Navegação e Determinação de Atitude em Aeronaves Através de Múltiplos Receptores GNSS

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Abstract

The development over recent decades of GNSS's, in particular the GPS, and augmentation systems, such as EGNOS and WAAS, made possible the use of this kind of navigation systems for applications in the civil aviation industry, while new developments are still expected in the coming decades, such as the GALILEO service. In order to use these now available systems, to full capacity, new applications are being developed. Among them aircraft attitude determination methods are especially prominent since they could replace the existing radio navigation aids systems and the expensive and complex, inertial navigation systems.

This paper presents a possible solution to the attitude determination problem using exclusively the estimated GNSS positioning of four on-board independent receivers. It also presents the models used in the navigation equation solution. The presented method evaluation was done in a computer simulation environment executed in MATLAB.

Keywords: Navigation, Attitude Determination, GNSS

1 Introduction

Over the last years global navigation satellite systems (GNSS) have become increasingly popular mainly because of the availability of an unrestricted GPS signal. Beside GPS, there are currently other alternative systems being developed and implemented, such as Russia's GLONASS, Galileo from the European Union and China's COMPASS; as there are also local augmentation systems for the GPS, like the European continent coverage by EGNOS, North-America's by WAAS and MSAS covering the Japan's archipelago. This acceptance, performance augmentation and the rise of new GNSSs, as well as integrated electronics evolution has led to a cost reduction and increased capability of the receivers.

GNSS is currently seen as the single most significant development in civil aviation since the introduction of radio, as it can complement the aircraft inertial navigation systems (INS) and also replace the expensive radio navigation aids which are also unavailable in large parts of the world. However, there is still much development to be made in future GNSS aeronautical applications. Among them, the precise attitude determination based on GNSS positioning could completely replace the complex and expensive INSs, applications such as the presented at [1] - in which the attitude is determined using a dual-band carrier phase tracking receiver with multiple antennas deployed across the aircraft - or the presented in [2] which uses a single antenna/receiver GPS complemented by aircraft aerodynamic information.

Although the GNSS measurements accuracy might be lower than the available for other marketed sensors, the use of GNSS receivers presents some important advantages: its accuracy does not degrade over time, it requires no maintenance (depending only on an external source: the GNSS satellite constellation), and it does not need any
ground reference [3].

In this paper a method is presented for aircraft attitude determination using the positioning solutions given by four independent GNSS receivers. The method for GNSS positioning (navigation) through the use of Extended Kalman Filtering is also presented.

2 GNSS Observables

The GNSS satellites continuously broadcast a binary coded message which can provide two kinds of measurements, depending on the receiver. If the receiver tracks the signal’s code, the signal’s transit time (between user and satellite) can be measured by comparing the time stamped received signal with its local copy. This transit time can be related with the apparent range (pseudorange) by being multiplied by the speed of light in the vacuum. Alternatively, if the receiver tracks the satellite signal’s carrier, by evaluating the difference between the tracked carrier and a locally generated carrier the carrier phase difference can be measured.

2.1 Code Phase Measurements

As stated above by tracking the GNSS code, a user u, can measure the pseudorange \( \rho^{(k)}_{u} \) between him and a satellite k. This measurement is called pseudorange because it is only an apparent range, as there is no way of ensuring that the receiver’s clock and satellite’s clock are synchronized. The pseudorange is defined as

\[
\rho^{(k)}_{u} = r^{(k)}_{u} + c \cdot (\delta t_{u} - \delta t^{(k)}) + I_{\rho,u} + T_{\rho,u} + \epsilon^{(k)}_{\rho,u}
\]

where \( r^{(k)}_{u} \) is the true range between u and k defined in (2), c is the speed of light, \( (\delta t_{u} - \delta t^{(k)}) \) is the desynchronization between the two clocks (where each member of the difference is a delay relative to a reference time), \( I_{\rho,u} \) is the ionospheric propagation delay, \( T_{\rho,u} \) is the tropospheric propagation delay (both measured in meters) and \( \epsilon^{(k)}_{\rho,u} \) is an error which aggregates the following: receiver’s thermal noise, multipath errors, code tracking errors and satellite position errors.

\[
r^{(k)}_{u} = \left\| x_{u} - x^{(k)} \right\| \tag{2}
\]

where \( x_{u} \) is the position vector of u and \( x^{(k)} \) the position vector of k.

