Aspects of Neutrino Physics and CP Violation

Pedro Miguel Lopes Loreto dos Santos
Under supervision of Margarida Nesbitt Rebelo
Dep. Physics, IST, Lisbon, Portugal

November 24, 2011

Abstract

A concise discussion on Majorana spinors is presented, leading to a review of the basic mechanisms of neutrino mass generation and the corresponding structure of the lepton mixing matrix, along with the different types of seesaw mechanism. We briefly review neutrino oscillation and double-β decay scheme, presenting the status of the current observations, including the novel results from T2K regarding θ_{13}. After explaining the theoretical conditions for producing a matter-antimatter asymmetry in the Universe, or more specifically the Sakharov's conditions, we explore one of them, CP violation, in the leptonic sector. Finally, we focus our attention in the creation of that asymmetry through leptogenesis, and we show that, in this scenario, all the qualitative ingredients are guaranteed once the seesaw mechanism is assumed to be the source of neutrino masses. For further details look at [1].

Keywords: neutrino physics, Majorana, CP violation, leptogenesis.

1 Introduction

The historic discovery of neutrino oscillations marks a turning point in particle and nuclear physics and implies that neutrinos have mass. This possibility was first suggested by theory in late seventies, where the smallness of neutrino mass was explained by seesaw mechanism [2–5]. The fact that neutrinos are indeed massive raises a question regarding its nature - they are either Dirac particles possessing distinctive antiparticles, or Majorana, neutral particles which are truly identical to their antiparticles [6]. On the one hand, massive Dirac neutrinos a realised in gauge theories in which the lepton charges, L_e, L_μ and L_τ, are not conserved, but a specific combination of the latter, e.g. $L = L_e+L_\mu+L_\tau$, is conserved; on the other hand, massive Majorana neutrinos arise if no lepton charge is conserved by the electroweak interactions.

From the problem of the nature of massive neutrinos also arise different possibilities on the way CP is violated. It is unambiguously estabilished that in the quark sector there is one CP violating phase [7]. In the context of Dirac neutrinos one would have possibly an analogous CP violating phase in the leptonic sector. There is, however, the possibility that instead of one CP violating phase, we have three CP violating phases [8] in the decoupling limit, in the case neutrinos are Majorana. This study adresses a rigorous analysis to this subject.

In the context of CP violation there is the puzzle of the baryon asymmetry of the Universe (BAU). The Standard Model contains all the ingredients to dynamically generate the observed BAU, but, yet, it fails to explain an asymmetry as large as the one observed (see, e.g. [9]), and for that reason new physics is called for. The new physics must [10], first, distinguish matter from antimatter in a more pronounced way than the weak interactions of SM do. Second, it should provide a departure from thermal equilibrium during the history of the universe, or modify the electroweak phase transition. It that context we present leptogenesis [11], which provides a very elegant explanation regarding this issue.

2 Majorana spinor

To obtain the Majorana spinor just consider a general mass term

$$\mathcal{L}_m = m\overline{\psi}\psi = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L), \tag{1}$$

where $\overline{\psi} = \psi^\dagger\gamma^0$, m is a real mass for simplicity and $\psi_{R/L} \equiv \frac{1}{2}(1 \pm \gamma_5)\psi$. The mass eigenstate is given by

$$\psi = \psi_L + \psi_R. \tag{2}$$

Taking $\psi_R = \psi_L^c$, the above Lagragian yields

$$\mathcal{L}_L = m_L(\overline{\psi_L^c}\psi_L + \overline{\psi_L}\psi_L^c) = m_L\overline{\chi}\chi, \tag{3}$$

and the mass eigenstates is the following self-conjugate fields

$$\chi = \psi_L + \psi_L^c, \chi^c = \chi. \tag{4}$$

The most general definition of a Majorana spinor yields

$$\chi = \exp(i\xi)\chi^c. \tag{5}$$

If we combinate Dirac and Majorana mass terms, then we have

$$2\mathcal{L}_{DM} = \overline{\Psi_L}\mathcal{M}\Psi_R + \overline{\Psi_R}\mathcal{M}^*\Psi_L, \tag{6}$$

where we introduced the following definitions: $\Psi_L \equiv (\psi_L, \psi_R^c)^T$, $\Psi_R \equiv (\psi_L^c, \psi_R)^T$ and

$$\mathcal{M} \equiv \begin{pmatrix} m_L^* & m_D \\ m_D & M_R \end{pmatrix}. \tag{7}$$

There are several characteristics of the Majorana fermions: they have no conserved charge (for that reason neutrinos can be Majorana - they lack electric charge), otherwise the mass term would not be invariant under $U(1)$; Majorana fermions are more 'economic', having in mind that we only need one independent spinor to build a mass term; a Majorana particle is its own antiparticle; Majorana mass terms are symmetric by definition, which means it is only meaningful in anticommuting fields; the most general mass term, defined in eq. (6) for a four-component fermion field in fact describes two Majorana particles with different masses and when one sets $m_L = M_R = 0$, one obtains the usual Dirac fermion.

