha5_A,21);
n=1;
for i=ETA_min:0.01:ETA_max
    for a=1:linhas_A
        G(a,n)=1./(massa*(Wn1^2-(w(a,1)).^2+sqrt(-1)*i*Wn1^2));
    end
    bb=n+1;
n=bb;
end
magnitude=20*log10(abs(G));

for j=1:21
    for k=1:(linhas_A-1)
        Aux22(k,j)=magnitude(k+1,j)-magnitude(k,j);
        Aux23(k,1)=y11(k+1,1)-y11(k,1);
    end
end

a=ampl;
b=linhas_A-1;
c=b-a+1;
dist=zeros(c,21);

for k=1:21
    m=1;
    for j=a:b
        dist(m,k)=abs(Aux22(j,k)-Aux23(j,1));
        dd=m+1;
        m=dd;
    end
end

for i=1:20
    Aux26(1,i)=sum((dist(1:(b-a),i)));
end

for i=1:20
    Aux25(1,i)=min(Aux26(1,i));
end

minimo = find(Aux25==min(Aux25(1,1:20)));
ETA_1=0.01*minimo+ETA_min;
G=1./(massa*(Wn1^2-w.^2+sqrt(-1)*ETA_1*Wn1^2));
magnitude=20*log10(abs(G));

% PLOT
figure(8)
plot(f(:,1),magnitude(:,1),'r'), hold on,
%legend
grid
xlabel('Frequency (Hz)')
ylabel('Magnitude [dB re. 1 m/N]')
title('Receptance','FontSize',16)
% PRF real
plot(x11,y11)

%