The linearization of (1) regarding the unknown variables, user position and clock offset, leads to

\[
\begin{bmatrix}
\rho^{(1)}_{u} \\
\vdots \\
\rho^{(M)}_{u}
\end{bmatrix} = z = H \cdot x + e_{p} \tag{3}
\]

where z is the observations vector containing M pseudoranges measured from M satellites, \( H \) the measurements matrix obtained through linearization, \( x = [x_{u_{1,3}} \ x_{\phi}]^{T} \) the vector of unknowns with \( x_{\phi} = \delta t_{u} - \delta t^{(k)} \) and \( e_{p} \) the total measurement error, traditionally called user equivalent range error (UERE) which for high-end receivers \( \sigma(e_{p}) = 0.5 \text{ m} \) [4].

2.2 Carrier Phase Measurements

Through the continuous tracking of the GNSS signal’s carriers a receiver u can measure the carrier phase difference \( \phi^{(k)}_{u} \), in units of cycles, between it and satellite k

\[
\phi^{(k)}_{u} = \lambda^{-1} \left[ \epsilon^{(k)}_{\phi,u} - I^{(k)}_{\phi,u} + T^{(k)}_{\phi,u} + f \left( \delta t_{u} - \delta t^{(k)} \right) \right] + N^{(k)}_{u} + \epsilon^{(k)}_{\phi,u} \tag{4}
\]

where \( \lambda \) and \( f \) are respectively the carrier’s wavelength and frequency, \( I^{(k)}_{\phi,u} \) is the ionospheric phase delay (presented with a minus sign in the equation - it’s actually a phase advance caused by the dispersive characteristic of the ionosphere), \( T^{(k)}_{\phi,u} \) the tropospheric phase delay, \( (\delta t_{u} - \delta t^{(k)}) \) the offset between the receiver’s and satellite’s clocks, \( N^{(k)}_{u} \) an integer carrier phase ambiguity and \( \epsilon^{(k)}_{\phi,u} \) the non-atmospheric phase measurement error (due to thermal noise, multipath, tracking errors and satellite position errors). In the absence of errors (clock bias, atmospheric and non-atmospheric errors) such a measurement when multiplied by the wavelength, can be interpreted as the range between satellite and receiver measured in a un-
known integer number $N$ of wavelengths plus a known fractional wavelength $\lambda \times \phi^{(k)}_u$.

The obvious disadvantage of this type of measurement is the introduction of the unknown parameter $N^{(k)}_u$ - one for each measurement. The advantage is that for high-end receivers $\sigma(\epsilon^{(k)}_{\phi,u}) \approx 0.025$ cycles (5 mm) [4].

With the introduction of a reference receiver, with a known position $x_r$, one can compute carrier phase single-difference $\phi^{(k)}_{ur} = \phi^{(k)}_u - \phi^{(k)}_r$, therefore reformulating the problem into relative positioning while also practically eliminating the atmospheric delays. By repeating this process for two different satellites one can also eliminate the receiver clock bias and achieve the double-difference carrier phase measurement

$$x_{ur} = \lambda^{-1} x_{ur}^{(k)} + N_{ur}^{(k)} + \epsilon_{\phi,ur}^{(k)}$$

(5)

where $(\cdot)_{ur} = (\cdot)_u - (\cdot)_r$, specifically

$$x_{ur}^{(kl)} = \left( x_u^{(k)} - x_u^{(l)} \right) - \left( x_r^{(k)} - x_r^{(l)} \right)$$

(6)

which through linearization with respect to the unknowns, $x_{ur}$ and $N$ leads to the measurements linear model

$$\begin{bmatrix} 
\phi^{(21)}_{ur} \\
\vdots \\
\phi^{(M1)}_{ur} 
\end{bmatrix} = y = G \cdot x_{ur} + N + \epsilon_{\phi}$$

(7)

where, for $M$ satellites, $y$ is the observations vector containing $M - 1$ double-difference carrier phase measurements; $G$ the measurements matrix obtained through linearization; $x_{ur} = x_u - x_r$ the unknown relative position vector; $N$ the integer ambiguities vector and $\epsilon_{\phi}$ the measurement error vector.