3 Minimal extension of the SM

With the evidence that neutrinos have mass, it urges to explain their smallness. A usual extension to the SM is, for instance, the consideration of a set of right-handed neutrino fields together with the seesaw mechanism type I, to help us understand the smallness of neutrino masses. With the introduction of these specific fields, we can also discuss whether neutrinos are Dirac particles or Majorana particles. They can of course be Dirac particles, like all charged fermions have to be. But there is also the possibility that neutrinos are their own antiparticles since they do not appear to have electric charge. The bottom line is that the Majorana neutrino mass is rather suggestive from the theoretical point of view, as well shall see by the naturalness of its implementation via seesaw mechanism and provides new physics at a higher scale, as is the case of leptogenesis. The prediction that this mass term does is the lepton number violation, $\Delta L \neq 0$, which can be violated, for instance, in processes such as neutrinoless double beta decay ($0\nu\beta\beta$) or same sign dilepton par production at colliders.

The three most popular and elegant ways to implement the required neutrino masses into the SM are via the seesaw mechanism type I - III. In all its realizations, the seesaw mechanism allows us to obtain an effective Majorana mass term as

$$m_\nu \nu_L^T C^{-1} \nu_L \tag{8}$$

where m_ν is

$$m_\nu \propto c_\nu \frac{v^2}{M}. \tag{9}$$

In this generic neutrino mass matrix we have the coupling of neutrinos, c_ν, the vacuum expectation value of the Higgs doublet, v, and a mass large mass $M \gg v$ (for instance, M_R in Type I and m_Δ^2/m^* in type II [8, 12–15]), which automatically induce neutrinos to be lighter than the charged fermions. Clearly, one would be able to lower m_ν even considering $M \simeq v$ (or even $M \ll v$) just by taking $y_\nu \ll 1$, however this is not natural, having in mind all the Yukawa coupling of other fermions.

To be more specific, in seesaw type I we consider the introduction of a right-handed $SU(2)$ singlet ν_R with large M_R, leading to the following effective Majorana mass term

$$m_{eff}^I \approx -m_D {M_R}^{-1} {m_D}^T. \tag{10}$$

with $(m_D)_{ij} = h_{ij}v$, being h_{ij} the Yukawa coupling between left- and right-handed neutrinos, and v the Higgs doublet vacuum expectation value (vev). In type II one introduces a heavy gauge $SU(2)$ triplet $\Delta = (\Delta^{++}, \Delta^+, \Delta^0)$, from which we obtain

$$m_{eff}^{II} \approx m_L^* \tag{11}$$

where $m_L^* = y_{\Delta ij}\frac{m^* v^2}{m_\Delta^2}$. In the case of type III, we also consider a triplet, as the seesaw type II does, but this time the triplet is fermionic and one of the doublets in the coupling is a Higgs doublet yielding the following result

$$m_{eff}^{III} \approx -y_T^T M_T^{-1} y_T v^2. \tag{12}$$

In summary, we have within the seesaw framework an effective theory at low energy scale with a Majorana mass term, which provides a window to new physics at high scale, taking in consideration the particles introduced. Clearly, one traded coupling y_ν between physical, observable particles, to the unknown y_D (or y_Δ or y_T) coupling and the unknown masses of those particles. However, one can relate it with another physics phenomenon. This is quite the same that was happening before the discovery of the W boson, where Fermi theory was describing low energy phenomena in an appropriate way.

Considering the low energy charged current interaction Lagrangian in the physical basis

$$\begin{aligned} & -\frac{g}{\sqrt{2}}\left(\overline{l_L^0}\gamma_\mu \nu_{L^0}\right) W^\mu + h.c \\ & \approx -\frac{g}{\sqrt{2}}\left(\overline{l_L}\gamma_\mu U \nu_L\right) W^\mu + h.c., \end{aligned} \tag{13}$$

U is the lepton mixing matrix, most of the times denominated Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, and is analogous to Cabbibo-Kabawashi-Maskawa (CKM) matrix in the quark sector. This matrix is usually presented in the following form

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \tag{14}$$

To obtain the number of physical phases of this matrix (the ones that cannot be rephased away), we should recall that, whereas the rephasing of a charged lepton fields

$$l_i \to l_i' = \exp(i\theta_i) l_i \tag{15}$$

with arbitrary θ_i's, leave the charged lepton mass terms $m_i \overline{l_i} l_i$ invariant, in the case of neutrinos, when we consider their Majorana nature, the rephasing

$$\nu_j \to \nu_j' = \exp(i\varphi_j)\nu_j \tag{16}$$

with arbitrary φ_j's is not allowed, since it would not keep the Majorana mass terms $\nu_{Lj}^T C^{-1} m_j \nu_{Lj}$ invariant. Furthermore, we also must have in mind that the n^2 parameters in the unitary mixing matrix can in principle be parametrised by $n(n-1)/2$ mixing angles and $n(n+1)/2$ phase. Taking in consideration 15 and 16, we see that if neutrinos are Dirac, then they have $(n-1)(n-2)/2$ physical phases, whereas if they are Majorana they have $(n)(n-1)/2$ unremoval phases. Particularising for the case of $n = 3$ massive neutrinos, we have one physical phase if neutrinos are Dirac, or three physical phases if they are Majorana. For this matrix we usually consider the standard parametrisation [16]

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}.$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and, without any loss of generality, all θ_{ij} in the first quadrant. Note that in the case neutrinos are Dirac, the matrix with factorisable fases α, β is the identity.

4 Oscillations, double-β decay and experimental situation

The 3×3 mass matrix of active neutrinos m_ν introduces several parameters to SM. Six parameters in principle show in our main source of experimental results, the neutrino oscillation: two mass-square splittings, three mixing angles and one 'Dirac' CP-violating phase. Furthermore, one should add the mass scale os neutrinos (i.e. the mass of the lightest neutrino), which does not appear in the oscillation probability and has to be derived from other types of experiments such as beta decay experiments or the cosmological observations, and also add the two Majorana phases.

Regarding neutrino oscillations, we see that it has roughly three stages: a neutrino source produces, via W exchange the charged lepton flavour i, plus an accompanying neutrino that, by definition, must be a ν_i; the neutrino propagates, in approximate vacuum, a distance L to a target/detector; it interacts again via charged current weak interaction and produces a second charged lepton l_j of flavour j. Considering the plane wave approximation, the probability P of $\nu_i \to \nu_j$ transitions at the distance $x = L$ and time $t = T$ of neutrino detection is the following

$$\begin{aligned} P_{\nu_i \to \nu_j}(L, E) &= |\langle \nu_j | \nu_i(T, L)\rangle|^2 \\ &= \left| \sum_\alpha U_{i\alpha}^* e^{i(p_\alpha L - E_\alpha T)} U_{j\alpha} \right|^2 \\ &= \sum_{i=1}^{3} |U_{i\alpha} U_{j\alpha}|^2 + \\ &+ 2\sum_{\alpha<\beta} \mathrm{Re}\left(U_{i\alpha} U_{j\alpha}^* U_{i\beta}^* U_{j\beta}\right) \cos\frac{\Delta m_{\alpha\beta}^2 L}{2E} \\ &+ 2\sum_{\alpha<\beta} \mathrm{Im}\left(U_{i\alpha} U_{j\alpha}^* U_{i\beta}^* U_{j\beta}\right) \sin\frac{\Delta m_{\alpha\beta}^2 L}{2E}. \end{aligned} \tag{17}$$

where $\Delta m_{\alpha\beta}^2 = m_\alpha^2 - m_\beta^2$. From the above equation, one clearly sees that oscilations are sensitive to the 'Dirac' phase δ, as well as to the mixing angles and mass differences. The current observational status regarding mixing angles and masses differences obtained from oscillation experiments is presented in table 1. It is clear that $\theta_{13} \neq 0$ is not an highly accurate measurement so far, but it may get better in the future. However, it is important to stress that these results have been viewed as the first reasonably robust experimental evidence that this angle is nonzero.

There are three more known methods to measure the remaining parameters: neutrinoless double β-decay

($0\nu\beta\beta$), which can only occur if neutrinos are Majorana; electron neutrino mass can be extracted from β-decay spectra; the sum of neutrino masses can be inferred from cosmology. From these methods, we see that the only one to be sensitive to Majorana phases is $0\nu\beta\beta$, whose decay is proportional to

$$\begin{aligned}|m_{ee}| &\equiv \left|\sum U_{ei}^2 m_i\right| \\ &= |\cos^2\theta_{13}(|m_1| cos^2\theta_{12} + |m_2| e^{2i\alpha}\sin^2\theta_{12}) + \\ &+ \sin^2\theta_{13}|m_3|e^{2i\beta}|. \end{aligned} \tag{18}$$

in the parametrisation similar to eq.(17). By the factorisation of the 'Majorana' phases, one can clearly see that by observing this process we are not only able to decide if neutrinos are Majorana, but we can also be able to determine one and only one combination of 'Majorana' phases, always depending on the experimental and theoretical uncertainties. Considering its dependence on the neutrino mass, which can be experimentally challenging, there are also good expectations to find better constraints on this measurement, having in mind that neutrino mixing angles θ_{ij} and neutrino mass-squared differences Δ_{ij}^2 can be accurately determined from oscillation experiments. It is also clear that cancellations among terms may occur due to those phases, thus, in principle, they can conspire so m_{ee} is zero, and in that circumstance no $0\nu\beta\beta$ occurs.