### 3 EKF GNSS Positioning

Consider the following discrete time dynamic system

$$x(k + 1) = f[k, x(k)] + v(k)$$

(8)

where $f$ is the state transition function, $x(k)$ the state vector and $v(k)$ a sequence of zero-mean white Gaussian process noise with covariance

$$E[v(k) v(k)^T] = Q(k)$$

(9)

Assuming measurements in the form of

$$z(k) = h[k, x(k)] + w(k)$$

(10)

where $h$ is the measurement function and $w(k)$ a sequence of zero-mean white Gaussian measurement noise with covariance

$$E[w(k) w(k)^T] = R(k)$$

(11)

Under the above conditions, with known matrices $Q$, $R$ and after linearization of the functions $f$ and $h$ (through evaluation of their Jacobian matrices $F$ and $H$), the first order Extended Kalman Filter (EKF) is able to estimate $x$ given the measurements $z$ by following the steps represented at Figure: 1 [5, 6].

![Flowchart of the Extended Kalman Filter](image_url)

**Fig. 1:** Flowchart of the Extended Kalman Filter [5]

where the following notation was used

$$\begin{align*}
\bar{x}_k^- &= \hat{x}(k \mid k - 1) \\
\bar{x}_k^+ &= \hat{x}(k \mid k) \\
P_k^- &= P(k \mid k) \\
P_k^+ &= P(k + 1 \mid k) \\
\tilde{z}_k &= h[\hat{x}(k \mid k - 1)]
\end{align*}$$

Given the appropriate modeling matrices, by defining the measurements vector elements the GNSS pseudoranges $z_k = \left[ \rho_1^{(k)}, \ldots, \rho_m^{(k)} \right]^T$ presented at Subsection: 2.1, the EKF can easily estimate the state vector $x$, containing the positioning solution and receiver clock bias (and any additional
states modeled) if the number of measurements is equal or greater than four \((M \geq 4)\) i.e. the solution for the four unknown variables (three positional and one temporal) requires a set of four or more equations.

This method is equally adapted to solve the relative positioning if given the double-difference carrier phase measurements described at Subsection: 2.2, but as this method of observation introduces one additional unknown for each measurement (integer ambiguities \(N\)), one must first correctly estimate them before using the EKF.

The following subsection presents the models of dynamics used in the thesis: a clock model and two alternative motion models.

### 3.1 Receiver Clock, Model T

The receiver clock offset relative to the GNSS time, was established by a two states model, \(x_{\phi,k}\) representing the clock offset (in meters) and \(x_{f,k}\) the clock frequency error (in meters per second). In this model both states present variations of the Brownian type, which is described in discrete-time by

\[
\begin{bmatrix}
    x_{\phi,k+1} \\
    x_{f,k+1}
\end{bmatrix} = F_{k,t} \begin{bmatrix}
    x_{\phi,k} \\
    x_{f,k}
\end{bmatrix} + \begin{bmatrix}
    u_{\phi,k} \\
    u_{f,k}
\end{bmatrix}
\]

with the state transition matrix

\[
F_{k,t} = \begin{bmatrix}
    1 & \Delta t \\
    0 & 1
\end{bmatrix}
\]

where \(\begin{bmatrix} u_{\phi,k} & u_{f,k} \end{bmatrix}^T\), were sequences of zero-mean white Gaussian process noise with covariance matrix

\[
Q_{k,t} = \begin{bmatrix}
    q_{\phi} \Delta t + \frac{q_{\phi}(\Delta t)^3}{4} & \frac{q_{\phi}(\Delta t)^2}{2} \\
    \frac{q_{\phi}(\Delta t)^2}{2} & q_{f} \Delta t
\end{bmatrix}
\]

Being the values \(q_{\phi} \approx \left(\frac{2\pi}{h}\right) \times c^2\) and \(q_{f} \approx (2\pi^2 h_{-2}) \times c^2\) defined in terms of the Allan variance parameters for a quartz temperature compensated crystal clock, \(h_{0} = 2 \times 10^{-19} \text{ (s}^2/\text{s})\) and \(h_{-2} = 2 \times 10^{-20} \text{ (s}^2/\text{s}^3)\) [7].