In the case of the investigation of a β-decay spectrum, usually the 'average electron neutrino mass' $m(\nu_e)$ is determined

$$m^2(\nu_e) \equiv \sum |U_{ei}|^2 {m_i}^2, \tag{19}$$

which is a coherent sum and therefore not sensitive to phases. One must also mention that cosmology can provide us an indirect measurement of neutrino masses. Indeed, cosmology is sensitive to the energy densities of the various components of the Universe, and specifically to the sum of the physical mass of neutrinos

$$\sum_{i=1}^{n} m_i. \tag{20}$$

These methods are able to provide three upper bounds regarding neutrino masses, but no positive detection of the mass combinations mentioned above was performed, as the following shows [17], [18], [19]

$$|m_{ee}| \lesssim (0.21 - 0.53) \text{ eV } (90\% \text{ C.L.}), \tag{21}$$

$$m^2(\nu_e) < 2.3 \text{ eV } (95\% \text{ C.L.}), \tag{22}$$

$$\sum m_i \lesssim 0.5\text{eV}. \tag{23}$$

Note that, as mentioned before, the limits set by experiments rely in the improvement of both observational techniques and larger data sets and are expected to drop in the future. For instance, the bounds set by $0\nu\beta\beta$ are expected to be lowered in the future, unless the nuclear physics of this process spoils this intent. Clearly the limit set by experiments involving β-decay date from late 90's and are still high, but it is expected that the KATRIN experiment can be sensitive to neutrino masses down to 0.2eV [20]. The cosmological probes are also expected to lower the limits, testing neutrino masses down to 0.1eV.

The other two remaining observables are the 'Majorana' phases, which are measurable only in processes that violate lepton number. At present time, the only such process that seems viable experimentally is neutrinoless double β-decay and it is sensitive to the particular combination of 'Majorana' phases in eq. (18).

5 CP violation in the leptonic sector

Oscillations are consequence of the existence of a non-trivial flavour mixing matrix, the PMNS matrix, and the fact that there seems to be no contraint on whether it is real or imaginary, instigates the study and the possibility of a CP non-invariance of the Lagrangian. Considering the following CP transformations which leaves the gauge interaction invariant

$$\begin{aligned} (\mathcal{CP})l_L^0(\mathcal{CP})^\dagger &= U'\gamma^0 C\overline{l_L^0}^T & (\mathcal{CP})l_R^0(\mathcal{CP})^\dagger &= V'\gamma^0 C\overline{l_R^0}^T \\ (\mathcal{CP})\nu_L^0(\mathcal{CP})^\dagger &= U'\gamma^0 C\overline{\nu_L^0}^T & (\mathcal{CP})\nu_R^0(\mathcal{CP})^\dagger &= W'\gamma^0 C\overline{\nu_R^0}^T \end{aligned} \tag{24}$$

in the case CP-invariance holds in the mass Lagrangian the following conditions must be satisfied

$$m_L = -U'^\dagger m_L^* U'^*, \tag{25}$$

$${M_R}^* = -W'^T M_R W', \tag{26}$$

$${m_D}^* = U'^\dagger m_D W', \tag{27}$$

$$m_l^* = U'^\dagger m_l V'. \tag{28}$$

Furthermore, when one considers only low energy effects, whose mass term is in eq. (8), then the condition

$$m_\nu^* = -U'^T m_\nu U', \tag{29}$$

must hold, in order to have CP invariance.

Considering the weak current interaction Lagrangian, we see the condition of CP invariance of the mixing matrix yields

$$U_{lj}^* = U_{lj}\rho_j, \qquad \rho_j = \frac{1}{i}\eta_{CP}(\chi_j) = \pm 1, \tag{30}$$

where $\eta_{CP}(\chi_j) = i\rho_j = \pm i$ is the CP parity of the Majorana neutrino χ_j. It can be seen that only relative CP parities are meaningful, since it depends on the a priori choice in the parametrisation, and not only on the Majorana phase.

Analogously to the quark sector, we can define conditions for CP invariance in a more meaningful manner. Considering the multiplication of two columns of U, one can define three of the so-called 'Majorana triangles'

$$\begin{aligned} U_{e1}U^*_{e2} + U_{\mu 1}U^*_{\mu 2} + U_{\tau 1}U^*_{\tau 2} = 0 \\ U_{e1}U^*_{e3} + U_{\mu 1}U^*_{\mu 3} + U_{\tau 1}U^*_{\tau 3} = 0 \\ U_{e2}U^*_{e3} + U_{\mu 2}U^*_{\mu 3} + U_{\tau 2}U^*_{\tau 3} = 0 \end{aligned} \tag{31}$$

In the complex plane, each term from the sums in (31) determines a vector. Since the sum is zero, it means that the start and the end point coincides, corresponding to three unitary triangles. From these triangles we can define the necessary and sufficient conditions for CP conservation: their common area $A = J/2$ vanishes; orientation of all Majorana triangles along the direction of the real or imaginary axis.

When one considers all imaginary parts of the 'quartets' to vanish, then as happens in the quark sector triangles, this is equivalent to the vanishing of the 'Dirac' phase. However, as mentioned before, this is not sufficient for CP conservation. One also requires ii) so the Majorana phases do not violate CP. If the three collapsed triangles are on the real axis, then the imaginary part of the rephasing invariants $\mathrm{Im}U_{ij}U^*_{ik} = 0$ for every possible combination, and obviously CP is conserved. Moreover, if one of these triangles is parallel to the imaginary axis, that means the Majorana neutrino χ_i and χ_k whose U elements were considered in that triangle, have opposite CP parities, but there exists no CP violation.