### 3.2 Receiver Motion Dynamics, Model PV

For the depiction of a “smooth” receiver motion, where the velocity is expected to change slowly, was established the PV model in which both the position \(x_{1,k}\) and velocity \(x_{2,k}\), are considered for each spatial dimension. Presented in its one-dimensional time-discrete form (where the velocity is modeled as a Brownian motion)

\[
\begin{bmatrix}
    x_{1,k+1} \\
    x_{2,k+1}
\end{bmatrix} = F_{k,pv} \begin{bmatrix}
    x_{1,k} \\
    x_{2,k}
\end{bmatrix} + \begin{bmatrix}
    0 \\
    u_{v,k}
\end{bmatrix}
\]

with state transition matrix

\[
F_{k,pv} = \begin{bmatrix}
    1 & \Delta t \\
    0 & 1
\end{bmatrix}
\]

and process noise with covariance matrix

\[
Q_{k,pv} = \begin{bmatrix}
    \frac{q_{\phi}(\Delta t)^3}{4} & \frac{q_{\phi}(\Delta t)^2}{2} \\
    \frac{q_{\phi}(\Delta t)^2}{2} & q_{f} \Delta t
\end{bmatrix}
\]

This model is integrated with the previous, clock model, achieving for the three spatial dimensions and one temporal the complete PV model

\[
\begin{bmatrix}
    x_{x,k+1} \\
    x_{z,k+1} \\
    x_{\phi,k+1} \\
    x_{f,k+1}
\end{bmatrix} = F_{k,\text{PVT}} \begin{bmatrix}
    x_{x,k} \\
    x_{z,k} \\
    x_{\phi,k} \\
    x_{f,k}
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    u_{f,k}
\end{bmatrix}
\]

with state transition matrix

\[
F_{k,\text{PVT}} = \begin{bmatrix}
    F_{k,pv} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\
    0_{2 \times 2} & F_{k,pv} & 0_{2 \times 2} & 0_{2 \times 2} \\
    0_{2 \times 2} & 0_{2 \times 2} & F_{k,pv} & 0_{2 \times 2} \\
    0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & F_{k,t}
\end{bmatrix}
\]

and process noise covariance matrix
with state transition matrix

\[
Q_{k, PV} = \begin{bmatrix}
Q_{k, pv} & 0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & Q_{k, pv} & 0_{2 \times 2} & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & Q_{k, pv} & 0_{2 \times 2} \\
0_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} & Q_{k, t}
\end{bmatrix}
\]  

(20)

3.3 Receiver Motion Dynamics, Model PVA

For the depiction of a “rougher” receiver motion, where the velocity is expected to change quickly due to large accelerations, the PVA model was established. A three state model in which the acceleration is also taken into account. Presented in its one-dimensional time-discrete form [7]

\[
\begin{bmatrix}
x_{1, k+1} \\
x_{2, k+1} \\
x_{3, k+1}
\end{bmatrix} = F_{k, pva} \begin{bmatrix}
x_{1, k+1} \\
x_{2, k+1} \\
x_{3, k+1}
\end{bmatrix} + \begin{bmatrix} 0 \\
0 \\
u_{a, k}
\end{bmatrix} 
\]

(21)

with state transition matrix

\[
F_{k, pva} = \begin{bmatrix}
1 & \Delta t & \frac{1}{2} \left( e^{-b \Delta t} + b \Delta t - 1 \right) \\
0 & 1 & \frac{1}{2} (1 - e^{-b \Delta t}) \\
0 & 0 & e^{-b \Delta t}
\end{bmatrix}
\]  

(22)

and process noise covariance matrix given by [8]

\[
Q_{k, pva} \approx \frac{1}{2} \left[ F_{k, pva} \times Q_{u, pva} + Q_{u, pva} \times F_{k, pva}^T \right] \Delta t
\]  

(23)

where \( Q_{u, pva} \) is the continuous-time process noise covariance matrix

\[
Q_{u, pva} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & a^2
\end{bmatrix}
\]  

(24)

The full PVA model, for three spatial dimensions and one temporal, is obtained in the same way presented in the previous section for the PV model.

4 Attitude Determination

For the purpose of GNSS positioning base attitude determination this method was developed.

Using a rotation matrix \( R_b^b \) (direction cosine matrix) a receiver position vector \( x^b \) in the b-frame (reference frame fixed to the aircraft body, centered at the receivers geometric center) can be transformed in the receiver position vector \( x^n \) in the n-frame (North-East-Down local frame, also centered at the receivers geometric center) by

\[
\begin{bmatrix}
x_{r1}^n \\
y_{r1}^n \\
z_{r1}^n
\end{bmatrix} = R_b^b \begin{bmatrix}
x_r^b \\
y_r^b \\
z_r^b
\end{bmatrix}
\]