We can also define CP conservation conditions considering the use of WB invariants, which are particularly useful from the point of view of model building. Let us now construct the following Hermitian matrices

$$\mathcal{A} = m_l m_l^\dagger, \tag{32}$$

$$\mathcal{B} = m_\nu^* m_\nu, \tag{33}$$

$$\mathcal{C} = m_\nu^* (m_l m_l^\dagger)^* m_\nu. \tag{34}$$

We can obtain a set of three necessary and sufficient conditions for CP violation by imposing a non-zero Jacobian (this procedure checks whether those objects are independent). By proceeding with this calculation we define the following possible set of independent CP-odd invariants [21]

$$\mathrm{Tr}[\mathcal{A}, \mathcal{B}]^3, \tag{35}$$

$$\mathrm{Tr}[\mathcal{A}, \mathcal{C}]^3, \tag{36}$$

$$\mathrm{Tr}([\mathcal{A}, \mathcal{B}]\mathcal{C}). \tag{37}$$

which obbey to Det $\left(\frac{\partial \mathcal{I}_i}{\partial \phi_j}\right) \neq 0$ and define the three independent conditions for CP conservation in the non-degenerate case as

$$\mathcal{I}_i = 0 \tag{38}$$

where $\mathcal{I}_i$ ($i = 1, 2, 3$) denote the invariants (35)-(37).

6 Leptogenesis

There is good evidence that the Universe has a predominance of matter over antimatter. To explain it, we shall present in this chapter one of the most simple and elegant models, Leptogenesis [11]. A beautiful aspect of this mechanism is the connection between baryon asymmetry and neutrino properties, which we have been analysing. One of the requirements for a successful baryogenesis through leptogenesis yields on the masses of neutrinos, and for that purpose we will not only present a qualitative study of leptogenesis, but also present a quantitative constraint on the mass of heavy neutrinos, which suggests the plausibility of the mechanism.

The observed Baryon Asymmetry of the Universe (BAU) is [22]

$$Y_{\Delta B} \equiv \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.75 \pm 0.23) \times 10^{-11}, \tag{39}$$

where s is the entropy. The basic idea of this baryogenesis via leptogenesis scenario is that scattering processes can produce enough heavy neutrinos at temperatures $T > M$ (where M is heavy neutrino mass). But when temperature drops below M, the equilibrium number density is suppressed and if the relevant interactions are CP violating, there can be created a net lepton L asymmetry in their out-of-equilibrium decays. This asymmetry is then partially transformed in the observed BAU through the SM $B + L$ violating processes, the so-called sphaleron interactions. Note that the $B - L$ symmetry of SM holds.

Under a leptogenesis scenario, the baryon asymmetry can be approximated as

$$Y_{\Delta B} \equiv \frac{n_b - n_{\bar{b}}}{s} \simeq \frac{135\zeta(3)}{4\pi^4 g_*} \epsilon \times \eta \times C. \tag{40}$$

The first factor is the equilibrium number density of N_1 divided by the entropy density at $T \gg M_1$ and is of $\mathcal{O}(4 \times 10^{-3})$ when we take the number of relativistic degrees of freedom g_* to be $\simeq 106.75$ in SM. The other three factors on the right-hand side represent the following physics aspects

1. factor ϵ is the CP asymmetry in N_1 decays. In every $1/\epsilon$ decays, there is one more L than there are $\overline{L}$'s;

2. factor η is the efficiency factor. We must consider the existence of inverse decays, 'wash out' processes and inefficiencies in N_1, that reduce the asymmetry by $0 < \eta < 1$;

3. factor C describes a further dilution of the asymmetry due to fast processes which redistribute the asymmetry in the lepton doublets. These include gauge, third generation Yukawa and $B + L$ violating non-perturbative processes. There can be a more accurate treatment to this factor, but for simplicity we will aproximate it by a single number.

Let us obtain a rough estimate of this asymmetry. The first parameter we shall compute is CP violation. Since we are considering the decay of a heavy neutrino N_1, CP violation manifests itself in a different decay rate to final states with particles and antiparticles. Such parameter is the following

$$\epsilon_{\alpha\alpha} \equiv \frac{\Gamma(N_1 \to \phi L_\alpha) - \Gamma(N_1 \to \overline{\phi L_\alpha})}{\Gamma(N_1 \to \phi L) + \Gamma(N_1 \to \overline{\phi L})} \tag{41}$$

where one recalls that, since N_1 is a Majorana fermion, $N_1 = \overline{N_1}$. The processes that contribute to the non-conservation of CP at leading order and next to leading order are reproduced in fig. 1. Due to CPT invariance and unitarity, there is no difference between the rate of some process and its CP-opposite. Therefore, it is important to include tree level and one-loop diagrams, and it is precisely from their interference that CP invariance arises.

Taking in consideration the following Lagrangian in terms of its Yukawa couplings, arising from seesaw mechanism type I (in the mass eigenbasis)

$$\mathcal{L} = -h^*_\beta \overline{L}_\beta \tilde{\phi} l_{R\beta} + \lambda^*_{\alpha k} \overline{L}_\alpha \phi^* N_k + \frac{1}{2}\overline{N}_j M_j N^c_j + h.c., \tag{42}$$

where $\alpha, \beta = e, \mu, \tau$, we can see [23] that

$$\epsilon = \frac{3M_1}{16\pi v^2 (\lambda^\dagger \lambda)_{11}} \mathrm{Im}(\sum_{\alpha,\beta} \lambda_{\alpha 1} m^*_{\alpha\beta} \lambda_{\beta 1}). \tag{43}$$

Regarding the out-of-equilibrium dynamics (η), we see that, in the most probable case, the thermal number density of N_1 is obtained independent of the initial conditions, and therefore we have a L symmetric situation at $T \sim M_1$. This is usually defined as the strong washout scenario. In this regime, at $T \sim M_1$, a thermal number density of N_1 is obtained independent of the initial conditions, and therefore we have a L symmetric situation, i.e. we know that any Y_L asymmetry in the production of N_1 is washed out. At this point, it is useful to introduce two parameters [24] of the order of the light neutrino masses $\tilde{m}$ and m_*, and which will represent, respectively, the decay rate Γ_D and the expansion rate $H(T = M_1)$ in an intuitive way

$$\tilde{m} \equiv \frac{8\pi v^2}{M_1}\Gamma_D = \frac{(\lambda^\dagger\lambda)_{11} v^2}{M_1}, \tag{44}$$

$$m_* \equiv \frac{8\pi v^2}{M_1} H(T = M_1) \simeq 1.1 \times 10^{-3}\text{eV}. \tag{45}$$

With these parameters, the N_1 decay out of equilibrium is defined as $\tilde{m} < m_*$. In the case $\tilde{m} \gtrsim \Delta m_{12}$, then this condition is not satisfied, thus the N_1 total decay is in equilibrium. Such range of parameters is referred to as 'strong washout'. The opposite, obviously less likely, case of decay out of equilibrium is the so-called 'weak washout'.

When the temperature drops below M_1, we see that the inverse decays, which initially washout the asymmetry (since they are faster than H), start being suppressed by a factor of $e^{-M_1/T}$, i.e. $\Gamma_{ID} \equiv \Gamma(\phi L \to N_1) \simeq \frac{1}{2}\Gamma_D e^{-M_1/T}$. At temperature T_F that inverse decays departs from equilibrium, and therefore we can estimate the efficiency factor η to be

$$\eta \simeq \frac{n_{N_1}(T_F)}{n_{N_1}(T \gg N_1)} \simeq e^{-M_1/T_F} \simeq \frac{m_*}{\tilde{m}}, \qquad \tilde{m} > m_*. \tag{46}$$

After the generation of L leaving B unchanged obtained from the out of equilibrium decay of heavy Majorana neutrinos, we should consider the sphaleron processes, which are $B + L$ violating (although $B - L$ conserving) and create a baryon asymmetry at a cost of decreasing lepton asymmetry. The final baryon asymmetry is the following [25]

$$Y_{\Delta B} \simeq Y_{\Delta(B-L)} \times \begin{cases} \frac{28}{79} & T > T_{EWPT}, \\ \\ \frac{12}{37} & T < T_{EWPT}. \end{cases} \tag{47}$$

and this is the result we consider for C_{sphal}. Putting all the ingredients together, we can estimate derive the following inequality

$$\sum_{\alpha,\beta} \frac{10^{-14} M_1(\text{GeV})\, m_{\alpha\beta}(\text{eV})}{(\lambda^\dagger\lambda)_{11}} \,\mathrm{Im}(\lambda_{\alpha 1}\lambda_{\beta 1}) \gtrsim 10^{-6} - 10^{-5}, \tag{48}$$

This condition is quite natural. More concretely, taking $m_i \sim 10^{-2}$eV, we see that for a real and imaginary part of $\lambda^\dagger\lambda$ of the same order, we have a very plausible lower limit for $M_1 \gtrsim 10^{10} - 10^{11}$ GeV.