(25)

for multiple receivers \( r \), where the rotation matrix from b-frame to n-frame is defined by the Euler angles: Yaw \( \psi \), Pitch \( \theta \) and Roll \( \phi \)

\[
R_b^b = \begin{bmatrix}
c\theta c\psi & -c\phi s\psi + s\phi s\theta c\psi & s\phi c\psi + c\phi s\theta s\psi \\
c\theta s\psi & c\phi c\psi + s\phi s\theta c\psi & -s\phi c\psi + c\phi s\theta s\psi \\
-s\theta & s\phi c\psi & c\phi c\theta
\end{bmatrix}
\]  

(26)

Given that the receivers’ coordinates in the b-frame are known from the start, and the n-frame coordinates are obtained from GNSS positioning estimates, then the rotation matrix can be estimated by

\[
\hat{R}_b^n = \left[ \begin{bmatrix}
x_{r1}^n \\
y_{r1}^n \\
z_{r1}^n
\end{bmatrix} \right] + \left[ \begin{bmatrix}
x_r^b \\
y_r^b \\
z_r^b
\end{bmatrix} \right]^T
\]

(27)

where \( \bullet \) represents estimation values and \([\bullet]^+\) the generalized inverse or pseudoinverse of a matrix.

The Euler angles can be extracted from the matrix elements \( R_b^n[\bullet, \bullet] \)

\[
\phi = \arctan 2 \left( R_b^n[2, 3], R_b^n[3, 3] \right)
\]

\[
\theta = -\arcsin \left( R_b^n[3, 1] \right)
\]

\[
\psi = \arctan 2 \left( R_b^n[2, 1], R_b^n[1, 1] \right)
\]

(28, 29, 30)

where \( \arctan 2 (y, x) \) is the four-quadrant inverse tangent.

Since the GNSS estimated position vectors are given in the e-frame (Earth-Centered Earth-Fixed...
frame of reference) $\tilde{x}^e$, the $n$-frame estimates were obtained through the use of a $R_n^e$ rotation matrix defined by the geodetic coordinates (latitude $\tilde{\phi}_c$ and longitude $\tilde{\lambda}_c$) of the $n$-frame origin position estimation $\tilde{c}^e$, which is the receivers $e$-frame position average.

$$\tilde{x}^n = R_n^e (\tilde{x}^e - \tilde{c}^e)$$ (31)

5 Simulation Results

In order to evaluate the presented attitude determination method the whole system was simulated in MATLAB. Simulations were made for two trajectories, one “smooth” called DP (Pacific Dynamic) which results are present in this paper and another “rougher” called DA (Aggressive Dynamic).

A set of four receivers was used, distributed as depicted in Figure: 2, with sampling frequency of 10 Hz and 16 satellites (GPS and Galileo combined) were used for position estimation.

5.1 Code Measurements for a Large Aircraft

For the distribution of the receivers over a large aircraft (Figure: 2 with $L = 73\text{ m}$ and $b = 79.8\text{ m}$, the AIRBUS A380 dimensions) and the model of measurements described in Section: 2.1 the following sub-subsections present the obtained simulation results.

5.1.1 Simulation DP-PV

Simulation where the PV model was used for position and velocity estimation of a DP trajectory/attitude, both depicted in Figure: 3.

By inspection of the above table one can observe that the positioning errors were (as usual for simple GNSS) much higher in the vertical direction, this is the result of an unfavorable geometry problem in that direction (high vertical dilution of precision). Caused by the fact that all the available satellites are positioned above the local horizontal plane.
Figure: 4 and Table: 2 show that the method was capable of, on average, estimate the aircraft attitude. Though this estimate is to “noisy” and could hardly be uses for precision applications.

### 5.1.2 Simulation DP-PV with EGNOS

Simulation where the PV model was used for position and velocity estimation of a DP trajectory/attitude through the use of EGNOS measurement corrections (mainly of the ionospheric delay).

<table>
<thead>
<tr>
<th>Linear Error</th>
<th>North</th>
<th>East</th>
<th>Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (m)</td>
<td>0.873</td>
<td>0.837</td>
<td>0.562</td>
</tr>
<tr>
<td>Percentile 95 (m)</td>
<td>0.810</td>
<td>0.743</td>
<td>1.083</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3D Error (m)</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE (m)</td>
<td>1.334</td>
</tr>
<tr>
<td>Percentile 95 (m)</td>
<td>1.302</td>
</tr>
</tbody>
</table>

Tab. 3: DP-PV positioning errors, averaged for the 4 receivers, EGNOS

The results presented on the table above show the increased performance of an EGNOS assisted system. The dramatic reduction of the vertical error (below horizontal errors) seems to show a high dependence of the vertical error on the ionospheric delays (corrected through EGNOS).

As expected the increase in position accuracy diminished the attitude errors, shown in Figure: 5 and Table: 4. This improvement is tightly related with the better vertical accuracy provided by EGNOS. Being the above results the better obtained through the use of code tracking measurements.

### 5.2 Code Measurements for a Small Aircraft

The above simulations were repeated for and aircraft with length $L = 11$ m and wing span $b = 7.62$ m, the Piper PA-24 dimensions.

#### 5.2.1 Simulation DP-PV with EGNOS

While this simulation (with measurements corrected by EGNOS) provided similar position errors to the ones on Sub-Subsection: 5.1.2, the attitude errors were completely unacceptable. Showing that for small aircraft, positioning errors in the decimeter order of magnitude compromise the attitude estimation.

<table>
<thead>
<tr>
<th>Angular Error</th>
<th>Yaw $\psi$</th>
<th>Pitch $\theta$</th>
<th>Roll $\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE ($^{\circ}$)</td>
<td>13,826</td>
<td>20,673</td>
<td>70,647</td>
</tr>
<tr>
<td>Percentile 95 ($^{\circ}$)</td>
<td>27,657</td>
<td>41,265</td>
<td>173,108</td>
</tr>
</tbody>
</table>

Tab. 5: DP-PV attitude errors, EGNOS, small aircraft

#### 5.2.2 Attitude Error Dependence on Aircraft Size

By defining a generalized aircraft with $b = L$, for several values of $L$ the above simulation was repeated. Through this it was possible to plot the attitude determination dependence on aircraft size.
The figure above shows an apparent inverse proportionality between the two.

5.3 Carrier Measurements

Simulations as the presented in Subsections: 5.1 and 5.2 were repeated for the carrier phase measurements model described in Subsection: 2.2. With the assumption that the integer ambiguity problem was correctly solved beforehand by unrelated methods.

5.3.1 Simulation DP-PV for a Large Aircraft

For the large aircraft simulation \((L = 73 \text{ m} \text{ and } b = 79,8 \text{ m})\) the following results were obtained:

<table>
<thead>
<tr>
<th>Linear Error</th>
<th>North</th>
<th>East</th>
<th>Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE ((m))</td>
<td>0,033</td>
<td>0,028</td>
<td>0,038</td>
</tr>
<tr>
<td>Percentile 95 ((m))</td>
<td>0,047</td>
<td>0,042</td>
<td>0,075</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3D Error ((m))</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE ((m))</td>
<td>0,058</td>
</tr>
<tr>
<td>Percentile 95 ((m))</td>
<td>0,084</td>
</tr>
</tbody>
</table>

Tab. 6: DP-PV positioning errors, with phase tracking, averaged for the 4 receivers

Where the centimetric character of the positioning accuracy can be observed, being largely inferior to the present in the code tracking simulations.

In Figure: 7 and Table: 7 is shown that a sub-angular accuracy is achieved, with errors at least one order of magnitude inferior to the ones presented in Sub-subsection: 5.1.2.

5.3.2 Simulation DP-PV for a Small Aircraft

For the small aircraft simulation \((L = 11 \text{ m} \text{ and } b = 7,62 \text{ m})\) the positioning results were as expected similar to those of the large aircraft, as positioning accuracy is unrelated to the receivers distance to each other.
The attitude estimation errors presented in the previous figure and table are considerably better than those for the same aircraft using EGNOS, which were unacceptable. More importantly is the fact that these errors (for the PIPER PA-24) were even smaller than the ones presented in the AIRBUS A380 simulation using EGNOS.

### 6 Conclusions

A method for GNSS based aircraft attitude determination was successfully developed and implemented. This was shown to be very sensitive to both the between receivers distance and GNSS positioning errors.

The simulations’ results revealed that the small aircraft attitude estimate was extremely inaccurate for the typical code tracking positioning errors. In this same condition (code tracking measurements) but for a large aircraft the accuracy increased significantly. Where the use of EGNOS corrections provided an angular accuracy three times lower than the non-EGNOS one, thereby showing its added-value.

The use of double-difference carrier phase measurements provided position solutions with an accuracy at centimeter level. This led to attitude accuracies in the small aircraft simulations lower than those in the code tracking, EGNOS aided, large aircraft simulations. This fact shows the particular importance of the positioning error in attitude determination and that for the real implementation of the method described, carrier phase tracking receivers should be used if capable of correctly estimate the phase measurements’ ambiguities.

### References