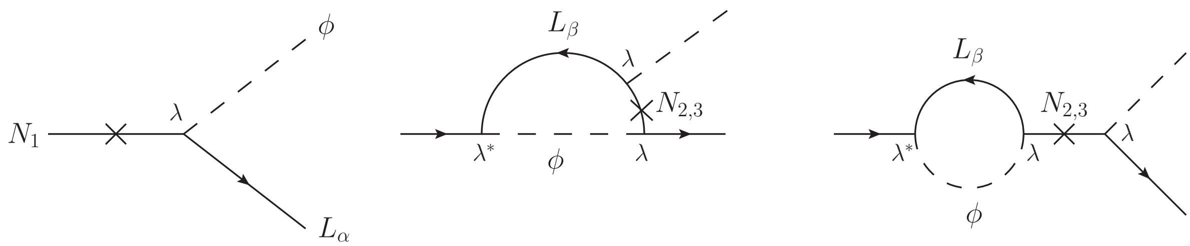

Figure 1: The diagrams contributing to the CP asymmetry $\epsilon_{\alpha\alpha}$. Note that in the case the intermediate neutrino would be N_1, the contribution would be real, therefore we do not have to account it. The flavour of the internal L_β is summed.

7 Conclusion

In this study we discussed a few subjects that arise when considering that the Standard Model neutrinos are massive. Being electrically neutral, massive neutrinos can either be Dirac or Majorana, and that distinctive features come from this fact. If neutrinos are Dirac particles, that means neutrinos adquire mass as a consequence of a small Yukawa coupling, but then we have no fulfilling explanation to the smallness of their masses. Moreover, we would only have one CP violating phase when considering three massive neutrinos. However, if neutrinos are Majorana, we would only have to rely on the heaviness of some particles (right-handed neutrinos in type I seesaw, scalar triplet in type II and fermion triplet in type III), to obtain small masses for neutrinos. In that case, with three light massive neutrinos, we have three phases.

We discussed the conditions for CP to hold. Those conditions can be presented in a more meaningful manner. For instance, we see that the extent of 'Dirac' violation is dependent on the area of the unitary triangles presented, whereas their orientation is meaningful in terms of 'Majorana' phases. If we look for a necessary and sufficient condition condition in a basis independent way for CP to hold, we just need the imaginary part of the following invariants, $\mathrm{Tr}[m_l m_l^\dagger, m_\nu^* m_\nu]^3$, $\mathrm{Tr}([m_l m_l^\dagger, m_\nu^* m_\nu] m_\nu^* (m_l m_l^\dagger)^* m_\nu)$, $\mathrm{Tr}[m_l m_l^\dagger, m_\nu^* (m_l m_l^\dagger)^* m_\nu]^3$ to vanish. Clearly, in the case of degenerate neutrinos this condition is relaxed, and we only need the imaginary part of the latter invariant to vanish.

In leptogenesis we show that it is attractive not only because all the required features are qualitatively present, but also because the quantitative constraints are plausibly satisfied, as we checked for only mild washout effects and $\tilde{m} \sim 0.01\mathrm{eV}$, suggested by the range of parameters still observationally unconstrained. Furthermore, we see that the required CP asymmetry is achieved for the majority of the seesaw parameter space.

References

[1] P. Loreto Santos, "Aspects of Neutrino Physics and CP Violation,". Dep. Physics. IST, Lisbon, Portugal.

[2] R. N. Mohapatra and G. Senjanovic, "Neutrino Mass and Spontaneous Parity Violation," *Phys.Rev.Lett.* **44** (1980) 912.

[3] S. Glashow, "The Future of Elementary Particle Physics," *NATO Adv.Study Inst.Ser.B Phys.* **59** (1980) 687. Preliminary version given at Colloquium in Honor of A. Visconti, Marseille-Luminy Univ., Jul 1979.

[4] M. Gell-Mann, P. Ramond, and R. Slansky *Proceedings of the Supergravity Stony Brook Workshop* (1980) . Eds. P. Van Niewenhuizen, D. Freeman (North-Holland, Amsterdam).

[5] T. Yanagida, "Horizontal Symmetry and Masses of Neutrinos," *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe* (1979) . Eds. A. Sawada, A. Sugamoto, KEK Report No. 79-18.

[6] E. Majorana, "Theory of the Symmetry of Electrons and Positrons," *Nuovo Cim.* **14** (1937) 171–184.

[7] M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction," *Prog.Theor.Phys.* **49** (1973) 652–657.

[8] J. Schechter and J. Valle, "Neutrino Masses in SU(2) x U(1) Theories," *Phys.Rev.* **D22** (1980) 2227.

[9] A. Dolgov, "NonGUT baryogenesis," *Phys.Rept.* **222** (1992) 309–386.

[10] A. D. Sakharov, "Violation of CP Invariance, c Asymmetry, and Baryon Asymmetry of the

Universe," *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32–35. [JETP Lett.5:24-27,1967].

[11] M. Fukugita and T. Yanagida, "Baryogenesis Without Grand Unification," *Phys.Lett.* **B174** (1986) 45.

[12] J. Schechter and J. Valle, "Neutrino Decay and Spontaneous Violation of Lepton Number," *Phys.Rev.* **D25** (1982) 774.

[13] G. Lazarides, Q. Shafi, and C. Wetterich, "Proton Lifetime and Fermion Masses in an SO(10) Model," *Nucl.Phys.* **B181** (1981) 287.

[14] R. N. Mohapatra and G. Senjanovic, "Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation," *Phys.Rev.* **D23** (1981) 165.

[15] C. Wetterich, "Neutrino Masses and the Scale of B-L Violation," *Nucl.Phys.* **B187** (1981) 343.

[16] **Particle Data Group** Collaboration, K. Nakamura *et al.*, "Review of particle physics," *J. Phys.* **G37** (2010) 075021.

[17] W. Rodejohann, "Neutrino-less Double Beta Decay and Particle Physics," `arXiv:1106.1334 [hep-ph]`.

[18] C. Kraus, B. Bornschein, L. Bornschein, J. Bonn, B. Flatt, *et al.*, "Final results from phase II of the Mainz neutrino mass search in tritium beta decay," *Eur.Phys.J.* **C40** (2005) 447–468, `arXiv:hep-ex/0412056 [hep-ex]`.

[19] K. Abazajian, E. Calabrese, A. Cooray, F. De Bernardis, S. Dodelson, *et al.*, "Cosmological and Astrophysical Neutrino Mass Measurements," `arXiv:1103.5083 [astro-ph.CO]`.

[20] C. Weinheimer, "Direct determination of Neutrino Mass from H-3 beta Spectrum," `arXiv:0912.1619 [hep-ex]`.

[21] H. K. Dreiner, J. S. Kim, O. Lebedev, and M. Thormeier, "Supersymmetric Jarlskog invariants: The neutrino sector," *Phys. Rev.* **D76** (2007) 015006, `arXiv:hep-ph/0703074`.

[22] **WMAP Collaboration** Collaboration, E. Komatsu *et al.*, "Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation," *Astrophys.J.Suppl.* **192** (2011) 18, `arXiv:1001.4538 [astro-ph.CO]`.

[23] S. Davidson, E. Nardi, and Y. Nir, "Leptogenesis," *Phys.Rept.* **466** (2008) 105–177, `arXiv:0802.2962 [hep-ph]`.

[24] W. Buchmuller and M. Plumacher, "Baryon asymmetry and neutrino mixing," *Phys.Lett.* **B389** (1996) 73–77, `arXiv:hep-ph/9608308 [hep-ph]`.

[25] S. Khlebnikov and M. Shaposhnikov, "The Statistical Theory of Anomalous Fermion Number Nonconservation," *Nucl.Phys.* **B308** (1988) 885–912.

[26] T. Schwetz, M. Tortola, and J. Valle, "Global neutrino data and recent reactor fluxes: status of three-flavour oscillation parameters," *New J.Phys.* **13** (2011) 063004, `arXiv:1103.0734 [hep-ph]`.

[27] **T2K Collaboration** Collaboration, K. Abe *et al.*, "Indication of Electron Neutrino Appearance from an Accelerator-produced Off-axis Muon Neutrino Beam," *Phys.Rev.Lett.* **107** (2011) 041801, `arXiv:1106.2822 [hep-ex]`.

[28] T. Schwetz, M. Tortola, and J. W. F. Valle, "Where we are on θ_{13}: addendum to 'Global neutrino data and recent reactor fluxes: status of three- flavour oscillation parameters'," `arXiv:1108.1376 [hep-ph]`.

8 Experimental summary

Parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5} eV^2]$	$7.59^{+0.20}_{-0.18}$	$7.24 - 7.99$	$7.09 - 8.19$
Δm^2_{31} $[10^{-3} eV^2]$	2.45 ± 0.09 $-(2.34^{+0.10}_{-0.09})$	$2.28 - 2.64$ $-(2.17 - 2.54)$	$2.18 - 2.73$ $-(2.08 - 2.64)$
$\sin^2 \theta_{12}$	$0.312^{+0.017}_{-0.015}$	$0.28 - 0.35$	$0.27 - 0.36$
$\sin^2 \theta_{23}$	0.51 ± 0.06 0.52 ± 0.06	$0.41 - 0.61$ $0.42 - 0.61$	$0.39 - 0.64$

$\sin^2 2\theta_{13}$ experimental results
Prior to T2K [26] (it accounts with solar and atmospheric neutrinos + KamLAND + Chooz + Palo Verde + + short-baseline experiments (Bugey4, ROVNO, Bugey3, Krasnoyarsk, ILL Gösgen) $0.040^{+0.035}_{-0.024}$, $0.051^{+0.035}_{-0.027}$ (best fit $\pm 1\sigma$) ≤ 0.105, ≤ 0.120 (2σ) ≤ 0.135, ≤ 0.150 (3σ)
T2K [27] $0.03 < \sin^2 2\theta_{13} < 0.28$ $0.04 < \sin^2 2\theta_{13} < 0.34$ at 90% Confidence level (C.L.) expected/observed number of $\nu_\mu \to \nu_e$ events - 1.5 ± 0.3 / 6 (2.5σ significance)

Table 1: Neutrino parameters summary. For Δm^2_{32}, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$ and $\sin^2 2\theta_{13}$, the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy. Dirac phase $\delta = 0$. The above results are presented in [28], while the ones below were released in last June [27].